

# Theoretical Aspects of Neutrino Oscillations<sup>\*</sup>

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## Abstract

I review some aspects concerning the physics of neutrino mixing and oscillations. I discuss in some detail the physical neutrino oscillations parameter space in the case of two and three family mixing, and briefly describe the current knowledge of neutrino mixing parameters according to the present solar, atmospheric, and reactor neutrino data. I also briefly comment on the possibility of solving the LNSD anomaly together with the solar and atmospheric ones. I conclude by emphasising that that even though in five to ten years time a lot will be learnt from the next round of neutrino experiments, a great deal about neutrino masses and neutrino mixing will remain unknown.

*Key words:* neutrino mixing, neutrino oscillations, neutrino puzzles, NuFACT'01

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## 1 Introduction and Motivation

There are, currently, three neutrino puzzles, two which strongly indicate the presence of physics beyond the standard model (SM). The oldest one – the solar neutrino puzzle [1] – is the fact that the flux of electron-type solar neutrinos measured at the Earth is significantly smaller than predicted by solar physics models. This deficit is confirmed by different experiments, which make use of very different techniques for detecting neutrinos. It is fair to say that, after the publication of the first results from SNO [2], there is very strong evidence for a flux of  $\nu_{\mu,\tau}$  coming from the Sun.

The second – the atmospheric neutrino puzzle [3] – is the fact that the flux of atmospheric muon-type neutrinos and antineutrinos differs significantly from

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theoretical predictions. This anomalous behaviour was first observed by the proton decay experiments IMB and Kamiokande, and later confirmed and firmly established by the SuperKamiokande experiment. The anomaly manifests itself more strongly in the ratio of the predicted  $\nu_e$  and  $\nu_\mu$  flux-ratio to the experimentally measured one, and the nontrivial angular dependency of the  $\nu_\mu$  flux. The latter is by far the most striking evidence for physics beyond the SM we have at the moment.

Finally, the LSND experiment [4], which studies neutrinos produced after pion and muon decays, has observed a 3-sigma excess of  $\bar{\nu}_e$ -like events from  $\mu^+$  decays. The LSND anomaly has not been confirmed or excluded by KARMEN [5] or other neutrino experiments. The situation will improve significantly with the advent of the MiniBoone [6] experiment, which is due to start taking data in 2002.

The neutrino puzzles are best solved by assuming that the neutrinos have mass, and that neutrino mass eigenstates and weak eigenstates differ, hence the neutrinos “oscillate.” The solar neutrino puzzle is best solved by assuming  $\nu_e \leftrightarrow \nu_{\mu,\tau}$  oscillations [7], while the atmospheric neutrino puzzle hints at quasi-maximal  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations [8]. The LSND anomaly requires  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations. There are, however, other exotic solutions to individual neutrino puzzles in the market, such as new neutrino–matter interactions, violation of Lorentz invariance and quantum mechanics, and neutrino decays, to name a few. Usually, these solutions are tailor-made to address a particular neutrino puzzle. It is fair to say that neutrino oscillations is the only hypothesis that can properly address *all* neutrino anomalies.

In this talk, I review some aspects of neutrino oscillations. In the next section, I discuss neutrino mixing and how it leads to neutrino flavour conversion, both in the absence and presence of a medium. I concentrate on two and three family mixing scenarios, and spend some time discussing the “physical parameter space.” In Sec. 3, I briefly review how neutrino oscillations solve the neutrino puzzles, and what the current data can and cannot tell us about the oscillation parameters. In Sec. 4, I discuss some of the issues that must be faced if the LSND anomaly is also to be explained by neutrino oscillations, and in Sec. 5 I conclude with an outlook regarding what we may expect to learn in the next few years and what will be left to do with future, more powerful, machines (neutrino factories?).

## 2 Neutrino Mixing and Oscillations

If neutrinos have mass, there is no reason for the mass eigenstates to coincide with the weak (also referred to as flavour) eigenstates. This not being the

case, the two bases are connected by a unitary matrix  $\nu_\alpha = U_{\alpha i} \nu_i$ , where  $\nu_\alpha$  are weak eigenstates ( $\alpha = e, \mu, \tau, \dots$ ) and  $\nu_i$  are mass eigenstates, with masses  $m_i$  ( $i = 1, 2, 3, \dots$ ).  $U_{\alpha i}$  will be referred to as the neutrino mixing matrix.<sup>2</sup> Note that one can choose to label the mass eigenstates *in ascending order of mass-squared*, with no loss of generality. This will be assumed, unless otherwise noted.

When neutrinos propagate in vacuum, it is very simple to calculate the probability that a neutrino with energy  $E_\nu$ , which is produced as a weak eigenstate  $\nu_\alpha$ , is detected a distance  $L$  from the source as a weak eigenstate  $\nu_\beta$ :

$$P(\nu_\alpha \rightarrow \nu_\beta)(L) \equiv P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \Re(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E_\nu} \right) - 2 \sum_{i < j} \Im(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left( \frac{\Delta m_{ij}^2 L}{2E_\nu} \right), \quad (1)$$

where  $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$  are the mass-squared differences. For antineutrinos, one simply has to replace  $U_{\alpha i} \leftrightarrow U_{\alpha i}^*$ , meaning that eq. (1) holds as long as the sign of the last term is flipped. One can define the different oscillation lengths

$$L_{\text{osc}}^{ij} = \pi \frac{4E_\nu}{\Delta m_{ij}^2} = \pi \left( \frac{E_\nu}{\text{GeV}} \right) \left( \frac{\text{eV}^2}{1.267 \Delta m_{ij}^2} \right) [\text{km}]. \quad (2)$$

Figure 1 depicts  $L_{\text{osc}}^{ij}$  as a function of  $\Delta m_{ij}^2$  for different values of  $E_\nu$ . When the neutrinos propagate in matter (of either constant or varying density) the situation is significantly altered, as will be discussed in the next two subsections.

## 2.1 Two Neutrino Mixing

If there are oscillation between only two neutrino states, the situation is rather simple. The neutrino mixing matrix is parametrised in terms of only one angle  $\theta$ , and one extra phase if the neutrinos are Majorana particles. This phase, however, is not observable in oscillation phenomena, and may be safely set to zero. Explicitly, if there is mixing only between  $\nu_e$  and  $\nu_x$ ,

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad (3)$$

<sup>2</sup> There is some controversy regarding how this matrix should be called. I will stir clear of this issue here.

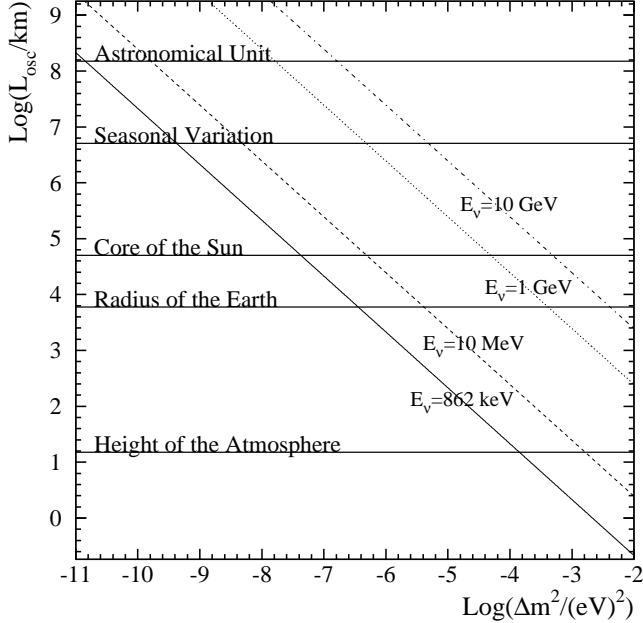


Fig. 1.  $L_{\text{osc}}^{ij}$  as a function of  $\Delta m_{ij}^2$ , for different values of the neutrino energy, namely,  $E_\nu=862$  keV ( ${}^7\text{Be}$  solar neutrino),  $E_\nu=10$  MeV (“typical”  ${}^8\text{B}$  solar neutrino),  $E_\nu=1$  GeV (“typical” atmospheric neutrino) and  $E_\nu=10$  GeV.

and

$$P_{ee}^{\text{vac}} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m_{12}^2 L}{4E_\nu} \right). \quad (4)$$

In the presence of neutrino–medium interactions, the neutrino propagation is modified [9]. These effects can be expressed in terms of a “matter potential,” such that neutrino propagation is governed by the following Schrödinger-like equation (in the weak basis),

$$i \frac{d}{dL} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \left[ \frac{\Delta m_{12}^2}{2E_\nu} \begin{pmatrix} \sin^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \cos^2 \theta \end{pmatrix} + \begin{pmatrix} V_{ee} & V_{ex} \\ V_{ex}^* & V_{xx} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}, \quad (5)$$

where  $V_{\alpha\beta}$  are the different matter potential functions, which are in principle a function of the position. If  $\nu_x$  is a linear combination of the other active neutrinos ( $\nu_\mu$  and  $\nu_\tau$ ), the matter potential matrix is such that  $V_{ex} = 0$  and  $V_{ee} - V_{xx} = \sqrt{2}G_F N_e$ ,<sup>3</sup> if the neutrinos propagate in “normal” matter with electron number density  $N_e$ , and only standard model (SM) interactions are taken into account. For antineutrinos,  $N_e$  is replaced by  $-N_e$  (which is the positron number density).

Two important issues can be addressed just by inspecting the general form of eq. (5). First of all, while eq. (5) is, in principle, not solvable, we can still construct useful relations between the different  $P_{\alpha\beta}$ . In particular, because the

<sup>3</sup> Note that the “Hamiltonian” in eq. (5) is defined up to a term proportional to the identity matrix, which does not yield any physically observable effect.

“Hamiltonian” is hermitian (this condition would break down, for example, in the presence of neutrino absorption by the medium or if the neutrinos decay),  $\sum_{\alpha} P_{\alpha\beta} = \sum_{\beta} P_{\alpha\beta} = 1$ . This implies, in the case of two neutrino oscillations, that only one  $P_{\alpha\beta}$  is independent. For example, one can express all  $P_{\alpha\beta}$  in terms of  $P_{ee}$ :  $P_{ex} = P_{xe} = 1 - P_{ee}$ ,  $P_{xx} = P_{ee}$ . It is curious to note that, in the case of two family oscillation,  $P_{ex} = P_{xe}$  (T-invariance) is guaranteed, while nothing, in principle, prevents  $P_{ee} \neq P_{\bar{e}\bar{e}}$  (matter induced CP-violation).

Second, it is interesting to define what is the “physical range” for the oscillation parameter  $\theta$ , *i.e.* what are the values of  $\theta$  which yield, if one measures  $P_{\alpha\beta}(L)$ , distinct results? Initially, because  $\theta$  is an angle, it need be defined only from 0 to  $2\pi$ . On top of this, it is trivial to check that the transformation  $\theta \rightarrow \pi - \theta$  is equivalent to  $\theta \rightarrow -\theta$ , in the sense that the differential equations eq. (5) transform in exactly the same way. This implies that choosing values of  $\theta = [0, \pi]$  is enough (this may also be seen by noting that eq. (5) depends only on  $2\theta$ , up to irrelevant terms proportional to the unit matrix). Furthermore, *if the matter potential matrix is diagonal*, the transformation  $\theta \rightarrow \pi - \theta$  modifies the differential equations in such a way that  $(\nu_e(L), \nu_x(L))_{\pi-\theta} = (\nu_e(L), -\nu_x(L))_{\theta}$ . Because  $P_{\alpha\beta}(L) = |\nu_{\beta}(L)|^2$  assuming the boundary condition is a pure  $\nu_{\alpha}$  state at  $L = 0$ ,  $\theta$  and  $\pi - \theta$  yield the same  $P_{\alpha\beta}(L)$ , and the physical parameter space can be safely restricted to  $[0, \pi/2]$ . This is *not* that case if the matter potential has off-diagonal terms and/or if the “weak” eigenstates at the production point differ from the “weak” eigenstates at the detection point. This is not the case within the SM, but in extensions to it one may indeed be able to distinguish  $\theta$  from  $\pi - \theta$ . Finally, in the absence of a matter potential (which is the case of vacuum oscillations), there is yet another “symmetry:”  $(\nu_e(L), \nu_x(L))_{\pi/2-\theta} = (\nu_e^*(L), -\nu_x^*(L))_{\theta}$ , such that the entire physical parameter space can be spanned by allowing  $\theta$  to vary from 0 to  $\pi/4$ . This can be explicitly seen in eq. (4).

The parameter space in the absence of physics beyond the SM is easy to interpret. In the region  $\theta = [0, \pi/4]$  (the light side)  $\nu_e$  is “predominantly light” ( $\nu_1$ ), while when  $\theta = [\pi/4, \pi/2]$  (the dark side)  $\nu_e$  is “predominantly heavy” ( $\nu_2$ ). While pure vacuum oscillations cannot tell the dark from the light side, matter effects break the degeneracy and allow one to explore the entire  $[0, \pi/2]$  range.

In many cases of interest, eq. (5) can be solved, at least approximately. If the neutrinos propagate in a constant electron number density,  $P_{ee}$  is easily calculable, and is given by eq. (4), where the mass-squared difference and the mixing angle are modified to “matter” parameters, given by

$$\cos 2\theta_M = \frac{\Delta_{12} \cos 2\theta - A}{\sqrt{\Delta_{12}^2 + A^2 - 2A\Delta_{12} \cos 2\theta + 4\Re(V_{ex})\Delta_{12} \sin 2\theta + 4|V_{ex}|^2}}, \quad (6)$$

$$\Delta_{12}^M = \sqrt{\Delta_{12}^2 + A^2 - 2A\Delta_{12} \cos 2\theta + 4\Re(V_{ex})\Delta_{12} \sin 2\theta + 4|V_{ex}|^2}, \quad (7)$$

where  $\Delta_{ij} \equiv \Delta m_{ij}^2/(2E_\nu)$  and  $A \equiv V_{ee} - V_{xx}$ . As discussed in the previous paragraph, the matter angle and frequency depend only on  $2\theta$ , and, if  $V_{ex} = 0$ , are invariant under  $\theta \rightarrow \pi - \theta$ .

Another very useful solution exists in the case of neutrinos propagating in a monotonically falling electron number density, as is the case of neutrinos produced in the Sun's core [10]:

$$P_{ee} = P_1 \cos^2 \theta + P_2 \sin^2 \theta - \cos 2\theta_M \sqrt{P_c(1 - P_c)} \sin 2\theta \cos(\Delta_{12}L + \phi_M), \quad (8)$$

where  $P_1 = 1 - P_2 = 1/2 + 1/2(1 - 2P_c) \cos 2\theta_M$ ,  $P_c$  is the ‘‘level crossing probability,’’  $\phi_M$  is a constant matter-induced phase, and the matter angle is to be computed at the production point. If the neutrino propagation inside the Sun is adiabatic,  $P_c = 0$ , while in the case of an exponential electron number density profile  $N_e(x) = N_0 \exp(-x/r_0)$ , the approximate form  $P_c = (\exp(-\gamma \sin^2 \theta) - \exp(-\gamma))/(1 - \exp(-\gamma))$ , where  $\gamma = 2\pi r_0 \Delta_{12}$ , is valid in a large portion of the parameter space [11]. Recently, a lot of progress has been made concerning the understanding of solar neutrino oscillations, including studies of whether the expressions above are valid for  $\theta > \pi/2$  [12,13] (the dark side of the parameter space), and how they should be understood and interpreted [14].

## 2.2 Three Neutrino Mixing

The three by three neutrino mixing matrix which connects  $\nu_e, \nu_\mu$ , and  $\nu_\tau$  to the mass eigenstates  $\nu_{1,2,3}$  is parametrised by three angles and one complex phase, plus two Majorana phases which are not detectable via neutrino oscillation experiments and can be safely set to zero. It is ‘‘traditional’’ to define the mixing angles  $\theta_{12,13,23}$  in the following way:

$$\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}, \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}, \quad \sin^2 \theta_{13} \equiv |U_{e3}|^2, \quad (9)$$

while

$$\Im(U_{e2}^* U_{e3} U_{\mu2} U_{\mu3}^*) \equiv \sin \theta_{12} \cos \theta_{12} \sin \theta_{23} \cos \theta_{23} \sin \theta_{13} \cos^2 \theta_{13} \sin \delta \quad (10)$$

defines the CP-odd phase  $\delta$ .

In the case of pure vacuum oscillations, the survival probabilities are given by eq. (1). It should be readily noted that, unlike the two-family case, if  $\delta \neq 0$  or  $\pi$ ,  $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$ , signalling CP-violation in the neutrino sector.<sup>4</sup>

If the neutrinos propagate inside a medium, the situation is (much) more complicated. In principle, one has to solve the Schrödinger-like equation (in the weak basis)

$$i \frac{d}{dL} \nu_\alpha = \left[ \sum_{i=2,3} \left( \frac{\Delta m_{1i}^2}{2E_\nu} \right) U_{\alpha i}^* U_{\beta i} + V_{\alpha\beta} \right] \nu_\beta, \quad \text{where } V_{\alpha\beta} = V_{\beta\alpha}^*. \quad (11)$$

On the other hand, because the ‘‘Hamiltonian’’ in eq. (11) is hermitian, the different oscillation probabilities are related. As before, the constraints  $\sum_\alpha P_{\alpha\beta} = 1$  and  $\sum_\beta P_{\alpha\beta} = 1$  arise from the unitary evolution of the quantum state. In the case of three family oscillation, they imply that there are only four independent  $P_{\alpha\beta}$ , which one may choose to be, say, the three ‘‘diagonal’’  $P_{\alpha\alpha}$  plus  $P_{e\mu}$ . The other five are linear combinations of these four. Explicitly,

$$\begin{aligned} P_{e\tau} &= 1 - P_{ee} - P_{e\mu}, & P_{\tau e} &= P_{\mu\mu} + P_{e\mu} - P_{\tau\tau}, \\ P_{\mu e} &= 1 + P_{\tau\tau} - P_{ee} - P_{\mu\mu} - P_{e\mu}, & & \\ P_{\mu\tau} &= P_{ee} + P_{e\mu} - P_{\tau\tau}, & P_{\tau\mu} &= 1 - P_{\mu\mu} - P_{e\mu}. \end{aligned} \quad (12)$$

Note that, unlike the two-family case,  $P_{\alpha\beta}$  may differ from  $P_{\beta\alpha}$ , even if the CP-odd phase  $\delta$  is ‘‘turned off’’ [15]. This means that the presence of the medium can induce not only matter CP-violation, but all matter T-violation. In order for this to happen, all that is required is that some  $V_{\alpha\beta}$  breaks T-invariance. This does not happen in the presence of a constant matter density, but may certainly happen, for example, for neutrinos produced inside the Sun [15]. Whether or not such effects are relevant for terrestrial neutrino beams has also been studied [16].

Also from eq. (11), it is possible to determine what is the ‘‘physical range’’ for the parameters  $\theta_{ij}$  and  $\delta$ . One can directly check, as discussed in the two-family case, that if  $V_{\alpha\beta} = 0$  for  $\alpha \neq \beta$ ,  $V_{\mu\mu} = V_{\tau\tau}$ , and the weak eigenstates are the same at the production and detection locations (as are the conditions in ‘‘normal’’ matter, including no interactions beyond the SM), all physically distinguishable values for the oscillation probabilities are probed if the three angles are allowed to vary in the range  $[0, \pi/2]$ , while  $\delta$  covers the full range  $[-\pi, \pi]$ . Note that, unlike the two-flavour case, there is no further reduction of the parameter space if the matter potential vanishes. In the presence of new physics effects that lead to a non-diagonal matter potential, a larger

<sup>4</sup> Of course, there is also T-violation,  $P_{\alpha\beta} \neq P_{\beta\alpha}$ , such that  $P_{\alpha\beta} = P_{\bar{\beta}\bar{\alpha}}$  (CPT).

parameter space is, in principle, required in order to describe all possible oscillation scenarios.

One final convention issue should be mentioned. Thanks to the current experimental data, it has become customary to redefine the order of the mass eigenstates if  $\Delta m_{12}^2 > \Delta m_{23}^2$ . Whenever this (an “inverted hierarchy”) happens, the mass-eigenstates are relabelled  $1 \rightarrow 3 \rightarrow 2$ , such that  $\Delta m_{13,23}^2 < 0$  and  $\Delta m_{12}^2 < |\Delta m_{23}^2|$ , always. This redefinition is done such that one can relate  $\theta_{12}$  with the “solar” angle,  $\theta_{23}$  with the “atmospheric” angle, and  $\theta_{13}$  with the “reactor” angle, as will be discussed briefly in the next section. Note that within this definition of the mass eigenstates (*i.e.*,  $m_1^2 < m_2^2$  and  $\nu_3$  is defined such that  $\Delta m_{12}^2 < |\Delta m_{23}^2|$ ), the sign of  $\Delta m_{23}^2$  becomes “physical,” and determines whether the neutrino masses are hierarchical, or whether  $m_1^2, m_2^2$  are quasi-degenerate ( $\Delta m_{12}^2 \ll m_2^2$ ) in an inverted hierarchy. It should be noted that oscillation experiments cannot distinguish whether all three mass eigenstates are quasi-degenerate or not.

Since we are now dealing with three-by-three matrices, simple exact solutions to eq. (11) do not exist, even in the case of a constant matter potential (see, however, [17]). However, Nature seems to have been kind, and a few approximations seem to be well justified, including  $\Delta m_{12}^2/|\Delta m_{23}^2| \ll 1$ <sup>5</sup> and  $\sin^2 \theta_{13} \ll 1$ . Under these circumstances many of the well known two neutrino results can be applied. A good rule of thumb is that, in the presence of “normal” matter (no new physics),  $\theta_{13}$  is modified to a matter  $\theta_{13}^M$ , given by eq. (6) with  $\theta \rightarrow \theta_{13}$  and  $\Delta_{12} \rightarrow \Delta_{13}$ ;  $\theta_{12}$  is also modified to a matter angle as defined in eq. (6) with  $\theta \rightarrow \theta_{12}$  and a modified matter potential:  $A \rightarrow A \times \cos^2 \theta_{13}$ ;  $\theta_{23}$  and  $\delta$  are not modified. For solar electron-type neutrinos (with  $E_\nu \lesssim 10$  MeV), for example,

$$P_{ee}^{3\nu} = \cos^4 \theta_{13} P_{ee}^{2\nu}(\theta \rightarrow \theta_{12}, A \rightarrow A \cos^2 \theta_{13}) + \sin^4 \theta_{13}, \quad (13)$$

where  $P_{ee}^{2\nu}$  is given by eq. (8), and  $\Delta_{13}$  effects are “averaged out” (see [9] and references therein).

### 3 Neutrino Oscillations in Action

Here, I briefly review what the experimental data has to say about the neutrino oscillation parameters if the neutrino puzzles are interpreted as evidence for neutrino oscillations. As mentioned in the introduction, I will ignore the yet unconfirmed LSND anomaly, and concentrate on addressing only the solar

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<sup>5</sup> see talk by Eligio Lisi [8] concerning the violation of this condition.



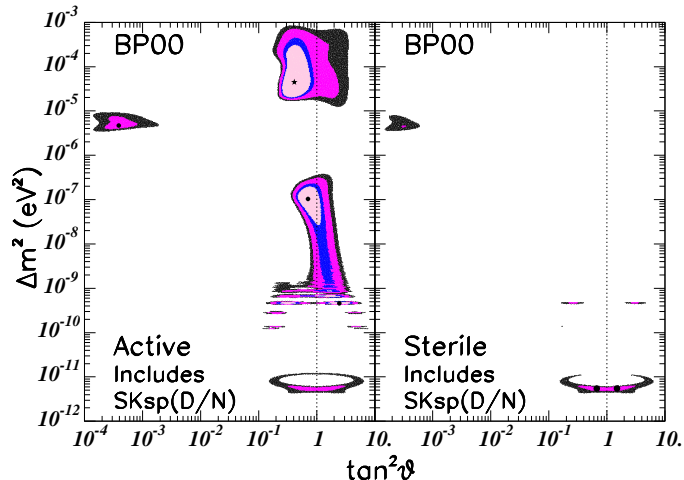


Fig. 2. Neutrino oscillation fit to the solar neutrino data, assuming  $\nu_\mu \leftrightarrow \nu_{\text{active}}$  (left) or  $\nu_\mu \leftrightarrow \nu_{\text{sterile}}$  (right). The contours correspond to 90%, 95%, 99%, and 99.7% (3 sigma) confidence levels. See [20] for details.

and atmospheric neutrino data, while satisfying the constraints imposed by reactor neutrino experiments.

First of all, reactor neutrino experiments [18] have, so far, failed to observe a depletion of the  $\bar{\nu}_e$ -flux produced by nuclear reactors. The survival probability

$$P_{\bar{e}\bar{e}} = 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta_{23}L}{2} \right) + O(\Delta_{12}L), \quad (14)$$

and, in light of the atmospheric and solar data,  $P_{\bar{e}\bar{e}} \sim 1$  implies  $\sin^2 2\theta_{13} \lesssim 0.1$  and  $\Delta m_{12}^2 \lesssim 10^{-3} \text{ eV}^2$  (reactor neutrinos have energies of several MeV, while the most recent experiments probed  $L$  of order 1 km). Second, the solar neutrino puzzle requires values of  $P_{ee}$  significantly different from 1, such that values of  $\sin^2 \theta_{13}$  significantly different from 1 are required (see eq. (13)). This, combined with the reactor bound, implies  $\sin^2 \theta_{13} \lesssim 0.1$ .

Because the atmospheric data requires  $|\Delta m_{23}^2| \sim \text{few} \times 10^{-3} \text{ eV}^2$ ,  $\Delta m_{23}^2$ -effects for solar neutrinos “average out”, and the solar neutrino oscillations are sensitive to mostly  $\Delta m_{12}^2$  and  $\theta_{12}$  (hence these are referred to as the solar parameters). Currently, disjointed regions of the parameter space satisfy the data quite well [7], and they are referred to as: the large mixing angle region, which contains  $0.2 \lesssim \tan^2 \theta_{12} \lesssim 3$ ,  $10^{-5} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 10^{-3} \text{ eV}^2$ , the low-mass-squared-just-so region, which contains  $0.1 \lesssim \tan^2 \theta_{12} \lesssim 10$ ,  $10^{-10} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 10^{-6} \text{ eV}^2$ , and the small mixing angle region, currently disfavoured with respect to the other two after the publication of the SNO [2,19,20] data, which contains  $10^{-4} \lesssim \tan^2 \theta_{12} \lesssim 10^{-3}$ ,  $10^{-6} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 10^{-5} \text{ eV}^2$ . These regions are depicted in fig. 3 [20].

Finally, the atmospheric data requires  $P_{\mu\mu}$  close to 0.5 for “large enough” values of  $L/E_\nu$ , while  $P_{ee}$  is roughly 1. The latter is easily satisfied given the reactor constraints on  $\Delta m_{12}^2$  and  $\sin^2 \theta_{13}$ , while the former determines the

values of  $|\Delta m_{23}^2|$  and  $\theta_{23}$ . Explicitly (ignoring matter effects),

$$P_{\mu\mu} = 1 - (\sin^2 2\theta_{23} \cos^4 \theta_{13} + \sin^2 \theta_{23} \sin^2 2\theta_{13}) \sin^2 \left( \frac{\Delta_{23} L}{2} \right) + O(\Delta_{12} L). \quad (15)$$

Note that “maximal mixing” corresponds to  $\sin^2 \theta_{23} \simeq 0.5/(1 - \sin^2 \theta_{13})$ . Detailed analyses of the atmospheric data in terms of neutrino oscillations constrain  $0.3 \lesssim \tan^2 \theta_{23} \lesssim 3$  and  $1 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m_{23}^2| \lesssim 6 \times 10^{-3} \text{ eV}^2$  [8].

#### 4 Including LSND – Four Neutrino Schemes

The LSND anomaly may be interpreted as evidence for  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  conversion. If the conversion mechanism is neutrino oscillations, the LSND result requires  $P_{\bar{\mu}\bar{e}} \sim 10^{-3}$  for neutrino energies of several tens of MeV and a baseline of roughly 30 metres [4]. In order to obtain a small enough oscillation length, values of  $\Delta m^2 \gtrsim 1 \text{ eV}^2$ , much larger than the values required in order to address the solar and atmospheric puzzles, are required. If all three neutrino puzzles are interpreted in terms of neutrino oscillations, three different  $\Delta m^2$  values are required, which implies the existence of (at least) four neutrino mass eigenstates. On the other hand, very precise LEP measurements at the  $Z^0$ -boson pole indicate that there are only three neutrino species that couple to the  $Z^0$ -boson. One is forced to conclude, therefore, that the LSND anomaly, combined with the solar and atmospheric puzzles, hints at the existence of a fourth neutrino, which is a SM singlet (hence referred to as “sterile,”  $\nu_s$ ).

There are two rather distinct ways of organising the four neutrino masses-squared in order to try to solve all the neutrino anomalies. One is the “3+1” scheme, which has four hierarchical masses-squared,  $m_1^2 < m_2^2 < m_3^2 < m_4^2$ , such that  $\Delta m_{12}^2$  is responsible for the solar anomaly,  $\Delta m_{23}^2 \simeq \Delta m_{13}^2$  for the atmospheric anomaly, and  $\Delta m_{34}^2 \simeq \Delta m_{24}^2 \simeq \Delta m_{14}^2$  for the LSND anomaly (there are other variations, such as an “inverted hierarchy” for the masses of  $\nu_{1,2,3}$ ). In this scheme, the  $\nu_4$  state turns out to be predominantly  $\nu_s$ , and the solar and atmospheric puzzles are interpreted in terms of (predominantly) active neutrino oscillations. The LSND anomaly, on the other hand, requires a small  $\nu_{e,\mu}$  component in  $\nu_4$ . A reasonable combined fit to all three puzzles exists, and is quite robust. The biggest obstacle to the 3+1 schemes is to satisfy bounds from terrestrial  $\nu_\mu$  and  $\nu_e$  oscillation experiments, which are sensitive to the large value of  $\Delta m^2$  required to solve the LSND anomaly. For more details, see the talk by Orlando Peres [21].

Another possibility is to “pair up” the neutrino masses, such that  $m_3^2 \sim m_4^2 \gg m_{1,2}^2$ . In this case,  $\Delta m_{12}^2$  is responsible for the solar anomaly,  $\Delta m_{34}^2$  for the atmospheric anomaly, and  $\Delta m_{13}^2 \simeq \Delta m_{14}^2 \simeq \Delta m_{23}^2 \simeq \Delta m_{24}^2$  for the LSND

anomaly (one variation is  $\Delta m_{12}^2$  responsible for the atmospheric anomaly and  $\Delta m_{34}^2$  for the solar one). These are referred to as “2+2” schemes. In this case, there are virtually no constraints from terrestrial neutrino oscillation searches but, instead, one has to worry about properly solving the solar and atmospheric neutrino puzzles.

The problem is easy to understand. It has to do with the fact that, in a 2+2 scheme, either solar  $\nu_e$  or atmospheric  $\nu_\mu$  oscillations have to be into a predominantly sterile state. Explicitly, let us assume  $U_{e3} = U_{e4} = 0$ , and  $U_{\mu1} = U_{\mu2} = 0$  (note that these conditions are weakly violated if one is to solve the LSND anomaly), such that

$$\nu_1 = \cos \vartheta \nu_e + \sin \vartheta (\cos \zeta \nu_\tau + \sin \zeta \nu_s), \quad (16)$$

$$\nu_2 = -\sin \vartheta \nu_e + \cos \vartheta (\cos \zeta \nu_\tau + \sin \zeta \nu_s), \quad (17)$$

$$\nu_3 = \cos \phi \nu_\mu + \sin \phi (-\sin \zeta \nu_\tau + \cos \zeta \nu_s), \quad (18)$$

$$\nu_4 = -\sin \phi \nu_\mu + \cos \phi (-\sin \zeta \nu_\tau + \cos \zeta \nu_s), \quad (19)$$

In this case,  $\vartheta$  ( $\phi$ ) would play the part of the solar (atmospheric) angle, while  $\zeta$  controls the “amount” of sterile neutrino in both the solar and the atmospheric sectors. It is easy to see that if  $\zeta = 0$ ,  $\nu_\mu$  ( $\nu_e$ ) oscillates into a pure  $\nu_s$  ( $\nu_\tau$ ) state, while the situation is reversed if  $\zeta = \pi/2$ .

The current atmospheric data strongly disfavors (at more than 99% confidence level [3])  $\zeta = 0$ , while the solar data disfavors  $\zeta = \pi/2$  [1] (this can also be seen in fig. 3). Detailed quantitative analysis [7,8,22] seem to indicate that, if all atmospheric and solar data are combined, acceptable fits can be found (the current best fit point is close to  $\sin^2 \zeta = 0.2$  [22]). It is curious to note that, according to [22], the point where the sterile component is evenly shared among the solar and atmospheric pairs ( $\zeta = \pi/4$ ) is strongly disfavored by the data. As more neutrino data accumulates, it is not impossible to imagine that 2+2 schemes will be severely constrained by the data (perhaps as much, or more, than the 3+1 schemes).

## 5 Conclusions and Outlook

It is easy to summarise the results of sec. 3:  $|\Delta m_{23}^2|$  and  $\theta_{23}$  have been rather well measured, while  $\Delta m_{12}^2$  and  $\theta_{12}$  have also been measured, but rather poorly. Nothing is known about the value of  $\theta_{13}$  (except that it is relatively small), the value of the CP-odd phase  $\delta$ , or the neutrino mass-hierarchy (the sign of  $\Delta m_{23}^2$  in the traditional parametrisation).

In the near future, the situation is bound to improve significantly, especially

in the solar sector. The on-going solar experiments (in particular SNO) will not only establish once and for all that neutrino conversion is the mechanism behind the solar neutrino problem, but will also improve our knowledge of the solar parameters. The near future experiments KamLAND [23] and Borexino [24] have the ability to go even further, by not only breaking the current degeneracy in the  $(\Delta m_{12}^2 \times \theta_{12})$ -plane but also determining the solar parameters with unprecedented precision (KamLAND in the case of the large mixing angle solution [23,25,26], Borexino in the case of the low-mass-squared-just-so solution [12,27]). Furthermore, it should also be noted that the KamLAND reactor experiment offers the first realistic opportunity to observe a “real” oscillatory pattern, while the Borexino solar neutrino experiment may establish, similar to what SNO will eventually accomplish, whether there are neutrinos other than  $\nu_e$  coming from the Sun [28].

In the “atmospheric sector,” the ongoing K2K experiment [29], the future MINOS experiment [30], and the CERN CNRS project [31] will confirm atmospheric neutrino oscillations, discover  $\tau$ -appearance, and extend slightly the sensitivity to  $\theta_{13}$ , on top of precisely measuring the atmospheric parameters  $|\Delta m_{23}^2|$  and  $\theta_{23}$ .

Finally, after the MiniBoone data [6] is collected and analysed, the issue of whether the LSND anomaly is indeed a consequence of new neutrino physics will be settled. We will then be able to decide whether there is indeed a light sterile neutrino, such that all three neutrino anomalies are explained by neutrino oscillations. It should be noted, as briefly discussed in the sec. 4, that the current neutrino data already places a significant amount of “pressure” on sterile neutrino oscillations, and the situation is bound to change as new experimental data from solar, reactor and long-baseline experiments becomes available (for an example, see [26]).

In five to ten years times, the neutrino mixing matrix will still be far from well known: most likely, one of the mixing angles will simply not have been observed (unless the long-baseline experiments “get lucky”), and whether or not there is a nontrivial CP-odd phase will remain unknown. Furthermore, we will be unable to say whether the neutrinos masses are hierarchical ( $m_1^2 < m_2^2 < m_3^2$ ), or whether the hierarchy is inverted ( $m_3^2 < m_1^2 \simeq m_2^2$ , such that  $m_2^2 - m_1^2 \ll m_3^2$ ).

These challenges have to be tackled by a new generation of neutrino experiments, which requires cleaner and much more intense neutrino beams, of different flavours (if possible, as is the case of a neutrino factory).

I conclude by mentioning that there are other ways of probing the neutrino mixing matrix and mass spectrum. In particular, the future observation of neutrinos from a nearby supernova will probably shed light on some of the

issues raised above [32], while continuing searches for neutrinoless double beta decay (which are the best probes of whether the neutrinos are Majorana or Dirac fermions) and a “kinematic” neutrino mass in the tritium beta-decay spectrum also add valuable information [33].

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