

CP Violation in the Minimal Supersymmetric Seesaw Model

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Abstract

We study CP violation in the lepton sector of the supersymmetric extension of the Standard Model with three generations of massive singlet neutrinos with Yukawa couplings Y_ν to lepton doublets, in a minimal seesaw model for light neutrino masses and mixing. This model contains six physical CP-violating parameters, namely the phase δ observable in oscillations between light neutrino species, two Majorana phases $\phi_{1,2}$ that affect $\beta\beta_{0\nu}$ decays, and three independent phases appearing in $Y_\nu Y_\nu^\dagger$, that control the rate of leptogenesis. Renormalization of the soft supersymmetry-breaking parameters induces observable CP violation at low energies, including T-odd asymmetries in polarized $\mu \rightarrow eee$ and $\tau \rightarrow \ell\ell\ell$ decays, as well as lepton electric dipole moments. In the leading-logarithmic approximation in which the massive singlet neutrinos are treated as degenerate, these low-energy observables are sensitive via $Y_\nu^\dagger Y_\nu$ to just one combination of the leptogenesis and light-neutrino phases. We present numerical results for the T-odd asymmetry in polarized $\mu \rightarrow eee$ decay, which may be accessible to experiment, but the lepton electric dipole moments are very small in this approximation. To the extent that the massive singlet neutrinos are not degenerate, low-energy observables become sensitive also to two other combinations of leptogenesis and light-neutrino phases, in this minimal supersymmetric seesaw model.

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1 Introduction

The solar [1, 2] and atmospheric [3] neutrino anomalies, which imply the existence of non-zero masses for the light neutrinos, provide the first experimental evidence for the existence of physics beyond the Standard Model (SM). A minimal extension of the SM includes three very heavy singlet neutrinos N_i^c , whose Yukawa couplings Y_ν to the light neutrinos explain naturally the smallness of their masses, via the seesaw mechanism [4]. At the same time, the electroweak scale must be stabilized against large radiative corrections. In particular, after introducing right-handed neutrinos, a quadratically-divergent contribution to the Higgs boson mass proportional to M_N^2 has to be cancelled. This is most commonly achieved by supersymmetrizing the theory, leading to the minimal supersymmetric extension of the Standard Model (MSSM) with singlet neutrinos.

Neutrino-flavour mixing originates from off-diagonal components in the Yukawa interaction $N^c Y_\nu L H_2$, in a basis where the charged-lepton and singlet-neutrino mass matrices are real and diagonal. Renormalization effects due to this interaction also induce flavour mixings in the soft supersymmetry-breaking slepton mass terms [5]. This may lead to observable rates for charged-lepton flavour-violating (LFV) processes such as $\mu \rightarrow e\gamma$, μ - e conversion in nuclei, $\mu \rightarrow eee$ and $\tau \rightarrow 3\ell$ [6, 7, 8, 9], where $\ell = e, \mu$ denotes a generic light charged lepton. LFV is also observable in principle in rare kaon decays, but at rates that are likely to be far below the current bounds [10].

In general, Y_ν is complex, leading to CP violation in neutrino oscillations and in the induced rare LFV processes, as well as in Majorana phases for the light neutrinos and in electric dipole moments for the charged leptons. The existence of CP violation in Y_ν is also required if the observed baryon asymmetry in the Universe originated in leptogenesis [11]. The purpose of this paper is to clarify the relations between these different manifestations of CP violation in the lepton sector, and to present numerical estimates of the T-odd CP-violating asymmetry A_T in $\mu \rightarrow eee$ decay, the electric dipole moments of the electron and muon. We argue that measurements of CP violation using charged leptons, combined with CP violation in the light-neutrino sector, in principle enable the leptogenesis phases to be extracted - within the framework of the minimal supersymmetric seesaw model.

If the solar-neutrino mass-squared difference Δm_{sol}^2 and the element U_{e3} of the Maki-Nakagawa-Sakita (MNS) neutrino-mixing matrix U are not too small, the CP-violating phase δ in U , which is analogous to the Cabbibo-Kobayashi-Maskawa (CKM) phase in the quark sector, can be measured via CP- and T-violating [12] observables in neutrino oscillations using a neutrino factory or possibly a low-energy neutrino superbeam. The recent SNO

result [2] encourages this possibility, since it further favours the large-mixing-angle (LMA) solution to the solar-neutrino deficit [13].

As mentioned above, processes that violate charged-lepton flavour can provide important complementary information on the leptonic CP-violating phases. These may be measured using intense sources of stopped muons. The SINDRUM II experiment is designed to be sensitive to $\mathcal{B}(\mu Ti \rightarrow eTi) \sim 10^{-14}$ [14], and the MECO project would be sensitive to $\mathcal{B}(\mu Al \rightarrow eAl) \sim 10^{-16}$ [15]. The experiment with the sensitivity $Br(\mu \rightarrow e\gamma) \sim 10^{-14}$ is proposed at PSI [16]. The PRISM project [6, 17] and the front ends of neutrino factories now under consideration at CERN [18] and elsewhere will provide beams of low-energy muons that are more intense by several orders of magnitude than any of the present facilities. This will enable the construction of stopped-muon experiments able to probe LFV processes with sensitivities $Br(\mu \rightarrow e\gamma) \sim 10^{-15}$, $Br(\mu \rightarrow eee) \sim 10^{-16}$. The latter sensitivity opens the way to measuring the T-odd, CP-violating asymmetry $A_T(\mu \rightarrow eee)$.

A measurement of the CP-violating electric dipole moment (EDM) of the muon with a sensitivity $d_\mu \sim 5 \times 10^{-26}$ e cm would also be possible [18]. However, because the Yukawa coupling constants Y_ν appear in the renormalization-group equations (RGEs) only in the Hermitian combination $Y_\nu^\dagger Y_\nu$, CP-violating phases are induced only in the off-diagonal terms of the slepton masses. This implies suppression of the EDMs of the electron and muon, whereas CP violation may occur in full strength in charged LFV processes, such as $\mu \rightarrow eee$ ¹.

Another arena to probe LFV is provided by rare τ decays. There has been some discussion in the literature of $\tau \rightarrow \ell\gamma$ decays [26], and one could in principle hope to measure CP-violating asymmetries in the various $\tau \rightarrow 3\ell$ decays. Another possibility is to search for LFV in sparticle decays [27, 28], e.g., $\tilde{\chi}_i^0 \rightarrow e\mu\tilde{\chi}_j^0$, where CP-violating asymmetries analogous to $A_T(\mu \rightarrow eee)$ can also be defined in principle. However, we do not investigate these possibilities further in this paper.

We concentrate here on CP-violating observables in the μ sector, assuming that the only sources of LFV and CP violation are the interactions with heavy singlet N^c neutrinos. We start by discussing general parametrisations of the Yukawa matrix Y_ν in terms of the high- as well as the low-energy observables, paying particular attention to the counting of physical degrees of freedom and their relations to CP-violating observables. Subsequently, we analyse the renormalization-group running of soft supersymmetry-breaking terms, assuming universal boundary conditions at the GUT scale. Our first objective in this analysis is to

¹In the light of very stringent constraints from electron, neutron and mercury EDMs [19, 20], we neglect the possible phases in diagonal soft supersymmetry-breaking terms throughout this paper. This is natural in mechanisms which generate only real soft terms, such as gravity- [21], gauge- [22], anomaly- [23], gaugino- [24] and radion-mediation [25] mechanisms.

demonstrate in principle the complementarity of the different observables, to see how all the CP-violating phases of the minimal seesaw model come into play, and to clarify the relationship of the observable phases to the phases appearing in leptogenesis. In the leading-logarithmic approximation, in which the heavy singlet neutrinos are treated as degenerate, this renormalization is sensitive to just one combination of the leptogenesis and light-neutrino phases, but two other combinations contribute beyond this approximation. We illustrate our results in a simple two-generation model. We then present numerical estimates of $Br(\mu \rightarrow e\gamma)$, $Br(\mu \rightarrow eee)$ and $A_T(\mu \rightarrow eee)$, taking into account the present knowledge of neutrino mixings and masses as well as bounds on sparticle masses. We find that the magnitude of A_T is in general anti-correlated with the rate of $\mu \rightarrow e\gamma$, and may be large in some models compatible with the experimental upper limit on $\mu \rightarrow e\gamma$ decay. If a cancellation occurs between different contributions to the $\mu - e - \gamma$ vertex, so that the box and penguin diagrams contributing to $\mu \rightarrow eee$ become comparable in magnitude, the T-odd asymmetry A_T in $\mu \rightarrow eee$ may be as large as $\sim 10\%$, while $Br(\mu \rightarrow eee)$ remains appreciable. However, the EDMs of the μ and e are rather small in the minimal seesaw model.

It is important to note that the neutrino-oscillation phase δ and the Majorana phases $\phi_{1,2}$ are completely independent of the three physical phases in the quantity $Y_\nu Y_\nu^\dagger$ that enters in leptogenesis calculations. On the other hand, A_T and the other renormalization-induced observables depend on mixtures of the light-neutrino and leptogenesis phases. Thus, neutrino factories and LFV measurements provide complementary information on the leptonic CP-violating phases. In particular, observation of A_T is possible even if CP violation in neutrino oscillations is unobservable, i.e., if either $\delta = 0$, $U_{e3} = 0$ or the solar-neutrino deficit is not explained by the LMA solution. However, in the minimal supersymmetric seesaw model, it is possible that a combination of CP-violating observables in the neutrino and charged-lepton sectors may provide constraints on the angles and phases responsible for leptogenesis.

Our work is organized as follows. In Section 2, we consider general parameterisations of the neutrino Yukawa couplings Y_ν and discuss CP violation in the minimal supersymmetric seesaw model. In Section 3, we give general formulae for the EDMs of the charged leptons, $\mu \rightarrow e\gamma$, and $\mu \rightarrow eee$, including the latter's T-odd asymmetry A_T . We present the results of the numerical analysis in Section 4. Finally, Section 5 is devoted to a discussion and our conclusions concerning the observability of the CP-violating phases in the minimal supersymmetric seesaw model.

2 CP Violation in the Lepton Sector of the Minimal Supersymmetric Seesaw Model

We consider the MSSM with three additional heavy singlet-neutrino superfields N_i^c , constituting the minimal supersymmetric seesaw model. The relevant leptonic part of its superpotential is

$$W = N_i^c (Y_\nu)_{ij} L_j H_2 - E_i^c (Y_e)_{ij} L_j H_1 + \frac{1}{2} N_i^c \mathcal{M}_{ij} N_j^c + \mu H_2 H_1, \quad (1)$$

where the indices i, j run over three generations and \mathcal{M}_{ij} is the heavy singlet-neutrino mass matrix. Taking account of the possible field redefinitions, this minimal supersymmetric seesaw model contains 21 parameters: 3 charged-lepton masses m_ℓ , 3 light-neutrino masses \mathcal{M}_ν^D , 3 heavy Majorana neutrino masses \mathcal{M}^D , 3 light-neutrino mixing angles $\theta_{ij} : 1 \leq i \neq j \leq 3$, 3 CP-violating light-neutrino mixing phases $\delta, \phi_{1,2}$ (the MNS phase and two Majorana phases), and 3 additional mixing angles and 3 more phases associated with the heavy-neutrino sector.

2.1 High-Energy Parametrization

In order to clarify the appearance and rôles of these parameters, we first analyze (1) in a basis where the charged leptons and the heavy neutrinos both have real and diagonal mass matrices:

$$\begin{aligned} (Y_e)_{ij} &= Y_{e_i}^D \delta_{ij}, \\ \mathcal{M}_{ij} &= \mathcal{M}_i^D \delta_{ij}, \end{aligned} \quad (2)$$

where $\mathcal{M}^D = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})$. *A priori*, the neutrino Yukawa-coupling matrix Y_ν has nine phases, which can be exposed by writing it in the form: $Y_\nu = Z^* Y_{\nu_k}^D X^\dagger \tilde{P}_1^*$, where $Y_{\nu_k}^D$ is diagonal and $\tilde{P}_1 = \text{diag}(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3})$. However, in the basis (2) one may redefine the left-handed lepton fields L_i , and thus rotate away the three phases in \tilde{P}_1 , which are unphysical. Thus the Yukawa-coupling matrix may be written in the form:

$$(Y_\nu)_{ij} = Z_{ik}^* Y_{\nu_k}^D X_{kj}^\dagger. \quad (3)$$

The matrix X is the analogue in the lepton sector of the quark CKM matrix, and thus it has only one physical phase. On the other hand, we can always write Z in the form

$$Z = P_1 \bar{Z} P_2, \quad (4)$$

where \overline{Z} is a CKM-type matrix with three real mixing angles and one physical phase, and $P_{1,2} = \text{diag}(e^{i\theta_{1,3}}, e^{i\theta_{2,4}}, 1)$ are diagonal matrices containing two phases each. Thus Z has 5 physical phases to add to that in X , and all six real mixing angles and six phase parameters in this basis are physical observables.

We now study the combination $Y_\nu Y_\nu^\dagger$ of the Yukawa couplings, which governs leptogenesis in this minimal seesaw model. It is straightforward to see from (3) that

$$Y_\nu Y_\nu^\dagger = P_1^* \overline{Z}^* (Y_\nu^D)^2 \overline{Z}^T P_1, \quad (5)$$

which depends on just three of the CP-violating phases, namely the two phases $\theta_{1,2}$ in P_1 and the single residual phase in \overline{Z} , as well as the three real mixing angles in \overline{Z} . This is consistent with the observation that, since the overall lepton number involves a sum over the light-lepton species (both charged leptons and light neutrinos), one would not expect leptogenesis to depend on the 6 MNS angles and phases.

On the other hand, as we discuss in more detail below, mixing and CP violation in the slepton sector of this minimal supersymmetric seesaw model is controlled by the combination $Y_\nu^\dagger Y_\nu$ of the neutrino Yukawa couplings, in the leading-logarithmic approximation where $M_{GUT} \gg M_{N_{1,2,3}}$. It is again straightforward to see from (3) that

$$Y_\nu^\dagger Y_\nu = X (Y_\nu^D)^2 X^\dagger. \quad (6)$$

Therefore, in this approximation, CP violation in charged LFV processes arises only from the one physical phase in the diagonalizing matrix X .

2.2 Low-Energy Parametrization

We now reconsider leptonic CP violation from a more familiar point of view, namely that of the effective low-energy theory obtained after the heavy neutrinos are decoupled. In this energy range, physics is described by the following effective superpotential:

$$W_{eff} = L_i H_2 \left(Y_\nu^T (\mathcal{M}^D)^{-1} Y_\nu \right)_{ij} L_j H_2 - E_i^c (Y_e)_{ij} L_j H_1, \quad (7)$$

where the effective light-neutrino masses are given in the basis (2) by

$$\mathcal{M}_\nu = Y_\nu^T (\mathcal{M}^D)^{-1} Y_\nu v^2 \sin^2 \beta, \quad (8)$$

where $v = 174$ GeV and as usual $\tan \beta = v_2/v_1$. The mass matrix \mathcal{M}_ν can be diagonalized by a unitary matrix U :

$$U^T \mathcal{M}_\nu U = \mathcal{M}_\nu^D, \quad (9)$$

where $\mathcal{M}_\nu^D = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. Since \mathcal{M}_ν is a symmetric matrix and contains in general six phases, U must also have 6 phases. It can be expressed in the form

$$U = \tilde{P}_2 V P_0, \quad (10)$$

where $P_0 = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, $\tilde{P}_2 = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ and V is the MNS matrix written in the CKM form:

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (11)$$

The phases in \tilde{P}_2 (10) can be removed by redefinition of the L_i fields, leading to a new basis in which

$$U = V P_0. \quad (12)$$

This differs from the basis (2) by the phase rotation \tilde{P}_2 . The new basis is appropriate if one works with the effective low-energy observables in the effective superpotential (7), e.g., for studying neutrino oscillations. Indeed, the mixing angles $\theta_{ij} : 1 \leq i \neq j \leq 3$, whose sin, cos we denote by s_{ij}, c_{ij} , are measurable in neutrino-oscillation experiments, as is the CP-violating MNS phase δ . One combination of two CP-violating Majorana phases $\phi_{1,2}$ is in principle measurable in $\beta\beta_{0\nu}$ experiments.

The physical interpretation of the Yukawa couplings in (1) is made more transparent in the basis (2), which does not contain the unphysical low-energy phases in \tilde{P}_2 that we rotated away in the previous paragraph. Note that one must change $X \rightarrow \tilde{P}_2 X$ if one works in the basis (12).

Our objective in this paper is to study CP-violating observables which are sensitive to different physical phases. For this purpose, we need a proper parametrization of the input parameters of the model. The most straightforward choice is to work in the basis (2) and to choose the physical observables in (2) and (3) as the input parameters. In this case, the physics is entirely transparent. However, the present experiments do not measure heavy-neutrino masses, their Yukawa-coupling and mixings directly. All the information we have on neutrinos comes from the low-energy neutrino-oscillation and $\beta\beta_{0\nu}$ experiments. If we choose the input parameters from (2), (3), we have to check every time that the induced \mathcal{M}_ν in (8) agrees with the experimental data. Instead, one can attempt to use the effective low-energy observables as an input.

To this end, we first rewrite the seesaw mechanism in the different form:

$$R \equiv \sqrt{\mathcal{M}^D}^{-1} Y_\nu U \sqrt{\mathcal{M}_\nu^D}^{-1} : R^T R = 1, \quad (13)$$

which is equivalent to (8). Starting with any given Y_ν and \mathcal{M}^D as input parameters, we obtain as outputs the seesaw-induced low-energy parameters \mathcal{M}_ν^D and U , and an auxiliary complex orthogonal matrix R . It is possible to choose different parameter sets for Y_ν and \mathcal{M}^D that give the same low energy effective \mathcal{M}_ν^D and U , but lead to different values for R .

One can turn the argument around [8], and parameterize the neutrino Yukawa-coupling matrix in terms of an arbitrary complex orthogonal matrix R' as follows:

$$Y'_\nu = \frac{\sqrt{\mathcal{M}^D} R' \sqrt{\mathcal{M}_\nu^D} U^\dagger}{v \sin \beta}. \quad (14)$$

We emphasize that the output Y'_ν in this parametrization is in the *low-energy* basis (12), and therefore contains *unphysical* phases. If one wants to use the induced Y'_ν to parametrize the superpotential (1), one should be careful to count correctly the physical degrees of freedom.

We now form the combinations $Y'_\nu Y'^{\dagger}_\nu$ and $Y'^{\dagger}_\nu Y'_\nu$ out of (14). In the first case, we obtain

$$Y'_\nu Y'^{\dagger}_\nu = \frac{\sqrt{\mathcal{M}^D} R' \mathcal{M}_\nu^D R'^{\dagger} \sqrt{\mathcal{M}^D}}{v^2 \sin^2 \beta}, \quad (15)$$

which contains three independent physical phases that are given entirely in terms of the parameters in the orthogonal matrix R' . This is consistent with (5), and the new parametrization therefore has not changed the counting of phases in $Y_\nu Y_\nu^\dagger$. On the other hand, we also obtain from (14)

$$Y'^{\dagger}_\nu Y'_\nu = U \frac{\sqrt{\mathcal{M}_\nu^D} R'^{\dagger} \mathcal{M}^D R' \sqrt{\mathcal{M}_\nu^D}}{v^2 \sin^2 \beta} U^\dagger. \quad (16)$$

This expression also appears to contain three phases, which are combinations of all the parameters in U and R' .

However, according to (6), $Y_\nu Y_\nu^\dagger$ is supposed to contain only one physical phase. What has happened? The answer is that physics has not changed, and thus two out of three phases in $Y'^{\dagger}_\nu Y'_\nu$ are unphysical. This is the case because we are working in the low-energy basis (12), and not in the basis (2). The three phases in \tilde{P}_2 , which were rotated away in defining U , appear now in Y'_ν . Instead of (6), we now have $Y'^{\dagger}_\nu Y'_\nu = \tilde{P}_2 X (Y_\nu^D)^2 X^\dagger \tilde{P}_2^*$. One overall phase is irrelevant, and the two unphysical relative phases in \tilde{P}_2 explain the faulty phase counting in (16).

In the following, we show explicitly that the unphysical phases in \tilde{P}_2 cancel out in the Jarlskog invariants which can be constructed using $Y'^{\dagger}_\nu Y'_\nu$. Therefore, in the leading-logarithmic approximation, all the CP-violating LFV observables depend only on the one physical phase in (16), which is a combination of the phases in U and R' . *Henceforward, we omit the superscript ', but one must still be careful to distinguish between the different bases.*

2.3 Relations to CP-Violating Observables

So far we have only considered the parametrization of the input neutrino parameters, which in general are complex, and the 6 resulting independent CP-violating phases. We now consider how physical observables depend on these various phases.

2.3.1 Leptogenesis

At present, our only experimental knowledge on CP violation in the lepton sector may be obtained from the baryon asymmetry of the Universe, assuming that this originated from leptogenesis. In leptogenesis scenarios, initial $B-L$ asymmetries ε^i appeared in decays of the heavy neutrinos N_i^c in the early Universe, as results of interferences between the tree-level and one-loop amplitudes for N_i^c decays. The L asymmetry in the decay of an individual species N_i^c is given in the supersymmetric case [29] by

$$\varepsilon^i = -\frac{1}{8\pi} \sum_l \frac{\text{Im} \left[\left(Y_\nu Y_\nu^\dagger \right)^{li} \left(Y_\nu Y_\nu^\dagger \right)^{li} \right]}{\sum_j |Y_\nu^{ij}|^2} \sqrt{x_l} \left[\text{Log}(1 + 1/x_l) + \frac{2}{(x_l - 1)} \right], \quad (17)$$

where $x_l = (M_{N_l}/M_{N_i})^2$ and both triangular and self-energy type loop diagrams are taken into account. This L asymmetry is converted into the observed baryon asymmetry by sphalerons acting before the electroweak phase transition. It is clear from (17) that the generated asymmetry depends only on the phases in $Y_\nu Y_\nu^\dagger$. Hence, according to the parametrization (14), the only phases entering in the calculation of the baryon asymmetry of the Universe are those in R . In order to demonstrate the feasibility of leptogenesis, it would be necessary to prove that at least one of the phases in R is non-zero. Moreover, as we shall see, at least one of the real part of the mixing angles in R must also be non-zero, and one would need to control other parameters, such as the heavy-neutrino mass spectrum, before being able to calculate the baryon asymmetry in terms of R , or vice versa.

2.3.2 CP Violation in Neutrino Oscillations

Measuring this is one of the main motivations for building neutrino factories. We assume that the real MNS mixing angles θ_{ij} and the mass-squared differences $\delta m_{ij}^2 \equiv m_{\nu_i}^2 - m_{\nu_j}^2$ are all non-vanishing, in which case the the MNS phase δ in (11) is in principle observable. It is, realistically, observable in long-baseline neutrino factory experiments if the LMA solution of the solar neutrino problem is correct. The Majorana phases $\phi_{1,2}$ do not affect neutrino

oscillations at observable energies, but do affect $\beta\beta_{0\nu}$ decay. The conventional nuclear $\beta\beta_{0\nu}$ experiments measure one combination of the light-neutrino masses m_{ν_i} and the Majorana phases $\phi_{1,2}$. As in the CKM case, one can introduce a Jarlskog invariant that characterizes the strength of CP violation in neutrino oscillations:

$$\begin{aligned} J_\nu &= \text{Im} \left[\left(\mathcal{M}_\nu^\dagger \mathcal{M}_\nu \right)_{12} \left(\mathcal{M}_\nu^\dagger \mathcal{M}_\nu \right)_{23} \left(\mathcal{M}_\nu^\dagger \mathcal{M}_\nu \right)_{31} \right] \\ &= \delta m_{12}^2 \delta m_{23}^2 \delta m_{31}^2 \text{Im} [V_{11} V_{12}^* V_{22} V_{21}^*]. \end{aligned} \quad (18)$$

One sees explicitly that the Majorana phases $\phi_{1,2}$ cancel out in J_ν .

It is clear from (8) that, from the high-energy point of view, δ depends on all the six independent phases in Y_ν , including those in the combinations $Y_\nu Y_\nu^\dagger$ and $Y_\nu^\dagger Y_\nu$. On the other hand, in the low-energy parametrization of (14), the phase δ is taken as an input parameter.

2.3.3 Renormalization of Soft Supersymmetry-Breaking Terms: Flavor-Changing Processes

In the minimal supersymmetric seesaw model, renormalization induces sensitivity to the neutrino Yukawa couplings Y_ν in the soft supersymmetry-breaking parameters in the slepton sector, in particular to the CP-violating phases in Y_ν . These may have measurable effects on several CP-violating lepton observables, including asymmetries in LFV decays, which are observable in rare μ and/or τ decays, and electric dipole moments. The μ electric dipole moment as well as rare μ decays may be measurable using slow or stopped muons produced at the front end of a neutrino factory. In this subsection, we concentrate on the flavor-changing processes, such as asymmetries in LFV decays, and we will discuss flavor-conserving processes, such as the electric dipole moment.

The soft supersymmetry-breaking terms in the leptonic sector of the minimal supersymmetric seesaw model are

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= \tilde{L}_i^\dagger (m_{\tilde{L}}^2)_{ij} \tilde{L}_j + \tilde{E}_i^{c*} (m_{\tilde{E}}^2)_{ij} \tilde{E}_j^c + \tilde{N}_i^{c*} (m_{\tilde{N}}^2)_{ij} \tilde{N}_j^c \\ &+ \left(\tilde{N}_i^c (A_N)_{ij} \tilde{L}_j H_2 - \tilde{E}_i^c (A_e)_{ij} \tilde{L}_j H_1 + \frac{1}{2} \tilde{N}_i^{c*} (B_N)_{ij} \tilde{N}_j^c \right. \\ &\left. + \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a + h.c. \right). \end{aligned} \quad (19)$$

We assume that the soft supersymmetry-breaking terms have universal boundary conditions at the GUT scale $M_{GUT} \sim 2 \times 10^{16}$ GeV:

$$\begin{aligned} (m_{\tilde{E}}^2)_{ij} &= (m_{\tilde{L}}^2)_{ij} = (m_{\tilde{N}}^2)_{ij} = m_0^2 \mathbf{1}, \\ m_{H_1}^2 &= m_{H_2}^2 = m_0, \end{aligned}$$

$$\begin{aligned}
(A_e)_{ij} &= A_0(Y_e)_{ij}, (A_\nu)_{ij} = A_0(Y_\nu)_{ij}, \\
M_1 &= M_2 = M_3 = m_{1/2}.
\end{aligned}
\tag{20}$$

At lower energies below M_{GUT} and above the heavy-neutrino mass scale M_N , which we assume to be $\ll M_{GUT}$, off-diagonal entries in Y_ν generate via the renormalization-group running off-diagonal entries in the effective soft supersymmetry-breaking terms. In the leading-logarithmic approximation the flavor-dependent parts of the soft supersymmetry-breaking terms are given by

$$\begin{aligned}
(\delta m_L^2)_{ij} &\approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e)_{ij} \log \frac{M_{GUT}}{M_N}, \\
(\delta m_E^2)_{ij} &\approx -\frac{1}{4\pi^2}(3m_0^2 + A_0^2)(Y_e Y_e^\dagger)_{ji} \log \frac{M_{GUT}}{M_N}, \\
(\delta A_e)_{ij} &\approx -\frac{1}{8\pi^2}A_0 Y_{ei}(3Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_{GUT}}{M_N}.
\end{aligned}
\tag{21}$$

Here, the Yukawa coupling constants are given at M_N , and then Y_e is diagonal. This means that m_E^2 remains diagonal in this approximation. Below M_N , the heavy neutrinos decouple, and the renormalization-group running is given entirely in terms of the MSSM particles and couplings, and is independent of Y_ν . We use in our numerical examples full numerical solutions to the one-loop renormalization-group equations, but the approximate analytical solutions (21) are useful for a qualitative analysis.

It is important to notice that, in the leading-logarithmic approximation (21), the only combination of neutrino Yukawa couplings entering the renormalization-group equations is $Y_\nu^\dagger Y_\nu$. This implies that CP-violating phases are induced only in the off-diagonal elements of $(m_L^2)_{ij}$ and $(A_e)_{ij}$, and further indicates that the lepton-flavour conserving but CP-violating observables like the electric dipole moments of charged leptons are naturally suppressed [20], while CP violation in the charged LFV processes should occur in full strength. This is analogous to CP violation in the quark sector of the Standard Model, which is also directly related to flavour-changing processes. As we saw earlier (6), the combination $Y_\nu^\dagger Y_\nu$ depends on just one CP-violating phase, namely that in the matrix X . Therefore, in the leading-logarithmic approximation, *all* slepton-induced observables are independent of the phases associated with leptogenesis, which are combinations of those in the matrices \bar{Z} and P_1 (6), in the high energy parametrization. On the other hand, in the low-energy parametrization, $Y_\nu^\dagger Y_\nu$ depends on one combination of the phases in U and R , as explained in subsection 2.2.

Since $Y_\nu^\dagger Y_\nu$ depends on only one physical phase, there is only one invariant for m_L^2 describing the strength of CP violation in any process induced by sleptons. By analogy with

the Standard Model quark sector, this can be taken to be [27]

$$J_{\tilde{L}} = \text{Im} \left[\left(m_{\tilde{L}}^2 \right)_{12} \left(m_{\tilde{L}}^2 \right)_{23} \left(m_{\tilde{L}}^2 \right)_{31} \right]. \quad (22)$$

Additional invariants including the A terms can be constructed:

$$J_{A_{12}} = \text{Im} \left[(A_e)_{12} \left(m_{\tilde{L}}^2 \right)_{23} \left(m_{\tilde{L}}^2 \right)_{31} \right] \quad (23)$$

and cyclic permutations, and similar invariants with two or three $(A_e)_{ij}$ factors. However, in this model they are all related to the basic invariant (22), and proportional to

$$\text{Im} \left[(Y_\nu^\dagger Y_\nu)^{12} (Y_\nu^\dagger Y_\nu)^{23} (Y_\nu^\dagger Y_\nu)^{31} \right] = \delta Y_{\nu 12}^2 \delta Y_{\nu 23}^2 \delta Y_{\nu 31}^2 \text{Im} [X_{11} X_{12}^* X_{22} X_{21}^*] \quad (24)$$

in the leading-logarithmic approximation (21). Here, $\delta Y_{\nu ij}^2 \equiv (Y_{\nu i}^D)^2 - (Y_{\nu j}^D)^2$.

The above analysis is modified when one includes in the renormalization-group running effects associated with the non-degeneracy of the heavy neutrinos: $M_{N_i} \neq M_{N_j}$. In this case, $(\delta m_{\tilde{L}}^2)_{ij}$ in (21) is replaced as follows: $(\delta m_{\tilde{L}}^2)_{ij} \rightarrow (\delta m_{\tilde{L}}^2)_{ij} + (\tilde{\delta} m_{\tilde{L}}^2)_{ij}$, and $(\tilde{\delta} m_{\tilde{L}}^2)_{ij}$ is given by

$$(\tilde{\delta} m_{\tilde{L}}^2)_{ij} \approx -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y_\nu^\dagger L Y_\nu)_{ij} : L \equiv \log \frac{M_N}{M_{N_i}} \delta_{ij}, \quad (25)$$

where M_N is now interpreted as the geometric mean of the heavy singlet-neutrino mass eigenvalues M_{N_i} . The first term in (25) contains the matrix factor

$$Y^\dagger L Y = X Y^D P_2 \bar{Z}^T L \bar{Z}^* P_2^* Y^D X^\dagger, \quad (26)$$

which induces some dependence on phases in $\bar{Z} P_2$. In the three-generation case, there are two independent entries in the traceless diagonal matrix L , so the renormalization induces in principle dependences on two new combinations of these phases, as well as the single phase in $Y_\nu^\dagger Y_\nu$. Thus low-energy observables become sensitive to all three leptogenesis phases. However, the dependences on the two extra phases are suppressed to the extent that $\log \frac{M_N}{M_{N_i}} \ll \log \frac{M_{GUT}}{M_N}$.

2.3.4 Renormalization of Soft Supersymmetry-Breaking Terms: Flavor-Conserving Processes

As mentioned above, since the CP-violating phases are in the off-diagonal components of the soft supersymmetry-breaking terms, the electric dipole moment of lepton is naturally

suppressed. The following is the lowest-order combination of the Yukawa couplings Y_ν and Y_e whose diagonal components have imaginary parts:

$$\begin{aligned} J_{\text{edm}}^{(i)} &= \text{Im} \left[\left[Y_e Y_\nu^\dagger Y_\nu \left[Y_e^\dagger Y_e, Y_\nu^\dagger Y_\nu \right] Y_\nu^\dagger Y_\nu \right]^{ii} \right] \\ &= 2Y_{e_i} \delta Y_{e_{jk}}^2 \delta Y_{\nu_{12}}^2 \delta Y_{\nu_{23}}^2 \delta Y_{\nu_{31}}^2 \text{Im} [X_{11} X_{12}^* X_{22} X_{21}^*] \sum_k \epsilon_{ijk} \end{aligned} \quad (27)$$

and the dominant contributions to the electric dipole moment are proportional to t^2 . Since $Y_\nu^\dagger Y_\nu$ in $J_{\text{edm}}^{(i)}$ comes from the radiative correction to the soft supersymmetry-breaking terms, the leading contribution to the electric dipole moment is proportional to $\log^3 \frac{M_{GUT}}{M_N}$ when $\log \frac{M_{GUT}}{M_N} \ll 4\pi$. The dependence on $Y_e^\dagger Y_e$ in $J_{\text{edm}}^{(i)}$ comes from the radiative correction in the soft supersymmetry-breaking terms or the tree-level mass matrix of the charged sleptons. In this subsection, we present the Jarlskog invariant for the soft supersymmetry-breaking terms contributing to the electric dipole moment in $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$. Also, we discuss cases where this approximation is invalid, namely when *i)* $\tan \beta \gg 1$, or *ii)* non-degeneracy between the heavy singlet neutrinos induces dependences of m_L^2 and A_e on phases in the product $\overline{Z}P_2$.

In order to evaluate the contribution to the electric dipole moment in $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$, we need the corrections to the soft supersymmetry-breaking terms at $\mathcal{O}(\log^2 \frac{M_{GUT}}{M_N})$, which are

$$\begin{aligned} (\delta^{(2)} m_L^2)_{ij} &\approx \frac{4}{(4\pi)^2} A_0^2 (3Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 3Y_e^\dagger Y_e Y_e^\dagger Y_e + \{Y_e^\dagger Y_e, Y_\nu^\dagger Y_\nu\})_{ij} \log^2 \frac{M_{GUT}}{M_N}, \\ (\delta^{(2)} m_E^2)_{ij} &\approx \frac{8}{(4\pi)^2} A_0^2 (3Y_e Y_e^\dagger Y_e Y_e^\dagger + Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger)_{ji} \log^2 \frac{M_{GUT}}{M_N}, \\ (\delta^{(2)} A_e)_{ij} &\approx 0. \end{aligned} \quad (28)$$

Here, we neglect irrelevant terms with a trace over flavor indices, or which are flavor-independent. The Yukawa couplings are evaluated at M_N . From these equations and (21), non-vanishing contributions to $J_{\text{edm}}^{(i)}$ arise from the following combinations of $\mathcal{O}(\log^3 \frac{M_{GUT}}{M_N})$:

$$\text{Im} \left[\delta A_e \delta A_e^\dagger \delta A_e \right]_{ii} = \frac{4}{(4\pi)^6} A_0^3 J_{\text{edm}}^{(i)} \log^3 \frac{M_{GUT}}{M_N}. \quad (29)$$

$$\text{Im} \left[\delta A_e Y_e^\dagger Y_e \delta^{(2)} m_L^2 \right]_{ii} = -\frac{12}{(4\pi)^6} A_0^3 J_{\text{edm}}^{(i)} \log^3 \frac{M_{GUT}}{M_N}, \quad (30)$$

In (30), the combination $Y_e^\dagger Y_e$ arises from the tree-level mass matrix of the charged sleptons. It is found from (29,30) that the electric dipole moments depend strongly on A_0 and less on m_0 .

²Similar studies for the electric dipole moment of neutron in the MSSM are done in [30], assuming that all CP violating phases come from the CKM matrix.

When $\tan\beta \gg 1$, (30) is proportional to $\tan^2\beta$ and (29) is not enhanced. On the other hand, terms such as

$$\text{Im} \left[\delta^{(2)} m_{\tilde{E}}^2 Y_e \delta^{(2)} m_{\tilde{L}}^2 \right], \quad (31)$$

are proportional to $\tan^3\beta$ and $\log^4 \frac{M_{GUT}}{M_N}$. Thus, they may make sizeable contributions to the electric dipole moments for $\tan\beta \gg 1$, even if $\log \frac{M_{GUT}}{M_N} \lesssim 4\pi$.

If the heavy neutrinos are not degenerate in mass, they induce dependences of the soft supersymmetry-breaking terms on phases in $\overline{Z}P_2$, as mentioned in the previous Section, which then contribute to the electric dipole moments. The Jarlskog invariant $J_{\text{edm}}^{(i)}$ depends on $Y_e^\dagger Y_e$, and this factor suppresses the electric dipole moment when $\tan\beta$ is small. In this case, the non-degeneracy of the heavy neutrinos may have a more important effect on the electric dipole moment. The corrections of $\mathcal{O}(\log \frac{M_{GUT}}{M_N} \log \frac{M_N}{M_{N_i}})$ are

$$\begin{aligned} (\tilde{\delta}^{(2)} m_{\tilde{L}}^2)_{ij} &\approx \frac{18}{(4\pi)^4} (m_0^2 + A_0^2) \{Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu\} \log \frac{M_{GUT}}{M_N}, \\ (\tilde{\delta}^{(2)} m_{\tilde{E}}^2)_{ij} &\approx 0, \\ (\tilde{\delta}^{(2)} A_e)_{ij} &\approx \frac{1}{(4\pi)^4} A_0 Y_e (11 \{Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu\} + 7 [Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu])_{ij} \log \frac{M_{GUT}}{M_N}. \end{aligned} \quad (32)$$

Here, we neglect terms with $Y_e^\dagger Y_e$ factors. The interesting point is that the second term in $\tilde{\delta}^{(2)} A_e$ can have imaginary parts in the diagonal components, and thus can contribute to the electric dipole moment³. Since phases in $\tilde{\delta}^{(2)} A_e$ arise from $\overline{Z}P_2$, we do not need three generations of leptons in order for $\tilde{\delta}^{(2)} A_e$ to have imaginary parts in the diagonal terms. The behaviour of this contribution will be discussed in the next subsection.

2.4 Two-Generation Model

We now demonstrate the interdependences of the above physical observables in a toy two-generation model. In this model, X has no physical phase while there may be one phase in Z . We parametrize the light- and heavy-neutrino masses and R as follows:

$$\begin{aligned} \mathcal{M}_\nu^D &= \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix}, & \mathcal{M}^D &= \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \\ R &= \begin{pmatrix} \cos(\theta_r + i\theta_i) & \sin(\theta_r + i\theta_i) \\ -\sin(\theta_r + i\theta_i) & \cos(\theta_r + i\theta_i) \end{pmatrix}. \end{aligned} \quad (33)$$

³Whilst the combination $(\tilde{\delta} A_e \delta m_{\tilde{L}}^2)_{ii}$ has an imaginary part, it does not contribute to the electric dipole moments, since $\text{Im}[(\delta A_e + \tilde{\delta} A_e)(\delta m_{\tilde{L}}^2 + \tilde{\delta} m_{\tilde{L}}^2)]_{ii} = 0$.

In this model, the leptogenesis invariant is $\text{Im} \left[\left(Y_\nu Y_\nu^\dagger \right)^{21} \left(Y_\nu Y_\nu^\dagger \right)^{21} \right] = 2 \times \text{Im} \left[\left(Y_\nu Y_\nu^\dagger \right)^{12} \right] \times \text{Re} \left[\left(Y_\nu Y_\nu^\dagger \right)^{12} \right]$, where

$$\begin{aligned} \text{Im} \left[\left(Y_\nu Y_\nu^\dagger \right)^{12} \right] &= \frac{(m_{\nu_1} + m_{\nu_2}) \sqrt{M_1 M_2}}{2v^2 \sin^2 \beta} \sinh 2\theta_i, \\ \text{Re} \left[\left(Y_\nu Y_\nu^\dagger \right)^{12} \right] &= -\frac{(m_{\nu_1} - m_{\nu_2}) \sqrt{M_1 M_2}}{2v^2 \sin^2 \beta} \sin 2\theta_r, \end{aligned} \quad (34)$$

so that

$$\text{Im} \left[\left(Y_\nu Y_\nu^\dagger \right)^{21} \left(Y_\nu Y_\nu^\dagger \right)^{21} \right] = -\frac{(m_{\nu_1}^2 - m_{\nu_2}^2) M_1 M_2}{2v^4 \sin^4 \beta} \sinh 2\theta_i \sin 2\theta_r. \quad (35)$$

As explained above, the phase θ_i in R controls leptogenesis, and the mixing angle θ_r must also be non-vanishing.

As concerns neutrino observables, we recall that there is no analogue of the MNS phase δ in this two-generation model. There is one CP-violating Majorana phase ϕ for the light neutrinos, but this does not contribute to leptogenesis, as we argued previously on general grounds and now see explicitly in (35).

We now consider the quantity $Y_\nu^\dagger Y_\nu$ which controls the renormalization of the soft supersymmetry-breaking terms in the leading-logarithmic approximation, in particular $(Y_\nu^\dagger Y_\nu)^{12}$ which has a non-zero imaginary part. For illustrational purposes, we assume that the light-neutrino mass matrix has maximal mixing:

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix} \quad (36)$$

where ϕ is a light-neutrino Majorana phase. In this case,

$$\begin{aligned} \text{Re} \left[\left(Y_\nu^\dagger Y_\nu \right)^{12} \right] &= -\frac{m_{\nu_2} + m_{\nu_1}}{4v^2 \sin \beta^2} (M_1 - M_2) \cos 2\theta_r \\ &\quad -\frac{m_{\nu_2} - m_{\nu_1}}{4v^2 \sin \beta^2} (M_1 + M_2) \cosh 2\theta_i, \end{aligned} \quad (37)$$

$$\begin{aligned} \text{Im} \left[\left(Y_\nu^\dagger Y_\nu \right)^{12} \right] &= \frac{\sqrt{m_{\nu_1} m_{\nu_2}}}{2v^2 \sin \beta^2} (M_1 + M_2) \cos \phi \sinh 2\theta_i \\ &\quad -\frac{\sqrt{m_{\nu_1} m_{\nu_2}}}{2v^2 \sin \beta^2} (M_1 - M_2) \sin \phi \sin 2\theta_r. \end{aligned} \quad (38)$$

We see that the imaginary part of the off-diagonal component depends both on the Majorana phase ϕ in U (36) and the phase θ_i in R . Even if it could be measured, and the neutrino mass eigenvalues $M_{1,2}, m_{\nu_{1,2}}$ were known, still only one combination of the angle factors $\theta_{r,i}$ entering in leptogenesis (35) would be known, and there would still be an ambiguity associated with the Majorana phase ϕ . In fact, no CP violation is induced by the renormalization (38) in

this simple two-generation model, since it is not possible to define the Jarlskog invariant $J_{\bar{L}}$ (22) and its analogues (23). Such invariants can be defined in a three-generation model, and CP-violating observables are demonstrably proportional to it, as we show in the next Section.

As commented in subsections 2.3.3 and 2.3.4, non-degeneracy between the heavy singlet neutrinos N_i induces, via renormalization, dependences of the entries of $m_{\bar{L}}^2$ and A_e on phases in the product $\bar{Z}P_2$. In the two-generation case, this dependence is on the one phase in P_2 , since \bar{Z} has no phases in this case. This makes changes in $\text{Arg}(m_{\bar{L}}^2)_{12}$ and $\text{Arg}(A_e)_{12}$, but these are suppressed to the extent that $\log \frac{M_N}{M_{N_i}} \ll \log \frac{M_{GUT}}{M_N}$. Moreover, these small changes are identical in the leading-logarithmic approximation. On the other hand, the corrections of the order of $\log \frac{M_N}{M_{N_i}} \log \frac{M_{GUT}}{M_N}$ to the phases of the diagonal terms in A_e may be sizeable. These are given in the two-generation model by

$$\begin{aligned} \text{Im} \left[[Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu]^{11} \right] &= \frac{\sqrt{m_{\nu_1} m_{\nu_2}} (m_{\nu_1} - m_{\nu_2}) M_1 M_2}{2v^4 \sin^4 \beta} \log \frac{M_2}{M_1} \cosh 2\theta_i \sin 2\theta_r \sin \phi, \\ &+ \frac{\sqrt{m_{\nu_1} m_{\nu_2}} (m_{\nu_1} + m_{\nu_2}) M_1 M_2}{2v^4 \sin^4 \beta} \log \frac{M_2}{M_1} \sinh 2\theta_i \cos 2\theta_r \cos \phi \\ \text{Im} \left[[Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu]^{22} \right] &= -\text{Im} \left[[Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu]^{11} \right]. \end{aligned} \quad (39)$$

We see that this effect vanishes if $M_2 = M_1$. Defining $M_2 = M_1(1+\delta)$, Eq.(39) grows with the dimensionless parameter δ (linearly, if δ is small) and is maximized when $\log[M_2/M_1] = 1$.

3 CP-Violating Observables in the Charged-lepton Sector

In this Section we discuss in more detail CP-violating and LFV observables in the charged-lepton sector. The slepton-mixing effects discussed in the previous Section generate LFV and CP-violating vertices involving charginos, which in turn induce effective non-renormalizable interactions, as we discuss in the following.

3.1 Chargino and Neutralino Interactions

The relevant neutralino and chargino interactions for leptons and sleptons are given by [7, 31]

$$\mathcal{L} = \bar{e}_i (N_{iAX}^L P_L + N_{iAX}^R P_R) \tilde{\chi}_A^0 \tilde{e}_X + \bar{e}_i (C_{iAX}^L P_L + C_{iAX}^R P_R) \tilde{\chi}_A^- \tilde{\nu}_X + h.c., \quad (40)$$

where $P_R = (1 + \gamma_5)/2$, $P_L = (1 - \gamma_5)/2$ and

$$N_{iAX}^L = -g \{ \sqrt{2} \tan \theta_W (O_N)_{A1} (U_e)_{X_{i+3}}^* + \frac{(m_e)_{ij}}{\sqrt{2} m_W \cos \beta} (O_N)_{A3} (U_e)_{X_j}^* \}, \quad (41)$$

$$N_{iAX}^R = -g\left[-\frac{1}{\sqrt{2}}\{(O_N)_{A2}^* + \tan\theta_W(O_N)_{A1}^*\}(U_e)_{Xi}^* + \frac{(m_e^\dagger)_{ij}}{\sqrt{2}m_W \cos\beta}(O_N)_{A3}^*(U_e)_{Xj+3}^*\right], \quad (42)$$

$$C_{iAX}^L = g\frac{(m_e)_{ij}}{\sqrt{2}m_W \cos\beta}(O_{CL})_{A2}^*(U_\nu)_{Xj}^*, \quad (43)$$

$$C_{iAX}^R = -g(O_{CR})_{A1}^*(U_\nu)_{Xi}^*. \quad (44)$$

In these expressions the matrices O_N , O_{CL} , O_{CR} , U_e and U_ν diagonalize the neutralino, left- and right-chargino, charged-slepton and sneutrino mass matrices, respectively. The indices run between $A = 1, \dots, 4$ for neutralinos, $A = 1, 2$ for charginos, $X = 1, \dots, 6$ for sleptons and $X = 1, 2, 3$ for sneutrinos. In our framework the complex phases appear only in U_e and U_ν .

3.2 LFV Muon Decays

The effective Lagrangian for polarized $\mu^+ \rightarrow e^+\gamma$ and $\mu^+ \rightarrow e^+e^+e^-$ decays is [31]

$$\begin{aligned} \mathcal{L} = & -\frac{4G_F}{\sqrt{2}}\{m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} \\ & + g_1(\bar{\mu}_R e_L)(\bar{e}_R e_L) + g_2(\bar{\mu}_L e_R)(\bar{e}_L e_R) \\ & + g_3(\bar{\mu}_R \gamma^\mu e_R)(\bar{e}_R \gamma_\mu e_R) + g_4(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_L \gamma_\mu e_L) \\ & + g_5(\bar{\mu}_R \gamma^\mu e_R)(\bar{e}_L \gamma_\mu e_L) + g_6(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_R \gamma_\mu e_R) + h.c.\}. \end{aligned} \quad (45)$$

Here A_L and A_R are the dimensionless photon-penguin couplings which induce $\mu^+ \rightarrow e_L^+ \gamma$ and $\mu^+ \rightarrow e_R^+ \gamma$, respectively, which also contribute to the $\mu^+ \rightarrow e^+e^+e^-$ process, and the $g_i : i = 1, \dots, 6$ are dimensionless four-fermion coupling constants which contribute only to $\mu^+ \rightarrow e^+e^+e^-$. Explicit expressions for $A_{L,R}$ and the g_i in terms of $N_{iAX}^{L,R}$, $C_{iAX}^{L,R}$ are lengthy [31], so we do not rewrite them here.

In the notation (45) the total $\mu^+ \rightarrow e^+\gamma$ branching ratio is given by

$$Br(\mu^+ \rightarrow e^+\gamma) = 384\pi^2 (|A_L|^2 + |A_R|^2), \quad (46)$$

and that of $\mu^+ \rightarrow e^+e^+e^-$ by

$$\begin{aligned} Br(\mu^+ \rightarrow e^+e^+e^-) = & 2(C_1 + C_2) + C_3 + C_4 + 32 \left\{ \log \frac{m_\mu^2}{m_e^2} - \frac{11}{4} \right\} (C_5 + C_6) \\ & + 16(C_7 + C_8) + 8(C_9 + C_{10}). \end{aligned} \quad (47)$$

The coefficients C_i appearing in (47) are functions of $A_{L,R}$ and g_i :

$$C_1 = \frac{|g_1|^2}{16} + |g_3|^2, \quad C_2 = \frac{|g_2|^2}{16} + |g_4|^2,$$

$$\begin{aligned}
C_3 &= |g_5|^2, \quad C_4 = |g_6|^2, \quad C_5 = |eA_R|^2, \quad C_6 = |eA_L|^2, \quad C_7 = \text{Re}(eA_R g_4^*), \\
C_8 &= \text{Re}(eA_L g_3^*), \quad C_9 = \text{Re}(eA_R g_6^*), \quad C_{10} = \text{Re}(eA_L g_5^*).
\end{aligned}
\tag{48}$$

In order for CP violation to appear in any process, interference between different terms in the amplitude for the process must occur. Therefore, all possible observables in $\mu \rightarrow e\gamma$ decays, such as differences between the $\mu^+ \rightarrow e^+\gamma$ and $\mu^- \rightarrow e^-\gamma$ rates, vanish in the leading order of perturbation theory. Moreover, the process $\mu^- \rightarrow e^-\gamma$ is not measurable with high accuracy because of the large backgrounds. However, when muons are polarized, a T-odd asymmetry for final-state particles in $\mu^+ \rightarrow e^+e^+e^-$ can be defined. Since CPT is conserved, the T-odd asymmetry measures the amount of CP violation in our model.

The muon polarization vector \vec{P} can be defined in the coordinate system in which the z axis is taken to be the direction of the electron momentum, the x axis the direction of the most energetic positron momentum, and the $(z \times x)$ plane defines the decay plane perpendicular to the y axis. It is necessary to introduce an energy cutoff for the more energetic positron: $E_1 < (m_\mu/2)(1 - \delta)$. We use $\delta = 0.02$ to optimize the T-odd asymmetry, following [31]. Assuming 100%-polarized muons the T-odd asymmetry is then defined by

$$A_T = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)} = \frac{3}{2Br(\delta = 0.02)} \{2.0C_{11} - 1.6C_{12}\}, \tag{49}$$

where $N(P_i > (<))0$ denotes the number of events with a positive (negative) P_i component for the muon polarization,

$$C_{11} = \text{Im}(eA_R g_4^* + eA_L g_3^*), \quad C_{12} = \text{Im}(eA_R g_6^* + eA_L g_5^*), \tag{50}$$

and the branching ratio for $\delta = 0.02$ is

$$\begin{aligned}
Br(\delta = 0.02) &= 1.8(C_1 + C_2) + 0.96(C_3 + C_4) + 88(C_5 + C_6) \\
&\quad + 14(C_7 + C_8) + 8(C_9 + C_{10}).
\end{aligned}
\tag{51}$$

It is known [31] that the asymmetry (49) may be large in SU(5) SUSY GUTs [32]. We study below whether this is also the case in the minimal supersymmetric seesaw model.

3.3 Electric Dipole Moments

The electric dipole moment of a generic lepton ℓ is defined as the coefficient d_ℓ of the interaction

$$\mathcal{L} = -\frac{i}{2}d_\ell \bar{\ell} \sigma_{\mu\nu} \gamma_5 \ell F^{\mu\nu}. \tag{52}$$

The current experimental bounds are $d_e < 4.3 \times 10^{-27}$ e cm for the electron [33], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}$ e cm for the muon [34], and $|d_\tau| < 3.1 \times 10^{-16}$ e cm for the τ [35]. An experiment has been proposed at BNL that could improve the sensitivity to d_μ down to $d_\mu \sim 10^{-24}$ e cm [36], and PRISM and neutrino factory experiments aim at sensitivities $d_\mu \sim 5 \times 10^{-26}$ e cm. These bounds will impose serious constraints on CP violation in the MSSM, as will prospective improvements in the sensitivity to d_e and d_τ .

In the MSSM, the d_ℓ receive contributions from chargino and neutralino loops:

$$d_l = d_l^{X^+} + d_l^{X^0}, \quad (53)$$

where [19, 20]

$$d_l^{X^+} = -\frac{e}{(4\pi)^2} \sum_{A=1}^2 \sum_{X=1}^3 \text{Im}(C_{lAX}^L C_{lAX}^{R*}) \frac{m_{\chi_A^+}}{m_{\tilde{\nu}_X}^2} A\left(\frac{m_{\chi_A^+}^2}{m_{\tilde{\nu}_X}^2}\right), \quad (54)$$

$$d_l^{X^0} = -\frac{e}{(4\pi)^2} \sum_{A=1}^4 \sum_{X=1}^6 \text{Im}(N_{lAX}^L N_{lAX}^{R*}) \frac{m_{\chi_A^0}}{M_{\tilde{l}_X}^2} B\left(\frac{m_{\chi_A^0}^2}{M_{\tilde{l}_X}^2}\right), \quad (55)$$

and the loop functions are given by

$$A(r) = \frac{1}{2(1-r)^2} \left(3 - r + \frac{2 \log r}{1-r} \right),$$

$$B(r) = \frac{1}{2(r-1)^2} \left(1 + r + \frac{2r \log r}{1-r} \right).$$

where the relevant chargino and neutralino couplings were given above. We do not consider in our analysis the possibility of CP violation in the chargino and neutralino mass matrices.

3.4 Rôle of the Jarlskog Invariants

We first present approximate formulae for the effective couplings in (45), in order to show the qualitative behaviours of the LFV processes and demonstrate the rôle of the Jarlskog invariants. Since $(\delta m_{\tilde{L}}^2)$ and (δA_e) are the only sources of off-diagonal components, the only non-negligible terms are A_R , g_4 and g_6 : other terms are suppressed by the electron or muon masses. For illustrative purposes in this subsection only, we assign to the soft supersymmetry-breaking parameters common value m_S at the electroweak scale:

$$M_2 = M_1 = \mu = (A_e)_{22}/Y_{e2} \equiv m_S, \quad ,$$

$$(m_{\tilde{L}}^2)_{ii} = (m_{\tilde{E}}^2)_{ii} \equiv m_S^2. \quad (56)$$

Assuming $m_S \gg m_Z$, we then find

$$A_R = 4.8 \times 10^{-5} \left(\frac{100\text{GeV}}{m_S} \right)^2 \times \left\{ \Delta_{21}^{\tilde{L}} - 0.64\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}} + 0.66\Delta_{21}^{A_e} - 0.40\Delta_{23}^{A_e}\Delta_{31}^{\tilde{L}} + \tan\beta(2.4\Delta_{21}^{\tilde{L}} - 1.12\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}}) \right\}, \quad (57)$$

$$g_4 = 6.4 \times 10^{-5} \left(\frac{100\text{GeV}}{m_S} \right)^2 \times \left\{ (1 - 0.24\sin 2\beta)\Delta_{21}^{\tilde{L}} + (-0.81 + 0.12(\sin 2\beta + \cos 2\beta))\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}} \right\}, \quad (58)$$

$$g_6 = -1.9 \times 10^{-5} \left(\frac{100\text{GeV}}{m_S} \right)^2 \times \left\{ (1 - 0.70\sin 2\beta)\Delta_{21}^{\tilde{L}} + (-0.43 + 0.35(\sin 2\beta + \cos 2\beta))\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}} \right\}, \quad (59)$$

where

$$\Delta_{ij}^{\tilde{L}} \equiv \left(\frac{(\delta m_{\tilde{L}}^2)_{ij}}{m_S^2} \right),$$

$$\Delta_{ij}^{A_e} \equiv \left(\frac{(\delta A_e)_{ij}/Y_{e_i}}{m_S} \right).$$

We remind that $\Delta_{21}^{\tilde{L}} = (\Delta_{12}^{\tilde{L}})^*$. The $\sin 2\beta$ and $\cos 2\beta$ dependences of g_4 and g_6 above are due to Z -penguin diagrams. In the branching ratio for $\mu \rightarrow 3e$, the contribution from A_R tends to dominate due to the phase-space integral. Then, assuming that A_R dominates in $Br(\mu^+ \rightarrow e^+e^+e^-)$, the T-odd asymmetry A_T is given by

$$A_T = \frac{\text{Im}[\Delta_{12}^{\tilde{L}}\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}}]}{|\Delta_{12}^{\tilde{L}}|^2} \frac{0.039 + 0.196 \tan\beta + 0.017/\tan\beta}{\left| (1 + 2.4 \tan\beta) - \frac{\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}}}{\Delta_{21}^{\tilde{L}}} (0.64 + 1.12 \tan\beta) \right|^2}, \quad (60)$$

where we have expanded $\sin 2\beta$ and $\cos 2\beta$ in terms of $\tan\beta$. Also, in writing (60), we have taken $\Delta_{21}^{A_e} = \Delta_{23}^{A_e} = 0$, for simplicity.

We see explicitly how A_T (60) depends on the Jarlskog invariant $J_{\tilde{L}}$ (22), and it is apparent how analogous invariants $J_{A_{12}}$, etc., with one or more $\Delta_{ij}^{\tilde{L}} \rightarrow \Delta_{ij}^{A_e}$ could also contribute. We see that A_T could in principle reach $\sim 10\%$. However, if $\text{Im}[\Delta_{12}^{\tilde{L}}\Delta_{23}^{\tilde{L}}\Delta_{31}^{\tilde{L}}] \ll |\Delta_{12}^{\tilde{L}}|^2$, as one might expect, or if $\tan\beta \gg 1$, A_T is suppressed. However, we stress that Eq.(60) is approximately correct only for (56) and cannot be used to predict A_T in more general cases which will be considered in the next Section.

Next, we present approximate formulae for the effective coupling in (52) in the specific case (56). Since relative phases between $(m_{\tilde{L}}^2)$, $(m_{\tilde{E}}^2)$ and (A_e) contribute to the electric dipole moments, non-vanishing contributions to d_l come only from slepton diagrams, not sneutrino diagrams. From the explicit formula (54), it is also clear that the sneutrino diagrams do not

give a non-vanishing value, since they depend on $(U_\nu)_{Xi}^*(U_\nu)_{Xi}$, and the CP-violating phases exist only in the mixing matrices of the sleptons, in our approximation.

Since it depends on the neutrino model which contribution is dominant in the electric dipole moments, as shown in the previous Section, we first show the general formula for $d_l^{\tilde{\chi}^0}$ in the limit (56):

$$\frac{d_l^{\tilde{\chi}^0}}{e} = \frac{g_Y^2}{(4\pi)^2} \frac{1}{m_S} \text{Im} \left[\sum_{N=1} \sum_{i_1, \dots, i_N} c_N \Delta_{li_1} \Delta_{i_1 i_2} \cdots \Delta_{i_N l} \right]. \quad (61)$$

Here, Δ_{ij} is the flavor-dependent part of the slepton mass matrix, which normalized by m_S^2 , and includes parts generated by the renormalization as well as tree-level parts. At least one of the Δ_{ij} in each product term involves the left-right mixing of a slepton. The coefficients c_i are:

$$c_1 = -\frac{1}{12}, \quad c_2 = \frac{1}{20}, \quad c_3 = -\frac{1}{30}, \quad c_4 = \frac{1}{42}, \quad c_5 = -\frac{1}{56}. \quad (62)$$

When the heavy singlet-neutrino masses are not almost degenerate, and corrections to the soft supersymmetry-breaking terms and the relative phases among (m_L^2) and (A_e) are generated at $\mathcal{O}(\log^2 \frac{M_{GUT}}{M_N})$, the approximate formula becomes

$$\begin{aligned} \frac{d_l^{\tilde{\chi}^0}}{e} &= \frac{g_Y^2}{(4\pi)^2} \frac{m_l}{m_S^2} \left(\frac{1}{42} \tan \beta^2 + \frac{1}{21} \tan \beta - \frac{1}{105} \right) \sum_j \text{Im} \left[\Delta_{lj}^{A_e} \frac{m_{l_j}^2}{m_S^2} \Delta_{jl}^{\tilde{L}} \right] \\ &\quad - \frac{1}{30} \frac{g_Y^2}{(4\pi)^2} \frac{m_l}{m_S^2} \sum_{j,k} \text{Im} \left[\Delta_{lj}^{A_e} \frac{m_{l_j}^2}{m_S^2} (\Delta_{kj}^{A_e})^* \Delta_{kl}^{A_e} \right], \end{aligned} \quad (63)$$

where the first term is proportional to (30) and the second to (29).

When the heavy singlet-neutrino masses are almost degenerate and the correction to $\text{Im}[A_e]_{ii}$, proportional to $\log \frac{M_{N_i}}{M_{N_j}} \log \frac{M_{GUT}}{M_N}$, is dominant in the electric dipole moments, the approximate formula is

$$\frac{d_l^{\tilde{\chi}^0}}{e} = -\frac{1}{12} \frac{g_Y^2}{(4\pi)^2} \frac{m_l}{m_S^2} \text{Im} \left[\Delta_{ll}^{A_e} \right]. \quad (64)$$

4 Numerical Analysis

4.1 Computational Procedure

We first fix the gauge couplings, charged-lepton and quark Yukawa couplings and $\tan \beta$ (which is a free parameter) at the scale M_Z , and then run them with the two-loop MSSM

renormalization-group equations up to the scale M_N . At M_N , we introduce the heavy singlet neutrinos, fixing their masses, the light-neutrino masses and mixings according to the oscillation data. We then choose the matrix R and calculate Y_ν according to (14). Subsequently, we run all the Yukawa-coupling matrices from M_N to M_{GUT} using the one-loop renormalization-group equations [7]. At M_{GUT} we assume universal boundary conditions (20) for the soft supersymmetry-breaking terms. We then run all the soft supersymmetry-breaking masses and Yukawa matrices back to M_N , where the heavy singlet neutrinos and sneutrinos decouple. The soft supersymmetry-breaking mass matrices at low energies are obtained finally by running all the MSSM parameters back down to M_Z . We use the electroweak symmetry-breaking conditions to fix the magnitude of the Higgs mixing parameter μ , taking its sign positive as motivated by $g_\mu - 2$. This sign is also consistent with the bounds from $b \rightarrow s\gamma$. Then we calculate the squark, slepton, chargino and neutralino mass matrices and finally LFV rates, the T-odd asymmetry A_T in polarized $\mu^+ \rightarrow e^+e^+e^-$ decays and the EDMs of the electron and muon, for chosen values of the input parameters.

4.2 Illustrative Results

In our numerical examples, we take [37] $\Delta m_{atm}^2 = 3.5 \times 10^{-3} \text{ eV}^2$ with $\sin \theta_{23} = 0.7$ for atmospheric neutrinos, and $\Delta m_{sol}^2 = 5.0 \times 10^{-5} \text{ eV}^2$ with $\tan^2 \theta_{12} = 0.45$ for solar neutrinos, corresponding to the LMA solution. Also, we fix $\tan^2 \theta_{13} = 0.055$, which is the largest value allowed by the global three-neutrino data fit. We parametrize the orthogonal matrix R in the form

$$R = \begin{pmatrix} \hat{c}_2 \hat{c}_3 & -\hat{c}_1 \hat{s}_3 - \hat{s}_1 \hat{s}_2 \hat{c}_3 & \hat{s}_1 \hat{s}_3 - \hat{c}_1 \hat{s}_2 \hat{c}_3 \\ \hat{c}_2 \hat{s}_3 & \hat{c}_1 \hat{c}_3 - \hat{s}_1 \hat{s}_2 \hat{s}_3 & -\hat{s}_1 \hat{c}_3 - \hat{c}_1 \hat{s}_2 \hat{s}_3 \\ \hat{s}_2 & \hat{s}_1 \hat{c}_2 & \hat{c}_1 \hat{c}_2 \end{pmatrix}, \quad (65)$$

where $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ are arbitrary complex angles. Using a generic R as input is crucial for neutrino phenomenology. We stress the following:

- (i) Because R is a complex orthogonal matrix, the values of its entries are not restricted to any small range, but rather are *exponential* functions of complex numbers. This implies via (14) that, for a suitable choice of $\hat{\theta}_{1,2,3}$, all the elements of Y_ν can be large even if one starts with small (for example hierarchical) neutrino masses;
- (ii) Large imaginary components are in general present in every entry of Y_ν . Since $Y_\nu^\dagger Y_\nu$ depends on the phases of both R and U , as seen in (16), sizeable CP-violating effects may be induced. However, if $R = 1$, only δ in U contributes to $Y_\nu^\dagger Y_\nu$.

We first calculate d_e and d_μ using (54) and (55), and study their possible ranges in our model by scanning over the allowed values of the free parameters. As expected, d_e and d_μ are very small. For degenerate right-handed neutrinos, typically d_μ does not exceed 10^{-30} e cm for the values of the parameters consistent with the bounds on $\mu \rightarrow e\gamma$, and is therefore many orders of magnitude below the sensitivity of any planned experiment. The approximate relation between muon and electron electric dipole moments in our model, $d_\mu/d_e \approx -m_\mu/m_e$, holds very well in this case. However, in the case of non-degenerate right-handed neutrinos, the $\log(M_{N_i}/M_{N_j})$ effects introduce a dependence of the electric dipole moments on the leptogenesis phases, as discussed previously. These new contributions may change the magnitude of dipole moments by a large factor, and d_μ can reach to 10^{-27} e cm. In particular, the sign of d_μ/d_e may be altered and the naive relation $d_\mu/d_e \approx -m_\mu/m_e$ may be violated by a large factor. However, d_e, d_μ still remain unobservable. Thus we conclude that, unlike in other supersymmetric models [38], the renormalization-induced electric dipole moments do not have experimental prospects at present.

On the other hand, the situation with the T-odd asymmetry in polarized $\mu^+ \rightarrow e^+e^+e^-$ decays can be different. As explained in the previous Section, the decay $\mu^+ \rightarrow e^+e^+e^-$ receives contributions from box diagrams and photonic penguin diagrams, with the latter usually dominating. However, if there are cancellations in the dipole-moment-type $\mu - e - \gamma$ vertex, the box and penguin contributions to $\mu^+ \rightarrow e^+e^+e^-$ may become comparable. In that case, if there are large CP-violating phases present in the slepton mass matrices, the T-odd asymmetry A_T can be large. This implies an anti-correlation between $Br(\mu \rightarrow e\gamma)$ and A_T : the latter can be large only if the former is suppressed.

To illustrate how such a cancellation might come about, consider the two-generation example described at the end of Section 2, which also applies in the full three-generation case if $m_{\nu_3} \ll m_{\nu_{1,2}}$: we assume for simplicity that also $m_{\nu_2} \ll m_{\nu_1}$. We see from (37) that $\text{Re}\left[\left(Y_\nu^\dagger Y_\nu\right)^{12}\right]$ is suppressed if

$$(M_1 + M_2) \cos 2\theta_r \approx (M_1 - M_2) \cosh 2\theta_i, \quad (66)$$

and that the smaller quantity $\text{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^{12}\right]$ is also suppressed if

$$(M_1 + M_2) \cos \phi \sinh 2\theta_i \approx (M_1 - M_2) \sin \phi \sin 2\theta_r. \quad (67)$$

Both the conditions (66,67) may in principle be satisfied simultaneously for suitably tuned values of θ_i, θ_r and ϕ .

In a general and more physical case, the cancellation in $Br(\mu \rightarrow e\gamma)$ is much more complicated and depends on all free parameters, namely the phases and mixing angles in

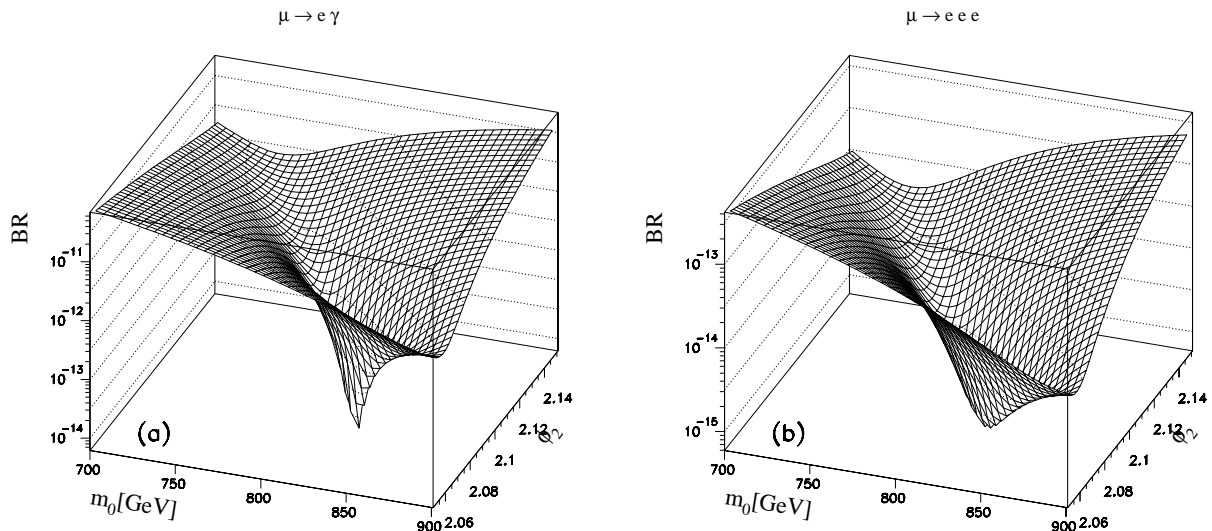


Figure 1: *Branching ratios for the decays (a) $\mu^+ \rightarrow e^+ \gamma$ and (b) $\mu^+ \rightarrow e^+ e^+ e^-$ as functions of the common soft mass m_0 and the Majorana phase ϕ_2 , for the fixed choice of neutrino parameters described in the text.*

the matrices U and R , the values of the light- and heavy- neutrino masses, the choice of the soft supersymmetry-breaking initial conditions, details of the renormalization-group running procedure, etc.. However, the fact that such cancellations occur is robust and qualitatively well understood. To give a representative numerical example, we choose hierarchical neutrino masses with $m_{\nu_1} = 0.028$ eV, $M_{N_1} = 1.2 \times 10^{15}$ GeV, $M_{N_2} = 1.5 \times 10^{15}$ GeV, $M_{N_3} = 3 \times 10^{14}$ GeV, $\delta = \pi/2$, $\phi_1 = -1$, $\hat{\theta}_1 = 0.3i$, $\hat{\theta}_2 = 0.5i$, $\hat{\theta}_3 = 0.1i$, and use the same neutrino oscillation parameters as above. We run the renormalization-group equations with right-handed neutrinos down to the scale $M_N = 3 \times 10^{14}$ GeV.

Fixing these parameters, we first choose $m_{1/2} = 500$ GeV, $A_0 = 0$ GeV and $\tan \beta = 20$ and scan over the remaining free parameters m_0 and ϕ_2 . In Fig. 1 we plot the branching ratios of the decays $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^+ e^-$ as functions of the common sfermion soft mass parameter m_0 and the Majorana phase ϕ_2 . For a large region in the plotted parameter space, $Br(\mu^+ \rightarrow e^+ \gamma)$ is below the present experimental bound, whilst $Br(\mu^+ \rightarrow e^+ e^+ e^-)$ does not at present impose any constraints. The branching ratios in Fig. 1 are correlated, implying that the photonic penguin diagrams are dominating also in $\mu^+ \rightarrow e^+ e^+ e^-$. For the plotted values, both rare decays should be observed in the planned experiments.

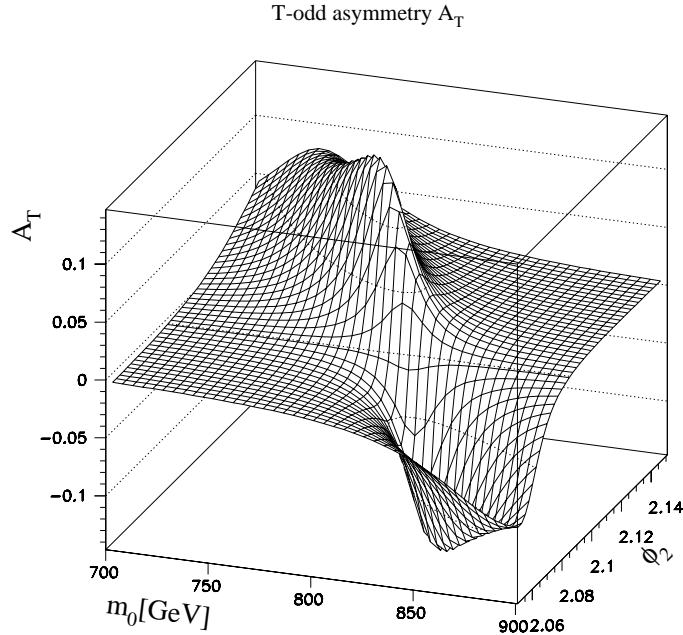


Figure 2: The T -odd asymmetry A_T in polarized $\mu^+ \rightarrow e^+e^+e^-$ decay for the same set of parameters as in Fig. 1.

The T -odd asymmetry A_T is plotted in Fig. 2 for the same set of parameters as in Fig. 1. Comparison with Fig. 1 shows that A_T is strongly anti-correlated with the branching ratios. For the region with the deepest cancellation, the T -odd asymmetry is negative. Whilst its absolute value may exceed 10%, the branching ratio of $\mu^+ \rightarrow e^+e^+e^-$ is relatively small in that region, making its precise determination difficult. However, for large positive A_T , $Br(\mu^+ \rightarrow e^+e^+e^-)$ exceeds the 10^{-14} level, implying that several hundred events could be observed in the planned experiments. Since the experimental sensitivity to A_T should scale as $1/\sqrt{N_{events}}$, these future experiments would be able to measure a non-zero value of A_T for large ranges of m_0 and ϕ_2 in Fig. 2.

Whilst the long-baseline oscillation experiments at neutrino factories will measure the phase δ in (11), we stress here again that the T -odd asymmetry A_T depends on all the phases in the matrices U and R . Thus different combinations of phases in the Yukawa matrix Y_ν are probed in the neutrino oscillation and stopped muon experiments. This point is made explicitly by the dependence of A_T on the Majorana phase ϕ_2 in Fig. 2.

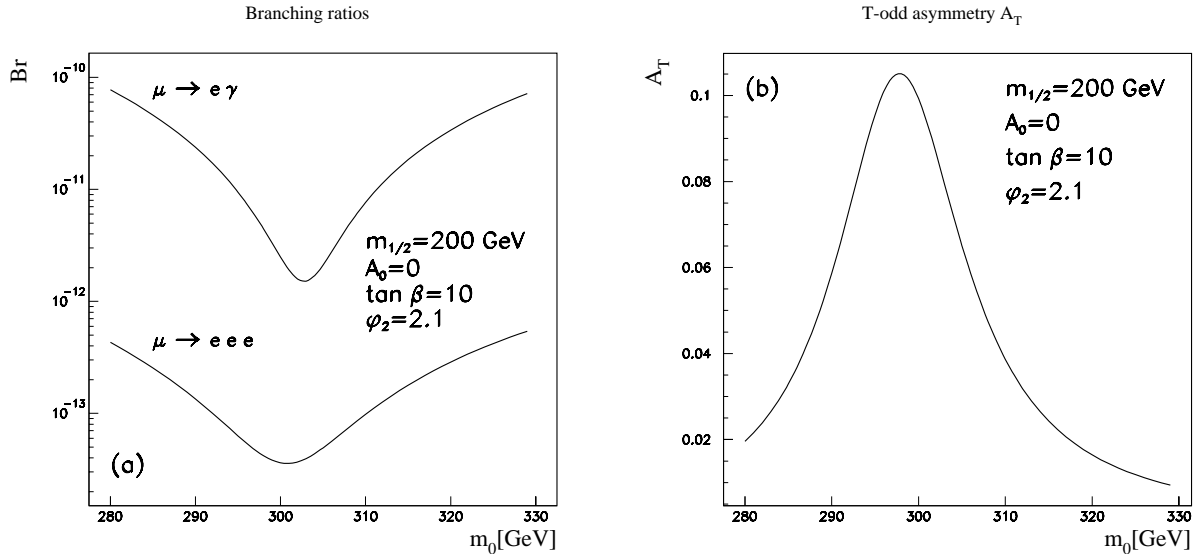


Figure 3: (a) Branching ratios for the decays $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^+ e^-$ and (b) the T-odd asymmetry A_T in $\mu^+ \rightarrow e^+ e^+ e^-$ decay, as functions of the common soft mass m_0 , for the fixed choice of neutrino parameters described in the text.

We have chosen the neutrino parameters in such a way that the absolute values of all the neutrino Yukawa couplings are close to unity. This induces large rates of LFV and CP violation. However, due to the cancellations in the photonic penguin diagrams, the induced $Br(\mu^+ \rightarrow e^+ \gamma)$ can be consistent with the current experimental bounds even for relatively small sparticle masses. In Fig. 3 we plot (a) $Br(\mu^+ \rightarrow e^+ \gamma)$ and $Br(\mu^+ \rightarrow e^+ e^+ e^-)$ and (b) A_T as functions of m_0 for fixed $m_{1/2} = 200$ GeV, $A_0 = 0$ GeV, $\tan \beta = 10$ and $\phi_2 = 2.1$. For a small region around $m_0 = 300$ GeV, $Br(\mu^+ \rightarrow e^+ \gamma)$ is below the present limit. At the same time, the T-odd asymmetry in $\mu^+ \rightarrow e^+ e^+ e^-$ may be as large as 10% for the allowed values of m_0 . The dependence of the branching ratios and the T-odd asymmetry on the Majorana phase ϕ_2 is demonstrated in Fig. 4. Again, large A_T is expected if the decay $\mu^+ \rightarrow e^+ \gamma$ is suppressed due to the cancellation.

Finally, we note that the branching ratio of the decay $\tau^+ \rightarrow \mu^+ \gamma$ *does not* have cancellations in the parameter region considered, as seen in Fig. 5. Therefore, in this scenario $Br(\tau^+ \rightarrow \mu^+ \gamma)$ might be just below the present experimental bound and discoverable at the LHC or the B factories. As the decays $\tau \rightarrow \ell \ell \ell$ are suppressed relative to $\tau^+ \rightarrow \mu^+ \gamma$, detailed quantitative studies of them are beyond the interest of the present work.

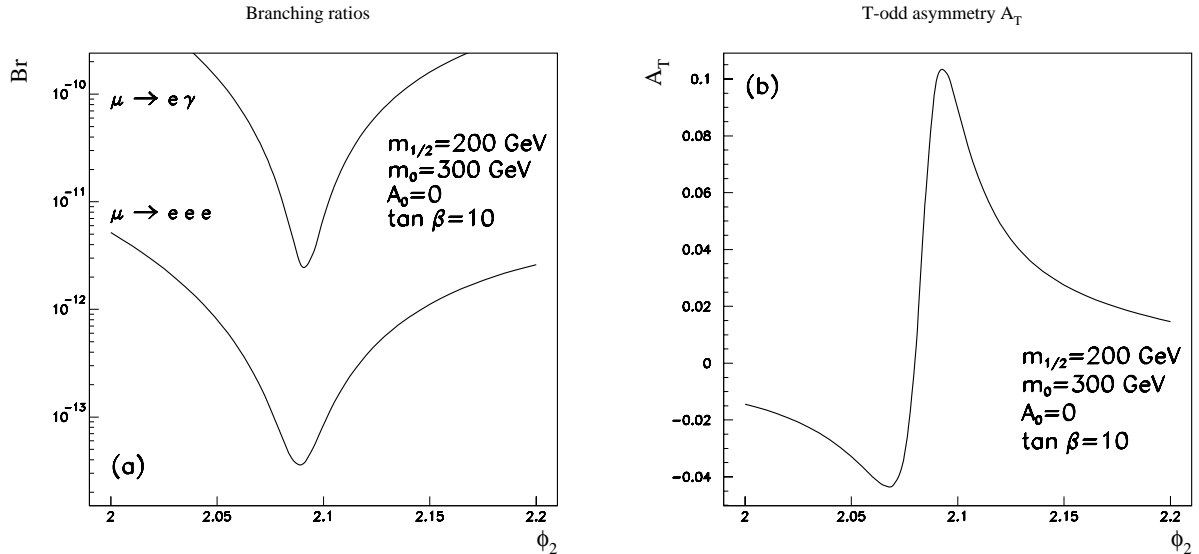


Figure 4: (a) Branching ratios of the decays $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^+ e^-$ and (b) the T-odd asymmetry A_T in $\mu^+ \rightarrow e^+ e^+ e^-$ decay, as functions of the Majorana phase ϕ_2 for $m_0 = 300$ GeV. All other parameters are fixed as in Fig. 3.

5 Discussion and Conclusions

We have seen in this paper that the T-odd asymmetry A_T in polarized $\mu^+ \rightarrow e^+ e^+ e^-$ decay may offer the best prospects for studying in the laboratory CP-violating effects in the minimal supersymmetric seesaw model. On the other hand, the electric dipole moments of the electron and muon are suppressed in the minimal supersymmetric seesaw scenario discussed here. This is because the CP-violating phases are induced by renormalization-group running only in the off-diagonal entries in the slepton mass matrices, as already discussed in [27]. The naive relation $d_\mu/d_e \approx -m_\mu/m_e$ holds very well in the case of degenerate right-handed neutrinos. In the case of non-degenerate right-handed neutrinos, logarithmic effects arising from $\log(M_{N_i}/M_{N_j})$ introduce a dependence on the leptogenesis phases. These new contributions may become dominant and the naive relation $d_\mu/d_e \approx -m_\mu/m_e$ is badly violated. However, the electric dipole moments still remain unobservable at the presently proposed experiments.

The possibility of measuring a non-zero A_T could have far-reaching consequences, since it provides complementary information on the CP-violating phases in the neutrino Yukawa matrix Y_ν . As has been discussed, A_T depends in the leading-logarithmic approximation on

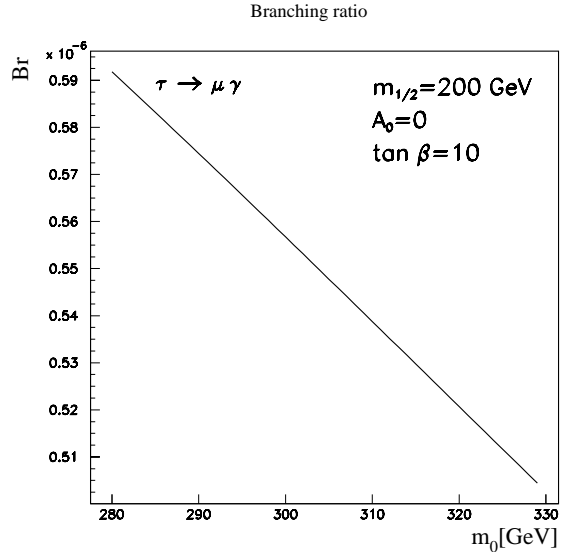


Figure 5: Branching ratio of the decay $\tau^+ \rightarrow \mu^+ \gamma$ as a function of m_0 . All other parameters are fixed as in Fig. 3.

a single combination of light-neutrino phases and the three phases in $Y_\nu^\dagger Y_\nu$ that contribute to leptogenesis, whereas the CP-violating phases in the light-neutrino effective mass matrix depend on the phases in $Y_\nu Y_\nu^\dagger$, that do not contribute to leptogenesis.

This is one reason why A_T may be observable even if CP violation is undetectable in neutrino oscillations. We recall also that the latter is in practice observable only if the neutrino masses and mixing angles are favourable. For example, if $U_{e3} \approx 0$ and/or Δm_{sol}^2 is small, as in the case of vacuum oscillations, CP violation is unobservable using neutrino factories. However, as seen in (14), (some of) the Yukawa couplings in Y_ν may still be large and imaginary, implying that A_T might be large.

On the other hand, a large value of A_T requires cancellations in the slepton-induced $\mu - e - \gamma^*$ vertex, which happens only in a restricted region of the parameter space. The asymmetry A_T is anti-correlated with the branching ratio of $\mu \rightarrow e\gamma$, and it can reach $\sim 10\%$ if $\mu \rightarrow e\gamma$ is suppressed. The asymmetry A_T may be measurable in planned high-intensity stopped-muon experiments, which aim at a sensitivity to $Br(\mu \rightarrow eee) \sim 10^{-16}$.

In the case of $\tau \rightarrow \mu\gamma$ the cancellation does not happen for the same parameters as in $\mu \rightarrow e\gamma$. Therefore, in the scenario considered in this paper $Br(\tau \rightarrow \mu\gamma)$ is large and can be observed at the LHC experiments.

It is interesting to review what we would learn if non-vanishing A_T were observed. The T-

odd asymmetry A_T is approximately proportional to the CP invariant (22). We recall that, if Y_ν has a hierarchical structure, the off-diagonal components of the left-handed slepton mass matrix are given by

$$\begin{aligned} \left(m_{\tilde{L}}^2\right)_{12} &\propto (Y_{\nu_3}^D)^2 X_{31} X_{32}^* + (Y_{\nu_2}^D)^2 X_{21} X_{22}^*, \\ \left(m_{\tilde{L}}^2\right)_{23} \left(m_{\tilde{L}}^2\right)_{31} &\propto (Y_{\nu_3}^D)^4 |X_{33}|^2 X_{32} X_{31}^*. \end{aligned}$$

The Jarlskog invariant $J_{\tilde{L}}$ may have a sizeable value if $(Y_{\nu_3}^D)^2 X_{31} X_{32}^*$ and $(Y_{\nu_2}^D)^2 X_{21} X_{22}^*$ are comparable. Thus, if non-vanishing A_T is observed, the generation structure in the neutrino Yukawa coupling may be constrained, as well as the CP-violating phase.

In conclusion: searching for CP violation in lepton-flavour-violating processes is a possibility that should not be neglected, since it provides information complementary to that provided by neutrino oscillation experiments. In particular, A_T may be measurable even if CP violation is unobservable in neutrino oscillations.

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