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Complete Cubic and Quartic Couplings of 16 and $\overline{16}$ in SO(10) Unification

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Abstract

A recently derived basic theorem on the decomposition of SO(2N) vertices is used to obtain a complete analytic determination of all SO(10) invariant cubic superpotential couplings involving $16_{\pm}$ semispinors of SO(10) chirality $\pm$ and tensor representations. In addition to the superpotential couplings computed previously using the basic theorem involving the 10, 120 and $\overline{126}$ tensor representations we compute here couplings involving the 1, 45 and 210 dimensional tensor representations, i.e., we compute the $\overline{16}_{\mp}16_{\pm}1$, $\overline{16}_{\mp}16_{\pm}45$ and $\overline{16}_{\mp}16_{\pm}210$ Higgs couplings in the superpotential. A complete determination of dimension five operators in the superpotential arising from the mediation of the 1, 45 and 210 dimensional representations is also given. The vector couplings $\overline{16}_{\pm}16_{\pm}1$, $\overline{16}_{\pm}16_{\pm}45$ and $\overline{16}_{\pm}16_{\pm}210$ are also analyzed. The role of large tensor representations and the possible application of results derived here in model building are discussed.

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1 Introduction

The group SO(10) is an interesting possible candidate for unification of interactions[1] and there has been considerable interest recently in investigating specific grand unified models based on this group. Thus SO(10) models have many desirable features allowing for all the quarks and leptons of one generation to reside in the irreducible 16 plet spinor representation of SO(10) and allowing for a natural splitting of Higgs doublets and Higgs triplets. Progress on the explicit computation of SO(10) couplings has been less dramatic. Thus while good initial progress occured in the early nineteen eightees in the introduction of oscillator techniques[2, 3, 4], there was little further progress on this front till recently when a technique was developed using the oscillator method which allows for the explicit computation of SO(2N) invariant couplings[5]. It was also shown in Ref.[5] that the new technique is specially useful in the analysis of couplings involving large tensor representations. Large tensor representations have already surfaced in several unified models based on SO(10)[6] and one needs to address the question of fully evaluating couplings involving the 16 plet of matter and Higgs with these tensors. In Ref.[5] a complete evaluation of the cubic superpotential involving the 16 plet of matter was given. Since $16 \times 16 = 10 + 120_a + 126_s$ the evaluations given in Ref.[5] involved $16 - 16 - 10$, $16 - 16 - 120$ and $16 - 16 - \overline{126}$ couplings.

In this paper we carry the analysis a step further and give a complete evaluation of the $\overline{16} - 16$ couplings which involve the SO(10) tensors 1, 45 and 210. Further, technically the couplings of $\overline{16}_{-}16_{+}$ are not necessarily the same as of $\overline{16}_{+}16_{-}$. Thus we give a full evaluation of the $\overline{16}_{\mp}16_{\pm}1$, $\overline{16}_{\mp}16_{\pm}45$ and the $\overline{16}_{\mp}16_{\pm}120$ couplings. An analysis of $\overline{16}_{\pm}16_{\pm}1$ vector couplings is also given. The analysis given here will have direct application in the further development of SO(10) unified models and in a fuller understanding of their detailed structure. We wish to point out that one may also use purely group theoretic methods to compute the Clebsch-Gordon co-efficients in the expansion of SO(10) invariant couplings. Such an approach was used in Ref.[7] to compute the E_6 couplings. Our approach is field theoretic and is specially suited for the computation of SO(2N) couplings. The outline of the rest of the paper is as follows: In Sec.2 we give a brief review of the basic theorem derived in Ref.[5] which is central to the computation of SO(2N) invariant couplings. In Sec.3 we use the basic theorem to compute the superpotential couplings cubic in fields involving $\overline{16}_{\mp}16_{\pm}$ and the 1 and 45 tensor fields. In Sec.4 a similar analysis is carried out using the 210 multiplet. In Sec.5

an analysis is given of the quartic couplings in the superpotential obtained from the elimination of the singlet, the 45 plet and the 210 plet of heavy Higgs fields from the cubic superpotential. Vector couplings are investigated in Sec.6. In Sec.7 the possible role of large tensor representations in model building is discussed. Conclusions are given in Sec.8. Some of the mathematical details are discussed in Appendices A and B.

2 Review of basic theorem for analysis of SO(2N) couplings

In this section we give a discussion of the oscillator method[2, 3, 4] together with a brief discussion of the basic theorem derived in Ref.[5] which is especially useful in evaluating SO(2N) gauge and Yukawa couplings involving large tensor representations of SO(2N). We begin by defining a set of five fermionic creation and annihilation operators b_i and $b_i^\dagger$ $(i = 1, ..., 5)$ obeying the anti-commutation rules

$$\{b_i, b_j^\dagger\} = \delta_i^j; \quad \{b_i, b_j\} = 0; \quad \{b_i^\dagger, b_j^\dagger\} = 0 \tag{1}$$

and represent the set of ten Hermitian operators Γ_μ $(\mu = 1, 2, .., 10)$ by

$$\Gamma_{2i} = (b_i + b_i^\dagger); \quad \Gamma_{2i-1} = -i(b_i - b_i^\dagger) \tag{2}$$

where Γ_μ define a rank-10 Clifford algebra,

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}. \tag{3}$$

and $\Sigma_{\mu\nu} = \frac{1}{2i}[\Gamma_\mu, \Gamma_\nu]$ are the 45 generators of SO(10) in the spinor representation. $\frac{1}{2}(1 \pm \Gamma_0)$ where $\Gamma_0 = i^5\Gamma_1\Gamma_2...\Gamma_{10}$ are the SO(10) chirality operators which split the 32-dimensional spinor Ψ into two inequivalent spinors through the relation

$$\Psi_{(\pm)} = \frac{1}{2}(1 \pm \Gamma_0)\Psi. \tag{4}$$

The semi-spinor $\Psi_{(+)}$ $(\Psi_{(-)})$ transforms as a 16 $(\overline{16})$ dimensional irreducible representation of SO(10). $\Psi_{(+)}$ $(\Psi_{(-)})$ contains $1 + \overline{5} + 10$ $(1 + 5 + \overline{10})$ in its SU(5) decomposition. In terms of their oscillator modes we can expand them as[2]

$$|\Psi_{(+)a}> = |0>\mathbf{M}_a + \frac{1}{2}b_i^\dagger b_j^\dagger|0>\mathbf{M}_a^{ij} + \frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{M}_{ai} \tag{5}$$

$$|\Psi_{(-)b}> = b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\mathbf{N}_b + \frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{N}_{bij} + b_i^\dagger|0>\mathbf{N}_b^i \tag{6}$$

where the SU(5) singlet state $|0>$ is such that $b_i|0>=0$. The subscripts $a,b=1,2,3$ are the generation indices. For the sake of completeness we identify the components of a 16 plet $|\Psi_{(+)a}>$ in terms of particle states so that

$$\begin{aligned}\mathbf{M}_a=\nu^c_{La};\quad \mathbf{M}_{a\alpha}=D^c_{La\alpha};\quad \mathbf{M}^{\alpha\beta}_a=\epsilon^{\alpha\beta\gamma}U^c_{La\gamma};\quad \mathbf{M}_{a4}=E^-_{La}\\ \mathbf{M}^{4\alpha}_a=U_{La\alpha};\quad \mathbf{M}_{a5}=\nu_{La};\quad \mathbf{M}^{5\alpha}_a=D_{La\alpha};\quad \mathbf{M}^{45}_a=E^+_{La}\end{aligned}\tag{7}$$

where $\alpha,\beta,\gamma=1,2,3$ are color indices and we adopt the convention that all particles are left handed(L).

Our main focus is the computation of the cubic and quartic couplings in the superpotential. As already mentioned in the introduction the couplings of the tensor fields 10, 120 and $\overline{126}$ with 16×16 have already been computed in Ref.[5] and here we focus on the couplings of the tensor fields 1, 45 and 210 with $\overline{16}\times 16$. Specifically the interactions of interest in the superpotential involving $16_\pm$ semispinors are of the form

$$\mathsf{W}^{(1)}_{-+}=h^{(1)}_{ab}<\widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b}>\Phi\tag{8}$$

$$\mathsf{W}^{(45)}_{-+}=\frac{1}{2!}h^{(45)}_{ab}<\widehat{\Psi}^*_{(-)a}|B\Sigma_{\mu\nu}|\widehat{\Psi}_{(+)b}>\Phi_{\mu\nu}\tag{9}$$

$$\mathsf{W}^{(210)}_{-+}=\frac{1}{4!}h^{(210)}_{ab}<\widehat{\Psi}^*_{(-)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}|\widehat{\Psi}_{(+)b}>\Phi_{\mu\nu\rho\lambda}\tag{10}$$

where

$$B=\prod_{\mu=odd}\Gamma_\mu=-i\prod_{k=1}^{5}(b_k-b_k^\dagger)\tag{11}$$

is an SO(10) charge conjugation operator, and

$$\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}=\frac{1}{4!}\sum_P(-1)^{\delta_P}\Gamma_{\mu_{P(1)}}\Gamma_{\nu_{P(2)}}\Gamma_{\rho_{P(3)}}\Gamma_{\lambda_{P(4)}}\tag{12}$$

with $\sum_P$ denoting the sum over all permutations and δ_P takes on the value 0 and 1 for even and odd permutations respectively. Semi-spinors $\Psi_{(\pm)}$ with a $\hat{}$ stands for chiral superfields. The result essential to the analysis of the above SO(2N) (N=5) invariant couplings is the theorem [5] that the vertex $\Gamma_\mu\Gamma_\nu\Gamma_\lambda..\Gamma_\sigma\ \Phi_{\mu\nu\lambda..\sigma}$ where $\Phi_{\mu\nu\lambda..\sigma}$ could be a large tensor representation, can be expanded in the following form

$$\begin{aligned}\Gamma_\mu\Gamma_\nu\Gamma_\lambda..\Gamma_\sigma\Phi_{\mu\nu\lambda...\sigma}&=b_i^\dagger b_j^\dagger b_k^\dagger...b_n^\dagger\Phi_{c_ic_jc_k...c_n}+\left(b_ib_j^\dagger b_k^\dagger...b_n^\dagger\Phi_{\overline{c}_ic_jc_k...c_n}+\ perms\right)\\ +\left(b_ib_jb_k^\dagger...b_n^\dagger\Phi_{\overline{c}_i\overline{c}_jc_k...c_n}+\ perms\right)&+...+\left(b_ib_jb_k...b_{n-1}b_n^\dagger\Phi_{\overline{c}_i\overline{c}_j\overline{c}_k...\overline{c}_{n-1}c_n}+\ perms\right)\\ &+b_ib_jb_k...b_n\Phi_{\overline{c}_i\overline{c}_j\overline{c}_k...\overline{c}_n}\end{aligned}\tag{13}$$

where we have introduced the notation $\Phi_{c_i} = \Phi_{2i} + i\Phi_{2i-1}$ and $\Phi_{\overline{c}_i} = \Phi_{2i} - i\Phi_{2i-1}$. This is extended immediately to define the quantity $\Phi_{c_i c_j \overline{c}_k ..}$ with an arbitrary number of barred and unbarred indices, where each c index can be expanded out so that $\Phi_{c_i c_j \overline{c}_k ..} = \Phi_{2ic_j \overline{c}_k ...} + i\Phi_{2i-1 c_j \overline{c}_k ..}$ etc.. Further the object $\Phi_{c_i c_j \overline{c}_k ... c_n}$ transforms like a reducible representation of SU(N) which can be further decomposed in its irreducible parts.

3 The 45-plet tensor coupling

We first present the result of the trivial $\overline{16} \times 16 \times 1$ couplings. Eq.(8) at once gives

$$\mathsf{W}^{(1)}_{-+} = ih^{(1)}_{ab}\left(\widehat{\mathbf{N}}^{\mathbf{T}}_a \widehat{\mathbf{M}}_b - \frac{1}{2}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_b + \widehat{\mathbf{N}}^{i\mathbf{T}}_a \widehat{\mathbf{M}}_{bi}\right)\mathsf{H} \tag{14}$$

where H is an SO(10) singlet. For eg. above $\widehat{\mathbf{N}}^{\mathbf{T}}_a$ represents the transpose of the chiral superfield, $\widehat{\mathbf{N}}_a$ etc.. A similar analysis gives $\mathsf{W}^{(1)}_{+-}$ and one has

$$\begin{aligned}\mathsf{W}^{(1)}_{+-} &= h^{(1)}_{ab} < \widehat{\Psi}^*_{(+)a}|B|\widehat{\Psi}_{(-)b} > \Phi \\ &= ih^{(1)}_{ab}\left(-\widehat{\mathbf{M}}^{\mathbf{T}}_a \widehat{\mathbf{N}}_b + \frac{1}{2}\widehat{\mathbf{M}}^{ij\mathbf{T}}_a \widehat{\mathbf{N}}_{bij} - \widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}^i_b\right)\mathsf{H}.\end{aligned} \tag{15}$$

To compute the $\overline{16} \times 16 \times 45$ couplings we expand the vertex $\Sigma_{\mu\nu}\Phi_{\mu\nu}$ using Eq.(12) where $\Phi_{\mu\nu}$ is the 45 plet tensor field

$$\Sigma_{\mu\nu}\Phi_{\mu\nu} = \frac{1}{i}\left(b_i b_j \Phi_{\overline{c}_i \overline{c}_j} + b^\dagger_i b^\dagger_j \Phi_{c_i c_j} + 2b^\dagger_i b_j \Phi_{c_i \overline{c}_j} - \Phi_{c_n \overline{c}_n}\right). \tag{16}$$

The reducible tensors that enter in the above expansion can be decomposed into their irreducible parts as follows

$$\Phi_{c_n \overline{c}_n} = \mathsf{h}; \quad \Phi_{c_i \overline{c}_j} = \mathsf{h}^i_j + \frac{1}{5}\delta^i_j \mathsf{h}; \quad \Phi_{c_i c_j} = \mathsf{h}^{ij}; \quad \Phi_{\overline{c}_i \overline{c}_j} = \mathsf{h}_{ij} \tag{17}$$

To normalize the SU(5) Higgs fields contained in the tensor $\Phi_{\mu\nu}$, we carry out a field redefinition

$$\mathsf{h} = \sqrt{10}\mathsf{H}; \quad \mathsf{h}_{ij} = \sqrt{2}\mathsf{H}_{ij}; \quad \mathsf{h}^{ij} = \sqrt{2}\mathsf{H}^{ij}; \quad \mathsf{h}^i_j = \sqrt{2}\mathsf{H}^i_j. \tag{18}$$

In terms of the normalized fields the kinetic energy of the 45 plet of Higgs $-\partial_A \Phi_{\mu\nu}\partial^A \Phi^\dagger_{\mu\nu}$ takes the form

$$\mathsf{L}^{45-Higgs}_{kin} = -\partial^A \mathsf{H}\partial_A \mathsf{H}^\dagger - \frac{1}{2!}\partial^A \mathsf{H}_{ij}\partial_A \mathsf{H}^\dagger_{ij} - \frac{1}{2!}\partial^A \mathsf{H}^{ij}\partial_A \mathsf{H}^{ij\dagger} - \partial_A \mathsf{H}^i_j \partial^A \mathsf{H}^{i\dagger}_j. \tag{19}$$

The terms in Eq.(18) are only exhibited for the purpose of normalization and the remaining supersymmetric parts are not exhibited as their normalizations are rigidly fixed relative to the parts given above[8]. Finally, straightforward evaluation of Eq.(9) using Eqs.(15-17) gives

$$\begin{aligned}\mathsf{W}^{(45)}_{-+} = \frac{1}{\sqrt{2}} h^{(45)}_{ab} [\sqrt{5}\left(\frac{3}{5}\widehat{\mathbf{N}}^{i\mathbf{T}}_{a}\widehat{\mathbf{M}}_{bi} + \frac{1}{10}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_{b} - \widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}_{b}\right)\mathsf{H}\\ + \left(-\widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}^{lm}_{b} + \frac{1}{2}\epsilon^{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}_{bk}\right)\mathsf{H}_{lm}\\ + \left(-\widehat{\mathbf{N}}^{\mathbf{T}}_{alm}\widehat{\mathbf{M}}_{b} + \frac{1}{2}\epsilon_{ijklm}\widehat{\mathbf{N}}^{i\mathbf{T}}_{a}\widehat{\mathbf{M}}^{jk}_{b}\right)\mathsf{H}^{lm}\\ +2\left(\widehat{\mathbf{N}}^{\mathbf{T}}_{aik}\widehat{\mathbf{M}}^{kj}_{b} - \widehat{\mathbf{N}}^{j\mathbf{T}}_{a}\widehat{\mathbf{M}}_{bi}\right)\mathsf{H}^{i}_{j}]. \end{aligned} \tag{20}$$

From Eq.(19) one finds that the $\overline{16}_N - 16_M - 45_H$ couplings consist of the following SU(5) invariant components: $5_N - \overline{5}_M - 1_H, \overline{10}_N - 10_M - 1_H, 1_N - 1_M - 1_H, 1_N - 10_M - \overline{10}_H, \overline{10}_N - \overline{5}_M - \overline{10}_H, \overline{10}_N - 1_M - 10_H, 5_N - 10_M - 10_H, \overline{10}_N - 10_M - 24_H$, and $5_N - \overline{5}_M - 24_H$ couplings. One can carry out a similar analysis for $\mathsf{W}^{(45)}_{+-}$ and one finds

$$\begin{aligned}\mathsf{W}^{(45)}_{+-} &= \frac{1}{2!} h^{(45)}_{ab} < \widehat{\Psi}^{*}_{(+)a}|B\Sigma_{\mu\nu}|\widehat{\Psi}_{(-)b} > \Phi_{\mu\nu}\\ &= \frac{1}{\sqrt{2}} h^{(45)}_{ab} [\sqrt{5}\left(\frac{3}{5}\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}^{i}_{b} + \frac{1}{10}\widehat{\mathbf{M}}^{ij\mathbf{T}}_{a}\widehat{\mathbf{N}}_{bij} - \widehat{\mathbf{M}}^{\mathbf{T}}_{a}\widehat{\mathbf{N}}_{b}\right)\mathsf{H}\\ &+ \left(-\widehat{\mathbf{M}}^{lm\mathbf{T}}_{a}\widehat{\mathbf{N}}_{b} + \frac{1}{2}\epsilon^{ijklm}\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}_{bjk}\right)\mathsf{H}_{lm}\\ &+ \left(-\widehat{\mathbf{M}}^{\mathbf{T}}_{a}\widehat{\mathbf{N}}_{blm} + \frac{1}{2}\epsilon_{ijklm}\widehat{\mathbf{M}}^{ij\mathbf{T}}_{a}\widehat{\mathbf{N}}^{k}_{b}\right)\mathsf{H}^{lm}\\ &+2\left(\widehat{\mathbf{M}}^{jk\mathbf{T}}_{a}\widehat{\mathbf{N}}_{bki} - \widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}^{j}_{b}\right)\mathsf{H}^{i}_{j}]. \end{aligned} \tag{21}$$

4 The 210-plet tensor coupling

We turn now to the computation of the $\overline{16} \times 16 \times 210$ couplings. Using Eq.(12) we decompose the vertex $\Gamma_\mu\Gamma_\nu\Gamma_\rho\Gamma_\lambda\Phi_{\mu\nu\rho\lambda}$ so that

$$\begin{aligned}\Gamma_\mu\Gamma_\nu\Gamma_\rho\Gamma_\lambda\Phi_{\mu\nu\rho\lambda} = 4b^\dagger_i b^\dagger_j b^\dagger_k b_l \Phi_{c_ic_jc_k\overline{c}_l} + 4b^\dagger_i b_j b_k b_l \Phi_{c_i\overline{c}_j\overline{c}_k\overline{c}_l} + b^\dagger_i b^\dagger_j b^\dagger_k b^\dagger_l \Phi_{c_ic_jc_kc_l}\\ + b_ib_jb_kb_l\Phi_{\overline{c}_i\overline{c}_j\overline{c}_k\overline{c}_l} - 6b^\dagger_i b^\dagger_j \Phi_{c_ic_jc_m\overline{c}_m} + 6b_ib_j\Phi_{\overline{c}_i\overline{c}_j\overline{c}_m c_m}\\ +3\Phi_{c_m\overline{c}_mc_n\overline{c}_n} - 12b^\dagger_i b_j\Phi_{c_i\overline{c}_jc_m\overline{c}_m} + 6b^\dagger_i b^\dagger_j b_k b_l \Phi_{c_ic_j\overline{c}_k\overline{c}_l}. \end{aligned} \tag{22}$$

The tensors that appear above can be decomposed into their irreducible parts as follows

$$
\begin{aligned}
\Phi_{c_m\bar{c}_m c_n\bar{c}_n} &= \mathsf{h};\quad \Phi_{\bar{c}_i\bar{c}_j\bar{c}_k\bar{c}_l} = \frac{1}{24}\epsilon_{ijklm}\mathsf{h}^m;\quad \Phi_{c_ic_jc_kc_l} = \frac{1}{24}\epsilon^{ijklm}\mathsf{h}_m\\
\Phi_{c_ic_jc_m\bar{c}_m} &= \mathsf{h}^{ij};\quad \Phi_{\bar{c}_i\bar{c}_j\bar{c}_m c_m} = \mathsf{h}_{ij};\quad \Phi_{c_i\bar{c}_j c_m\bar{c}_m} = \mathsf{h}^i_j + \frac{1}{5}\delta^i_j\mathsf{h}\\
\Phi_{c_ic_j\bar{c}_k\bar{c}_l} &= \mathsf{h}^{ij}_{kl} + \frac{1}{3}\left(\delta^i_l\mathsf{h}^j_k - \delta^i_k\mathsf{h}^j_l + \delta^j_k\mathsf{h}^i_l - \delta^j_l\mathsf{h}^i_k\right) + \frac{1}{20}\left(\delta^i_l\delta^j_k - \delta^i_k\delta^j_l\right)\mathsf{h}\\
\Phi_{c_ic_jc_k\bar{c}_l} &= \mathsf{h}^{ijk}_l + \frac{1}{3}\left(\delta^k_l\mathsf{h}^{ij} - \delta^j_l\mathsf{h}^{ik} + \delta^i_l\mathsf{h}^{jk}\right)\\
\Phi_{\bar{c}_i\bar{c}_j\bar{c}_k c_l} &= \mathsf{h}^l_{ijk} + \frac{1}{3}\left(\delta^l_k\mathsf{h}_{ij} - \delta^l_j\mathsf{h}_{ik} + \delta^l_i\mathsf{h}_{jk}\right)
\end{aligned}
\tag{23}
$$

where h, h^i, h_i, h^{ij}, h_{ij}, h^i_j, h^{ijk}_l; h^i_{jkl} and h^{ij}_{kl} are the 1-plet, 5-plet, $\overline{5}$-plet, 10-plet, $\overline{10}$-plet, 24-plet, 40-plet, $\overline{40}$-plet, and 75-plet representations of SU(5), respectively. We carry out a field redefinition such that

$$
\begin{aligned}
\mathsf{h} &= 4\sqrt{\frac{5}{3}}\mathsf{H};\quad \mathsf{h}^i = 8\sqrt{6}\mathsf{H}^i;\quad \mathsf{h}_i = 8\sqrt{6}\mathsf{H}_i\\
\mathsf{h}^{ij} &= \sqrt{2}\mathsf{H}^{ij};\quad \mathsf{h}_{ij} = \sqrt{2}\mathsf{H}_{ij};\quad \mathsf{h}^i_j = \sqrt{2}\mathsf{H}^i_j\\
\mathsf{h}^{ijk}_l &= \sqrt{\frac{2}{3}}\mathsf{H}^{ijk}_l;\quad \mathsf{h}^i_{jkl} = \sqrt{\frac{2}{3}}\mathsf{H}^i_{jkl};\quad \mathsf{h}^{ij}_{kl} = \sqrt{\frac{2}{3}}\mathsf{H}^{ij}_{kl}.
\end{aligned}
\tag{24}
$$

Now the kinetic energy for the 210 dimensional Higgs field is $-\partial_A\Phi_{\mu\nu\rho\lambda}\partial^A\Phi^\dagger_{\mu\nu\rho\lambda}$, which in terms of the redefined fields takes the form

$$
\begin{aligned}
\mathsf{L}^{210-Higgs}_{kin} &= -\partial_A\mathsf{H}\partial^A\mathsf{H}^\dagger - \partial_A\mathsf{H}^i\partial^A\mathsf{H}^{i\dagger} - \partial_A\mathsf{H}_i\partial^A\mathsf{H}_{i\dagger}\\
&-\frac{1}{2!}\partial_A\mathsf{H}^{ij}\partial^A\mathsf{H}^{ij\dagger} - \frac{1}{2!}\partial_A\mathsf{H}_{ij}\partial^A\mathsf{H}^\dagger_{ij} - \partial_A\mathsf{H}^i_j\partial^A\mathsf{H}^{i\dagger}_j\\
&-\frac{1}{3!}\partial_A\mathsf{H}^{ijk}_l\partial^A\mathsf{H}^{ijk\dagger}_l - \frac{1}{3!}\partial_A\mathsf{H}^l_{ijk}\partial^A\mathsf{H}^{l\dagger}_{ijk} - \frac{1}{2!}\frac{1}{2!}\partial_A\mathsf{H}^{ij}_{kl}\partial^A\mathsf{H}^{ij\dagger}_{kl}.
\end{aligned}
\tag{25}
$$

Evaluation of Eq.(10) using Eq.(21) and the normalization of Eq.(23) gives

$$
\begin{aligned}
\mathsf{W}^{(210)}_{-+} = i\sqrt{\frac{2}{3}}h^{(210)}_{ab}[&\frac{1}{2}\sqrt{\frac{5}{2}}\left(\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b + \frac{1}{10}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_b + \frac{1}{5}\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\right)\mathsf{H}\\
&+\frac{\sqrt{3}}{4}\left(\widehat{\mathbf{N}}^{\mathbf{T}}_{alm}\widehat{\mathbf{M}}_b + \frac{1}{6}\epsilon_{ijklm}\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}^{jk}_b\right)\mathsf{H}^{lm}\\
&-\frac{\sqrt{3}}{4}\left(\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}^{lm}_b + \frac{1}{6}\epsilon^{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}_{bk}\right)\mathsf{H}_{lm}\\
&-\frac{\sqrt{3}}{2}\left(\widehat{\mathbf{N}}^{j\mathbf{T}}_a\widehat{\mathbf{M}}_{bi} + \frac{1}{3}\widehat{\mathbf{N}}^{\mathbf{T}}_{aik}\widehat{\mathbf{M}}^{kj}_b\right)\mathsf{H}^i_j
\end{aligned}
$$

$$+\frac{1}{6}\epsilon_{ijklm}\widehat{\mathbf{N}}_a^{i\mathbf{T}}\widehat{\mathbf{M}}_b^{jn}\mathsf{H}_n^{klm}+\frac{1}{6}\epsilon^{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{ain}\widehat{\mathbf{M}}_{bj}\mathsf{H}^n_{klm}$$
$$+\frac{1}{4}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}_b^{kl}\mathsf{H}^{ij}_{kl}+\widehat{\mathbf{N}}_a^{\mathbf{T}}\widehat{\mathbf{M}}_{bi}\mathsf{H}^i+\widehat{\mathbf{N}}_a^{i\mathbf{T}}\widehat{\mathbf{M}}_b\mathsf{H}_i]. \tag{26}$$

We note that $\overline{16}_N-16_M-210_H$ couplings have the SU(5) invariant structure consisting of $1_N-1_M-1_H, \overline{10}_N-10_M-1_H, 5_N-\overline{5}_M-1_H, \overline{10}_N-1_M-10_H,$ $5_N-10_M-10_H, 1_N-10_M-\overline{10}_H, \overline{10}_N-\overline{5}_M-\overline{10}_H, 5_N-\overline{5}_M-24_H, \overline{10}_N-10_M-24_H,$ $5_N-10_M-40_H, \overline{10}_N-\overline{5}_M-\overline{40}_H, \overline{10}_N-10_M-75_H, 1_N-\overline{5}_M-5_H, 5_N-1_M-\overline{5}_H$. An analysis similar to that for Eq.(25) gives $\mathsf{W}^{(210)}_{+-}$

$$\mathsf{W}^{(210)}_{+-}=\frac{1}{4!}h^{(210)}_{ab}<\widehat{\Psi}^*_{(+)a}|B\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}|\widehat{\Psi}_{(-)b}>\Phi_{\mu\nu\rho\lambda}$$
$$=i\sqrt{\frac{2}{3}}h^{(210)}_{ab}[-\frac{1}{2}\sqrt{\frac{5}{2}}\left(\widehat{\mathbf{M}}_a^{\mathbf{T}}\widehat{\mathbf{N}}_b+\frac{1}{10}\widehat{\mathbf{M}}_a^{ij\mathbf{T}}\widehat{\mathbf{N}}_{bij}+\frac{1}{5}\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}^i_b\right)\mathsf{H}$$
$$-\frac{\sqrt{3}}{4}\left(\widehat{\mathbf{M}}_a^{\mathbf{T}}\widehat{\mathbf{N}}_{blm}+\frac{1}{6}\epsilon_{ijklm}\widehat{\mathbf{M}}_a^{ij\mathbf{T}}\widehat{\mathbf{N}}^k_b\right)\mathsf{H}^{lm}$$
$$+\frac{\sqrt{3}}{4}\left(\widehat{\mathbf{M}}_a^{lm\mathbf{T}}\widehat{\mathbf{N}}_b+\frac{1}{6}\epsilon^{ijklm}\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}_{bjk}\right)\mathsf{H}_{lm}$$
$$+\frac{\sqrt{3}}{2}\left(\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}^j_b+\frac{1}{3}\widehat{\mathbf{M}}_a^{jk\mathbf{T}}\widehat{\mathbf{N}}_{bki}\right)\mathsf{H}^i_j$$
$$+\frac{1}{12}\epsilon_{ijklm}\widehat{\mathbf{M}}_a^{ij\mathbf{T}}\widehat{\mathbf{N}}^n_b\mathsf{H}^{klm}_n-\frac{1}{12}\epsilon^{ijklm}\widehat{\mathbf{M}}^{\mathbf{T}}_{an}\widehat{\mathbf{N}}_{bij}\mathsf{H}^n_{klm}$$
$$-\frac{1}{4}\widehat{\mathbf{M}}_a^{kl\mathbf{T}}\widehat{A\mathbf{N}}_{bij}\mathsf{H}^{ij}_{kl}-\widehat{\mathbf{M}}^{\mathbf{T}}_{ai}\widehat{\mathbf{N}}_b\mathsf{H}^i-\widehat{\mathbf{M}}_a^{\mathbf{T}}\widehat{\mathbf{N}}^i_b\mathsf{H}_i]. \tag{27}$$

We note that the couplings of $\mathsf{W}^{(210)}_{-+}$ are in general not the same as in $\mathsf{W}^{(210)}_{+-}$. Thus some of the terms have signs which are opposite in the two sets. Further, we note that there are in general two ways in which the 40 plet and the $\overline{40}$ plet can contract with the matter fields. For the case of $\mathsf{W}^{(210)}_{-+}$ one of the 40 plet tensor index contracts with the tensor index in of the 10 plet of matter and similarly one of the tensor index on the $\overline{40}$ contracts with the tensor index in the $\bar{10}$ of the $\overline{16}$ (see Eq.25). However, in the $\mathsf{W}^{(210)}_{+-}$ couplings this is not the case. Here one of the tensor index of 40 plet contracts with the tensor index in of the 5 plet of matter and similarly one of the tensor index in $\overline{40}$ contracts with the tensor index in the $\bar{5}$ plet of matter (see Eq.26).

5 Quartic Couplings of the form $\overline{16}$ 16 $\overline{16}$ 16

In phenomenological analyses one generally needs more than one Higgs representations. Hence to keep the analysis very general we not only keep the generational indices but also allow for mixing among Higgs representations. To that end, we assume several Higgs representations of the same kind: $\Phi_{\mathcal{X}}, \Phi_{\mu\nu\mathcal{Y}}, \Phi_{\mu\nu\rho\lambda\mathcal{Z}}$. Consider the superpotential

$$\mathsf{W}^{(16\times\overline{16})} = \mathsf{W}^{(16\times\overline{16})}_{Higgs} + \mathsf{W}^{(16\times\overline{16})}_{mass} \tag{28}$$

where

$$\mathsf{W}^{(16\times\overline{16})}_{Higgs} = \mathsf{W}^{(1)'}_{-+} + \mathsf{W}^{(45)'}_{-+} + \mathsf{W}^{(210)'}_{-+} \tag{29}$$

and

$$\mathsf{W}^{(16\times\overline{16})}_{mass} = \frac{1}{2}\Phi_{\mathcal{X}}\mathcal{M}^{(1)}_{\mathcal{X}\mathcal{X}'}\Phi_{\mathcal{X}'} + \frac{1}{2}\Phi_{\mu\nu\mathcal{Y}}\mathcal{M}^{(45)}_{\mathcal{Y}\mathcal{Y}'}\Phi_{\mu\nu\mathcal{Y}'} + \frac{1}{2}\Phi_{\mu\nu\rho\lambda\mathcal{Z}}\mathcal{M}^{(210)}_{\mathcal{Z}\mathcal{Z}'}\Phi_{\mu\nu\rho\lambda\mathcal{Z}'}. \tag{30}$$

The terms $\mathsf{W}^{(1)'}_{-+}$, $\mathsf{W}^{(45)'}_{-+}$, $\mathsf{W}^{(210)'}_{-+}$ in Eq.(28) are the same as those given by Eqs.(8), (9), and (10) except that the tensors Φ, $\Phi_{\mu\nu}$, and $\Phi_{\mu\nu\rho\lambda}$ are replaced by $f^{(1)}_{\mathcal{X}}\Phi_{\mathcal{X}}$, $f^{(45)}_{\mathcal{Y}}\Phi_{\mu\nu\mathcal{Y}}$, and $f^{(210)}_{\mathcal{Z}}\Phi_{\mu\nu\rho\lambda\mathcal{Z}}$, respectively. We next eliminate $\Phi_{\mathcal{X}}, \Phi_{\mu\nu\mathcal{Y}}, \Phi_{\mu\nu\rho\lambda\mathcal{Z}}$ as superheavy dimension-5 operators using the F-flatness conditions:

$$\frac{\partial\mathsf{W}^{(16\times\overline{16})}}{\partial\Phi_{\mathcal{X}}} = 0; \quad \frac{\partial\mathsf{W}^{(16\times\overline{16})}}{\partial\Phi_{\mu\nu\mathcal{Y}}} = 0; \quad \frac{\partial\mathsf{W}^{(16\times\overline{16})}}{\partial\Phi_{\mu\nu\rho\lambda\mathcal{Z}}} = 0. \tag{31}$$

The above leads to

$$\mathsf{W}^{(\overline{16}\times16)}_{dim-5} = \mathcal{I}_1 + \mathcal{I}_{45} + \mathcal{I}_{210}. \tag{32}$$

$\mathcal{I}_1$, $\mathcal{I}_{45}$ and $\mathcal{I}_{210}$ can be computed quite straightforwardly by integrating out the heavy SO(10) singlet, 45 and 210 plets fields in the superpotential. Details are given in Appendix A. We record here the results.

$$\begin{aligned}\mathcal{I}_1 = \frac{1}{2}\lambda^{(1)}_{ab,cd}[&-\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_b\widehat{\mathbf{N}}^{\mathbf{T}}_{ckl}\widehat{\mathbf{M}}^{kl}_d + 4\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{\mathbf{T}}_{cjk}\widehat{\mathbf{M}}^{jk}_d - 4\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{j\mathbf{T}}_c\widehat{\mathbf{M}}_{dj}\\ &+4\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{\mathbf{T}}_{cij}\widehat{\mathbf{M}}^{ij}_d - 8\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{i\mathbf{T}}_c\widehat{\mathbf{M}}_{di} - 4\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{\mathbf{T}}_c\widehat{\mathbf{M}}_d]\end{aligned} \tag{33}$$

$$\begin{aligned}\mathcal{I}_{45} = &\left(-4\lambda^{(45)}_{ad,cb} + 11\lambda^{(45)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{j\mathbf{T}}_c\widehat{\mathbf{M}}_{dj} + 8\left(\lambda^{(45)}_{ad,cb} + \lambda^{(45)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bj}\widehat{\mathbf{N}}^{\mathbf{T}}_{cik}\widehat{\mathbf{M}}^{kj}_d\\ &+\left(4\lambda^{(45)}_{ad,cb} - 7\lambda^{(45)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{\mathbf{T}}_{cjk}\widehat{\mathbf{M}}^{jk}_d + \left(4\lambda^{(45)}_{ad,cb} + \lambda^{(45)}_{ab,cd}\right)\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{\mathbf{T}}_{cij}\widehat{\mathbf{M}}^{ij}_d\\ &+\frac{1}{4}\lambda^{(45)}_{ab,cd}[-8\epsilon^{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}_{bk}\widehat{\mathbf{N}}^{\mathbf{T}}_{clm}\widehat{\mathbf{M}}_d - 8\epsilon_{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}^{ij}_b\widehat{\mathbf{N}}^{k\mathbf{T}}_c\widehat{\mathbf{M}}^{lm}_d - 16\widehat{\mathbf{N}}^{\mathbf{T}}_{aik}\widehat{\mathbf{M}}^{kj}_b\widehat{\mathbf{N}}^{\mathbf{T}}_{cjl}\widehat{\mathbf{M}}^{li}_d\\ &+3\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_b\widehat{\mathbf{N}}^{\mathbf{T}}_{ckl}\widehat{\mathbf{M}}^{kl}_d + 24\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{i\mathbf{T}}_c\widehat{\mathbf{M}}_{di} - 20\widehat{\mathbf{N}}^{\mathbf{T}}_a\widehat{\mathbf{M}}_b\widehat{\mathbf{N}}^{\mathbf{T}}_c\widehat{\mathbf{M}}_d]\end{aligned} \tag{34}$$

$$\mathcal{I}_{210} = -\frac{1}{24}[4\left(-18\lambda^{(210)}_{ad,cb} - 25\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{j\mathbf{T}}_c\widehat{\mathbf{M}}_{dj} + 16\left(\lambda^{(210)}_{ad,cb} + 5\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_a\widehat{\mathbf{M}}_{bj}\widehat{\mathbf{N}}^{\mathbf{T}}_{cik}\widehat{\mathbf{M}}^{kj}_d$$

$$
\begin{aligned}
&+12\left(-2\lambda^{(210)}_{ad,cb}+3\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{i\mathbf{T}}_{a}\widehat{\mathbf{M}}_{bi}\widehat{\mathbf{N}}^{\mathbf{T}}_{cjk}\widehat{\mathbf{M}}^{jk}_{d}+4\left(-6\lambda^{(210)}_{ad,cb}+\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}_{b}\widehat{\mathbf{N}}^{\mathbf{T}}_{cij}\widehat{\mathbf{M}}^{ij}_{d}\\
&\quad+\left(8\lambda^{(210)}_{ad,cb}+25\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}^{ij}_{b}\widehat{\mathbf{N}}^{\mathbf{T}}_{ckl}\widehat{\mathbf{M}}^{kl}_{d}+8\left(8\lambda^{(210)}_{ad,cb}+\lambda^{(210)}_{ab,cd}\right)\widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}_{b}\widehat{\mathbf{N}}^{i\mathbf{T}}_{c}\widehat{\mathbf{M}}_{di}]\\
&+4\lambda^{(210)}_{ab,cd}\{-\epsilon^{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{aij}\widehat{\mathbf{M}}_{bk}\widehat{\mathbf{N}}^{\mathbf{T}}_{clm}\widehat{\mathbf{M}}_{d}-\epsilon_{ijklm}\widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}^{ij}_{b}\widehat{\mathbf{N}}^{k\mathbf{T}}_{c}\widehat{\mathbf{M}}^{lm}_{d}-2\widehat{\mathbf{N}}^{\mathbf{T}}_{aik}\widehat{\mathbf{M}}^{kj}_{b}\widehat{\mathbf{N}}^{\mathbf{T}}_{cjl}\widehat{\mathbf{M}}^{li}_{d}\\
&\qquad+5\widehat{\mathbf{N}}^{\mathbf{T}}_{a}\widehat{\mathbf{M}}_{b}\widehat{\mathbf{N}}^{\mathbf{T}}_{c}\widehat{\mathbf{M}}_{d}\}]
\end{aligned}
\tag{35}
$$

where

$$
\begin{aligned}
\lambda^{(1)}_{ab,cd}&=h^{(1)}_{ab}h^{(1)}_{cd}f^{(1)}_{\mathcal{X}}\left[\left(\mathcal{M}^{(1)}+\mathcal{M}^{(1)\mathbf{T}}\right)^{-1}\left\{\mathcal{M}^{(1)}\left(\mathcal{M}^{(1)}+\mathcal{M}^{(1)\mathbf{T}}\right)^{-1}-\mathbf{1}\right\}\right]_{\mathcal{X}\mathcal{X}'}f^{(1)}_{\mathcal{X}'}\\
\lambda^{(45)}_{ab,cd}&=h^{(45)}_{ab}h^{(45)}_{cd}f^{(45)}_{\mathcal{Y}}\left[\left(\mathcal{M}^{(45)}+\mathcal{M}^{(45)\mathbf{T}}\right)^{-1}\left\{\mathcal{M}^{(45)}\left(\mathcal{M}^{(45)}+\mathcal{M}^{(45)\mathbf{T}}\right)^{-1}-\mathbf{1}\right\}\right]_{\mathcal{Y}\mathcal{Y}'}f^{(45)}_{\mathcal{Y}'}\\
\lambda^{(210)}_{ab,cd}&=h^{(210)}_{ab}h^{(210)}_{cd}f^{(210)}_{\mathcal{Z}}\left[\left(\mathcal{M}^{(210)}+\mathcal{M}^{(210)\mathbf{T}}\right)^{-1}\left\{\mathcal{M}^{(210)}\left(\mathcal{M}^{(210)}+\mathcal{M}^{(210)\mathbf{T}}\right)^{-1}-\mathbf{1}\right\}\right]_{\mathcal{Z}\mathcal{Z}'}f^{(210)}_{\mathcal{Z}'}.
\end{aligned}
\tag{36}
$$

The exact same technique can be used to compute the quartic couplings of the form $[1616]_{10}[1616]_{10}$, $[1616]_{120}[1616]_{120}$, and $[1616]_{\overline{126}}[1616]_{126}$ arising from the elimination of the 10 plet, the 120 plet and the $\overline{126}$ plet of heavy Higgs using the cubic couplings already derived in Ref.[5]. Similarly one can compute $[\overline{1616}][\overline{1616}]$ and $[1616][\overline{1616}]$ couplings using the technique above.

6 Vector Couplings

For the construction of couplings of vector fields with $16_\pm$ plets it is natural to consider the couplings of the 1 and 45 vector fields as abelian and Yang-Mills gauge interactions. However, one cannot do the same for the $\overline{16}_\pm 16_\pm 210$ couplings. These couplings cannot be treated as gauge couplings as there are no corresponding Yang-Mills interactions for the 210 plet. For this reason we focus here first on the computation of the gauge couplings of the 1 and 45 plet of vector fields. The supersymmetric kinetic energy and gauge couplings of the chiral superfield $\hat{\phi}$ can be written in the usual superfield notation

$$\int d^4\theta\ tr(\hat{\phi}^\dagger e^{g\hat{V}}\hat{\phi}) \tag{37}$$

where $\hat{V}$ is the Lie valued vector superfield. Similarly the supersymmetric Yang-Mills part of the Lagrangian can be gotten from

$$\int d^2\theta\ tr(W^\alpha W_\alpha)) + \int d^2\bar{\theta}\ tr(\overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}) \tag{38}$$

where W_α is the field strength chiral spinor superfield. Since supersymmetry does not play any special role in the analysis of SO(10) Clebsch-Gordon co-efficients, we will display in the analysis here only the parts of the Lagrangian relevant for our discussion. Thus the interactions of the 16_+ of fermions with gauge vectors for the 1 and 45 plet cases are given by

$$\mathsf{L}^{(1)}_{++} = g^{^{(1)}}_{ab} < \Psi_{(+)a}|\gamma^0\gamma^A|\Psi_{(+)b} > \Phi_A \tag{39}$$

$$\mathsf{L}^{(45)}_{++} = \frac{1}{i}\frac{1}{2!} g^{^{(45)}}_{ab} < \Psi_{(+)a}|\gamma^0\gamma^A\Sigma_{\mu\nu}|\Psi_{(+)b} > \Phi_{A\mu\nu} \tag{40}$$

where $\gamma^A(A, B = 0-3)$ spans the Clifford algebra associated with the Lorentz group, g's are the gauge coupling constants, and Φ_A and $\Phi_{A\mu\nu}$ are gauge tensors of dimensionality 1 and 45, respectively. Similarly one defines $\mathsf{L}^{(1)}_{--}$, $\mathsf{L}^{(45)}_{--}$ with Ψ_+ replaced by Ψ_- in Eqs.(38) and (39).

We first present the result of the trivial $\overline{16} \times 16 \times 1$ couplings. Eqs.(38) and (12) at once give

$$\mathsf{L}^{(1)}_{++} = g^{^{(1)}}_{ab} \left(\overline{\mathbf{M}}_a\gamma^A\mathbf{M}_b + \frac{1}{2}\overline{\mathbf{M}}_{aij}\gamma^A\mathbf{M}^{ij}_b + \overline{\mathbf{M}}^i_a\gamma^A\mathbf{M}_{bi}\right)\mathsf{G}_A. \tag{41}$$

The barred matter fields are defined so that $\overline{\mathbf{M}}_{ij} = \mathbf{M}^\dagger_{ij}\gamma^0$ etc.

A similar analysis gives $\mathsf{L}^{(1)}_{--}$ and one has

$$\begin{aligned}\mathsf{L}^{(1)}_{--} &= g^{^{(1)}}_{ab} < \Psi_{(-)a}|\gamma^0\gamma^A|\Psi_{(-)b} > \Phi_A \\ &= g^{^{(1)}}_{ab} \left(\overline{\mathbf{N}}_a\gamma^A\mathbf{N}_b + \frac{1}{2}\overline{\mathbf{N}}^{ij}_a\gamma^A\mathbf{N}_{bij} + \overline{\mathbf{N}}_{ai}\gamma^A\mathbf{N}^i_b\right)\mathsf{G}_A.\end{aligned} \tag{42}$$

We next discuss the couplings of the 45 plet gauge tensor $\Phi_{A\mu\nu}$ whose decomposition in terms of reducible SU(5) tensors can be written similar to Eq.(15). This can be further reduced into irreducible parts similar to Eq.(16) by

$$\Phi_{Ac_n\bar{c}_n} = \mathsf{g}_A; \quad \Phi_{Ac_i\bar{c}_j} = \mathsf{g}^i_{Aj} + \frac{1}{5}\delta^i_j\mathsf{g}_A; \quad \Phi_{Ac_ic_j} = \mathsf{g}^{ij}_A; \quad \Phi_{A\bar{c}_i\bar{c}_j} = \mathsf{g}_{Aij} \tag{43}$$

and normalized so that

$$\mathsf{g}_A = 2\sqrt{5}\mathsf{G}_A; \quad \mathsf{g}_{Aij} = \sqrt{2}\mathsf{G}_{Aij}; \quad \mathsf{g}^{ij}_A = \sqrt{2}\mathsf{G}^{ij}_A; \quad \mathsf{g}^i_{Aj} = \sqrt{2}\mathsf{G}^i_{Aj}. \tag{44}$$

The kinetic energy for the 45-plet is given by $-\frac{1}{4}\mathcal{F}^{AB}_{\mu\nu}\mathcal{F}_{AB\mu\nu}$, where $\mathcal{F}^{AB}_{\mu\nu}$ is the 45 of SO(10) field strength tensor. In terms of the redefined fields, 45-plet's kinetic energy takes the form

$$\mathsf{L}^{45-gauge}_{kin} = -\frac{1}{2}\mathcal{G}_{AB}\mathcal{G}^{AB\dagger} - \frac{1}{2!}\frac{1}{2}\mathcal{G}^{ABij}\mathcal{G}^{ij\dagger}_{AB} - \frac{1}{2!}\frac{1}{2}\mathcal{G}^{ABi}_j\mathcal{G}^j_{ABi} \tag{45}$$

As mentioned in the beginning of this section we do not exhibit the gaugino and D terms needed for supersymmetry since their normalization is fixed relative to terms exhibited in Eq.(44). Using Eqs.(39), (15) and the above normalizations we find

$$\begin{aligned}\mathsf{L}^{(45)}_{++} = g^{(45)}_{ab}[\sqrt{5}\left(-\frac{3}{5}\overline{\mathbf{M}}^i_a\gamma^A\mathbf{M}_{bi} + \frac{1}{10}\overline{\mathbf{M}}_{aij}\gamma^A\mathbf{M}^{ij}_b + \overline{\mathbf{M}}_a\gamma^A\mathbf{M}_b\right)\mathsf{G}_A \\ +\frac{1}{\sqrt{2}}\left(\overline{\mathbf{M}}_a\gamma^A\mathbf{M}^{lm}_b + \frac{1}{2}\epsilon^{ijklm}\overline{\mathbf{M}}_{aij}\gamma^A\mathbf{M}_{bk}\right)\mathsf{G}_{Alm} \\ -\frac{1}{\sqrt{2}}\left(\overline{\mathbf{M}}_{alm}\gamma^A\mathbf{M}_b + \frac{1}{2}\epsilon_{ijklm}\overline{\mathbf{M}}^i_a\gamma^A\mathbf{M}^{jk}_b\right)\mathsf{G}^{lm}_A \\ +\sqrt{2}\left(\overline{\mathbf{M}}_{aik}\gamma^A\mathbf{M}^{kj}_b + \overline{\mathbf{M}}^j_a\gamma^A\mathbf{M}_{bi}\right)\mathsf{G}^i_{Aj}].\end{aligned} \tag{46}$$

A similar analysis gives

$$\begin{aligned}\mathsf{L}^{(45)}_{--} = \frac{1}{i}\frac{1}{2!}g^{(45)}_{ab} < \Psi_{(-)a}|\gamma^0\gamma^A\Sigma_{\mu\nu}|\Psi_{(-)b} > \Phi_{A\mu\nu} \\ = g^{(45)}_{ab}[\sqrt{5}\left(\frac{3}{5}\overline{\mathbf{N}}_{ai}\gamma^A\mathbf{N}^i_b - \frac{1}{10}\overline{\mathbf{N}}^{ij}_a\gamma^A\mathbf{N}_{bij} - \overline{\mathbf{N}}_a\gamma^A\mathbf{N}_b\right)\mathsf{G}_A \\ +\frac{1}{\sqrt{2}}\left(\overline{\mathbf{N}}^{lm}_a\gamma^A\mathbf{N}_b + \frac{1}{2}\epsilon^{ijklm}\overline{\mathbf{N}}_{ai}\gamma^A\mathbf{N}_{bjk}\right)\mathsf{G}_{Alm} \\ -\frac{1}{\sqrt{2}}\left(\overline{\mathbf{N}}_a\gamma^A\mathbf{N}_{blm} + \frac{1}{2}\epsilon_{ijklm}\overline{\mathbf{N}}^{ij}_a\gamma^A\mathbf{N}^k_b\right)\mathsf{G}^{lm}_A \\ -\sqrt{2}\left(\overline{\mathbf{N}}^{jk}_a\gamma^A\mathbf{N}_{bki} + \overline{\mathbf{N}}_{ai}\gamma^A\mathbf{N}^j_b\right)\mathsf{G}^i_{Aj}].\end{aligned} \tag{47}$$

We discuss now the 210 vector multiplet. This vector mutiplet is not a gauge multiplet with the usual Yang-Mills interactions. This makes the multiplet rather pathological and it cannot be treated in a normal fashion. Specifically Eq.(37) is not valid for this case in any direct fashion. However, for the sake of completeness, we present here the SO(10) globally invariant couplings corresponding to Eq.(40). Thus we we have

$$\mathsf{L}^{(210)}_{++} = \frac{1}{4!}g^{(210)}_{ab} < \Psi_{(+)a}|\gamma^0\gamma^A\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]}|\Psi_{(+)b} > \Phi_{A\mu\nu\rho\lambda}. \tag{48}$$

To compute the couplings we carry out expansions similar to Eqs.(21) and (22) and to normalize the fields we carry out a field redefinition

$$\begin{aligned}\mathsf{g}_A = 4\sqrt{\frac{10}{3}}\mathsf{G}_A; \quad \mathsf{g}^i_A = 8\sqrt{6}\mathsf{G}^i_A; \quad \mathsf{g}_{Ai} = 8\sqrt{6}\mathsf{G}_{Ai} \\ \mathsf{g}^{ij}_A = \sqrt{2}\mathsf{G}^{ij}_A; \quad \mathsf{g}_{Aij} = \sqrt{2}\mathsf{G}_{Aij}; \quad \mathsf{g}^i_{Aj} = \sqrt{2}\mathsf{G}^i_{Aj} \\ \mathsf{g}^{ijk}_{Al} = \sqrt{\frac{2}{3}}\mathsf{G}^{ijk}_{Al}; \quad \mathsf{g}^i_{Ajkl} = \sqrt{\frac{2}{3}}\mathsf{G}^i_{Ajkl}; \quad \mathsf{g}^{ij}_{Akl} = \frac{\sqrt{2}}{\sqrt{3}}\mathsf{G}^{ij}_{Akl}\end{aligned} \tag{49}$$

so that the 210-plet's kinetic energy $-\frac{1}{4}\mathcal{F}^{AB}_{\mu\nu\rho\lambda}\mathcal{F}_{AB\mu\nu\rho\lambda}$ takes the form

$$\begin{aligned}\mathsf{L}^{210-gauge}_{kin} &= -\frac{1}{2}\mathcal{G}_{AB}\mathcal{G}^{AB\dagger} - \frac{1}{2}\mathcal{G}^{i}_{AB}\mathcal{G}^{ABi\dagger} - \frac{1}{2!}\frac{1}{2}\mathcal{G}^{ij}_{AB}\mathcal{G}^{ABij\dagger}\\ &-\frac{1}{2!}\frac{1}{2}\mathcal{G}^{i}_{ABj}\mathcal{G}^{ABj}_{i} - \frac{1}{3!}\frac{1}{2}\mathcal{G}^{ijk}_{ABl}\mathcal{G}^{ABijk\dagger}_{l} - \frac{1}{2!}\frac{1}{2!}\frac{1}{4}\mathcal{G}^{ij}_{ABkl}\mathcal{G}^{ABkl}_{ij}.\end{aligned} \tag{50}$$

As discussed above, the 210 vector multiplet is not a gauge multiplet and thus the quantity $\mathcal{G}_{AB}$ is just an ordinary curl. Using Eqs.(47), (21) and the normalizations of Eq.(49) one can compute $\mathsf{L}^{(210)}_{++}$. One finds

$$\begin{aligned}\mathsf{L}^{(210)}_{++} = \frac{1}{\sqrt{6}}g^{(210)}_{ab}[&\sqrt{5}\left(\overline{\mathbf{M}}_{a}\gamma^{A}\mathbf{M}_{b} - \frac{1}{10}\overline{\mathbf{M}}_{aij}\gamma^{A}\mathbf{M}^{ij}_{b} + \frac{1}{5}\overline{\mathbf{M}}^{i}_{a}\gamma^{A}\mathbf{M}_{bi}\right)\mathsf{G}_{A}\\ &+\frac{\sqrt{3}}{2}\left(-\overline{\mathbf{M}}_{a}\gamma^{A}\mathbf{M}^{lm}_{b} + \frac{1}{6}\epsilon^{ijklm}\overline{\mathbf{M}}_{aij}\gamma^{A}\mathbf{M}_{bk}\right)\mathsf{G}_{Alm}\\ &+\frac{\sqrt{3}}{2}\left(-\overline{\mathbf{M}}_{alm}\gamma^{A}\mathbf{M}_{b} + \frac{1}{6}\epsilon_{ijklm}\overline{\mathbf{M}}^{i}_{a}\gamma^{A}\mathbf{M}^{jk}_{b}\right)\mathsf{G}^{lm}_{A}\\ &+\sqrt{3}\left(-\overline{\mathbf{M}}^{j}_{a}\gamma^{A}\mathbf{M}_{bi} + \frac{1}{3}\overline{\mathbf{M}}_{aik}\gamma^{A}\mathbf{M}^{kj}_{b}\right)\mathsf{G}^{i}_{Aj}\\ &-\frac{1}{3}\epsilon^{ijklm}\overline{\mathbf{M}}_{ain}\gamma^{A}\mathbf{M}_{bj}\mathsf{G}^{n}_{Aklm} + \frac{1}{3}\epsilon_{ijklm}\overline{\mathbf{M}}^{i}_{a}\gamma^{A}\mathbf{M}^{jn}_{b}\mathsf{G}^{klm}_{An}\\ &-\frac{1}{2}\overline{\mathbf{M}}_{aij}\gamma^{A}\mathbf{M}^{kl}_{b}\mathsf{G}^{ij}_{Akl} + 2\overline{\mathbf{M}}^{i}_{a}\gamma^{A}\mathbf{M}_{b}G_{Ai} + 2\overline{\mathbf{M}}_{a}\gamma^{A}\mathbf{M}_{bi}\mathsf{G}^{i}_{A}].\end{aligned} \tag{51}$$

A similar analysis gives

$$\begin{aligned}\mathsf{L}^{(210)}_{--} &= \frac{1}{4!}g^{(210)}_{ab} < \Psi_{(-)a}|\gamma^{0}\gamma^{A}\Gamma_{[\mu}\Gamma_{\nu}\Gamma_{\rho}\Gamma_{\lambda]}|\Psi_{(-)b} > \Phi_{A\mu\nu\rho\lambda}\\ &= \frac{1}{\sqrt{6}}g^{(210)}_{ab}[\sqrt{5}\left(\overline{\mathbf{N}}_{a}\gamma^{A}\mathbf{N}_{b} - \frac{1}{10}\overline{\mathbf{N}}^{ij}_{a}\gamma^{A}\mathbf{N}_{bij} + \frac{1}{5}\overline{\mathbf{N}}_{ai}\gamma^{A}\mathbf{N}^{i}_{b}\right)\mathsf{G}_{A}\\ &+\frac{\sqrt{3}}{2}\left(\overline{\mathbf{N}}^{lm}_{a}\gamma^{A}\mathbf{N}_{b} - \frac{1}{6}\epsilon^{ijklm}\overline{\mathbf{N}}_{ai}\gamma^{A}\mathbf{N}_{bjk}\right)\mathsf{G}_{Alm}\\ &+\frac{\sqrt{3}}{2}\left(\overline{\mathbf{N}}_{a}\gamma^{A}\mathbf{N}_{blm} - \frac{1}{6}\epsilon_{ijklm}\overline{\mathbf{N}}^{ij}_{a}\gamma^{A}\mathbf{N}^{k}_{b}\right)\mathsf{G}^{lm}_{A}\\ &+\sqrt{3}\left(-\overline{\mathbf{N}}_{ai}\gamma^{A}\mathbf{N}^{j}_{b} + \frac{1}{3}\overline{\mathbf{N}}^{jk}_{a}\gamma^{A}\mathbf{N}_{bki}\right)\mathsf{G}^{i}_{Aj}\\ &+\frac{1}{6}\epsilon^{ijklm}\overline{\mathbf{N}}_{an}\gamma^{A}\mathbf{N}_{bij}\mathsf{G}^{n}_{Aklm} + \frac{1}{6}\epsilon_{ijklm}\overline{\mathbf{N}}^{ij}_{a}\gamma^{A}\mathbf{N}^{n}_{b}\mathsf{G}^{klm}_{An}\\ &-\frac{1}{2}\overline{\mathbf{N}}^{kl}_{a}\gamma^{A}\mathbf{N}_{bij}\mathsf{G}^{ij}_{Akl} + 2\overline{\mathbf{N}}_{a}\gamma^{A}\mathbf{N}^{i}_{b}G_{Ai} + 2\overline{\mathbf{N}}_{ai}\gamma^{A}\mathbf{N}_{b}\mathsf{G}^{i}_{A}].\end{aligned} \tag{52}$$

Supersymmetrizations of Eqs.(51) and (52) requires that we deal with a massive vector multiplet and this topic will be dealt with elsewhere[10].

7 Possible role of large tensor representations in model building

Most of the model building in SO(10) has occured using small Higgs representations[9] and large representations are generally avoided as they lead to non-perturbative physics above the grand unified scale. However, for the purposes of physics below the grand unified scale, the existence of non-perturbativity above the unfied scale is not a central concern since the region above this scale in any case cannot be fully understood without taking into account quantum gravity effects. Thus there is no fundamental reason not to consider model building which allows for couplings with large tensor representations. Indeed large tensor representations have some very interesting and desirable features. Thus, for example, if the $\overline{126}$ develops a VEV in the direction of $\overline{45}$ of $SU(5)$ one can get the ratio 3:1 in the "22" element of the lepton vs. the down quark sector in a natural fashion as desired in the Georgi-Jarlskog textures[11]. A similar 3:1 ratio also appears in the 120 plet couplings. Because of this feature the tensor representations 120 and $\overline{126}$ have already appeared in several analyses of lepton and quark textures[6]. Further, it was pointed out in Ref.[5] that the tensor representation $\overline{126}$ may also play a role in suppressing proton decay arising from dimension five operators in supersymmetric models. This is so because couplings involving $\overline{126}$ plet of Higgs to 16 plet of matter do not give rise to dimension five operators. The result derived here including the computation of cubic and quartic couplings may find application also in the study of neutrino masses and mixings. Thus, for example, one may consider contributions to the neutrino mass (N) and to the up quark mass (U) from the contraction $[16_a\overline{16}_H]_{45}[16_b\overline{16}_H]_{45}$. From Eq.(33) we find that a contribution to N arises from the fifth term in the bracket of Eq.(33) while the contribution to U arises from the second term in the bracket of Eq.(33). Now comparing the above with Eq.(8) of Ref.[5] for the 10 plet Higgs coupling which gives a N:U ratio of 1:1 we find that the two couplings refered to above in Eq.(33) give N:U=3:8 in agreement with Ref.[12]. Regarding the 210 dimensional tensor, such a mutiplet could play a role in the quark-lepton and neutrino mass textures. The role of a 210 dimensional vector multiplet is less clear. One possible way it may surface in low energy physics is as a condensate field. However, this topic needs further exploration. A more detailed discussion of model building including large tensor representations is under investigation.

8 Conclusion

In this paper we have given a complete determination of the $SO(10)$ invariant couplings $\overline{16}_{\mp} - 16_{\pm} - 1$, $\overline{16}_{\mp} - 16_{\pm} - 45$ and $\overline{16}_{\mp} - 16_{\pm} - 210$ in the superpotential in their $SU(5)$ decomposed form. Further, we have computed all the allowed quartic interactions in the superpotential of the type $\overline{16}_{\mp}16_{\pm}\overline{16}_{\mp}16_{\pm}$. We also exhibited a technique which is much simpler and involves elimination of heavy fields in cubic couplings in their SU(5) decomposed form. These techniques can be directly applied to the computation of quartic couplings of the type $16_{+}16_{+}16_{+}16_{+}$ using cubic couplings involving $16_{+}16_{+}$ with the 10, 120 and $\overline{126}$ tensor multiplets which have already been computed in the work of Ref.[5]. An analysis of vector couplings involving the SO(10) vector mutiplets 1, 45 and 210 was also given. In all of our analysis we have made explicit use of the theorem developed in Ref[5] on the decomposition of SO(10) vertices which allows the complete determination of the couplings with large tensor representations. It would be very straightforward now to expand all the $SU(5)$ invariants in terms of $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariants using the particle assignments given by Eq.(7) and an example of this is given in Appendix B. We also discussed in this paper some interesting features of large tensor representations and the role they may play in future model building.

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10 Appendix A

We expand here on the technique for the elimination of heavy fields for the case when the fields belong to a large tensor representation. There are infact three approaches one can use in affecting this elimination. The first one is the direct approach where one eliminates the heavy large Higgs representation in its SO(10) form. While this is the most straightforward approach the disadvantage is that the analysis of dimension 4 operators cannot be directly made use of and one has to carry out the entire computation from scratch. An alternative possibility is that one utilizes the result of computations of dimension 4 operators already done to compute dimension five operators. In this case, however, since all the heavy Higgs fields are in their SU(5) irreducible representations the elimination of such fields would involve cross cancellations which are quite delicate. Thus, for example, in its SU(5) decomposition $210 = 1 + 5 + \bar{5} + 10 + \overline{10} + 24 + 40 + \overline{40} + 75$ and elimination of these involve cancellations between the 10 and the 40 plet contributions, between the $\overline{10}$ and the $\overline{40}$ plet contributions, and between the 1, 24 and 75 plet contributions. Such cancellations make the analysis tedious once again. It turns out that there is yet a third possibility which is to derive the dimension 4 operators in SU(5) decomposition leaving the SU(5) fields in their reducible form where possible, i.e., to use Eq.(13) without further reduction of the tensor fields in their irreducible components. Thus, for example, in this case one would carry out the following SU(5) decomposition of the SO(10) tensor, $210 = 5 + \bar{5} + 50 + \overline{50} + 100$ where $50, \overline{50}, 100$ are reducible SU(5) representations. After computing the dimension 4 operators in terms of these tensors one eliminates them. This method has the advantage of having the cancellations of the second approach already built in. We give now more details of the three approaches.

We begin by discussion of the first approach where one eliminates the heavy fields in the superpotential before one carries out an SU(5) decomposition. Here on using the flatness conditions one finds

$$\mathcal{I}_1 = 2\lambda^{(1)}_{ab,cd} < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} > \tag{53}$$

$$\begin{aligned}\mathcal{I}_{45} = -\frac{1}{2}\lambda^{(45)}_{ab,cd}[&< \widehat{\Psi}^*_{(-)a}|B\Gamma_\mu\Gamma_\nu|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B\Gamma_\mu\Gamma_\nu|\widehat{\Psi}_{(+)d} > \\ &-10 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} >]\end{aligned} \tag{54}$$

Expansion in oscillator modes gives

$$\begin{aligned}\mathcal{I}_{45} = \lambda^{(45)}_{ab,cd}[&-4 < \widehat{\Psi}^*_{(-)a}|Bb_ib_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_i^\dagger b_j^\dagger|\widehat{\Psi}_{(+)d} > \\ &+4 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_j^\dagger b_i|\widehat{\Psi}_{(+)d} > \\ &-4 < \widehat{\Psi}^*_{(-)a}|Bb_n^\dagger b_n|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} > \\ &+5 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} >].\end{aligned} \tag{55}$$

A similar analysis for the 210 plet field gives

$$\begin{aligned}
\mathcal{I}_{210} = \frac{1}{288}\lambda^{(210)}_{ab,cd}[&< \widehat{\Psi}^*_{(-)a}|B\Gamma_\mu\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B\Gamma_\mu\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\widehat{\Psi}_{(+)d} > \\
&-52 < \widehat{\Psi}^*_{(-)a}|B\Gamma_\mu\Gamma_\nu|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B\Gamma_\mu\Gamma_\nu|\widehat{\Psi}_{(+)d} > \\
&+240 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} >] \\
= -\frac{1}{18}\lambda^{(210)}_{ab,cd}[&8 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j b_k b_l|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_j^\dagger b_k^\dagger b_l^\dagger b_i|\widehat{\Psi}_{(+)d} > \\
&-6 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j^\dagger b_k b_l|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_k^\dagger b_l^\dagger b_i b_j|\widehat{\Psi}_{(+)d} > \\
&-2 < \widehat{\Psi}^*_{(-)a}|Bb_i b_j b_k b_l|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_i^\dagger b_j^\dagger b_k^\dagger b_l^\dagger|\widehat{\Psi}_{(+)d} > \\
&+24 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_j^\dagger b_n^\dagger b_n b_i|\widehat{\Psi}_{(+)d} > \\
&-12 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j^\dagger|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_n^\dagger b_n b_i b_j|\widehat{\Psi}_{(+)d} > \\
&-12 < \widehat{\Psi}^*_{(-)a}|Bb_i b_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_i^\dagger b_j^\dagger b_n^\dagger b_n|\widehat{\Psi}_{(+)d} > \\
&-6 < \widehat{\Psi}^*_{(-)a}|Bb_m^\dagger b_m|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_n^\dagger b_n|\widehat{\Psi}_{(+)d} > \\
&-6 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_m^\dagger b_n^\dagger b_n b_m|\widehat{\Psi}_{(+)d} > \\
&+18 < \widehat{\Psi}^*_{(-)a}|Bb_i b_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_i^\dagger b_j^\dagger|\widehat{\Psi}_{(+)d} > \\
&-18 < \widehat{\Psi}^*_{(-)a}|Bb_i^\dagger b_j|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_j^\dagger b_i|\widehat{\Psi}_{(+)d} > \\
&+24 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|Bb_n^\dagger b_n|\widehat{\Psi}_{(+)d} > \\
&-15 < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} >< \widehat{\Psi}^*_{(-)c}|B|\widehat{\Psi}_{(+)d} >]. \qquad (56)
\end{aligned}$$

Although this is the most straigtforward technique, one has to carry out the entire analysis ab initio which can be very labor intensive for the case of large tensor representations.

We discuss now the second approach where one decomposes the large tensor representations in its irreducible SU(5) components and utilizes the results of the cubic superpotential already computed to derive dimension five operators. For illustration we consider the elimination of the 45 plet in the $\overline{16} - 16 - 45$ coupling and for simplicity we consider only one generation of Higgs. We begin by displaying the 45 plet mass term in terms of its irreducible SU(5) components

$$\frac{1}{2}\mathcal{M}^{(45)}\Phi_{\mu\nu}\Phi_{\mu\nu} = \frac{1}{2}\mathcal{M}^{(45)}\left[\mathsf{H}^{ij}\mathsf{H}_{ij} - \mathsf{H}^i_j\mathsf{H}^j_i - \mathsf{H}^2\right]. \qquad (57)$$

The superpotential is given by

$$\mathsf{W}^{(45)}_{-+} = J^{(1/45)}\mathsf{H} + J^{(\overline{10}/45)ij}\mathsf{H}_{ij} + J^{(10/45)}_{ij}\mathsf{H}^{ij} + J^{(24/45)j}_i\mathsf{H}^i_j \qquad (58)$$

where

$$J^{(1/45)} = \sqrt{\frac{5}{2}} h_{ab}^{(45)} \left(\frac{3}{5} \widehat{\mathbf{N}}_a^{i\mathbf{T}} \widehat{\mathbf{M}}_{bi} + \frac{1}{10} \widehat{\mathbf{N}}_{aij}^{\mathbf{T}} \widehat{\mathbf{M}}_b^{ij} - \widehat{\mathbf{N}}_a^{\mathbf{T}} \widehat{\mathbf{M}}_b \right)$$

$$J^{(\overline{10}/45)lm} = \frac{h_{ab}^{(45)}}{\sqrt{2}} \left(-\widehat{\mathbf{N}}_a^{\mathbf{T}} \widehat{\mathbf{M}}_b^{lm} + \frac{1}{2} \epsilon^{ijklm} \widehat{\mathbf{N}}_{alm}^{\mathbf{T}} \widehat{\mathbf{M}}_{bk} \right)$$

$$J_{lm}^{(10/45)} = \frac{h_{ab}^{(45)}}{\sqrt{2}} \left(-\widehat{\mathbf{N}}_{alm}^{\mathbf{T}} \widehat{\mathbf{M}}_b + \frac{1}{2} \epsilon_{ijklm} \widehat{\mathbf{N}}_a^{i\mathbf{T}} \widehat{\mathbf{M}}_b^{jk} \right)$$

$$J_i^{(24/45)j} = \sqrt{2} h_{ab}^{(45)} \left(\widehat{\mathbf{N}}_{aik}^{\mathbf{T}} \widehat{\mathbf{M}}_b^{kj} - \widehat{\mathbf{N}}_a^{j\mathbf{T}} \widehat{\mathbf{M}}_{bi} \right). \tag{59}$$

Eliminating the irreducible SU(5) heavy Higgs fields through F-flatness conditions taking care of the tracelessnes condition for H_i^j one gets

$$\mathsf{I}_{45} = \frac{1}{10\mathcal{M}^{(45)}} [5J^{(1/45)} J^{(1/45)} - 20 J^{(\overline{10}/45)ij} J_{ij}^{(10/45)} + 5 J_i^{(24/45)j} J_j^{(24/45)i} - J_m^{(24/45)m} J_n^{(24/45)n}]. \tag{60}$$

I_{45} computed above is the same as $\mathcal{I}_{45}$ given by Eq.(34) using the direct method with $\frac{h_{ab}^{(45)} h_{cd}^{(45)}}{\mathcal{M}^{(45)}}$ replaced by $-4\lambda_{ab,cd}^{(45)}$. As pointed out in the beginning of this appendix one has cancellations in this procedure between the contributions arising from elimination of the 1 plet and the 24 plet. Such cancellations become more abundant for the 210 plet case. Thus for this case it is more convenient to decompose the 210 plet into reducible SU(5) tensors. We begin by exhibiting the mass term for this case

$$\frac{1}{2} \mathcal{M}^{(210)} \Phi_{\mu\nu\rho\lambda} \Phi_{\mu\nu\rho\lambda} = \mathcal{M}^{(210)} \left[\frac{1}{16} \mathsf{K}^{ijkl} \mathsf{K}_{ijkl} + \frac{1}{4} \mathsf{K}_i^{jkl} \mathsf{K}_{jkl}^i + \frac{3}{16} \mathsf{K}_{ij}^{kl} \mathsf{K}_{kl}^{ij} \right] \tag{61}$$

where K^{ijkl}, K_{ijkl}, K_i^{jkl}, K_{jkl}^i and K_{kl}^{ij} are the 5plet, $\bar{5}$plet, 50plet, $\overline{50}$plet and 100plet representations of SU(5). As before we keep only one generation of Higgs. The superpotential $\mathsf{W}_{-+}^{210)}$ in this case may be written as

$$\mathsf{W}_{-+}^{210)} = J_{ijkl}^{(\bar{5}/210)} \mathsf{K}^{ijkl} + J^{(5/210)ijkl} \mathsf{K}_{ijkl} + J_{ijk}^{(50/210)l} \mathsf{K}_l^{ijk} + J_l^{(\overline{50}/210)ijk} \mathsf{K}_{ijk}^l + J_{ij}^{(50/210)} \mathsf{K}_n^{ijn} + J^{(\overline{50}/210)ij} \mathsf{K}_{ijn}^n + J_{kl}^{(100/210)ij} \mathsf{K}_{ij}^{kl} + J_i^{(100/210)j} \mathsf{K}_{jn}^{in} + J^{(100/210)} \mathsf{K}_{mn}^{mn} \tag{62}$$

where

$$J_{ijkl}^{(\bar{5}/210)} = \frac{h_{ab}^{(210)}}{24} < \widehat{\Psi}_{(-)a}^* | B b_i^\dagger b_j^\dagger b_k^\dagger b_l^\dagger | \widehat{\Psi}_{(+)b} >$$

$$
\begin{aligned}
J^{(5/210)ijkl} &= \frac{h^{(210)}_{ab}}{24} < \widehat{\Psi}^*_{(-)a}|Bb_ib_jb_kb_l|\widehat{\Psi}_{(+)b} > \\
J^{(50/210)l}_{ijk} &= \frac{h^{(210)}_{ab}}{6} < \widehat{\Psi}^*_{(-)a}|Bb^\dagger_ib^\dagger_jb^\dagger_kb_l|\widehat{\Psi}_{(+)b} > \\
J^{(\overline{50}/210)jkl}_i &= -\frac{h^{(210)}_{ab}}{6} < \widehat{\Psi}^*_{(-)a}|Bb^\dagger_ib_jb_kb_l|\widehat{\Psi}_{(+)b} > \\
J^{(50/210)}_{ij} &= -\frac{h^{(210)}_{ab}}{4} < \widehat{\Psi}^*_{(-)a}|Bb^\dagger_ib^\dagger_j|\widehat{\Psi}_{(+)b} > \\
J^{(\overline{50}/210)ij} &= \frac{h^{(210)}_{ab}}{4} < \widehat{\Psi}^*_{(-)a}|Bb_ib_j|\widehat{\Psi}_{(+)b} > \\
J^{(100/210)kl}_{ij} &= \frac{h^{(210)}_{ab}}{4}\widehat{\Psi}^*_{(-)a}|Bb^\dagger_ib^\dagger_jb_kb_l|\widehat{\Psi}_{(+)b} > \\
J^{(100/210)j}_i &= \frac{h^{(210)}_{ab}}{2} < \widehat{\Psi}^*_{(-)a}|Bb^\dagger_ib_j|\widehat{\Psi}_{(+)b} > \\
J^{(100/210)} &= -\frac{h^{(210)}_{ab}}{8} < \widehat{\Psi}^*_{(-)a}|B|\widehat{\Psi}_{(+)b} > .
\end{aligned}
\tag{63}
$$

Eliminating the reducible SU(5) Higgs fields through the F-flatness condition we get

$$
\begin{aligned}
\mathsf{I}_{210} = -\frac{1}{3\mathcal{M}^{(210)}}[4J^{(100/210)ij}_{kl}J^{(100/210)kl}_{ij} &+ 8J^{(100/210)mi}_{mj}J^{(100/210)j}_i \\
+8J^{(100/210)mn}_{mn}J^{(100/210)} &+ 3J^{(100/210)j}_iJ^{(100/210)i}_j \\
+J^{(100/210)m}_mJ^{(100/210)n}_n &+ 16J^{(100/210)m}_mJ^{(100/210)} \\
+40J^{(100/210)}J^{(100/210)} &+ 48J^{(5/210)ijkl}J^{(\overline{5}/210)}_{ijkl} \\
+12J^{(\overline{50}/210)ijk}_lJ^{(50/210)l}_{ijk} &+ 12J^{(\overline{50}/210)mij}_mJ^{(50/210)}_{ij} \\
+12J^{(\overline{50}/210)ij}J^{(50/210)m}_{ijm} &+ 12J^{(\overline{50}/210)ij}J^{(50/210)}_{ij}].
\end{aligned}
\tag{64}
$$

One may now check that I_{210} derived above coincides with $\mathcal{I}_{210}$ given by Eq.(56) using the direct method when we make the identification $\frac{h^{(210)}_{ab}h^{(210)}_{cd}}{\mathcal{M}^{(210)}}$ with $-4\lambda^{(210)}_{ab,cd}$.

11 Appendix B

In this appendix we expand some of the SO(10) interactions in the familiar particle notation and exhibit the differences between some of the $\overline{16}-16-45$ and the $\overline{16}-16-210$ couplings. We start by looking at the gauge interactions of the 24 plet of SU(5) in $\overline{16}-16-45$ coupling. We can read this off from the last term in Eq.(45). Disregarding the front factor, this term is of the form

$$
\mathcal{L}_{24/45} = g^{(45)}_{ab}\left(\overline{\mathbf{M}}_{aik}\gamma^A\mathbf{M}^{kj}_b + \overline{\mathbf{M}}^j_a\gamma^A\mathbf{M}_{bi}\right)\mathsf{G}^i_{Aj} \tag{65}
$$

An expansion of Eq.(65) using the SM particle states defined by Eq.(7) gives

$$\mathcal{L}_{24/45} = g_{ab}^{(45)} \sum_{x=1}^{8} \left[\overline{U}_a \gamma^A \mathbf{V}_A^x \frac{\lambda_x}{2} U_b + \overline{D}_a \gamma^A \mathbf{V}_A^x \frac{\lambda_x}{2} D_b \right]$$
$$+ g_{ab}^{(45)} \sum_{y=1}^{3} \left[\begin{pmatrix} \overline{\nu} & \overline{E}^- \end{pmatrix}_{aL} \gamma^A \mathbf{W}_A^y \frac{\tau_y}{2} \begin{pmatrix} \nu \\ E^- \end{pmatrix}_{bL} + \begin{pmatrix} \overline{U} & \overline{D} \end{pmatrix}_{aL} \gamma^A \mathbf{W}_A^y \frac{\tau_y}{2} \begin{pmatrix} U \\ D \end{pmatrix}_{bL} \right]$$
$$+ g_{ab}^{(45)} \sqrt{\frac{3}{5}} [-\frac{1}{2} \left(\overline{E}_{aL}^- \gamma^A \mathbf{B}_A E_{bL}^- + \overline{\nu}_{aL} \gamma^A \mathbf{B}_A \nu_{bL} \right) + \frac{1}{6} \left(\overline{U}_{aL}^- \gamma^A \mathbf{B}_A U_{bL} + \overline{D}_{aL} \gamma^A \mathbf{B}_A D_{bL} \right)$$
$$+ \frac{2}{3} \overline{U}_{aR} \gamma^A \mathbf{B}_A U_{bR} - \frac{1}{3} \overline{D}_{aR} \gamma^A \mathbf{B}_A D_{bR} - \overline{E}_{aR}^- \gamma^A \mathbf{B}_A E_{bR}^-]$$
$$+ ... \tag{66}$$

where $\mathbf{V}_A^x$ is an SU(3) octet of gluons, $\mathbf{W}_A^y$ is an SU(2) isovector of intermediate bosons, $\mathbf{B}_A$ is the hypercharge boson, τ_y and λ_x are the usual Pauli and Gell-Mann matrices, and the dots stand for the couplings of the lepto-quark/diquark bosons to fermions. The above result, of course, contains the SM interactions. Next, let us look at the vector interaction of the 24 plet of SU(5) in the $\overline{16} - 16 - 210$ coupling. This can be read off from Eq.(51) and one has

$$\mathcal{L}_{24/210} = g_{ab}^{(210)} \left(-\overline{\mathbf{M}}_a^j \gamma^A \mathbf{M}_{bi} + \frac{1}{3} \overline{\mathbf{M}}_{aik} \gamma^A \mathbf{M}_b^{kj} \right) \mathsf{G}_{Aj}^i$$
$$= \frac{1}{3} \frac{g_{ab}^{(210)}}{g_{ab}^{(45)}} \mathcal{L}_{24/45} - \frac{4}{3} \{ g_{ab}^{(210)} \sum_{x=1}^{8} \overline{D}_{aR} \gamma^A \mathbf{V}_A^x \frac{\lambda_x}{2} D_{bR}$$
$$+ g_{ab}^{(210)} \sqrt{\frac{3}{5}} \left[-\frac{1}{2} \left(\overline{E}_{aL}^- \gamma^A \mathbf{B}_A E_{bL}^- + \overline{\nu}_{aL} \gamma^A \mathbf{B}_A \nu_{bL} \right) - \frac{1}{3} \overline{D}_{aR} \gamma^A \mathbf{B}_A D_{bR} \right]$$
$$+ g_{ab}^{(210)} \sum_{y=1}^{3} \begin{pmatrix} \overline{\nu} & \overline{E}^- \end{pmatrix}_{aL} \gamma^A \mathbf{W}_A^y \frac{\tau_y}{2} \begin{pmatrix} \nu \\ E^- \end{pmatrix}_{bL} + ... \} \tag{67}$$

Eq.(67) shows that the 24 plet of SU(5) couplings in $\overline{16} - 16 - 210$, unlike the case of the 24 plet couplings in $\overline{16} - 16 - 45$, do not contain the same exact interactions as in the Standard Model, as for example, the octet of color vector bosons $\mathbf{V_A^x}$ has both vector and axial vector interactions.

References

[1] H. Georgi, in Particles and Fields (edited by C.E. Carlson), A.I.P., 1975; H. Fritzch and P. Minkowski, Ann. Phys. **93**(1975)193.

[2] R.N. Mohapatra and B. Sakita, Phys. Rev.**D21**(1980)1062.

[3] F. Wilczek and A. Zee, Phys. Rev. **D25**(1982)553.

[4] S. Nandi, A. Stern, E.C.G. Sudarshan, Phys. Rev. **D26**(1982)1653.

[5] P. Nath and Raza M. Syed, Phys. Lett. **B506**(2001)68; hep-ph/0103165

[6] H. Georgi and D.V. Nanopoulos, Nucl. Phys. **B159**(1979)16; J.A. Harvey, P. Ramond, and D.B. Reiss, Phys. Lett. **B92**(1980)309; J.A. Harvey, D.B. Reiss and P. Ramond, Nucl. Phys. **B199**(1982)223; S.P. Martin, Phys. Rev. **46**(1992)2769; K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett.**70**(1993)2845; C.S. Aulakh, A. Melfo, A. Rasin and G. Senjanovic, Phys. Lett. **B459**(1999)557; C. Aulakh, B. Bajc, A. Melfo, A. Rasin and G. Senjanovic, Nucl.Phys.**B597**(2001)89; M-C. Chen and K.T. Mahanthappa, Phys. Rev. **D62**(2000)055001; hep-ph/0106093.

[7] G.W. Anderson and T. Blazek, J. Math. Phys. **41** (2000)8170; hep-ph/0101349.

[8] P. Nath, R. Arnowitt and A.H. Chamseddine, "Applied N=1 Supergravity", (World Scientific, singapore, 1984); J.Wess and J. Bagger, "Supersymmetry and Supergravity", (Princeton University Press, Princeton NJ, 1992).

[9] For a recent discussion of the status of models of this type see, J.C Pati, hep-ph/0106082; T. Blazek, R. Dermisek, and S. Raby, hep-ph/0107097.

[10] P. Nath and Raza M. Syed, work in progress.

[11] H. Georgi and C. Jarlskog, Phys. Lett. **B86**(1979)297.

[12] K.S. Babu and S.M. Barr, Phys. Rev. Lett. 85(2000)1170.