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Measurement of the Lifetime Difference of B_d Mesons: Possible and Worthwhile?

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Abstract

We estimate the decay width difference $\Delta\Gamma_d/\Gamma_d$ in the B_d system including $1/m_b$ contributions and next-to-leading order QCD corrections, and find it to be around 0.3%. We explicitly show that the time measurements of an untagged B_d decaying to a single final state isotropically can only be sensitive to quadratic terms in $\Delta\Gamma_d/\Gamma_d$, and hence the use of at least two different final states is desired. We discuss such pairs of candidate decay channels for the final states and explore the feasibility of a $\Delta\Gamma_d/\Gamma_d$ measurement through them. With tagged decays to CP eigenstates, it is possible to have measurements sensitive to linear terms in $\Delta\Gamma_d/\Gamma_d$ with only one final state. The measurement of this width difference is essential for an accurate measurement of $\sin(2\beta)$ at the LHC. The nonzero width difference may also be used to resolve a twofold discrete ambiguity in the B_d–$\bar{B}_d$ mixing phase, and hence its measurement is crucial for identifying new physics effects in the mixing. We also derive an upper bound on the value of $\Delta\Gamma_d/\Gamma_d$ in the presence of new physics, and point out some differences in the phenomenology of width differences in the B_s and B_d systems.

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1 Introduction

Within the standard model (SM), the difference in the decay widths of B_d mesons is CKM-suppressed with respect to that in the B_s system. A rough estimate leads to

$$\frac{\Delta\Gamma_d}{\Gamma_d} \sim \frac{\Delta\Gamma_s}{\Gamma_s} \cdot \lambda^2 \approx 0.5\% \quad , \tag{1}$$

where $\lambda = 0.225$ is the sine of Cabibbo angle, and we have taken $\Delta\Gamma_s/\Gamma_s \approx 15\%$ [1, 2, 3]. Here $\Gamma_{d(s)} = (\Gamma_L + \Gamma_H)/2$ is the average decay width of the light and heavy $B_{d(s)}$ mesons (B_L and B_H respectively). We denote these decay widths by Γ_L, Γ_H respectively, and define $\Delta\Gamma_{d(s)} \equiv \Gamma_L - \Gamma_H$. No experimental measurement of $\Delta\Gamma_d$ is currently available. Moreover, no motivation for its measurement (other than just measuring another number to check against the SM prediction) has been discussed, and hence the study of the lifetime difference between B_d mesons has hitherto been neglected as compared to that in the B_s system. The phenomenology of the lifetime difference between B_s mesons has been explored in detail in [4, 5].

With the possibility of experiments with high time resolution and high statistics, it is worthwhile to have a look at this quantity and make a realistic estimate of the possibility of its measurement. At LHCb for example, the proper time resolution is expected to be as good as $\Delta\tau \approx 0.03$ ps. This indeed is a very small fraction of the B_d lifetime ($\tau_{B_d} \approx 1.5$ ps [6]), so the time resolution is not a limiting factor in the accuracy of the measurement, the statistical error plays the dominant role. Taking into account the estimated number of B_d produced — for example the number of reconstructed $B_d \to J/\psi\, K_S$ events at the LHC is expected to be 5×10^5 ([7] table 3) — the measurement of the lifetime difference does not look too hard at first glance. Naively, one may infer that if the number of relevant events with the proper time of decay measured with the precision $\Delta\tau$ is N, then the value of $\Delta\Gamma_d/\Gamma_d$ is measured with an accuracy of $1/\sqrt{N}$. With a sufficiently large number of events N, it should be possible to reach the accuracy of 0.5% or better.

The measurement of $\Delta\Gamma_d/\Gamma_d$ is in reality harder than what the above naive expectation may suggest, since most of the quantities that involve the lifetime difference are quadratic in the small quantity $\Delta\Gamma_d/\Gamma_d$. In fact, as we shall explicitly show in this paper, the time measurements in the decays of an untagged B_d to a single final state are sensitive only to $(\Delta\Gamma_d/\Gamma_d)^2$. This implies that in order to discern two different lifetimes, the measurements need to have an accuracy of $(\Delta\Gamma_d/\Gamma_d)^2 \sim 2.5 \times 10^{-5}$, which is beyond the reach of the currently planned experiments.

However, the combination of lifetimes measured in two different untagged decay channels may be sensitive to linear terms in $\Delta\Gamma_d/\Gamma_d$. We explore three pairs of such untagged measurements in this paper: (i) lifetime measurements through decays to self-tagging (e.g. semileptonic) final states and to CP eigenstates, (ii) CP even and odd components in the decay mode $B_d \to J/\psi K^*(K_s\pi^0)$, and (iii) time-dependent untagged asymmetry between $B_d \to J/\psi K_S$ and $B_d \to J/\psi K_L$.

The conventional "gold-plated" decays for β measurement, $J/\psi K_S$ and $J/\psi K_L$, neglect the lifetime difference while determining $\sin(2\beta)$. For an accurate determination of β, the systematic errors due to $\Delta\Gamma_d/\Gamma_d$ need to be taken into account. Moreover, if the lifetime difference is neglected, the ambiguity $\beta \leftrightarrow (\pi/2-\beta)$ remains unresolved. We show that measurable quantities that are sensitive to the lifetime difference resolve this discrete ambiguity. This is indeed a strong motivation for the measurement of the small lifetime difference $\Delta\Gamma_d$.

In order to resolve this ambiguity in the B_d–$\bar{B}_d$ mixing phase, the theoretical uncertainties on $\Delta\Gamma_d$ need to be minimized. Therefore, we start by presenting in Sec. 2 a detailed calculation of $\Delta\Gamma_d$, including $1/m_b$ contributions and next-to-leading order (NLO) QCD corrections. The NLO precision in the width difference $\Delta\Gamma_d$ is also essential for obtaining a proper matching of the Wilson coefficients to the matrix elements of local operators from the lattice gauge theory.

The rest of the paper is organized as follows. In Sec. 3 we explicitly demonstrate the quadratic dependence on $\Delta\Gamma_d/\Gamma_d$ of quantities measurable through untagged B decays to a single final state. We explore the combinations of decay modes that can measure quantities linear in $\Delta\Gamma_d/\Gamma_d$ and can help resolving the discrete ambiguity in β. We calculate the corrections due to $\Delta\Gamma_d$ as well as the CP violation in K–$\bar{K}$ mixing to the measurement of $\sin(2\beta)$ through $B_d \to J/\psi K_S$, and also indicate the possibility of the $\Delta\Gamma_d$ measurement through tagged decays to CP eigenstates. In Sec. 4, we point out important differences in the upper bounds on $\Delta\Gamma_s$ and $\Delta\Gamma_d$ in the presence of new physics, and elaborate on the possibility of resolution of the discrete ambiguities in the mixing phases through them. We summarize our findings in Sec. 5.

2 Next-to-leading order estimation of $\Delta\Gamma_d$

2.1 Basic definitions

We briefly recall the basic definitions: in the Wigner–Weisskopf approximation the oscillation and the decay of a general linear combination of the neutral flavour eigenstates B_d and $\bar{B}_d$, $a|B_d\rangle + b|\bar{B}_d\rangle$, is described by the time-dependent Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} a \\ b \end{pmatrix}. \tag{2}$$

Here $\mathbf{M}$ and $\mathbf{\Gamma}$ are 2×2 Hermitean matrices. CPT invariance leads to the conditions $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. Exact CP invariance would imply $M_{21} = M_{12}$ and $\Gamma_{21} = \Gamma_{12}$ (a phase choice, namely $\mathcal{CP}|B_d\rangle = -|\bar{B}_d\rangle, \mathcal{CP}|\bar{B}_d\rangle = -|B_d\rangle$ is made). Independent of the choice of the unphysical phases, CP invariance (in mixing) would imply $\mathrm{Im}(M_{21}^*\Gamma_{21}) = 0$.

The mass eigenstates, the light B_L and the heavy B_H, are given by

$$|B_{L,H}\rangle = p|B_d\rangle \pm q|\bar{B}_d\rangle \tag{3}$$

with the normalization condition $|q|^2 + |p|^2 = 1$. Only the magnitude $|q/p|$ is measurable, the phase of this quantity is unphysical and can be fixed arbitrarily by convention.

The mass difference and the width difference between the physical states are defined by

$$\Delta m = M_H - M_L, \quad \Delta\Gamma = \Gamma_L - \Gamma_H \ , \tag{4}$$

such that $\Delta m > 0, \Delta\Gamma_d > 0$ in the SM. The real and imaginary parts of the eigenvalue equations are the following:

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = (4|M_{21}|^2 - |\Gamma_{21}|^2), \tag{5}$$
$$\Delta m \Delta\Gamma = -4\mathrm{Re}(M_{21}^* \Gamma_{21}). \tag{6}$$

With the help of the CP-violating parameter δ

$$\delta \equiv \frac{-2\ \mathrm{Im}(M_{21}^*\Gamma_{21})}{(\Delta m)^2 + |\Gamma_{21}|^2} = |p|^2 - |q|^2 = \langle B_L|B_H\rangle, \tag{7}$$

The effect of CP violation due to mixing on the mass difference Δm and on the lifetime difference $\Delta\Gamma$ may be explicitly shown:

$$(\Delta m)^2 = \frac{4|M_{21}|^2 - \delta^2|\Gamma_{21}|^2}{1+\delta^2} \tag{8}$$
$$(\Delta\Gamma)^2 = \frac{4|\Gamma_{21}|^2 - 16\delta^2|M_{21}|^2}{1+\delta^2} \ . \tag{9}$$

In the limit of exact CP invariance ($\delta = 0$) the mass eigenstates coincide with the CP eigenstates, $\mathcal{CP}|B_H\rangle = -|B_H\rangle$ and $\mathcal{CP}|B_L\rangle = +|B_L\rangle$ and the mass difference and width difference are given by $\Delta m = 2|M_{21}|, \Delta\Gamma = 2|\Gamma_{21}|$. However, even with a non-zero δ, taking into account that δ is constrained by the upper bound $|\delta| \leq |\Gamma_{21}|/(2|M_{21}|)$ and $\Gamma_{21}/M_{21} = \mathcal{O}(m_b^2/m_t^2)$, we can write

$$\Delta m = 2|M_{21}|\left[1 + \mathcal{O}\left(\frac{m_b^4}{m_t^4}\right)\right], \quad \Delta\Gamma = -\frac{2\mathrm{Re}(M_{21}^*\Gamma_{21})}{|M_{21}|}\left[1 + \mathcal{O}\left(\frac{m_b^4}{m_t^4}\right)\right]. \tag{10}$$

We shall neglect the terms of $\mathcal{O}(m_b^4/m_t^4) \sim 10^{-6}$ in our calculations.

2.2 Method of calculation

In the following we consider the two off-diagonal elements M_{21} and Γ_{21}, which correspond respectively to the dispersive and the absorptive part of the transition amplitude from B_d

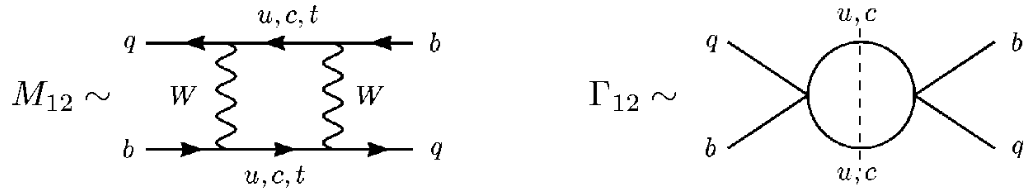

Figure 1: Schematic representation of Feynman diagrams for $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$.

to $\bar{B}_d$. We follow the method of [2, 3] which was used there in the B_s–$\bar{B}_s$ system (see also [8, 9]).

Within the SM the well-known box diagram is the starting point of the calculations. M_{21} is related to the real part of this diagram (see Fig. 1). The important QCD corrections are most easily implemented with the help of the standard operator product expansion. Because of the dominance of the top quark contribution, M_{21} can be described by a local $\Delta B = 2$ Hamiltonian below the m_W scale:

$$M_{21} = \frac{1}{2M_{B_d}} \langle \bar{B}_d | \mathcal{H}_{eff}^{\Delta B=2} | B_d \rangle \left[1 + O\left(\frac{m_b^2}{m_W^2} \right) \right] , \tag{11}$$

$$\mathcal{H}_{eff}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} (V_{tb}^* V_{td})^2 C^Q(m_t, m_W, \mu) Q(\mu) + \text{H.c.} , \tag{12}$$

$$Q = (\bar{b}_i d_i)_{V-A} (\bar{b}_j d_j)_{V-A} . \tag{13}$$

The Wilson coefficient C^Q contains the short-distance physics. It is known up to NLO precision [10]. The hadronic matrix element $\langle \bar{B}_d | Q(\mu \approx m_b) | B_d \rangle$ will be discussed below.

In the standard model, Γ_{21} is related to the imaginary part of the box diagram. Via the optical theorem it is fixed by the real intermediate states. Therefore, only the box diagrams with internal c and u quarks contribute (see Fig. 1). In contrast to the B_s–$\bar{B}_s$ case where the intermediate $c\bar{c}$ contribution is the dominating one, because of its CKM factor $(V_{cb}^* V_{cs})^2$, over the $u\bar{u}$, the $c\bar{u}$ and the $u\bar{c}$ contribution (see Sec. 4.1), in the B_d–$\bar{B}_d$ all four contributions have to be taken into account. In the effective theory where we integrate out the W boson, Γ_{21} is given by:

$$\Gamma_{21} = \frac{1}{2M_{B_d}} \langle \bar{B}_d | \text{Im}\, i \int d^4x\, T\, \mathcal{H}_{eff}^{\Delta B=1}(x)\ \mathcal{H}_{eff}^{\Delta B=1}(0) | B_d \rangle , \tag{14}$$

where

$$\begin{aligned} \mathcal{H}_{eff}^{\Delta B=1} = {} & \frac{G_F}{\sqrt{2}} \Big(V_{ub}^* V_{ud} \sum_{i=1,2} C_i Q_i^{uu} + V_{cb}^* V_{ud} \sum_{i=1,2} C_i Q_i^{cu} + V_{ub}^* V_{cd} \sum_{i=1,2} C_i Q_i^{uc} + \\ & + V_{cb}^* V_{cd} \sum_{i=1,2} C_i Q_i^{cc} - V_{tb}^* V_{td} \sum_{i=3}^{6} C_i Q_i \Big). \end{aligned} \tag{15}$$

The operators are (i,j denote color indices)

$$
\begin{aligned}
Q_1^{qq'} &= (\bar{b}_i q_j)_{V-A}(\bar{q}'_j d_i)_{V-A}, \quad Q_2^{qq'} = (\bar{b}_i q_i)_{V-A}(\bar{q}'_j d_j)_{V-A}, && (16)\\
Q_3 &= (\bar{b}_i d_i)_{V-A}(\bar{q}_j q_j)_{V-A}, \quad\quad Q_4 = (\bar{b}_i d_j)_{V-A}(\bar{q}_j q_i)_{V-A}, && (17)\\
Q_5 &= (\bar{b}_i d_i)_{V-A}(\bar{q}_j q_j)_{V+A}, \quad\quad Q_6 = (\bar{b}_i d_j)_{V-A}(\bar{q}_j q_i)_{V+A}. && (18)
\end{aligned}
$$

The penguin operators $Q_3 - Q_6$ have small Wilson coefficients and are therefore suppressed with respect to the four-quark operators – which all have the same two Wilson coefficients C_1 and C_2. In the leading logarithmic approximation we have:

$$
C_\pm = C_2 \pm C_1, \quad C_\pm(\mu) = \left(\frac{\alpha(M_W)}{\alpha(\mu)}\right)^{\frac{\gamma_\pm^{(0)}}{2\beta_0}} C_\pm(M_W), \quad C_\pm(M_W) = 1 \quad , \tag{19}
$$

where $\beta_0 = (11N - 2f)/3 = 23/3$ and $\gamma_\pm^{(0)} = \pm 6(1 \pm N)/N$. The coefficients to NLO precision can be found in [11].

Because there is another short-distance scale, the bottom quark mass, the operator product of two $\Delta B = 1$ operators can be expanded in inverse powers of the bottom quark mass scale in terms of local $\Delta B = 2$ operators:

$$
\begin{aligned}
\Gamma_{21} &= \frac{1}{2M_{B_d}} \langle \bar{B}_d | \mathrm{Im}\, i \int d^4x T \mathcal{H}_{eff}^{\Delta B=1}(x) \mathcal{H}_{eff}^{\Delta B=1}(0) | B_d \rangle && (20)\\
&= \sum_n \frac{E_n}{m_b^n} \langle \bar{B}_d | \mathcal{O}_n^{\Delta B=2}(0) | B_d \rangle \quad . && (21)
\end{aligned}
$$

These matching equations fix the values of the $\Delta B = 2$ Wilson coefficients E_n. The corresponding four quark operators $\mathcal{O}_n$ are the following: The operators Q and Q_S,

$$
\begin{aligned}
Q &= (\bar{b}_i d_i)_{V-A}(\bar{b}_j d_j)_{V-A}, && (22)\\
Q_S &= (\bar{b}_i d_i)_{S-P}(\bar{b}_j d_j)_{S-P}, && (23)
\end{aligned}
$$

represent the leading order contributions. Their matrix elements are given in terms of the bag parameters, B and B_S, the mass of the B_d meson M_{B_d}, and its decay constant f_{B_d}:

$$
\begin{aligned}
\langle \bar{B}_d | Q | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 2 \frac{N+1}{N} B, && (24)\\
\langle \bar{B}_d | Q_S | B_d \rangle &= -f_{B_d}^2 M_{B_d}^2 \frac{M_{B_d}^2}{(\bar{m}_b + \bar{m}_d)^2} \frac{2N-1}{N} B_S \quad . && (25)
\end{aligned}
$$

In the naive factorization approximation, B and B_S are fixed by $B = B_S = 1$. Reliable lattice calculations for B and B_S are already available [12]. We note that to NLO precision one has to distinguish between the pole mass m_b and the running quantity

$$
\bar{m}_b(\mu) = m_b \left[1 - \frac{\alpha_s}{\pi} \left(ln \frac{\mu^2}{m_b^2} + \frac{4}{3}\right)\right] \tag{26}
$$

using the $\overline{MS}$ scheme.

The $1/m_b$ corrections are given by the operators

$$
\begin{aligned}
R_1 &= \frac{m_d}{m_b}(\bar{b}_i d_i)_{S-P}(\bar{b}_j d_j)_{S+P}, &(27)\\
R_2 &= \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho d_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) d_j), &(28)\\
R_3 &= \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho d_i)(\bar{b}_j (1-\gamma_5) d_j), &(29)\\
R_4 &= \frac{1}{m_b}(\bar{b}_i (1-\gamma_5) i D_\mu d_i)(\bar{b}_j \gamma^\mu (1-\gamma_5) d_j), &(30)\\
R_0 &= Q_S + \frac{1}{2} Q + \tilde{Q}_S, &(31)
\end{aligned}
$$

where $\tilde{Q}_S$ has the "interchanged" color structure as compared to Q_S. There are also "color-interchanged" operators $\tilde{R}_i$ and $\tilde{Q}$ corresponding to R_i and Q. We note that these $1/m_b$ operators are not independent, the relations between them are in fact the equations of motion.

The matrix elements of these operators within the B_s–$\bar{B}_s$ system were estimated in [2] using naive factorization, which means that all the corresponding bag factors were set to 1. For the B_d–$\bar{B}_d$ system the analogous results are:

$$
\begin{aligned}
\langle \bar{B}_d | R_0 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \left(\frac{N+1}{N}\right)\left(1 - \frac{M_{B_d}^2}{m_b^2}\right) , &(32)\\
\langle \bar{B}_d | R_1 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \frac{m_d}{m_b}\frac{2N+1}{N} = 0 , &(33)\\
\langle \bar{B}_d | \tilde{R}_1 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \frac{m_d}{m_b}\frac{N+2}{N} = 0 , &(34)\\
\langle \bar{B}_d | R_2 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \left(\frac{M_{B_d}^2}{m_b^2} - 1\right)\frac{1-N}{N} = -\langle \bar{B}_d | \tilde{R}_2 | B_d \rangle , &(35)\\
\langle \bar{B}_d | R_3 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \left(\frac{M_{B_d}^2}{m_b^2} - 1\right)\frac{2N+1}{2N} , &(36)\\
\langle \bar{B}_d | \tilde{R}_3 | B_d \rangle &= f_{B_d}^2 M_{B_d}^2 \left(\frac{M_{B_d}^2}{m_b^2} - 1\right)\frac{N+2}{2N} , &(37)\\
\langle \bar{B}_d | R_4 | B_d \rangle &= -f_{B_d}^2 M_{B_d}^2 \left(\frac{M_{B_d}^2}{m_b^2} - 1\right) , &(38)\\
\langle \bar{B}_d | \tilde{R}_4 | B_d \rangle &= -f_{B_d}^2 M_{B_d}^2 \left(\frac{M_{B_d}^2}{m_b^2} - 1\right)\frac{1}{N} . &(39)
\end{aligned}
$$

Henceforth we shall neglect terms proportional to m_d/m_b; the other terms proportional to $(M_{B_d}^2/m_b^2) - 1$ are of order Λ_{QCD}/m_b.

In the matrix elements $\langle R_i \rangle$ (eqs. (32)–(39)), we use the pole mass m_b. There is a subtlety involved here: as discussed in [3], there are terms of order α_s and of leading power in m_b in the matrix element of R_0 to NLO precision. In view of the relation (31), it is not surprising that there are such terms. In the scheme – which was used in [3] and which is also used here – these terms are subtracted in the matrix element $\langle R_0 \rangle$ while taking into account the leading NLO contribution. Then the $\langle R_0 \rangle$ matrix element is still of a subleading nature. The specific subtraction scheme for the factorized matrix elements $\langle R_i \rangle$ corresponds to the use of the pole mass in eqs. (32)–(39). Of course this specific choice for the matrix elements has to be taken into account if the NLO results are combined with a lattice calculation of the $\langle R_i \rangle$.

There is an additional remark in order. We estimate Γ_{21} by the cut of the partonic diagrams. The underlying assumption of local quark-hadron duality can be verified in the B_s–$\bar{B}_s$ system, in the simultaneous limit of large N and of small velocity [1], therefore one expects no large duality violations. In the B_d–$\bar{B}_d$ system the small velocity argument fails since the $u\bar{u}$, $u\bar{c}$ and $c\bar{u}$ intermediate states contribute significantly, and the larger number of light intermediate states leads to a larger energy release. We follow ref. [2] and make the assumption that the duality violations in the $B_d - \bar{B}_d$ system are also not larger than 10%. In order to test this assumption one should include all corrections up to that accuracy.

2.3 Analytical results

In this section, we present an analytic expression for Γ_{21} including $1/m_b$, penguin and NLO corrections. If one takes into account the error inherent in the naive factorization approach to the matrix elements of the subleading operators R, it seems to be a reasonable approximation to keep at least all terms up to an accuracy of $10^{-2}\ \Gamma_{21}^{leading}$. We keep also higher order terms in order to check the accuracy of our approximation.

In the effective theory of the $\Delta B = 2$ transitions the matrix elements of the $1/m_b$ operators (R) are formally suppressed by a factor of the order of 0.1 with respect to those of the leading operators Q and Q_S. The natural variable $z = m_c^2/m_b^2$ also formally introduces a suppression factor of approximately 0.1. The NLO contribution has formally an extra suppression factor $(\alpha_s/4\pi)$ of order 0.01. Within the effective theory of the $\Delta B = 1$ Hamiltonian, the combination $K' = C^{peng} C^{dom}$ and $K'' = C^{peng} C^{peng}$ are suppressed by almost a factor 0.01 and 10^{-4} respectively, with respect to the combination $K = C^{dom} C^{dom}$, where C^{peng} denotes the Wilson coefficients of the penguin operators $Q_3 ... Q_6$ and C^{dom} that of the dominating operators $Q_{1\,(2)}^{qq'}$. The contribution due to K'' therefore can be safely neglected. Schematically our analytical result for Γ_{21} has the following form:

$$\begin{aligned}
\Gamma_{21} &= K\,\langle \mathcal{Q} \rangle && (40)\\
&+ K\,\langle R \rangle\,(O(1) + O(z) + O(z^2) + \{O(z^3)\}) && (41)\\
&+ K'\,\langle \mathcal{Q} \rangle\,(O(1) + O(z) + \{O(z^2)\}) && (42)\\
&+ K'\,\langle R \rangle\,(O(1) + \{O(z^2)\} && (43)
\end{aligned}$$

$$+ \quad \alpha_s/(4\pi)\ K\,\langle \mathcal{Q}\rangle\, O(1)\quad , \tag{44}$$

where $\mathcal{Q}$ represents the leading order operators Q and Q_S. The terms inside the curly brackets are the ones that we calculate only to estimate the errors. In the presentation of the results the following combinations of the Wilson coefficients are used:

$$K_1 = 3C_1^2 + 2C_1C_2, \quad K_2 = C_2^2, \quad K_3 = C_1^2, \quad K_4 = C_1C_2 \tag{45}$$

$$K_1' = 2(3C_1C_3 + C_1C_4 + C_2C_3), \quad K_2' = 2C_2C_4, \tag{46}$$

$$K_3' = 2(3C_1C_5 + C_1C_6 + C_2C_5 + C_2C_6), \tag{47}$$

and the common factor of $[-G_F^2 m_b^2/(24\pi M_{B_d})]$ is implicit in the following equations (48), (49), (51), (53).

In the leading log approximation we calculate the $Q_1^{qq'}$ and the $Q_2^{qq'}$ contributions to Γ_{21}. By extracting the absorptive parts of the $c\bar{c}, u\bar{c}, c\bar{u}$ and $u\bar{u}$ intermediate states, we can find the off-diagonal element. For this leading contribution (40), after replacing $V_{ub}^*V_{ud}$ by the unitarity relation, we get to all orders in z:

$$\begin{aligned}
\Gamma_{21}^{leading} &= (V_{tb}^*V_{td})^2 \left[\left(K_1 + \frac{1}{2}K_2\right)\langle Q\rangle + (K_1 - K_2)\langle Q_S\rangle\right] \\
&+ (V_{cb}^*V_{cd})(V_{tb}^*V_{td})\Big[(3z(K_1+K_2) - 3z^2K_2 - z^3(K_1-K_2))\langle Q\rangle \\
&\qquad +(6z^2(K_1-K_2) - 4z^3(K_1-K_2))\langle Q_S\rangle\Big] \\
&+ (V_{cb}^*V_{cd})^2\Big\{\sqrt{1-4z}\left[\left(K_1 + \frac{1}{2}K_2\right) - z(K_1+2K_2)\right]\langle Q\rangle \\
&\qquad +\sqrt{1-4z}(1+2z)(K_1-K_2)\langle Q_s\rangle \\
&\qquad +\left((1-z)^2\left[-(2K_1+K_2) + z(K_2-K_1)\right] + \left[K_1 + \frac{1}{2}K_2\right]\right)\langle Q\rangle \\
&\qquad +\left[(1-z)^2(2+4z)(K_2-K_1) - (K_2-K_1)\right]\langle Q_S\rangle\Big\}\quad .
\end{aligned} \tag{48}$$

The $1/m_b$ corrections to the operators $Q_1^{qq'}$ and $Q_2^{qq'}$ give [see the term (41)]

$$\begin{aligned}
\Gamma_{21}^{1/m_b} &= (V_{tb}^*V_{td})^2\left[-2\left(K_1 - \frac{1}{2}K_2\right)\langle R_2\rangle - 2K_1\langle R_1\rangle + 2K_2\langle R_4\rangle\right] \\
&+ (V_{cb}^*V_{cd})(V_{tb}^*V_{td})\Big[-12z^2K_1(\langle R_1\rangle - 2\langle R_3\rangle) \\
&\qquad + 6z^2K_2(\langle R_2\rangle + 4\langle R_3\rangle + 2\langle R_4\rangle)\Big] \\
&+ \Big\{K\langle R\rangle O(z^3)\Big\}.
\end{aligned} \tag{49}$$

The term in curly brackets in (49) can be written as

$$\begin{aligned}
\{...\} &= (V_{cb}^*V_{cd})(V_{tb}^*V_{td}) \times \\
&\quad [4z^3K_1(2\langle R_1\rangle - \langle R_2\rangle - 6\langle R_3\rangle) - 4z^3K_2(\langle R_2\rangle + 6\langle R_3\rangle + 2\langle R_4\rangle)] \\
&+ (V_{cb}^*V_{cd})^2 \times \\
&\quad [12z^3K_1(2\langle R_1\rangle - \langle R_2\rangle - 6\langle R_3\rangle) - 12z^3K_2(\langle R_2\rangle + 6\langle R_3\rangle + 2\langle R_4\rangle)]
\end{aligned} \tag{50}$$

The penguin contributions [terms (42), (43)] are

$$\begin{aligned}
\Gamma_{21}^{peng} &= (V_{tb}^* V_{td})^2 \Big[\Big(K_1' + \frac{1}{2} K_2' \Big) \langle Q \rangle + (K_1' - K_2') \langle Q_S \rangle \\
&\qquad + (-2\langle R_2 \rangle - 2\langle R_1 \rangle) K_1' + (\langle R_2 \rangle + 2\langle R_4 \rangle) K_2' \Big] \\
&+ (V_{cb}^* V_{cd})(V_{tb}^* V_{td})(3zK_1' + 3zK_2' - 3zK_3')\langle Q \rangle \\
&\quad + \{ K' \, \langle \mathcal{Q} \rangle \, O(z^2) + K' \, \langle R \rangle \, O(z) \} \quad , \qquad (51)
\end{aligned}$$

where the terms in curly brackets (and the lower order ones) may be written as

$$\begin{aligned}
\{...\} &= (V_{cb}^* V_{cd})(V_{tb}^* V_{td}) \times \\
&\quad [4z^3 K_1 (2\langle R_1 \rangle - \langle R_2 \rangle - 6\langle R_3 \rangle) - 4z^3 K_2 (\langle R_2 \rangle + 6\langle R_3 \rangle + 2\langle R_4 \rangle)] \\
&+ (V_{cb}^* V_{cd})^2 \times \\
&\quad [12z^3 K_1 (2\langle R_1 \rangle - \langle R_2 \rangle - 6\langle R_3 \rangle) - 12z^3 K_2 (\langle R_2 \rangle + 6\langle R_3 \rangle + 2\langle R_4 \rangle)] \quad . \qquad (52)
\end{aligned}$$

The NLO QCD correction $\Gamma_{21}^{NLO} = \alpha_s/(4\pi) \; K \, \langle \mathcal{Q} \rangle$ [term (44)] is found from [3] by taking the limit $z \to 0$ of their results[5]:

$$\begin{aligned}
\Gamma_{21}^{NLO} &= \frac{\alpha_s(m_b)}{4\pi} (V_{tb}^* V_{td})^2 \Big\{ \left[\frac{109}{6} K_3 - \frac{254}{9} K_4 - \left(\frac{\pi^2}{3} + \frac{115}{18} \right) K_2 \right] \langle Q \rangle \\
&\quad + \left[\left(10K_3 + \frac{20}{3} K_4 + \frac{8}{3} K_2 \right) \ln\left(\frac{\mu_2}{m_b} \right) - (34K_4 + 10K_2) \ln\left(\frac{\mu_1}{m_b} \right) \right] \langle Q \rangle \\
&\quad - \left[\frac{40}{3} K_3 + \frac{272}{9} K_4 - \left(\frac{8\pi^2}{3} - \frac{152}{9} \right) K_2 \right] \langle Q_S \rangle \\
&\quad + \left[\left(32K_3 - \frac{64}{3} K_4 + \frac{32}{3} K_2 \right) \ln\left(\frac{\mu_2}{m_b} \right) - (16K_4 + 16K_2) \ln\left(\frac{\mu_1}{m_b} \right) \right] \langle Q_S \rangle \\
&\quad - \left[\frac{2}{27} + \frac{2}{9} \ln\left(\frac{\mu_1}{m_b} \right) + \frac{1}{3} \frac{C_8}{C_2} \right] K_2 (\langle Q \rangle - 8\langle Q_S \rangle) \Big\} \; . \qquad (53)
\end{aligned}$$

The explicit μ_1 and μ_2 dependence in (53) cancels against the μ dependence of the Wilson coefficients of the hamiltonian $\mathcal{H}_{eff}^{\Delta B=1}$ (15) and the μ dependence of the matrix elements of the $\Delta B = 2$ operators at the order in α_s we take into account. For a proper matching with lattice evaluations of these matrix elements it is important to note that the results in (53) are based on the NDR scheme, with the choice of γ_5 and the evanescent operators as given in eqs. (13)–(15) of [3].

The net Γ_{21} is

$$\Gamma_{21} = \Gamma_{21}^{leading} + \Gamma_{21}^{1/m_b} + \Gamma_{21}^{peng} + \Gamma_{21}^{NLO} \quad , \qquad (54)$$

with the implicit multiplicative factor of $[-G_F^2 m_b^2/(24\pi M_{B_d})]$.

[5]We add only the leading contribution of the NLO QCD corrections for the term $(V_{tb}^* V_{td})^2$. The leading terms of the contributions for the terms $(V_{cb}^* V_{cd})(V_{td}^* V_{td})$ and $(V_{cb}^* V_{cd})^2$ cancel out through the GIM mechanism.

2.4 Numerical results

Let us now calculate the numerical value of $\Delta\Gamma_d$. From eq. (10), $\Delta\Gamma_d$ can be approximately written as

$$\Delta\Gamma_d \approx -2|M_{21}|\mathrm{Re}\frac{\Gamma_{21}}{M_{21}} = -\Delta m \ \mathrm{Re}\frac{\Gamma_{21}}{M_{21}} \ . \tag{55}$$

where M_{21} [see eq. (11)] is given by

$$M_{21} = \frac{G_F^2 M_W^2 \eta_B}{(4\pi)^2 (2M_{B_d})}(V_{tb}^* V_{td})^2 S_0(x_t)\langle Q\rangle. \tag{56}$$

Here $x_t = \bar{m}_t^2/M_W^2$, η_B is the QCD correction factor and S_0 is the Inami–Lim function:

$$S_0(x) = x\left(\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2}\right) - \frac{3}{2}\left(\frac{x}{1-x}\right)^3 \log x \ . \tag{57}$$

Using the results obtained in the previous section, we can write down the width difference (normalized to the average width) in the form

$$\begin{aligned}
\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_d} = & \left(\frac{\Delta m}{\Gamma}\right)_d \mathcal{K} \times \\
& \left[G^{tt} + \frac{5}{8}\frac{B_S}{B}\frac{M_{B_d}^2}{\bar{m}_b^2} G_S^{tt} + \frac{3}{8}\frac{1}{B} G_{1/m}^{tt}\right. \\
& + \mathrm{Re}\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \cdot \left(G^{ct} + \frac{5}{8}\frac{B_S}{B}\frac{M_{B_d}^2}{\bar{m}_b^2} G_S^{ct} + \frac{3}{8}\frac{1}{B}\{G_{1/m}^{ct}\}\right) \\
& \left. + \ \mathrm{Re}\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)^2 \cdot \left(G^{cc} + \frac{5}{8}\frac{B_S}{B}\frac{M_{B_d}^2}{\bar{m}_b^2} G_S^{cc}\right)\right] \ .
\end{aligned} \tag{58}$$

The superscripts $\{tt, ct, cc\}$ correspond to the terms in the expression for $\Delta\Gamma_d$ (54) that involve the CKM factors $\{(V_{td}V_{tb}^*)^2, (V_{cd}V_{cb}^*)(V_{td}V_{tb}^*), (V_{cd}V_{cb}^*)^2\}$ respectively. The subscript S denotes the contribution from the operator Q_S, and the subscript $1/m$ denotes the terms that give the $1/m_b$ corrections. The normalizing factor $\mathcal{K} \equiv (4\pi m_b^2)/(3M_W^2 \eta_B S_0(x_t))$ and the value of $(\Delta m/\Gamma)_d$ may be taken from experiments: $x_d \equiv (\Delta m/\Gamma)_d = 0.73 \pm 0.03$ [6]. The form of eq. (58) can bring out important features of the dependence of $\Delta\Gamma_d$ on various parameters, as we shall see below. In contrast to the B_s system, this representation is preferable, because within the leading term the CKM dependence cancels out and the value of $(\Delta m/\Gamma)_d$ may be taken from experiments: $x_d \equiv (\Delta m/\Gamma)_d = 0.73 \pm 0.03$ [6].

A remark about the penguin contributions is in order. We only include the interference of the penguin operators $C_3 ... C_6$ with the leading operators C_1 and C_2. At the NLO, this approximation can be made consistent (in the sense of scheme independence) by counting the Wilson coefficients $C_3 ... C_6$ as of order α_s. These Wilson coefficients are modified at NLO through the mixing of C_1 and C_2 into $C_3 ... C_6$. For C_1 and C_2 we use the complete NLO values. Since the contribution due to C_8 starts only at the NLO level, we only have to use the LO value for that Wilson coefficient. We stress that if one uses the consistent NLO approximation just described, the corresponding LO approximation includes no penguin contributions and uses the LO values for C_1 and C_2.

The choice of the b-quark mass at LO is ambiguous (it may be taken to be the pole mass or the running mass at one or two loop level); we take it to be the running mass in the MS scheme to leading order in m_b.

We use the following values of parameters to estimate $\Delta\Gamma_d$:

$$
\begin{aligned}
M_{B_d} = 5.28 \text{ GeV} \ , \ m_b = 4.8 \text{ GeV} \ , \ m_c = 1.4 \text{ GeV} \ , \\
\bar{m}_b(m_b) = 4.4 \text{ GeV} \ , \ \bar{m}_t(m_b) = 167 \text{ GeV} \ .
\end{aligned}
\tag{59}
$$

To the NLO precision [we use here the NDR scheme to get $\eta_B(m_b) = 0.846$ and include the NLO Wilson coefficients [13] and the corrections computed in eqs. (49),(51)], we get (in units of 10^{-3})

$$
\begin{aligned}
\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_d} = & \ 1.00 + 5.15\frac{B_S}{B} - 1.38\frac{1}{B} \\
& -\frac{\cos\beta}{R_t}\left(1.07 + 0.29\frac{B_S}{B} + 0.02\frac{1}{B} + \left\{0.005 - 0.021\frac{B_S}{B} + 0.003\frac{1}{B}\right\}\right) \\
& +\frac{\cos 2\beta}{R_t^2}\left(0.02 - 0.06\frac{B_S}{B} + \left\{-0.01\frac{1}{B}\right\}\right) \quad .
\end{aligned}
\tag{60}
$$

Let us perform a conservative estimate of the error on the value of $\Delta\Gamma_d/\Gamma_d$ that we obtain here. The errors arise from the uncertainties in the values of the CKM parameters, the bag parameters and the mass of the b quark. There are also errors from the scale dependence, the breaking of the naive factorization approximation, and the neglected higher order terms in the z expansion.

In the SM, we have

$$
\cos\beta/R_t = 1.03 \pm 0.08 \quad , \quad \cos 2\beta/R_t^2 = 0.87 \pm 0.15 \ ,
\tag{61}
$$

where we have taken the values of the CKM parameters from the global fit [14]. The leading term on the first line in (60) is independent of the CKM elements. The quantity $\cos\beta/R_t$ is known to an accuracy of about 10% and appears in (60) with a coefficient ~ 0.2 relative to the leading term. The quantity $\cos 2\beta/R_t^2$, although known to only about 20%, appears with a very small coefficient ($\sim 10^{-2}$) as compared to the leading term in (60). The net error due to the uncertainty in the CKM elements is thus approximately only 2%, i.e. about $\pm 0.06 \times 10^{-3}$.

We estimate the effect of the uncertainties in the bag factors by computing (60) with three sets of values of the bag parameters. The numerical results are as shown in Table 1.

	LO	A	B	C
B	1.0	0.90	0.83	1.0
B_S	1.0	0.75	0.84	1.0
$\Delta\Gamma_d/\Gamma_d$	6.3×10^{-3}	2.4×10^{-3}	3.1×10^{-3}	3.3×10^{-3}

Table 1: The numerical value of $\Delta\Gamma_d/\Gamma$ for different values of the bag parameters. The column LO (C) shows the leading (next-to-leading) order result with factorization, i.e. $B = B_S = 1$. The values of the bag factors in column A are taken from [3] and the ones in column B from the (preliminary) results in an unquenched ($N_f = 2$) lattice calculation by the JLQCD collaboration [12].

From the table, and using the uncertainties on the values of the bag parameters as given in [15], we conservatively estimate the corresponding uncertainty in the value of $\Delta\Gamma_d/\Gamma_d$ due to bag factors to be approximately $\pm 0.4 \times 10^{-3}$. The uncertainty in the value of $\bar{m}_b = 4.4 \pm 0.2$ also leads to an error of $\pm 0.5 \times 10^{-3}$. The uncertainty due to the scale μ_1 dependence is estimated to be ${}^{+0.4}_{-1.0} \times 10^{-3}$ (where μ_1 is varied between $2m_b$ and $m_b/2$ following the common convention). The error due to the input value of x_d is 0.1×10^{-3}.

The errors due to the breaking of the naive factorization assumption (which was made in the calculation of the matrix elements of the $1/m_b$ operators) are hard to quantify. Assuming an error of 30% in the R matrix elements (as in [15]), we estimate the error due to this source to be $\pm 0.3 \times 10^{-3}$.

Table 1 also gives the LO value of $\Delta\Gamma_d/\Gamma_d$ in the factorization approximation. We observe that the NLO corrections significantly decrease the value of $\Delta\Gamma_d/\Gamma_d$ as computed at LO, and that there effectively is no real $\alpha_s/4\pi$ suppression of the NLO contribution, as one naively expects. Therefore higher-order terms in the z expansion become important. While we estimate the error due the z expansion in the $1/m_b$ and the penguin contributions from the terms in curly brackets in (60) to be less than $\pm 0.05 \times 10^{-3}$, the issue of higher order terms in the NLO contribution (53) is more subtle. Here in (53) we have only calculated the coefficient of $(V_{tb}^* V_{td})^2$, which includes all the terms of the order z^0. We do not know anything a priori about the contribution of the z^1 terms – a complete NLO calculation is necessary for that. However, we may estimate the error due to the z^1 terms by looking at the corresponding expansion in the B_s system. In B_s system, the magnitude of the z^1 terms is as much as 40% of the magnitude of the z^1 terms. We note that the uncertainty due to the z^1 term is even higher in one of the terms within the B_s system. Thus, a complete NLO calculation is definitely desirable in order to reduce this error and give a reliable value of $\Delta\Gamma_d/\Gamma_d$. We then conservatively take the error in the NLO contribution due to the terms of z^1 and higher order to be 50%, which results in the estimation of the net error in $\Delta\Gamma_d/\Gamma_d$ due to these terms to be $\pm 0.6 \times 10^{-3}$.

Our net estimation for the width difference (with conservative error estimates) is

$$\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right)_{B_d} = (3.1^{+1.0}_{-1.4}) \times 10^{-3} \quad . \tag{62}$$

We have taken the central value to be the one obtained from the latest preliminary (unquenched) results from lattice calculations [12]. The dominating theoretical errors are the scale dependence and the terms in Γ_{21}^{NLO} that are of the order of z^1 or higher.

3 Measurement of $\Delta\Gamma_d/\Gamma_d$

It is not possible to find a final state to which the decay of B_d involves only one of the decay widths Γ_L and Γ_H. Indeed, since the B_d–$\bar{B}_d$ mixing phase (2β) is large, the CP eigenstates are appreciably different from the lifetime eigenstates. The decay rate to a CP eigenstate therefore involves both the lifetimes. The semileptonic decays are flavor-tagging, and hence also involve both the lifetimes in equal proportion.

We start by concentrating on the untagged measurements, i.e. the measurements in which the (Δmt) oscillations are cancelled out. When the production asymmetry between B_d and $\bar{B}_d$ is zero (as is the case at the B factories), this corresponds to not having to determine whether the decaying meson was B_d or $\bar{B}_d$. Restricting ourselves to untagged measurements is a way of getting rid of tagging inefficiencies and mistagging problems.

In this section, we show that the time measurements of the decay of an untagged B_d to a single final state can only be sensitive to quadratic terms in $\Delta\Gamma_d/\Gamma_d$. This would imply that, for determining $\Delta\Gamma_d/\Gamma_d$ using only one final state, the accuracy of the measurement needs to be $(\Delta\Gamma_d/\Gamma_d)^2 \sim 10^{-5}$. This indicates the necessity of combining measurements from two different final states to be sensitive to a quantity linear in $\Delta\Gamma_d/\Gamma_d \sim 0.3 \times 10^{-2}$. We discuss three pairs of candidate channels for achieving this task. We also indicate how these can resolve the discrete ambiguity in β. Finally, we point out the extent of systematic error in the conventional measurement of β due to the neglect of the width difference, and show how the tagged $B_d \to J/\psi K_S$ mode can also measure $\Delta\Gamma_d/\Gamma_d$ by itself.

3.1 Quadratic sensitivity to $\Delta\Gamma_d/\Gamma_d$ of untagged measurements

The non-oscillating part of the proper time distribution of the decay of B_d can be written in the most general form as

$$f(t) = \frac{1}{2}\left[(1+b)e^{-\Gamma_L t} + (1-b)e^{-\Gamma_H t}\right] \quad . \tag{63}$$

The non-oscillating part can also be looked upon as the untagged measurement.

For an isotropic decay, the only information available from the experiment is the time t. This information may be *completely* encoded in terms of the (infinitely many) time moments

$$\langle t^n \rangle \equiv \frac{\int t^n f(t) dt}{\int f(t) dt} \quad . \tag{64}$$

Expanding in powers of $\Delta\Gamma_d/\Gamma_d$, we get

$$\langle t^n \rangle = \frac{n!}{(\Gamma_d)^n} \left[1 - \frac{n\, b}{2} \frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2 \right] \right] \quad . \tag{65}$$

Defining the *effective untagged lifetime* as $\tau_b \equiv \frac{1}{\Gamma_d}\left(1 - \frac{b}{2}\frac{\Delta\Gamma_d}{\Gamma_d}\right)$, all the available information (64) is encoded in

$$\langle t^n \rangle = n! (\tau_b)^n \left[1 + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2 \right] \right] \quad . \tag{66}$$

Thus, when the accuracy of the lifetime measurement is less than $(\Delta\Gamma_d/\Gamma_d)^2$, only the combination τ_b of Γ_d, $\Delta\Gamma_d$ and b may be measured through a single final state. This measurement is insensitive to b (to this order) and hence incapable of even discerning the presence of two distinct lifetimes ($b = 0$ and $b = 1$ would correspond to the presence of only a single lifetime involved in the decay.) In particular, in order to determine $\Delta\Gamma_d/\Gamma_d$, the lifetime measurement through the semileptonic decay needs to be more accurate than $(\Delta\Gamma_d/\Gamma_d)^2 \sim 10^{-5}$. This task is beyond the capacity of the currently planned experiments.

Combining time measurements from two different final states, however, can enable us to measure quantities linear in $\Delta\Gamma_d/\Gamma_d$. Indeed, for two final states with different values b (say b_1 and b_2), we can measure

$$\frac{\tau_{b_1}}{\tau_{b_2}} = 1 + \frac{b_2 - b_1}{2} \frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2 \right] \quad . \tag{67}$$

In the next subsections, we discuss pairs of decay channels that can measure this quantity (67) that is linear in $\Delta\Gamma_d/\Gamma_d$.

3.2 Decay widths in semileptonic and CP-specific channels

Let us first develop the formalism that will be applicable for all the decays that we shall consider below. When the width difference is taken into account, the decay rate of an initial B_d to a final state f is given as follows. Let $A_f \equiv \langle f | B_d \rangle$, $\bar{A}_f \equiv \langle f | \bar{B}_d \rangle$, and

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} \ , \tag{68}$$

where p and q are as defined in (3). Using the CP-violating parameter δ^d as defined in (7), we get

$$\left| \frac{q}{p} \right| = \sqrt{\frac{1 - \delta^d}{1 + \delta^d}} \approx 1 - \delta^d \quad . \tag{69}$$

The approximation here is valid since we have $|\delta^d| \sim |\Delta\Gamma_d/\Delta m_d| \lesssim 10^{-2}$. Henceforth, we shall only consider terms linear in δ^d.

The decay rate of an initial tagged B_d or $\bar{B}_d$ to a final state f is given by [5]:

$$\begin{aligned}
\Gamma(B_d(t) \to f) \quad &= \mathcal{N}_f\, |A_f|^2 \, \frac{1+|\lambda_f|^2}{2}\, e^{-\Gamma_d t} \times \\
&\left[\cosh\frac{\Delta\Gamma_d\, t}{2} + \mathcal{A}_{\rm CP}^{\rm dir}\cos(\Delta m t) + \mathcal{A}_{\Delta\Gamma}\sinh\frac{\Delta\Gamma_d\, t}{2} + \mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta m t)\right], \quad (70)\\
\Gamma(\bar{B}_d(t) \to f) \quad &= \mathcal{N}_f\, |\bar{A}_f|^2 \, \frac{1+|\lambda_f|^2}{2}\, e^{-\Gamma_d t} \times \\
&\left[\cosh\frac{\Delta\Gamma_d\, t}{2} - \mathcal{A}_{\rm CP}^{\rm dir}\cos(\Delta m t) + \mathcal{A}_{\Delta\Gamma}\sinh\frac{\Delta\Gamma_d\, t}{2} - \mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta m t)\right]. \quad (71)
\end{aligned}$$

where the CP asymmetries are defined as

$$\mathcal{A}_{\rm CP}^{\rm dir} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \qquad \mathcal{A}_{\rm CP}^{\rm mix} = -\frac{2\,{\rm Im}\lambda_f}{1+|\lambda_f|^2} \quad \text{and} \quad \mathcal{A}_{\Delta\Gamma} = -\frac{2\,{\rm Re}\lambda_f}{1+|\lambda_f|^2}, \tag{72}$$

and $\mathcal{N}_f$ is a time-independent normalization factor.

In the case of semileptonic decays, $f \equiv \{D\ell^+\nu\}$, so that $\bar{A}_f = 0$ and hence $\lambda_f = 0$. The time evolution (70) then becomes

$$\begin{aligned}
\Gamma(B_d(t) \to f) \ &\propto \ e^{-\Gamma_d t}\left[\cosh\frac{\Delta\Gamma_d\, t}{2} + \cos(\Delta m\, t)\right], & (73)\\
&\propto \ e^{-\Gamma_L t} + e^{-\Gamma_H t} + \text{ oscillating terms }, & (74)
\end{aligned}$$

so that for semileptonic decays, we have $b_{SL} = 0$. Note that $b = 0$ is true for all self-tagging modes, so that all the arguments below for semileptonic modes hold true also for all the self-tagging decay modes.

For the decays to CP eigenstates that proceed only through tree processes (and have zero or negligible penguin contribution), we have $\lambda_f = \pm(1-\delta^d)e^{-2i\beta}$ (the two signs "+" and "−" correspond to CP-even and CP-odd final states respectively). Then (70) gives

$$\begin{aligned}
\Gamma(B_d(t) \to f) \ &\propto \ e^{-\Gamma_d t}\left[\cosh\frac{\Delta\Gamma_d\, t}{2} \mp \cos(2\beta)\sinh\frac{\Delta\Gamma_d\, t}{2} \pm \sin(2\beta)\sin(\Delta m\, t)\right], & (75)\\
&\propto \ e^{-\Gamma_L t}(1 \pm \cos(2\beta)) + e^{-\Gamma_H t}(1 \mp \cos(2\beta)) + \text{ oscillating terms }, & (76)
\end{aligned}$$

where we have neglected the small corrections due to δ^d. Thus, for CP eigenstates, we have $b_{CP+} = +\cos(2\beta)$ and $b_{CP-} = -\cos(2\beta)$.

The ratio between the two lifetimes $\tau_{CP\pm}$ and τ_{SL} is then

$$\frac{\tau_{SL}}{\tau_{CP\pm}} = 1 \pm \frac{\cos(2\beta)}{2}\frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2\right] \ . \tag{77}$$

The measurement of these two lifetimes should be able to give us a value of $\Delta\Gamma_d$, since $|\cos(2\beta)|$ will already be known to a good accuracy by that time.

Note that it is also possible to measure the ratio of the lifetimes τ_{CP-} and τ_{CP+}:

$$\frac{\tau_{CP-}}{\tau_{CP+}} = 1 + \cos(2\beta)\frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2\right] \quad . \tag{78}$$

Although the deviation of the ratio from 1.0 in this case is larger by a factor of 2, using the effective semileptonic lifetime instead of one of the CP eigenstates would still be the favoured method. This is because the CP specific decay modes of B_d (e.g. $J/\psi K_{S(L)}, D^+D^-$) have smaller branching ratios than the semileptonic modes. In addition, the "semileptonic" data sample may be enhanced by including the self-tagging decay modes (e.g. $D_s^{(*)+}D^{(*)-}$) that also have large branching ratios. After 5 years of LHC, we should have about 5×10^5 events of $J/\psi K_S$, whereas the number of semileptonic decays at LHCb alone that will be directly useful in the lifetime measurements is expected to be more than 10^6 per year, even with conservative estimates of efficiencies.

The accurate measurement of the ratio of lifetimes also resolves the discrete ambiguity $\beta \leftrightarrow \pi/2 - \beta$ that stays when β is determined through the measurement of $\sin(2\beta)$. This is explained in detail in Sec. 4.2.

3.3 Transversity angle distribution in $B_d \to J/\psi K^*$

The decays $B_d \to VV$ (where VV is a flavour-blind final state consisting of two vector mesons) take place both through CP-even and CP-odd channels. Since the angular information is available here in addition to the time information, these decay modes are not subject to the constraints of the theorem in Sec. 3.1, and quantities sensitive linearly to $\Delta\Gamma_d/\Gamma_d$ can be obtained through a single final state. This cancels out many systematic uncertainties, and hence these modes can be extremely useful as long as the direct CP violation is negligible, and we can disentangle the CP-even and CP-odd final states from each other. This separation can indeed be achieved through the transversity angle distribution ([16]–[18]).

We illustrate the procedure with the example of $B_d \to J/\psi(\ell^+\ell^-)K^*(K_S\pi^0)$. The most general amplitude for the decay $B \to J/\psi K^*$ is given in terms of the polarizations $\epsilon_{J/\psi}, \epsilon_{K^*}$ of the two vector mesons:

$$A(B_d \to J/\psi K^*) = A_0 \left(\frac{m_{K^*}}{E_{K^*}}\right) \epsilon^{*L}_{J/\psi}\epsilon^{*L}_{K^*} - \frac{A_\parallel}{\sqrt{2}}\, \epsilon^{*T}_{J/\psi} \cdot \epsilon^{*T}_{K^*} - i\frac{A_\perp}{\sqrt{2}}\, \epsilon^*_{J/\psi} \times \epsilon^*_{K^*} \cdot \hat{\mathbf{p}} \quad , \tag{79}$$

where E_{K^*} is the energy of the K^* and $\hat{\mathbf{p}}$ the unit vector in the direction of K^* in the J/ψ rest frame. The superscripts L and T represent the longitudinal and transverse components respectively. Since the direct CP violation in this mode is negligible, the amplitudes A_0 and $A_\parallel$ are CP-even, whereas $A_\perp$ is CP-odd. Let us define the angles as follows. Let the x axis be the direction of K^* in the J/ψ rest frame, and the z axis be perpendicular to the decay plane of $K^* \to K_S\pi^0$, with the positive y direction chosen such that $p_y(K_S) \geq 0$. Then we define (θ, φ) as the decay direction of ℓ^+ in the J/ψ rest frame and ψ as the angle made by K_S with the x axis in the K^* rest frame.

Here θ is the transversity angle, *i.e.* the angular distribution in θ can separate CP-even and CP-odd components of the final state. The angular distribution is given by [19]

$$\frac{d\Gamma[B_d \to J/\psi(\ell^+\ell^-)K^*(K_S\pi^0)]}{d\cos\theta} = \frac{3}{8}|\mathcal{A}_+(t)|^2(1+\cos^2\theta) + \frac{3}{4}|\mathcal{A}_-(t)|^2\sin^2\theta \qquad (80)$$

where $|\mathcal{A}_+(t)|^2 \equiv |A_0(t)|^2 + |A_\parallel(t)|^2$ is the CP-even component and $|\mathcal{A}_-(t)|^2 \equiv |A_\perp(t)|^2$ the CP-odd one. These two components can be separated from the angular distribution (80) through a likelihood fit or through the method of angular moments [19, 20][6].

The time evolutions of the CP-even and CP-odd components are given by

$$\begin{aligned} |\mathcal{A}_+(t)|^2 &= |\mathcal{A}_+(0)|^2 \left[\cos^2\beta\, e^{-\Gamma_L t} + \sin^2\beta\, e^{-\Gamma_H t} + e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\right] \quad , &(81)\\ |\mathcal{A}_-(t)|^2 &= |\mathcal{A}_-(0)|^2 \left[\sin^2\beta\, e^{-\Gamma_L t} + \cos^2\beta\, e^{-\Gamma_H t} - e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\right] \quad . &(82)\end{aligned}$$

These are the same as the time evolutions in (76). The difference in the untagged lifetimes of the two components,

$$\frac{\tau_{CP-}}{\tau_{CP+}} = 1 + \cos(2\beta)\frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2\right] \quad , \qquad (83)$$

is linear in the lifetime difference $\Delta\Gamma_d$. In addition to the measurement of $\Delta\Gamma_d/\Gamma_d$, this channel can also resolve the discrete ambiguity in β (see Sec. 4.2).

The disentanglement of the CP-even and CP-odd components from the angular distribution is a statistically efficient process [20]. In fact, in the B_s system, the angular distribution of $B_s \to J/\psi(\ell^+\ell^-)\phi(K^+K^-)$ can be used for determining the lifetime difference $\Delta\Gamma_s$, and is the preferred mode for measuring this quantity.

The mode $J/\psi K^*$ suffers from the presence of a π^0 in the final state, which may be missed by the detector, thus introducing a source of systematic error that needs to be minimized.

3.4 Untagged asymmetry between $B \to J/\psi K_S$ and $J/\psi K_L$

Two of the decay modes of B_d that have been well explored experimentally (because of their usefulness in measuring β) are $B \to J/\psi K_S$ and $J/\psi K_L$. Here we show that the time-dependent asymmetry between the decay rates of these modes is a quantity linear in $\Delta\Gamma_d/\Gamma_d$, and therefore within the domain of experimental feasibility.

[6] In [18] we suggested to use the CP-odd–CP-even interference in the decay $B \to J/\psi K^*$ to disentangle measure the value of $\Delta\Gamma_d/\Gamma_d$. However, it involves tagged measurements in addition to two- or three-angle distributions, and hence is not as attractive as the untagged measurements described here. We show the analysis explicitly in the appendix A where we also discuss the resolution of the discrete ambiguity in β through this mode.

Let us define

$$A(B_d \to J/\psi K_S) = A_S, \quad A(\bar{B}_d \to J/\psi K_S) = \bar{A}_S,$$

$$A(B_d \to J/\psi K_L) = A_L, \quad A(\bar{B}_d \to J/\psi K_L) = \bar{A}_L,$$

so that using

$$|K_S\rangle = (1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle \ , \quad |K_L\rangle = (1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle \ , \tag{84}$$

we can write (with the phase convention $\mathrm{Arg}(q/p) = 0$)

$$\begin{aligned} A_S \quad &= A_L = \quad A e^{i\beta}(1 + a_p e^{i\theta} e^{i\Delta\gamma})(1+\epsilon) \ , \\ \bar{A}_S \quad &= -\bar{A}_L = \quad A e^{-i\beta}(1 + a_p e^{i\theta} e^{-i\Delta\gamma})(1-\epsilon) \ , \end{aligned} \tag{85}$$

where $a_p e^{i\theta} e^{i\Delta\gamma}$ is the ratio of contributions that involve the CKM factors $V_{cb}^* V_{cs}$ and $V_{tb}^* V_{td}$ respectively. The latter contribution (penguin) is highly suppressed with respect to the former one (tree): the value of a_p is less than a percent. Here θ is the strong phase and $\Delta\gamma \equiv \mathrm{Arg}(V_{tb}^* V_{ts}/V_{cb}^* V_{cd}) \approx -0.015$ in the SM. From (68), (69) and (85), we get

$$\lambda_S = -\lambda_L \approx -(1-\delta^d) e^{-2i\beta}(1 - 2\epsilon - 2i \sin\Delta\gamma \ a_p e^{i\theta}) \approx -e^{-2i\beta}(1 - 2\bar{\epsilon}) \ , \tag{86}$$

where $2\bar{\epsilon} \equiv 2\epsilon + \delta^d$ (here δ^d is as defined in (7)). The term involving a_p is neglected since it is proportional to the product of two small quantities, a_p and $\sin\Delta\gamma$.

When the production asymmetry between B_d and $\bar{B}_d$ is zero (as is the case at the B factories), the untagged rate of decay is

$$\begin{aligned} \Gamma[B_{un} \to J/\psi K_S(K_L)] \ \approx \ & \mathcal{N} |A_S|^2 (1 - 2\mathrm{Re}(\bar{\epsilon})) e^{-\Gamma_d t} \times \\ & \left[\cosh\left(\frac{\Delta\Gamma_d t}{2}\right) + \mathcal{A}_{\Delta\Gamma_d} \sinh\left(\frac{\Delta\Gamma_d t}{2}\right)\right] . \end{aligned} \tag{87}$$

The only difference between the decay to K_S and that to K_L is the sign of $\mathcal{A}_{\Delta\Gamma}$:

$$\mathcal{A}_{\Delta\Gamma}(K_S) = -\mathcal{A}_{\Delta\Gamma}(K_L) = \cos(2\beta) - 2 \ \mathrm{Im}(\bar{\epsilon}) \sin(2\beta) \ . \tag{88}$$

The untagged time-dependent asymmetry between $B_{un} \to J/\psi K_S$ and K_L is

$$A(K_L, K_S) \ \equiv \ \frac{\Gamma(B_{un}(t) \to J/\psi K_S) - \Gamma(B_{un}(t) \to J/\psi K_L)}{\Gamma(B_{un}(t) \to J/\psi K_S) + \Gamma(B_{un}(t) \to J/\psi K_L)} \tag{89}$$

$$= \ \cos(2\beta) \tanh\left(\frac{\Delta\Gamma_d t}{2}\right) [1 - 2 \ \mathrm{Im}(\bar{\epsilon}) \tan(2\beta)] \tag{90}$$

$$\approx \ \cos(2\beta) \tanh\left(\frac{\Delta\Gamma_d t}{2}\right) \ . \tag{91}$$

Thus, the measurement of this asymmetry will enable us to determine $\Delta\Gamma_d$, given sufficient statistics and a measurement of $\sin 2\beta$.

The factor limiting the accuracy of the above asymmetry is the measurement of $\Gamma(B_{un}(t) \to J/\psi K_L)$. The determination of this quantity requires the knowledge of the decay widths of K_S and K_L, in at least one of their decay channels. Although the width of K_S is known to 0.1%, the current accuracy in the width of K_L is only about 0.8%. The statistical error may decrease by a factor of 3–4 when the complete set of KTeV data is analysed, but the systematic errors are expected to dominate and one may have to wait for future kaon experiments to give us a measurement of $\Delta\Gamma_d$ through these channels. This is an example where the accurate measurement in the B system is dependent on an accurate measurement in the K system.

This $B \to J/\psi K_{S(L)}$ analysis can also be applied for $B \to \phi K_{S(L)}$, although the branching ratio, and hence the number of events, in the case of $B \to \phi K_{S(L)}$ would be much smaller.

3.5 Effect on the measurement of $\sin(2\beta)$

The time-dependent CP asymmetry measured through the "gold-plated" mode $B_d \to J/\psi K_S$ is [21, 22]

$$\mathcal{A}_{CP} = \frac{\Gamma[\bar{B}_d(t) \to J/\psi K_S] - \Gamma[B_d(t) \to J/\psi K_S]}{\Gamma[\bar{B}_d(t) \to J/\psi K_S] + \Gamma[B_d(t) \to J/\psi K_S]} \tag{92}$$

$$\approx \sin(\Delta m_d t)\sin(2\beta) \quad , \tag{93}$$

which is valid when the lifetime difference, the direct CP violation, and the mixing in the neutral K mesons is neglected. As the accuracy of this measurement increases, the corrections due to these factors will need to be taken into account. Keeping only linear terms in the small quantities $\bar{\epsilon}, a_p, \Delta\gamma, \Delta\Gamma$, we get

$$\mathcal{A}_{CP} = \sin(\Delta m t)\sin(2\beta)\left[1 - \sinh\left(\frac{\Delta\Gamma_d t}{2}\right)\cos(2\beta)\right] \tag{94}$$

$$+2\mathrm{Re}(\bar{\epsilon})\left[-1 + \sin^2(2\beta)\sin^2(\Delta m t) - \cos(\Delta m t)\right] \tag{95}$$

$$+2\mathrm{Im}(\bar{\epsilon})\cos(2\beta)\sin(\Delta m t) \quad . \tag{96}$$

The first term in (94) represents the standard approximation used (93) and the correction due to the lifetime difference $\Delta\Gamma_d$. The rest of the terms [(95) and (96)] are corrections due to the CP violation in B–$\bar{B}$ and K–$\bar{K}$ mixings. Note that the corrections due to the direct CP violation in the decay of $B_d \to J/\psi K_S$ (those involving a_p) are absent to this leading order.

In the future experiments that aim to measure β to an accuracy of 0.005 [7], the correction terms need to be taken into account. With $\bar{\epsilon} \approx 2 \times 10^{-3}$ and $\Delta\Gamma_d t \sim \Delta\Gamma_d/\Gamma_d \approx 3 \times 10^{-3}$, the corrections due to $\Delta\Gamma_d$ will form a major part of the systematic error, which can be taken care of by a simultaneous fit to $\sin(2\beta), \Delta\Gamma_d$ and $\bar{\epsilon}$. The BaBar collaboration tries to measure the coefficient of $\cos(\Delta m t)$ in (95), while neglecting the other correction terms [23].

When the measurements are accurate enough to measure the $\cos(\Delta mt)$ term, the rest of the terms would also have come within the domain of measurability.

3.6 Tagged measurements

Until now, we have discussed only the untagged measurements. Taking into account the oscillating part of the time evolution of the decay rate, we have the decay rate in general as

$$g(t) = f(t) + Ce^{-\Gamma_d t} \sin(\Delta mt + \Phi) \ , \tag{97}$$

where $f(t)$ is the untagged decay rate as defined in (63), C a constant and Φ a phase. The lifetime of the oscillating part is an additional lifetime measurement, which opens up the possibility of being able to determine $\Delta\Gamma_d/\Gamma_d$ through only one final state (and without angular distributions as in Sec. 3.3).

In the case of the semileptonic decays, this strategy fails since the semileptonic width measured with the untagged sample is

$$\Gamma_{SL} = \frac{(\Gamma_L + \Gamma_H)\Gamma_L\Gamma_H}{(\Gamma_L)^2 + (\Gamma_H)^2} = \Gamma_d \frac{1 - \frac{1}{4}\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right)^2}{1 + \frac{1}{4}\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right)^2} \tag{98}$$

so that

$$\Gamma_{SL}/\Gamma_d = 1 + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2\right] \quad . \tag{99}$$

Thus the semileptonic decays provide sensitivity only to quadratic terms in $\Delta\Gamma_d/\Gamma_d$.

However, the untagged lifetime measured through the decay to a CP eigenstate is

$$\tau_{CP\pm} \approx \frac{1}{\Gamma_d}\left(1 \mp \frac{\cos(2\beta)}{2}\frac{\Delta\Gamma_d}{\Gamma_d}\right) \quad , \tag{100}$$

so that it differs from the lifetime of the oscillating part ($\tau_d \equiv 1/\Gamma_d$) by terms linear in $\Delta\Gamma_d/\Gamma_d$. Thus, the tagged measurements of a CP-even or CP-odd final state (D^+D^-, $J/\psi K_S$, $J/\psi K_L$, etc.) can measure $\Delta\Gamma_d/\Gamma_d$ by themselves.

The mistag fraction is the main limiting factor on the accuracy of this measurement, and the tagging efficiency limits the number of events available. It is indeed possible that the τ_d measurement through the semileptonic decays will be more accurate than that through the oscillating part of the CP-specific final state. This then reduces to the method suggested in Sec. 3.2. For further experimental details on a tagged measurement of $\Delta\Gamma_d/\Gamma_d$ we refer the reader to reference [24].

4 Lifetime differences in B_s and B_d systems

The calculations of the lifetime difference in B_d (as performed here) and in the B_s system (as in [2, 3]) run along similar lines. However, there are some subtle differences involved, due to the values of the different CKM elements involved, which have significant consequences. In particular, whereas the upper bound on the value of $\Delta\Gamma_s$ (including the effects of new physics) is the value of $\Delta\Gamma_s(SM)$ [25], the upper bound on $\Delta\Gamma_d$ involves a multiplicative factor in addition to $\Delta\Gamma_d(SM)$. Also, whereas the difference in lifetimes of CP-specific final states in the B_s system cannot resolve the discrete ambiguity in the B_s–$\bar{B}_s$ mixing phase, the corresponding measurement in the B_d system can resolve the discrete ambiguity in the B_d–$\bar{B}_d$ mixing phase. Let us elaborate on these two differences in this section.

4.1 Upper bounds on $\Delta\Gamma_{d(s)}$ in the presence of new physics

For convenience, let us define $\Theta_q \equiv \mathrm{Arg}(\Gamma_{21})_q$, $\Phi_q \equiv \mathrm{Arg}(M_{21})_q$, where $q \in \{d, s\}$. Then we can write

$$\Delta\Gamma_q = -2|\Gamma_{21}|_q \cos(\Theta_q - \Phi_q) \ . \tag{101}$$

Since the contribution to Γ_{21} comes only from tree diagrams, we expect the effect of new physics on this quantity to be very small and we neglect it. We therefore take $|\Gamma_{21}|_q$ and Θ_q to be unaffected by new physics. On the other hand, the mixing phase Φ_q appears from loop diagrams and can therefore be very sensitive to new physics.

Let us first consider the B_s system. Here Γ_{21} may be written in the form

$$\Gamma_{21}(B_s) = -\mathcal{N}[(V_{cb}^* V_{cs})^2 f(z,z) + 2(V_{cb}^* V_{cs})(V_{ub}^* V_{us}) f(z,0) + (V_{ub}^* V_{us})^2 f(0,0)] \tag{102}$$

where $\mathcal{N}$ is a positive normalization constant and $f(x,y)$ are the hadronic factors that do not depend on the CKM matrix elements. In the limit $z \equiv m_c^2/m_b^2 \to 0$, we get $f(z,z) = f(z,0) = f(0,0)$. Since the f's are smooth functions of z and $z \approx 0.1$, the actual values of all f's involved in (102) are well approximated by $f(0,0)$ to an accuracy of about 30% (this may be seen explicitly by computing the f's numerically). Thus, all the f's have similar magnitude. On the other hand, the CKM elements involved in (102) obey the hierarchy $(V_{cb}^* V_{cs})^2 \sim \lambda^4$, $(V_{cb}^* V_{cs})(V_{ub}^* V_{us}) \sim \lambda^6$, $(V_{ub}^* V_{us})^2 \sim \lambda^8$. The term involving $(V_{cb}^* V_{cs})^2$ then dominates in (102), and we can write

$$\Gamma_{21}(B_s) = -\mathcal{N}(V_{cb}^* V_{cs})^2 f(z,z)[1 + O(\lambda^2)] \ . \tag{103}$$

Since the f's are real positive functions, we have $\Theta_s \approx \pi + \mathrm{Arg}(V_{cb}^* V_{cs})^2$. Then,

$$\Delta\Gamma_s = 2|\Gamma_{21}|_s \cos[\mathrm{Arg}(V_{cb}^* V_{cs})^2 - \Phi_s] \ . \tag{104}$$

In SM, $\Phi_s = \mathrm{Arg}(V_{tb}^* V_{ts})^2$, therefore the argument of the cosine term in (104) is given by $\mathrm{Arg}[(V_{cb}^* V_{cs})^2/(V_{tb}^* V_{ts})^2] = -2\Delta\gamma \approx 0.03$. Thus in SM, we have

$$\Delta\Gamma_s(SM) = 2|\Gamma_{21}|_s \cos(2\Delta\gamma) \ . \tag{105}$$

The effect of new physics on $\Delta\Gamma_s$ can then be bounded by giving an upper bound on $\Delta\Gamma_s$:

$$\Delta\Gamma_s \leq \frac{\Delta\Gamma_s(SM)}{\cos(2\Delta\gamma)} \approx \Delta\Gamma_s(SM) \quad . \tag{106}$$

Thus, the value of $\Delta\Gamma_s$ can only decrease in the presence of new physics [25].

In the case of the B_d system, the situation is slightly different. As in the B_s case, we can write

$$\Gamma_{21}(B_d) = -\mathcal{N}[(V_{cb}^* V_{cd})^2 f(z,z) + 2(V_{cb}^* V_{cd})(V_{ub}^* V_{ud}) f(z,0) + (V_{ub}^* V_{ud})^2 f(0,0)] \tag{107}$$

where the normalizing factor $\mathcal{N}$ and the hadronic factors f are the same as in the B_s case in the limit of the U-spin symmetry (see [26]). U-spin breaking is known to be at the 15% level, which is sufficiently small for our purpose. All the f's are thus of similar magnitude. The CKM elements involved in (107) do not obey a hierarchy similar to the B_s case: we have $(V_{cb}^* V_{cd})^2 \sim (V_{cb}^* V_{cd})(V_{ub}^* V_{ud}) \sim (V_{ub}^* V_{ud})^2 \sim \lambda^6$. Then no single term in (107) can dominate. We can, however, use the unitarity of the CKM matrix[7] to rearrange (107) in the form

$$\begin{aligned}\Gamma_{21}(B_d) \;\; = \;\; & -\mathcal{N}\Big[(V_{cb}^* V_{cd})^2 [f(z,z) - 2f(z,0) + f(0,0)] \\ & + 2(V_{cb}^* V_{cd})(V_{tb}^* V_{td})[f(0,0) - f(z,0)] + (V_{tb}^* V_{td})^2 f(0,0)\Big] \quad .\end{aligned} \tag{108}$$

Note that in the limit of $z \to 0$, all the factors f are identical and hence the coefficients of $(V_{cb}^* V_{cd})^2$ and $(V_{cb}^* V_{cd})(V_{tb}^* V_{td})$ vanish. The last term in (108) is then left over as the dominating one, and we get

$$\Gamma_{21}(B_d) \approx -\mathcal{N}(V_{tb}^* V_{td})^2 f(0,0) \; . \tag{109}$$

The finite value of $z \approx 0.1$ may give corrections of more than 30% to this value. Numerically we get $\mathcal{F}_{ct} \equiv 2[f(0,0) - f(z,0)]/f(0,0) \approx 0.2$–$0.3$ and $\mathcal{F}_{cc} \equiv [f(z,z) - 2f(z,0) + f(0,0)]/f(0,0) \approx 0.01$–$0.02$. We therefore neglect the $\mathcal{F}_{cc}$ term to write

$$\Gamma_{21}(B_d) \approx -\mathcal{N}(V_{tb}^* V_{td})^2 f(0,0) \left[1 + \mathcal{F}_{ct} \frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right] \quad . \tag{110}$$

Defining $\delta f \equiv \mathcal{F}_{ct}(V_{cb}^* V_{cd})/(V_{tb}^* V_{td})$, we get $\Theta_d = \pi + \mathrm{Arg}(V_{tb}^* V_{td})^2 + \mathrm{Arg}(1+\delta f)$. Using (101), we then have

$$\Delta\Gamma_d \approx 2|\Gamma_{21}|_d \cos[\mathrm{Arg}(V_{tb}^* V_{td})^2 - \Phi_d + \mathrm{Arg}(1+\delta f)] \quad . \tag{111}$$

[7]We note that this assumption of the unitarity for a three-generation CKM matrix is quite general, because most popular new physics models, including supersymmetric models, preserve the three-generation CKM unitarity. The present CKM values, constrained from various experiments, are completely consistent with the unitarity for the three-generation CKM matrix. Moreover, one can show that the non-unitary effects within the three-generation CKM, which can stem from the fourth generation or E(6)-inspired models with one singlet down-type quark, are $\lesssim \lambda^4$, once we assume a Wolfenstein-type hierarchical structure for the extended CKM matrix.

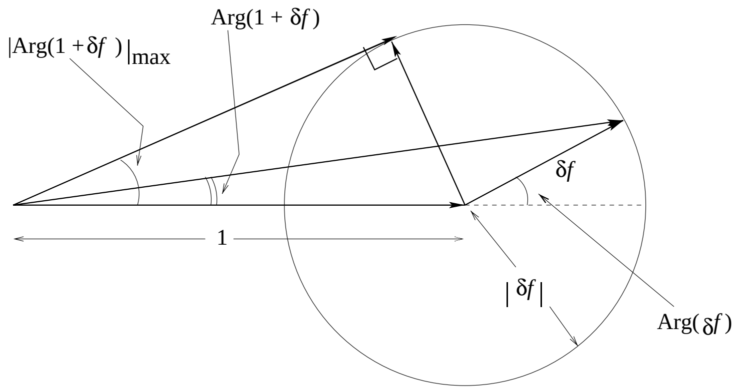

Figure 2: The geometrical proof of $|\mathrm{Arg}(1+\delta f)| \leq \sin^{-1}(|\delta f|)$

In SM, $\Phi_d = \mathrm{Arg}(V_{tb}^* V_{td})^2$, so that

$$\Delta\Gamma_d(SM) \approx 2|\Gamma_{21}|_d \cos[\mathrm{Arg}(1+\delta f)] \quad . \tag{112}$$

Using the fit obtained in [14], we have $|(V_{cb}^* V_{cd})/(V_{tb}^* V_{td})| < 1.35$ (95% C.L.). Then $|\delta f| < 0.4$ and we can use the geometrical relation (see Fig. 2):

$$|\mathrm{Arg}(1+\delta f)| \leq \sin^{-1}(|\delta f|) \tag{113}$$

to get $|\mathrm{Arg}(1+\delta f)| < 0.4$.

In the same spirit as in the B_s case, we can put an upper bound on $\Delta\Gamma_d$ in the presence of new physics:

$$\Delta\Gamma_d \leq \frac{\Delta\Gamma_d(SM)}{\cos[|\mathrm{Arg}(1+\delta f)|]} \leq 1.1\ \Delta\Gamma_d(SM) \quad . \tag{114}$$

Thus, in the case of the B_d system also, we have an upper bound (which may go down with more accurate information about the CKM elements) analogous to the one in the B_s system. The reasons behind the existence of these two upper bounds differ, however. Whereas in the B_s case it is due to the hierarchy in the CKM elements, in the B_d case it is due to the smallness of the hadronic terms $\mathcal{F}_{ct}$ and $\mathcal{F}_{cc}$. Note that whereas unitarity was not needed in the B_s case, the assumption that $(\Gamma_{21})_q$ is unaffected by new physics is required in both the cases.

In the limit $z \to 0$, from (103) and (109), we have

$$\frac{\Delta\Gamma_d}{\Delta\Gamma_s} \approx \left| \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cs}} \right|^2 \quad . \tag{115}$$

This is modified because of the finite value of z, the numerical value being approximately 0.03.

4.2 Discrete ambiguity in the mixing phase

The B_d–$\bar{B}_d$ mixing phase Φ_d is efficiently measured through the decay modes $J/\psi K_s$ and $J/\psi K_L$. If we take the new physics effects into account, the time-dependent asymmetry (92) is $\mathcal{A}_{CP} = \sin(\Delta M_d t)\sin(\Phi_d)$, which reduces to (93) in the SM, where $\Phi_d = 2\beta$. The measurement of $\sin(\Phi_d)$ still allows for a discrete ambiguity $\Phi_d \leftrightarrow \pi - \Phi_d$. Whenever a discrete ambiguity in β is referred to ($\beta \leftrightarrow \pi/2 - \beta$) in this paper (or in the literature), strictly speaking we are talking about the discrete ambiguity $\Phi_d \leftrightarrow \pi - \Phi_d$. In this section, we shall use the notation Φ_d instead of 2β in order to illustrate the comparison with the corresponding quantities in the B_s system.

Getting rid of the above discrete ambiguity is a way of uncovering a possible signal of new physics[8]. Ways to get rid of this ambiguity have been suggested in literature, using the comparison of CP asymmetries in $J/\psi K_S$ and $\pi\pi$ [28], time dependent CP asymmetries in $B_s \to \rho K_S$ [29] and in $B_s \to \pi K,\ KK$ [30], angular distributions and U-spin symmetry arguments [31], or cascade decays $B \to D \to K$ [32]. The measurement of Φ_d through the measurements involving $\Delta\Gamma_d$ is unique in the sense that it uses only untagged measurements.

In Sections 3.2 and 3.3, we have seen that the ratio of two effective lifetimes can enable us to measure the quantity $\Delta\Gamma_{obs(d)} \equiv \cos(2\beta)\Delta\Gamma_d/\Gamma$. In the presence of new physics, this quantity is in fact (see eq. 101)

$$\Delta\Gamma_{obs(d)} = -2(|\Gamma_{21}|_d/\Gamma_d)\cos(\Phi_d)\cos(\Theta_d - \Phi_d) \quad . \tag{116}$$

Solving (116) gives two solutions for $\Phi_d \in [0,\pi]$ in general: Φ_{1d} and Φ_{2d} such that $\tan(\Phi_{1d}) + \tan(\Phi_{2d}) = \tan(\Theta_d)$. As long as $\tan(\Theta_d) \neq 0$ (as is the case in the B_d system), $\sin(\Phi_{1d}) \neq \sin(\Phi_{2d})$. Therefore, only one of the solutions will correspond to the value of $\sin(\Phi_d)$ obtained through $\mathcal{A}_{CP}(J/\psi K_s)$ and will give the actual value of Φ_d. Combining the measurements of $\mathcal{A}_{CP}(J/\psi K_s)$ and $\Delta\Gamma_{obs(d)}$ thus gets rid of the discrete ambiguity in principle. In practice, this means knowing $|\Gamma_{21}|_d$ theoretically to a high precision and having to measure $\Delta\Gamma_{obs(d)}$ to sufficient accuracy to be able to distinguish between Φ_{1d} and Φ_{2d}. A complete NLO calculation is needed for the former. The latter may be achieved at the LHC using the effective lifetimes of decays to semileptonic final states and to $J/\psi K_s$.

Let us contrast this case with that in the B_s system. The corresponding time-dependent asymmetry is measured through the modes $J/\psi\phi$ or $J/\psi\eta^{(\prime)}$, which give the value of $\sin(\Phi_s)$, and therefore leave the discrete ambiguity $\Phi_s \leftrightarrow \pi - \Phi_s$ unresolved. The ratio of two effective lifetimes in the B_s system can enable us to measure the quantity

$$\begin{aligned} \Delta\Gamma_{obs(s)} &\equiv \cos(\Phi_s)\Delta\Gamma_s/\Gamma \\ &= -2|\Gamma_{21}|_s/\Gamma_s\cos(\Phi_s)\cos(\Theta_s - \Phi_s) \quad . \end{aligned} \tag{117}$$

[8] In SM, the value of Φ_d must match with the phase of the $b \to d$ penguin. However, the direct measurement of the latter phase is not theoretically clean [27], so the preferred way is to compare the measured value of Φ_d with the value of 2β determined through a fit for all the CKM parameters [14].

Since $\Theta_s \approx \pi + \mathrm{Arg}(V_{cb}^* V_{cs})^2 \approx \pi$, we have

$$\Delta\Gamma_{obs(s)} \approx 2|\Gamma_{21}|_s/\Gamma_s \cos^2(\Phi_s) \ . \tag{118}$$

This measurement thus still has the same discrete ambiguity $\Phi_s \leftrightarrow \pi - \Phi_s$ as in the $J/\psi\phi$ (or $J/\psi\eta^{(\prime)}$) case, and the discrete ambiguity in the B_s system is not resolved.

5 Summary and conclusions

It has been known for many years that the B_d system is a particularly good place to test the standard model explanation of CP violation through the unitary CKM matrix. The phase 2β involved in the $B_d - \bar{B}_d$ mixing is large, and hence the CP violation is expected to be larger in the B_d system in general, as compared to the K or the B_s system. This feature has already been exploited in various methods for extracting α, β and γ, the angles of the unitarity triangle, by measuring CP-violating rate asymmetries in the decays of neutral B_d mesons to a variety of final states. In particular, the precise measurement of $\sin(2\beta)$ from the theoretically clean decay modes $B_d(t) \to J/\psi K_S(K_L)$ is a test of the SM, as well as the opportunity to search for the presence of physics beyond the standard model.

The two mass eigenstates of the neutral B_d system — B_H and B_L — have slightly different lifetimes: the lifetime difference is less than a percent. At the present accuracy of measurements, this lifetime difference $\Delta\Gamma_d$ can well be ignored. As a result, the measurement and the phenomenology of $\Delta\Gamma_d$ has been neglected so far, as compared to the lifetime difference in the B_s system for example. However, with the possibility of experiments with high time resolution and high statistics, such as the electronic asymmetric B factories of BaBar, BELLE, and hadronic B factories of CDF, LHC and BTeV, this quantity starts becoming more and more relevant.

Taking the effect of $\Delta\Gamma_d$ into account is important in two aspects. On one hand, it affects the accurate measurements of crucial quantities like the CKM phase β and therefore must be measured in order to estimate and correct the error due to it. On the other hand, the nonzero value of $\Delta\Gamma_d$ can resolve the discrete ambiguity in the measurement of β, which stays unresolved through $B_d(t) \to J/\psi K_S(K_L)$ if $\Delta\Gamma_d$ is ignored. Thus in addition to being the measurement of a well-defined physical quantity which can be compared with the theoretical prediction, the value of $\Delta\Gamma_d$ is crucial for getting a firm grip on our understanding of CP violation. It is therefore worthwhile to have a look at this quantity and make a realistic estimation of the possibility of its measurement, as we do in this paper.

We estimate $\Delta\Gamma_d/\Gamma_d$ including $1/m_b$ contributions and next-to-leading order QCD corrections. We keep terms upto an accuracy of 1% of the leading order contribution, and upto z^0 terms in the NLO contribution. We find that adding these corrections decreases the value of $\Delta\Gamma_d/\Gamma_d$ computed at the leading order by almost a factor of two. We get the final result as

$\Delta\Gamma_d/\Gamma_d = (3.1^{+1.0}_{-1.4}) \times 10^{-3}$, where for the central value we have used the preliminary values for the bag factors from the JLQCD collaboration. A conservative error estimation gives the approximate errors due to the uncertainties in the values of parameters as $\pm 0.06 \times 10^{-3}$ from the CKM parameters, $\pm 0.4 \times 10^{-3}$ from the bag parameters, $\pm 0.5 \times 10^{-3}$ from the mass of the b quark, and $\pm 0.1 \times 10^{-3}$ from the measured value of x_d. The breaking of naive factorization contributes an error of approximately $\pm 0.3 \times 10^{-3}$, and the error due to the z-expansion in the $1/m_b$ and penguin contributions is $\pm 0.05 \times 10^{-3}$. The major sources of error are the scale dependence ($^{+0.4}_{-1.0} \times 10^{-3}$) and the z^1 and higher order terms in the NLO contribution ($\pm 0.6 \times 10^{-3}$). The last error is more subtle, and we have used the corresponding expansion in the B_s system to estimate it. This error can be reduced significantly if a complete NLO calculation is performed.

The most obvious way of trying to measure the lifetime difference is through the semileptonic decays, however it runs into major difficulties. If only the non-oscillating (untagged) part of the time evolution of the decay is considered, we indeed have a combination of two exponential decays with different lifetimes. However, as we show in this paper, there is no observable quantity here that is linear in $\Delta\Gamma_d/\Gamma_d$. The time measurements allow us to determine the quantity $\tau_{SL} \equiv (1/\Gamma_d)[1 + \mathcal{O}(\Delta\Gamma_d/\Gamma_d)^2]$. This decay mode is thus sensitive only to quantities quandratic in $\Delta\Gamma_d/\Gamma_d$. So this method would involve measuring a quantity as small as $(\Delta\Gamma_d/\Gamma_d)^2 \sim 10^{-5}$, which is not practical. The lifetime of the oscillating part is also $1/\Gamma_d$, so adding the information from the oscillating part of the time evolution does not help at all. This problem arises for all self-tagging decays. Therefore, though self-tagging decays of B_d have significant branching ratios, they cannot by themselves be expected to give a measurement of $\Delta\Gamma_d/\Gamma_d$.

The time evolutions of B_d decaying into CP eigenstates also involve both the lifetimes, since the $B_d - \bar{B}_d$ mixing phase (2β) is large, which implies that the CP eigenstates are appreciably different from the lifetime eigenstates. As a result, it is not possible to find a final state to which the decay of B_d involves only one of the decay widths Γ_L and Γ_H. The non-oscillating part of the time evolution of decays to CP eigenstates gives a quantity $\tau_{CP_\pm} \equiv (1/\Gamma_d)[1 \pm (\cos(2\beta)/2)\Delta\Gamma_d/\Gamma_d + \mathcal{O}(\Delta\Gamma_d/\Gamma_d)^2]$, but the quantities Γ_d and $\Delta\Gamma_d$ cannot be separately determined through this measurement, and sensitivity to $(\Delta\Gamma_d/\Gamma_d)^2$ is necessary. (Indeed, we explicitly prove a general theorem that shows that, for isotropic decays of B_d to any final state, the untagged measurements can only be sensitive to $(\Delta\Gamma_d/\Gamma_d)^2$.)

The oscillating part of the time evolution to CP eigenstates has a lifetime $1/\Gamma_d$ (to an accuracy of $\mathcal{O}(\Delta\Gamma_d/\Gamma_d)^2$). Therefore, if this lifetime is measured accurately, it can be combined with the measurement of $\tau_{CP_\pm}$ through the untagged part to get a measurement linear in $\Delta\Gamma_d/\Gamma_d$. However, the need for tagging, and consequent mistagging errors, reduce the efficiency of this method.

A viable option, perhaps the most efficient among the ones considered here, is to compare the measurements of the untagged lifetimes τ_{SL} and $\tau_{CP_\pm}$. Since τ_{SL} is in fact the lifetime for all self-tagging decays, and the branching ratios for self-tagging decays of B_d are much larger than the decays to CP eigenstates, we expect that the most useful combination will be the measurement of τ_{SL} through self-tagging decays and that of τ_{CP_+} through $B_d \to J/\psi K_S$.

The untagged asymmetry between $B_d \to J/\psi K_S$ and $B_d \to J/\psi K_L$ is a particular case of using the combination of measurements of τ_{CP_+} and τ_{CP_-}, which we analyze in detail. The effects of CP violation in the mixing and decay of B_d, as well as the indirect CP violation in the K system has been taken into account. The limiting factor for the utility of this method is the poorly known width of K_L, which may be improved through the future kaon experiments. This is one of the cases where the accurate measurement of a quantity in B system is dependent on the accurate measurements of a quantity in K system.

Since the theorem referred to above — about a single untagged decay being sensitive only to $(\Delta\Gamma_d/\Gamma_d)^2$ — applies only to isotropic decays, decays of the type $B \to VV$ can still be used by themselves to determine quantities linear in $\Delta\Gamma_d/\Gamma_d$. A promising example is $B_d \to J/\psi(\to \ell^+\ell^-)\ K^*(\to K_s\pi^0)$. The CP-odd and CP-even components in the final state can be disentangled through the transversity angle distribution, and both τ_{CP_+} and τ_{CP_-} can be determined through the same decay. Since there is only one final state, many systematic errors are reduced. The only undesirable feature of this decay mode is the presence of π^0 in the final state, which may be missed, especially in the hadronic machines. The three angle distribution of the same decay mode can also be used to obtain $\Delta\Gamma_d/\Gamma_d$ through the interference between CP-even and CP-odd final states. The three angle method, described in the appendix, is however not as efficient as the single angle distribution, since one has to use tagged decays and more number of parameters need to be fitted.

We also point out the interlinked nature of the accurate measurements of β and $\Delta\Gamma_d/\Gamma_d$ through the conventional gold-plated decay. In the future experiments that aim to measure β to an accuracy of 0.005 or better, the corrections due to $\Delta\Gamma_d$ will form the major part of the systematic error, which can be taken care of by a simultaneous fit to $\sin(2\beta), \Delta\Gamma_d$ and $\bar{\epsilon}$, a combination of CP violation in mixing in the B_d and K system.

All the combinations of untagged decay modes discussed here involve measuring the quantity $(\cos(2\beta)/2)\Delta\Gamma_d/\Gamma_d$, wherein the value of $\Delta\Gamma_d/\Gamma_d$ also depends on β. The complete dependence on β is of the form $\cos(2\beta)\cos(\Theta_d - 2\beta)$, where Θ_d is the phase of Γ_{21}. This form is not invariant under $\beta \leftrightarrow \pi/2 - \beta$, so that the discrete ambiguity in β that stays in its usual determination through $\sin(2\beta)$ is resolved. Note that this feature is unique to the B_d system — in the B_s system for example, this ambiguity would still stay unresolved since the corresponding value of Θ_s vanishes. In the three angle distribution in $B_d \to J/\psi(\to \ell^+\ell^-)\ K^*(\to K_s\pi^0)$ as discussed in the appendix, the dependence on β has another form, again not invariant under $\beta \leftrightarrow \pi/2 - \beta$, and hence the discrete ambiguity in β can be resolved.

It is known that, if $(\Gamma_{21})_s$ is unaffected by new physics, then the value of $\Delta\Gamma_s$ in the B_s system is bounded from above by its value as calculated in the SM. In the B_d system, this statement does not strictly hold true. However, if $(\Gamma_{21})_d$ is unaffected by new physics and the unitarity of the 3×3 CKM matrix holds, then an upper bound on the value of $\Delta\Gamma_d$ may be derived as $\Delta\Gamma_d \leq 1.1\Delta\Gamma_d(SM)$.

With the high statistics and accurate time resolution of the upcoming experiments, the measurement of $\Delta\Gamma_d$ seems to be in the domain of measurability. And given the rich phenomenology that comes with it, it is certainly a worthwhile endeavor.

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A CP-odd–CP-even interference in $B \to J/\psi K^*$

For completeness, we also discuss some further opportunities to measure $\Delta\Gamma_d/\Gamma_d$ with the help of the decay $B \to J/\psi(\ell^+\ell^-)K^*(K_S\pi^0)$ — in addition to the favoured one discussed in Section 3.3. Here we use the tagged measurements and multiple-angle distributions. The angular resolution at CDF as well as LHC is expected to be accurate enough so that the efficiency of this method is limited mainly by tagging.

The complete angular distribution in the three physical angles θ, φ, ψ is given as [16, 19]:

$$\frac{d^3\Gamma[B_d(t) \to J/\psi(\to l^+l^-)K^*(\to K_S\pi^0)]}{d\cos\theta\; d\varphi\; d\cos\psi} \propto \frac{9}{32\pi}\Big[\, 2|A_0(t)|^2\cos^2\psi(1-\sin^2\theta\cos^2\varphi)$$
$$+\sin^2\psi\{|A_\parallel(t)|^2(1-\sin^2\theta\sin^2\varphi)+|A_\perp(t)|^2\sin^2\theta - \mathrm{Im}\;(A_\parallel^*(t)A_\perp(t))\sin 2\theta\sin\varphi\}$$
$$+\frac{1}{\sqrt{2}}\sin 2\psi\{\;\mathrm{Re}\;(A_0^*(t)A_\parallel(t))\sin^2\theta\sin 2\varphi + \mathrm{Im}\;(A_0^*(t)A_\perp(t))\sin 2\theta\cos\varphi\;\}\,\Big]\;. \tag{119}$$

The time evolutions of the coefficients of the six angular terms are

$$|A_0(t)|^2 \;=\; |A_0(0)|^2\left[\cos^2\beta\; e^{-\Gamma_L t}+\sin^2\beta\; e^{-\Gamma_H t}+e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\right] \tag{120}$$

$$
\begin{aligned}
|A_\parallel(t)|^2 &= |A_\parallel(0)|^2 \left[\cos^2\beta\ e^{-\Gamma_L t} + \sin^2\beta\ e^{-\Gamma_H t} + e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\right] && (121)\\
|A_\perp(t)|^2 &= |A_\perp(0)|^2 \left[\sin^2\beta\ e^{-\Gamma_L t} + \cos^2\beta\ e^{-\Gamma_H t} - e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\right] && (122)\\
\mathrm{Re}\{A_0^*(t)A_\parallel(t)) &= |A_0(0)||A_\parallel(0)|\cos(\delta_2-\delta_1)\Big[\cos^2\beta\ e^{-\Gamma_L t} + \sin^2\beta\ e^{-\Gamma_H t} \\
&\qquad + e^{-\Gamma_d t}\sin(\Delta M_d t)\sin(2\beta)\Big] && (123)\\
\mathrm{Im}\{A_\parallel^*(t)A_\perp(t)\} &= |A_\parallel(0)||A_\perp(0)|\Big[e^{-\Gamma_d t}\left\{\sin\delta_1\cos(\Delta M_d t) - \cos\delta_1\sin(\Delta M_d t)\cos(2\beta)\right\} \\
&\qquad - \frac{1}{2}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos\delta_1\sin(2\beta)\Big]\ , && (124)\\
\mathrm{Im}\{A_0^*(t)A_\perp(t)\} &= |A_0(0)||A_\perp(0)|\Big[e^{-\Gamma_d t}\left\{\sin\delta_2\cos(\Delta M_d t) - \cos\delta_2\sin(\Delta M_d t)\cos(2\beta)\right\} \\
&\qquad - \frac{1}{2}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos\delta_2\sin(2\beta)\Big]\ , && (125)
\end{aligned}
$$

where $\delta_1 = \mathrm{Arg}(\mathrm{A}_\parallel^*(0)\mathrm{A}_\perp(0))$, and $\delta_2 = \mathrm{Arg}(\mathrm{A}_0^*(0)\mathrm{A}_\perp(0))$. Note that even before reaching the precision to be able to separate Γ_H and Γ_L, the above can already measure the value of $\sin(2\beta)$ through the time evolutions (120)–(122) and the value of $\sin\delta_1, \sin\delta_2$ through (123,124,125). The discrete ambiguity $\beta \leftrightarrow \pi/2 - \beta$ would remain unresolved in the absence of the lifetime separation, since the sign of $\cos\delta_{1(2)}$, and hence the sign of $\cos(2\beta)$, is undetermined. This sign may be determined in the following manner. The non-oscillating parts of (124) and (125) are

$$
\begin{aligned}
\mathcal{C}_1 &\equiv [\mathrm{Im}\{A_\parallel^*(t)A_\perp(t)\}]_{NO} = -\frac{1}{2}|A_\parallel(0)||A_\perp(0)|\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos\delta_1\sin(2\beta)\ , && (126)\\
\mathcal{C}_2 &\equiv [\mathrm{Im}\{A_0^*(t)A_\perp(t)\}]_{NO} = -\frac{1}{2}|A_0(0)||A_\perp(0)|\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos\delta_2\sin(2\beta)\ , && (127)
\end{aligned}
$$

which are also the non-oscillating parts of the corresponding terms for the charge conjugate decay $\bar{B}_d \to J/\psi K^*(\to K_S\pi^0)$. The signs of the quantities $\mathcal{C}_{1(2)}$ are the same as the sign of $\cos\delta_{1(2)}$, since $\Gamma_L > \Gamma_H$ and $\sin(2\beta) > 0$. This in turn establishes the sign of $\cos(2\beta)$ through (124), (125). Note that, in the absence of any B_d–$\bar{B}_d$ production asymmetry, the non-oscillating parts of (124,125) are exactly the quantities measured if the initial B meson was not tagged. Then the determination of the signs of $\mathcal{C}_{1(2)}$ would need neither tagging nor time measurements.

Note that there is in principle no need to determine both $\mathcal{C}_1$ and $\mathcal{C}_2$. Moreover, since it is sufficient to determine the sign of only one of $\cos\delta_{1(2)}$, the use of either (124) or (125) is sufficient. In fact, let us show that under certain circumstances, the ambiguity in β may be resolved without having to measure the angle ψ at all. We only need the two-angle distribution [16]

$$
\begin{aligned}
\frac{d^3\Gamma[B_d \to (\ell^+\ell^-)_{J/\psi}(K_S\pi^0)_{K^*}]}{d\cos\theta\ d\varphi\ dt} &= \frac{3}{8\pi}[|A_0|^2(1-\sin^2\theta\cos^2\varphi) + |A_\parallel|^2(1-\sin^2\theta\sin^2\varphi) \\
&+ |A_\perp|^2\sin^2\theta - \mathrm{Im}\ (A_\parallel^* A_\perp)\sin 2\theta\sin\varphi]\ , && (128)
\end{aligned}
$$

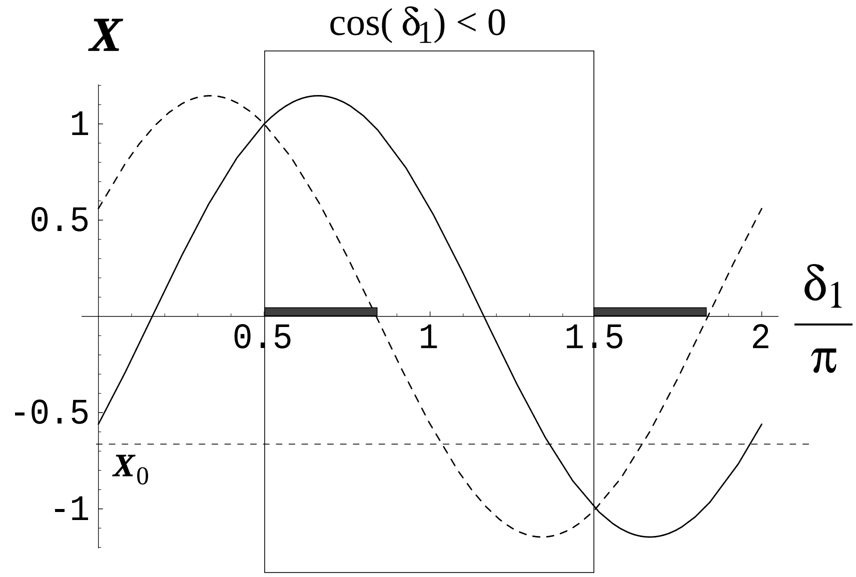

Figure 3: The ambiguities in the solution of eq. (129). The solid (dashed) curve stands for $\cos(2\beta) = +(-)0.8$. The region inside the box corresponds to $\cos\delta_1 < 0$. The intersections of the horizontal line $X = X_0$ with the curves represent the fourfold discrete ambiguity. If δ_1 is in the shaded region, the sign of $\cos\delta_1$ determines the sign of $\cos(2\beta)$.

with the time evolutions of the terms given by eqs. (120)–(124). The first three equations determine $\sin(2\beta)$, and the oscillating part of (124) further determines the value of δ_1 upto a fourfold discrete ambiguity in general, twofold due to the sign of $\cos(2\beta)$ and twofold due to the oscillatory nature of the time evolution. This can be seen in Fig. 3, which shows two curves

$$X = \bar{\Gamma}\sin(\delta_1) - \Delta m \cos(\delta_1)\cos(2\beta) \tag{129}$$

with different signs of $\cos(2\beta)$, and the corresponding four solutions for $X = X_0$. The non-oscillatory part of (124), i.e. the sign of $\mathcal{C}_1$, determines the sign of $\cos\delta_1$, thus selecting the region inside or outside the box in the figure. If both the solutions corresponding to this sign of $\cos\delta_1$ correspond to the same sign of $\cos(2\beta)$, then the sign of $\cos(2\beta)$ is determined and the discrete ambiguity is resolved. This happens when the actual value of δ_1 lies in the shaded region of Fig. 3. This region corresponds to

$$\frac{\pi}{2} \leq \delta_1 \leq \frac{\pi}{2} + 2\theta \quad , \quad \frac{3\pi}{2} \leq \delta_1 \leq \frac{3\pi}{2} + 2\theta \quad , \text{ where } \quad \theta = \tan^{-1}\left(\frac{\Delta m|\cos(2\beta)|}{\bar{\Gamma}}\right) \quad . \tag{130}$$

With $\Delta m/\bar{\Gamma} \approx 0.7$ and $|\cos(2\beta)| \approx 0.8$, the region covers a fraction $1/\pi \approx 32\%$ of the total range of δ_1. In the remaining parameter space, the complete three-angle distribution (119) needs to be used.

The additional use of (123), (125) makes sure that this discrete ambiguity is absent. The three-angle distribution is thus a reliable way of getting the sign of $\cos(2\beta)$, and hence resolving the discrete ambiguity in β. Using

$$\int dt\ \mathcal{C}_i \approx -\frac{1}{2}|A_X(0)||A_\parallel(0)|\frac{\Delta\Gamma}{\Gamma_d^2}\cos\delta_i\sin(2\beta) \quad , \tag{131}$$

the value of $\Delta\Gamma$ can also be determined.

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