Loitering Phase in Brane Gas Cosmology

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Abstract

Brane Gas Cosmology (BGC) is an approach to M-theory cosmology in which the initial state of the Universe is taken to be small, dense and hot, with all fundamental degrees of freedom near thermal equilibrium. Such a starting point is in close analogy with the Standard Big Bang (SBB) model. The topology of the Universe is assumed to be toroidal in all nine spatial dimensions and is filled with a gas of p-branes. The dynamics of winding modes allow, at most, three spatial dimensions to become large, thus explaining the origin of our macroscopic $3 + 1$-dimensional Universe. Here we conduct a detailed analysis of the loitering phase of BGC. We do so by including into the equations of motion that describe the dilaton gravity background some new equations which determine the annihilation of string winding modes into string loops. Specific solutions are found within the model that exhibit loitering, i.e. the Universe experiences a short phase of slow contraction during which the Hubble radius grows larger than the physical extent of the Universe. As a result the brane problem (generalized domain wall problem) in BGC is solved. The initial singularity and horizon problems of the SBB scenario are solved without relying on an inflationary phase.

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1 Motivation and Introduction

The necessity to search for alternatives to the Standard Big Bang (SBB) scenario is driven by a significant number of problems within the theory such as the horizon, flatness, structure formation and cosmological constant problems. Although inflationary models have managed to address many of these issues, all current formulations remain incomplete. In particular, the present models of inflation suffer from the fluctuation, super-Planck scale physics, initial singularity and cosmological constant problems, as discussed in [1]. Furthermore, several theorems have appeared which show that de Sitter spacetime, the simplest example of an inflationary Universe, cannot be a classical solution to supergravity theories [2, 3]. As supergravity is the low energy limit of M-theory, it does not seem clear that a pure de Sitter model of inflation will arise naturally within the theory.

M-theory is currently our best candidate for a quantum theory of gravity. As such, the theory should provide the correct description of physics in regions of space with high energies and large curvature scales similar to those found in the initial conditions of the Universe. Therefore, it is only natural to incorporate string and M-theory into models of cosmology. [1] It is our hope that M-theory will provide answers to the pending questions of cosmology while preserving the triumphs of the SBB model ultimately leading to a complete description of the Universe.

"Brane Gas Cosmology" (BGC) which is presented in [6] and based on the earlier work of [7] is one example of a cosmological scenario motivated by M-theory. The BGC model is relatively simple and starts out in close analogy with the Standard Big Bang scenario. According to [6], the initial state of the Universe is small, dense, hot and with all fundamental degrees of freedom in approximate thermal equilibrium. [2] For simplicity, the background spatial geometry is assumed to be toroidal, and the Universe is filled with a hot gas of p-branes, the fundamental objects appearing in string theories.

These branes may wrap around the cycles of the torus (winding modes), they can have a center-of-mass motion along the cycles (momentum modes) or they may simply fluctuate in the bulk space (oscillatory modes). By symmetry, we assume equal numbers of winding and anti-winding modes. As the Universe tries to expand, the winding modes become heavy

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[1]For a review of some string theory motivated cosmological models see [4]. A recent proposal, not covered in the review, is the "Ekpyrotic" scenario introduced in [5].

[2]Note that mathematically it is not possible for the Universe to be in thermal equilibrium as the FRW cosmological model does not possess a time-like Killing vector field. However, it is true that the Universe has been very nearly in thermal equilibrium. Obviously, the departures from equilibrium make things interesting!

and halt the expansion [8]. Spatial dimensions can only dynamically decompactify if the winding modes can disappear, and this is only possible (for string winding modes) in $3 + 1$ dimensions [7]. Thus, BGC may provide an explanation for the observed number of large spatial dimensions. However, by causality at least one winding mode per Hubble volume will be left behind, leading to the *brane problem* for BGC [6], a problem analogous to the domain wall problem of standard cosmology.

The present paper provides a simple solution to the brane problem: the winding modes will halt the expansion of the spatial sections, and lead to a phase of slight contraction (*loitering* [9, 10]) during which the Hubble radius becomes larger than the spatial sections and hence all remaining winding modes can annihilate in the large $3 + 1$ dimensions. We supplement the equations for the dilaton gravity background of BGC [11, 8] by equations which describe the annihilation of string winding modes into string loops. A study of these equations demonstrates that solutions exist in which the winding modes force the Universe to contract for a short time and enter a "loitering" phase. During the phase of contraction the number density of the remaining winding modes increases and the winding and anti-winding modes begin to annihilate. The winding branes appear as solitons (analogous to cosmic strings) in the bulk space. The annihilation of winding and antiwinding modes (analogous to cosmic string intersections) leads to the production of string loops which have the same equation of state as cold matter.

Brane Gas Cosmology is a simple nonsingular model which addresses some of the problems of the SBB scenario, simultaneously providing a dynamical resolution to the dimensionality problem of string theory. In this respect it is very different from other attempts to incorporate M-theory into cosmology, such as "brane world" scenarios in which our Universe is located on the world volume of a 3-brane. In our opinion, brane world models suffer from a lack of cosmological motivation. All current models require that the extra dimensions be compactified by hand. Although this is a serious problem from a cosmological viewpoint it is rarely, if ever, discussed.

The organization of this paper is as follows. We begin with a brief review of the Brane Gas model in Section 2. Our concrete starting point is presented, followed by a derivation of the equation of state for a gas of branes and an analysis of the background dynamics. This is followed (in Section 3) by a discussion of the dilaton gravity equations in the presence of a brane gas, of the benefits of loitering, and of attractor solutions. In Section 4, we supplement the system of equations with equations which describe the annihilation of string winding modes into string loops, and based on this we provide a detailed analysis of a

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loitering solution. We use both numerical and analytical methods to study the solutions.

We conclude, in Section 5, with a brief summary and a few conjectures concerning supersymmetry breaking, the effective breaking of T-duality, and dilaton mass generation in the late Universe.

2   Brane Gases

The Brane Gas scenario of [6] is formulated within the context of eleven-dimensional M-theory compactified on S^1. This leads to ten-dimensional, type II-A string theory, whose low-energy effective action is that of supersymmetrized dilaton gravity.

Since M-theory admits the graviton, 2-branes and 5-branes as fundamental degrees of freedom, the compactification on S^1 leads to 0-branes, strings (1-branes), 2-branes, 4-branes, 5-branes, 6-branes and 8-branes in the ten-dimensional Universe. The remaining nine spatial dimensions are assumed to be toroidal (of radius R). The Universe starts out hot, dense and near thermal equilibrium. It is filled with a gas of all the branes (wrapped and unwrapped) which appear in the spectrum of the theory.

2.1   Brane Gas Equation of State:

In this section we derive the equation of state of the brane gas described above. The gas consists of all the branes of spatial dimension p. Contributions of the winding, momentum and oscillatory modes are treated separately below.

The total action for the above model is the sum of the bulk effective action and the action of all the branes in the gas. The low-energy bulk effective action is

$$S_{bulk} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} e^{-2\phi} \left[R + 4G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} \right], \qquad (1)$$

where G is the determinant of the background metric $G_{\mu\nu}$, ϕ is the dilaton, H denotes the field strength corresponding to the bulk antisymmetric tensor field $B_{\mu\nu}$, and κ is determined by the ten-dimensional Newton constant.

Fluctuations of each of the p-branes are described by the Dirac-Born-Infeld (DBI) action [12] and are coupled to the ten-dimensional action via delta function sources. The DBI action is

$$S_p = T_p \int d^{p+1}\zeta e^{-\phi} \sqrt{-det(g_{mn} + b_{mn} + 2\pi\alpha' F_{mn})}, \qquad (2)$$

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where T_p is the tension of the brane, g_{mn} is the induced metric on the brane, b_{mn} is the induced antisymmetric tensor field, and F_{mn} the field strength tensor of gauge fields A_m living on the brane. The constant $\alpha' \sim l_{st}^2$ is given by the string length scale l_{st}.

The induced metric on the brane g_{mn}, (with indices $m, n, ...$ denoting spacetime dimensions parallel to the brane), is determined by the background metric $G_{\mu\nu}$ and by scalar fields ϕ_i (not to be confused with the dilaton ϕ) living on the brane. The indices $i, j, ...$ denote dimensions transverse to the brane and the ϕ_i describe the fluctuations of the brane in the transverse directions. In the string frame, the tension of a p-brane, appearing in front of the action (2), is given by

$$T_p = \pi g_s (4\pi^2 \alpha')^{-(p+1)/2} , \tag{3}$$

where g_s is the string coupling constant. We assume small string coupling so that the fluctuations of the branes will be small. We choose to work with conformal time η in a background metric of the form

$$G_{\mu\nu} = a(\eta)^2 diag(-1, 1, \ldots, 1) , \tag{4}$$

where $a(\eta)$ is the cosmological scale factor.

Assuming that the transverse fluctuations of the brane and the gauge fields on the brane are small, it is possible to expand the brane action as

$$
\begin{aligned}
S_p &= T_p \int d^{p+1}\zeta a(\eta)^{p+1} e^{-\phi} \\
&\times e^{12 tr log(1 + \partial_m \phi_i \partial_n \phi_i + a(\eta)^{-2} 2\pi \alpha' F_{mn})} \\
&= T_p \int d^{p+1}\zeta a(\eta)^{p+1} e^{-\phi} \\
&\times (1 + 12(\partial_m \phi_i)^2 - \pi^2 \alpha'^2 a^{-4} F_{mn} F^{mn}) .
\end{aligned}
\tag{5}
$$

The first term inside the parentheses in the last line represents the brane winding modes, the second term corresponds to the transverse fluctuations, and the third term relates to brane matter. In the low-energy limit, the transverse fluctuations of the brane are described by a free scalar field action, and the longitudinal fluctuations are given by a Yang-Mills theory. The induced equation of state which describes the second and third terms has pressure $p \geq 0$.

We are now ready to compute the equation of state for the brane gases for various p. There are three types of modes that we will need to consider. First, there are the winding modes. The background space is T^9, and hence a p-brane can wrap around any set of p toroidal directions. These modes are related by T-duality to the momentum modes corresponding to

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the center of mass motion of the branes. Finally, the modes corresponding to fluctuations of the branes in the transverse directions are (in the low-energy limit) described by the scalar fields on the brane, ϕ_i. There are also bulk matter fields and brane matter fields.

Let us begin by considering the winding modes. From equation (5), one can compute the equation of state for a winding p-brane:

$$\tilde{p} = w_p \rho \quad \text{with} \quad w_p = -pd, \tag{6}$$

where d is the number of spatial dimensions, and $\tilde{p}$ and ρ represent the pressure and energy density, respectively.

Both fluctuations of the branes and brane matter are described by free scalar and gauge fields living on the brane. These can be viewed as particles in the directions transverse to the brane and thus the equation of state is just that of ordinary matter:

$$\tilde{p} = w\rho \quad \text{with} \quad 0 \le w \le 1. \tag{7}$$

From the action (5) we see that the energy in the winding modes will be

$$E_p(a) \sim T_p a(\eta)^p, \tag{8}$$

where the constant of proportionality is dependent on the number of branes. Note that the energy in the winding modes increases with the expansion of the Universe in contrast to the energy of the brane fluctuations and brane matter.

2.2   Background Equations of Motion:

The background equations of motion are calculated from the variation of (1) with the metric (4) and were derived (in the absence of H) in [8] (see also [11]). It is convenient to define

$$\lambda(t) = log(a(t)), \tag{9}$$

and to use the "shifted" dilaton

$$\varphi = 2\phi - d\lambda, \tag{10}$$

which absorbs the space volume factor. Assuming for simplicity the isotropic case (all $a_i = a$ and therefore $\lambda_i = \lambda$) the background equations are

$$-d\dot{\lambda}^2 + \dot{\varphi}^2 = e^\varphi E \tag{11}$$

$$\ddot{\lambda} - \dot{\varphi}\dot{\lambda} = 12e^\varphi P \tag{12}$$

$$\ddot{\varphi} - d\dot{\lambda}^2 = 12e^\varphi E, \tag{13}$$

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where E and P denote the total energy and pressure, respectively. Both sources E and P are made up from contributions of all the branes in the gas. We have already calculated these contributions in the previous section:

$$
\begin{aligned}
E &= \sum_p E_p^w + E^{nw} \\
P &= \sum_p w_p E_p^w + w E^{nw} ,
\end{aligned}
\tag{14}
$$

where the superscripts w and nw stand for the winding modes and the non-winding modes, respectively. Here the contributions of the non-winding modes of all branes are combined into one term. The constants w_p and w are given by (6) and (7). Each E_p^w is the sum of the energies of all of the winding branes with dimension p.

2.3 Brane Gas Cosmology:

We now turn our attention to the model of Brane Gas Cosmology proposed in [6]. The first important aspect of BGC is that it is free of the initial cosmological singularity of the SBB model. This can be explained by the T-duality symmetry of string theory as was first demonstrated in [7]. As the Universe contracts, the T-duality self-dual fixed point $(R = 1)$ is reached at some temperature T less than the limiting Hagedorn temperature. As the background continues to contract, the temperature *decreases* according to the T-duality equation for temperature,

$$
T(1R) = T(R) .
\tag{15}
$$

Thus, there is no physical singularity as R approaches 0.

Let us now examine the de-compactification mechanism which leads to three large spatial dimensions [6, 7]. Recall, that our starting point is a Universe with nine spatial dimensions all equal and near the self-dual point, $R = 1$. The Universe is in thermal equilibrium, and by symmetry we argue that there are equal numbers of winding and anti-winding modes.

As the Universe begins to expand (symmetrically in all directions), λ increases and the total energy in the winding modes increases according to equation (8). Note that the energy in the winding modes of the branes with the largest value of p increases the fastest. Therefore branes with the largest value of p will have an important effect first. As we have already explained, the winding modes will prevent further expansion until they have annihilated.

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A simple counting argument demonstrates that the world-volumes of two p-branes will probably intersect in at most $2p + 1$ spatial dimensions. [3] Clearly, in $d = 9$ spatial dimensions the winding branes with $p = 8, 6, 5$ and 4 will have no problems interacting and self-annihilating. However, branes with smaller values of p will allow a hierarchy of dimensions to expand. Since the energy of the branes with the largest value of p is greatest, the 2-branes will first allow a 5-dimensional torus to expand. Within this T^5, the strings (1-branes) will allow a T^3 subspace to become large. Hence, this mechanism provides a solution to the dimensionality problem of string theory and may explain the origin of our large $3 + 1$-dimensional Universe. [4]

3   A Loitering Universe

Despite the ability of winding modes to self-annihilate in a distinguished number of dimensions, causality demands that there is at least one winding mode per Hubble volume remaining [13]. In our macroscopic four-dimensional spacetime, the wrapped branes with $p \geq 2$ will appear as domain walls. The presence of such topological defects would result in a cosmological disaster, as even one wall per Hubble volume today will overclose the Universe if the tension of the brane is larger than the electroweak scale. [5] This "domain wall problem" [14] is common in cosmological scenarios based on quantum field theories which admit domain wall solutions.

As a solution of this "brane problem" in BGC the authors of [6] suggested a phase of cosmological loitering. If at some stage in the evolution of the Universe the Hubble radius becomes larger than the spatial extent of the Universe, there is no causal obstruction for all winding modes to annihilate. Loitering allows for previously causally disconnected regions of the Universe to communicate, and therefore provides a solution to the horizon problem of the SBB model.

Within the context of dilaton gravity, cosmological solutions which exhibit a loitering

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[3]For example, consider two particles (0-branes) moving through a space of dimension d. These particles will definitely interact (assuming the space is periodic) if $d = 1$, whereas they probably will not find each other in a space with $d > 1$.

[4]By large we mean large compared to the string scale. Without inflation we are not able to solve the entropy problem of standard cosmology, namely to produce a Universe large enough to contain our present observed Universe.

[5]Even strings have large tensions at low temperatures and will therefore contribute too much to the energy density of the Universe.

phase appear rather naturally due to the presence of winding modes. To see this, we will examine the phase space of solutions to the background equations (11) - (13). The phase space of solutions for general p was discussed in [8], and a numerical plot of the full phase space is given below.

By defining $l = \dot{\lambda}$ and $f = \dot{\varphi}$, the background EOM equations (11) - (13) simplify to two first order differential equations:

$$\dot{l} = pl^2 2 + lf - \frac{pf^2}{2d}, \tag{16}$$

$$\dot{f} = dl^2 2 + f^2 2. \tag{17}$$

Notice that for positive energy density E, equation (11) implies that $\dot{\varphi}$ will never change sign. We are interested in studying the initial conditions with $\dot{\varphi} < 0$. If $\dot{\varphi} \not< 0$ the boosting effect of the dilaton on λ will invalidate the adiabatic approximation used in the derivation of the EOM. Furthermore, growing φ together with expanding λ implies the growth of the effective coupling $\exp(\phi)$ in contradiction with a weak coupling assumption [8].

We will therefore consider solutions to the background EOM with initial conditions corresponding to an expanding Universe $\dot{\lambda} > 0$ with $\dot{\varphi} < 0$. Note that positivity of E imposes another restriction (see equation (11)):

$$|l| < 1\sqrt{d}|f|. \tag{18}$$

Solutions with initial conditions described above are driven towards the $l = 0$ line in the f vs. l phase space at a finite value of f (see Fig. 1).

There are three special lines in the phase space which correspond to straight line trajectories and pass through the origin:

$$l\dot{f} = lf = \pm 1\sqrt{d}, \, pd. \tag{19}$$

Notice that the line $l/f = -1/\sqrt{d}$ is a repeller and solutions which start out in the energetically allowed region are pushed away from the line, cross the f-axis and then approach the attractor line $l/f = p/d$ to the origin. Near the $l = 0$ line, equations (16) and (17) may be approximated by

$$\dot{l} \simeq -p2df^2, \, \dot{f} \simeq f^2 2. \tag{20}$$

When a solution crosses the f-axis, l changes sign. This means that the Universe begins to contract. Since $f \to 0$ only as $t \to \infty$ the solution never crosses the l-axis.
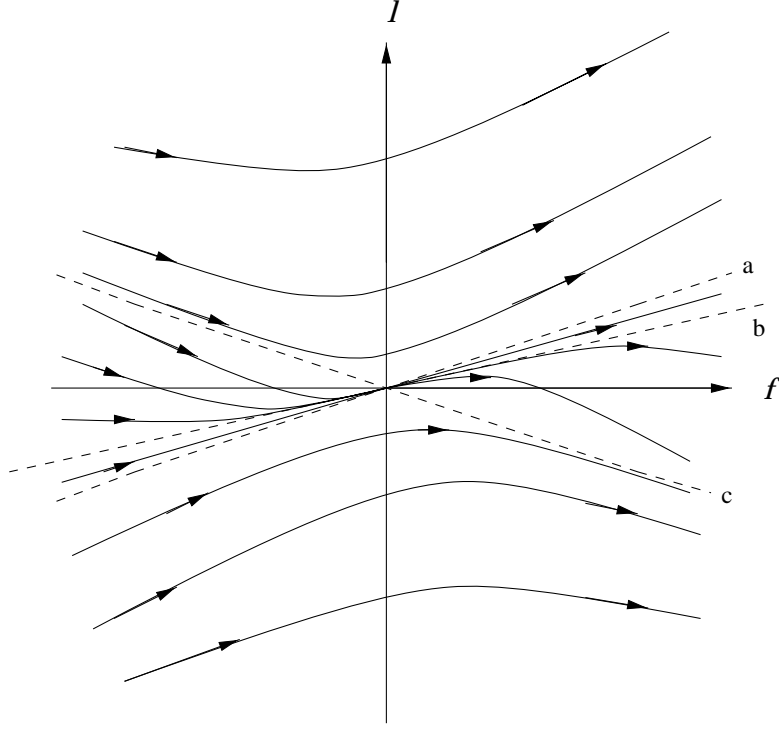
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Figure 1: Phase space trajectories of the solutions of the background equations (11 - 13) for the values $p = 2$ and $d = 9$. The energetically allowed region lies near the $l = 0$ axis between the special lines a and c, which are the lines given by $l/f = \pm 1/\sqrt{d}$. The trajectory followed in the scenario investigated in this paper starts out in the upper left quadrant close to the special line c (corresponding to an expanding background), crosses the $l = 0$ axis at some finite value of f (at this point entering a contracting phase), and then approaches the loitering point $(l, f) = (0, 0)$ along the phase space line b which corresponds to $l/f = p/d$.

Note that this analysis assumes that the winding modes are not decaying into loops. The above provides an accurate description of the early Universe, before winding modes have self-annihilated. We will examine the case of loop production and the late time evolution of the Universe in Section 4.

4 Unwinding and Loop Production

We now wish to extend the analysis presented in Section 3 in order to study the late time behavior of the Universe, i.e. to include the effects of winding mode annihilation and loop production.

Recall that after the winding modes have annihilated, a three-dimensional subspace will grow large. In what follows we will therefore take $d = 3$. The strings in the theory are the last branes to unwind which implies at late times that we should consider the case of $p = 1$. When the winding strings self-annihilate they create loops in the $3 + 1$-dimensional universe.

We now set up the equations describing the unwinding and corresponding loop production. They are analogous to the corresponding equations for cosmic strings in an expanding Universe. First, note that the energy density ρ_w in winding strings can be expressed in terms of the string tension μ and the number $\tilde{\nu}(t)$ of winding modes per "Hubble" volume t^3 as

$$\rho_w(t) = \mu \tilde{\nu}(t) \, t^{-2} \, . \tag{21}$$

Since loops are produced by the intersection of two winding strings, the rate of loop production is proportional to $\tilde{\nu}^2$:

$$\frac{dn(t)}{dt} = c \tilde{\nu}(t)^2 \, t^{-4} \, , \tag{22}$$

where $n(t)$ is the number density of loops and c is a proportionality constant expected to be of the order 1. The energy density in the winding modes decreases both due to the expansion of space and due to the decay into loops:

$$\frac{d\rho_w(t)}{dt} + 2l \, \rho_w(t) = -c' \mu t \frac{dn(t)}{dt} = -cc' \mu \tilde{\nu}(t)^2 \, t^{-3} \, , \tag{23}$$

where c' is a constant which relates the mean radius $R = c't$ of a string loop to its length. Without loop production ($c = 0$), the energy density ρ_w redshifts corresponding to the equation of state $p = -13\rho$. This explains the coefficient of the Hubble damping term in (23). [6] Inserting equation (21) into the energy conservation equation (23), we obtain an

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[6] For more on strings in an expanding Universe see, e.g. [15].

equation for $\tilde{\nu}(t)$:

$$\frac{d\tilde{\nu}(t)}{dt} = 2\tilde{\nu}(t^{-1} - l) - cc't^{-1}\tilde{\nu}^2 \, . \tag{24}$$

In addition to $\rho_w(t)$, we will also require information about the energy density in loops, $\rho_l(t)$. The energy density in loops obeys the conservation equation

$$\frac{d\rho_l(t)}{dt} + 3l\,\rho_l(t) = cc'\mu\tilde{\nu}(t)^2\,t^{-3} \, . \tag{25}$$

As in equation (23), the second term on the left hand side of the equation represents the decrease in the density due to Hubble expansion, with the coefficient reflecting the equation of state $p = 0$ of a gas of static loops, and the term on the right hand side representing the energy transfer from winding modes to loops. Without loop production, $\rho_l(t)$ would scale as

$$\rho_l(t) = g(t)e^{-3(\lambda(t)-\lambda_0)} \, , \tag{26}$$

with $g(t)$ constant. Here $\lambda_0 = \lambda(t_0)$, where t_0 is some initial time, and $g(t)$ is a function which obeys the equation

$$\frac{dg(t)}{dt} = cc'\mu t^{-3}\tilde{\nu}^2 e^{3(\lambda(t)-\lambda_0)} \, . \tag{27}$$

Using the expressions for $\rho_w(t)$ and $\rho_l(t)$ as sources for the energy density E and pressure P in equations (11) - (13) we can obtain background equations analogous to equations (16) and (17):

$$\dot{l} = lf + \frac{1}{2}l^2 - \frac{1}{6}f^2 + \frac{1}{6}ge^{\varphi+3\lambda_0} \, , \tag{28}$$

$$\dot{f} = \frac{1}{2}f^2 + \frac{3}{2}l^2 \, . \tag{29}$$

(Recall that $p = 1$ and $d = 3$ in the above equations.)

Equations (24), (27), (28) and (29), along with the equations $l = \dot{\lambda}$ and $f = \dot{\varphi}$ provide six first-order differential equations which fully describe the Universe during the process of loop production. These provide us with the initial conditions required for a numerical analysis. Note that the $c = 0$ case corresponds to no loop production and the background equations reduce to the previous equations (16) and (17).

Figure 2 demonstrates the behavior of a typical numerical solution to the EOM having initial conditions in the energetically allowed region of the phase space, taking into account the effects of loop production. Recall that when no loops are produced (see Fig. 1) the
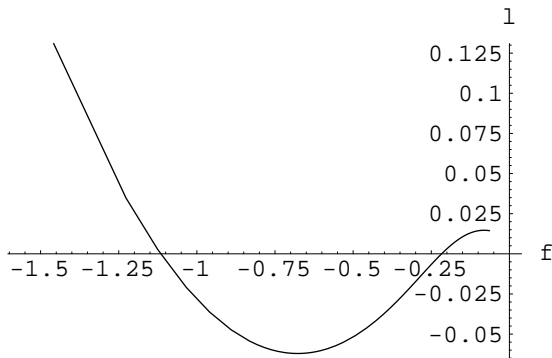
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Figure 2: A solution of the background equations (28) and (29) including the effects of loop production. This depicts a typical solution which starts in the energetically allowed region of the phase space. The solution crosses the $l = 0$ axis at some finite value of f (at which point the Universe enters a contracting phase), and then crosses the $l = 0$ line a second time when the winding modes have fully annihilated. At this point the Universe begins to expand, is matter dominated and the dilaton is assumed to become massive.

solutions of interest cross the $l = 0$ line only once and approach the origin of the phase space as $t \to \infty$. When the decay of the winding modes is taken into account, the solutions are pushed back over the $l = 0$ line as in Fig. 2.

In more detail, the dynamics of our loitering solution is depicted in Figures 3, 4, 5 and 6. Figure 3 shows the time evolution of the Hubble expansion rate $H = l$. Note that since we have set Newton's constant $G = 1$ in our background equations, time is measured in Planck time units. By comparing with the value of $a(t)$ from Fig. 4, we see that

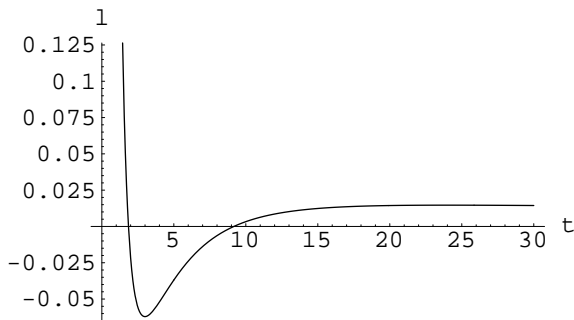$$H^{-1}(t) \gg a(t) \tag{30}$$

Figure 3: The time evolution of $H = l$. The loitering phase begins when $l(t)$ crosses the $l = 0$ line for the first time and ends when $l(t)$ crosses back over the $l = 0$ line.
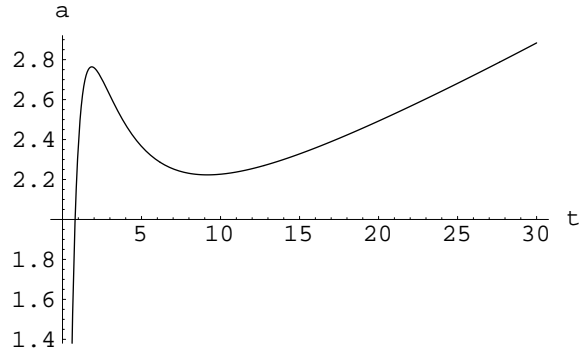
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Figure 4: The time evolution of the scale factor a. By comparing this plot with Fig. 3 we see that the loitering phase lasts long enough to allow all winding modes to self-annihilate in the large three-dimensional Universe.
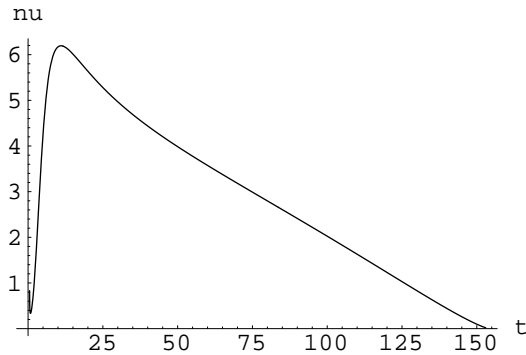

Figure 5: Time evolution of $\tilde{\nu}$. Initially, $\tilde{\nu}$ increases as the Universe contracts. The winding modes begin to self-annihilate ($\tilde{\nu}$ decreases) and eventually vanish ($\tilde{\nu} \to 0$).

during the loitering phase. Keeping in mind that the initial spatial size of the tori is Planck scale, it follows immediately from equation (30) and from the time duration of the loitering phase that loitering lasts sufficiently long to allow causal communication over the entire spatial section. This is reflected in Fig. 5 which shows that the winding modes completely annihilate by the end of the loitering phase, after which $g(t)$ tends to a constant (Fig 6).

Our modified picture is as follows: the Universe begins to expand until the winding modes become too massive and force the expansion to stop and contraction to begin. This corresponds to the solution in Fig. 2 crossing over the $l = 0$ axis and signals the beginning of the loitering phase. The loitering phase ends when the solution is pushed back over the $l = 0$ line due to the decay of the winding modes. From this point on the Universe begins to expand again.

Soon after the solutions cross the $l = 0$ line for the second time, our equations become
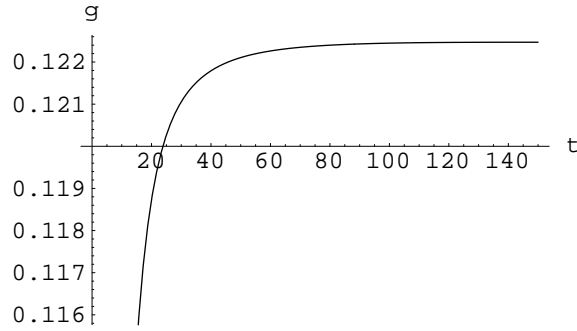
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Figure 6: Time evolution of g. Note that g goes to a constant when $\tilde{\nu}$ goes to zero.

singular. Soon after the winding modes vanish ($\tilde{\nu}(t) \to 0$), all the fields l, f, λ and φ in our equations of motion blow up. This singularity does not concern us however since it can be eliminated by the introduction of a simple potential $V(\phi)$ used to freeze the dilaton at the moment loop production is exhausted. The massless dilaton does not appear in nature and therefore such a potential is required in any string theory-motivated cosmological model at late times. The precise mechanism responsible for dilaton mass generation is unknown, although it is often suspected that this mechanism will coincide with the breaking of supersymmetry. From this analysis we are lead to the conjecture that dilaton mass generation may coincide with the elimination of winding modes. We will comment more on this below.

For the time being, let us assume that the dilaton has frozen at the value $C_\phi = \phi(t_{freeze})$ and therefore $\dot{\phi} = \ddot{\phi} = 0$. We will also assume that this occurs when the winding modes have vanished, $\tilde{\nu}(t) \to 0$ and hence the number of loops has reached a constant so that $g(t) \to C_g$. Now the EOM simplify greatly. By fixing the dilaton in equations (28) and (29) we can derive an equation for the scale factor (after shifting back to the true dilaton ϕ):

$$\dot{a} - C_\gamma a^{-\frac{1}{2}} = 0 \,, \tag{31}$$

where C_γ is a constant given by

$$C_\gamma = \sqrt{\frac{C_g}{12}} \, e^{C_\phi + \frac{3}{2}\lambda_0} \,. \tag{32}$$

The most general solution to equation (31) is

$$a(t) = \left(\frac{3C_\gamma}{2}\right)^{\left(\frac{2}{3}\right)} \left(t^2 - 2Ct + C^2\right)^{\left(\frac{1}{3}\right)} \,, \tag{33}$$

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where C is an integration constant. For the value $C = 0$ or for large values of t (late times), the scale factor grows as

$$a(t) \sim t^{\frac{2}{3}},\tag{34}$$

which is exactly the behavior for a matter dominated Universe.

Our interpretation is that the winding modes look like solitons in the $3 + 1$-dimensional Universe. The self-annihilation of these winding modes corresponds to the creation of matter in the Universe and the scale factor evolves appropriately. In our equations, the loops are modelled as static. In reality, the loops will oscillate and decay by emitting (mostly) gravitational radiation, thus producing a radiation dominated Universe.

Let us return to the issue of SUSY breaking and dilaton mass generation. One thing which appears inevitable within the context of this model is the spontaneous "breaking" of T-duality in the large four-dimensional Universe. This is most easily understood once all of the winding modes have self-annihilated since it is impossible to create new ones. It would cost too much energy for a brane to wrap around the large dimensions. Thus, the state of the system is not symmetric under T-duality, and in the absence of string winding modes and for fixed dilaton, our background equations reduce to those of Einstein's General Relativity which do not exhibit the $R \leftrightarrow 1/R$ symmetry of string theory.

It is also interesting to note that there seems to be a relation between the amount of supersymmetry in a theory and the presence of T-duality. Using a specific example in [16], Aspinwall and Plesser show that T-duality can be broken by nonperturbative effects in string coupling. Furthermore, a holonomy argument is given to show that T-dualities should only be expected when large amounts of supersymmetry are present. It seems very likely that the dynamics in the BGC scenario will cause SUSY to break. This result seems to be in agreement with the possibility of dilaton mediated SUSY breaking occurring simultaneously with the breaking of T-duality.

Considering the above evidence we are inclined to hypothesize about the possible relations between supersymmetry breaking and the breaking of T-duality, as well as dilaton mass generation and the vanishing of winding modes.

5   Conclusions and Speculations

In this paper we have conducted a detailed analysis of a loitering phase in the model of Brane Gas Cosmology presented in [6]. The concrete starting point of BGC is M-theory

compactified on S^1 which gives ten-dimensional, type-IIA string theory. We assume toroidal topology in all nine spatial dimensions. The initial conditions for the Universe include a small, hot, dense gas of the p-branes in the theory. These fundamental degrees of freedom are assumed to be in thermal equilibrium. T-duality ensures that the initial singularity of the SBB model is not present in this scenario.

We compute the equation of state for the brane gas system and the background equations of motion. We study the solutions which initiate in the energetically allowed region of the phase space. The Universe tries to expand until winding modes force the expansion to stop and a phase of slow contraction (loitering) to begin. Loitering provides a solution to the brane problem first discussed in [6]. It also provides a solution to the horizon problem of the SBB model without requiring an inflationary phase. [7]

The counting argument of [6] demonstrates that winding modes will allow a hierarchy of dimensions in the T^9 to grow large. When the string winding modes self-annihilate we are left with a large T^3 subspace, simultaneously explaining the origin of our $3 + 1$-dimensional Universe and solving the dimensionality problem of string theory.

Branes wrapped around the cycles of the torus appear as solitons in the early Universe. They are topological defects (domain walls for $p \geq 2$). When the winding modes and anti-winding modes self-annihilate, matter is produced and the Universe begins to expand again. We hypothesize that winding mode annihilation corresponds to dilaton mass generation. We also believe there may be a relation between SUSY breaking and the breaking of T-duality, although we cannot provide any direct evidence for this. Once winding states have vanished, we cannot map momentum modes into winding modes via T-duality. The breaking of T-duality requires further study.

Particle phenomenology demands compactification on Calabi-Yau three-folds if the four-dimensional low energy effective theory is to have $N = 1$ supersymmetry. Calabi-Yau compactifications will be studied in future works, However, in the context of early Universe cosmology it is not reasonable to require $N = 1$ supersymmetry. Toroidal backgrounds, such as the one considered here, are compatible with maximal supersymmetry.

Brane Gas Cosmology provides a method of incorporating string and M-theory into cosmology which is an alternative to popular "brane world" scenarios. In our opinion, BGC has the advantage over brane world scenarios in that its foundations are analogous to those

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[7]Here we do not address the structure formation problem or the flatness problem of the SBB model. Both of these are solved by an inflationary phase. It is likely that the brane gas scenario will require something like inflation in order to produce a Universe which is large enough to contain the known Hubble radius.

of the Standard Big Bang model. In the BGC model the Universe starts out small, hot and dense, with no initial singularity. All the current versions of brane world scenarios embedded in string theories rely on the compactification of the extra dimensions by hand. In our opinion this is a considerable problem which is often overlooked. A dynamical mechanism in BGC leads naturally to four large space-time dimensions.

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