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# Muon $g-2$ and Implications for Supersymmetry

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## Abstract

A brief review is given of the implications of the recent Brookhaven result on the muon anomaly ( $a_\mu$ ) for supersymmetry. We focus mainly on the implications of the recent results for the minimal supergravity unified model. We show that the observed difference implies the existence of sparticles most of which should become observable at the Large Hadron Collider. Further, as foreseen in works prior to the Brookhaven experiment the sign of the difference between experimental prediction of  $a_\mu$  and its Standard Model value determines the sign of the Higgs mixing parameter  $\mu$ . The  $\mu$  sign has important implications for the direct detection of dark matter. Implications of the Brookhaven result for other low energy phenomena are also discussed.

## 1 Introduction

In this talk we give a brief discussion of the recent developments in the analyses of the muon anomaly. First, we will discuss the recent Brookhaven National Laboratory (BNL) result on  $a_\mu[1]$  ( $a = (g - 2)/2$  where  $g$  is the gyromagnetic ratio) and its Standard Model prediction. We then discuss the supersymmetric electro-weak effects on  $a_\mu$ . We will also discuss briefly the effects of extra dimensions on  $a_\mu$ . Finally we will discuss the

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implications of the BNL result for the direct detection of supersymmetry. The anomalous moment is a sensitive probe of new physics since

$$a_l^{new-physics} \sim \frac{m_l^2}{\Lambda^2} \quad (1)$$

Thus  $a_\mu$  is more sensitive to new physics relative to  $a_e$  even though  $a_e$  is more accurately determined[2] since  $\frac{a_\mu}{a_e} \sim 4 \times 10^4$ . Regarding the experimental determination of  $a_\mu$  one has first the classic CERN experiment of 1977[3] which gave  $a_\mu^{exp} = 11659230(84) \times 10^{-10}$ . The error in this measurement was reduced by a factor of 2 in 1998 by the BNL experiment[4] which gave  $a_\mu^{exp} = 11659205(46) \times 10^{-10}$ , and the same error was further reduced by a factor of 3 by the most recent BNL result[1], i.e.,

$$a_\mu^{exp} = 11659203(15) \times 10^{-10} \quad (2)$$

The Standard Model contribution consists of several parts[5]

$$a_\mu^{SM} = a_\mu^{qed} + a_\mu^{EW} + a_\mu^{hadronic} \quad (3)$$

where the qed correction is computed to order  $\alpha^5$ [5]

$$a_\mu^{qed} = 11658470.57(.29) \times 10^{-10} \quad (4)$$

and  $a_\mu^{EW}$  including the one loop[6] and the two loop[5] Standard Model electro-weak correction is

$$a_\mu^{EW} = 15.2(0.4) \times 10^{-10} \quad (5)$$

The most difficult part of the analysis relates to the hadronic contribution. It consists of several parts: the  $\alpha^2$  hadronic vacuum polarization contribution, the  $\alpha^3$  hadronic correction, and the light-by-light contribution. The  $\alpha^2$  hadronic vacuum polarization contribution can be related to observables. Specifically one can write

$$a_\mu^{had}(vac.pol.) = \left(\frac{1}{4\pi^3}\right) \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_h(s) \quad (6)$$

where  $\sigma_h(s) = \sigma(e^+e^- \rightarrow hadrons)$  and  $K(s)$  is a kinematical factor. The integral in Eq.(6) is dominated by the low energy part, i.e., the part up to 2 GeV, which correspondingly is also very sensitive to errors in the input data. In the evaluations of Eq.(6)

one uses a combination of experimental data at low energy and a theoretical (QCD) extrapolation in the high energy tail. The analysis of  $a_\mu^{had}(vac.pol.)$  is the most contentious part of the analysis. In computing the difference  $a_\mu^{exp} - a_\mu^{SM}$ , BNL used the result of Davier and Høker[7], i.e.,  $a_\mu^{had}(vac.pol.) = 692.4(6.2) \times 10^{-10}$ . However, other estimates have appeared more recently and we will mention these later. The  $\alpha^3$  hadronic correction can also be related to observables but is generally small with a correspondingly small error[8], i.e.,  $\Delta a_\mu^{had}(vac.pol.) = -10.1(.6) \times 10^{-10}$ . The light-by-light hadronic correction is the second most contentious part of  $a_\mu^{SM}$ . This part cannot be related to any observables and is thus a purely theoretical construct. In the free quark model it evaluates to a positive contribution. However, more realistic analyses give a negative contribution[9], i.e.,  $\Delta a_\mu^{had}(light - by - light) = -8.5(2.5) \times 10^{-10}$ . This result which is though more reliable than the result from the free quark model, still has a degree of model dependence. Overall, however,  $\Delta a_\mu^{had}(light - by - light)$  is not the controlling factor in interpreting the BNL result unless, of course, its sign is reversed. The total result then is  $a_\mu^{had}(total) = a_\mu^{had}(vac.pol.) + \Delta a_\mu^{had}(vac.pol.) + \Delta a_\mu^{had}(light - by - light)$  which gives

$$a_\mu^{hadronic} = 673.9(6.7) \times 10^{-10} \quad (7)$$

Together one finds,

$$a_\mu^{SM} = 11659159.7(6.7) \times 10^{-10} \quad (8)$$

and a 2.6 sigma deviation of experiment from theory,

$$a_\mu^{exp} - a_\mu^{SM} = 43(16) \times 10^{-10}. \quad (9)$$

After the new  $g - 2$  result from Brookhaven became available, there have been several reanalyses of the hadronic uncertainty[10, 11, 12, 13, 14]. Thus, e.g., the analysis of Ref.[11] gives  $\Delta = 33.3(17.1)$  and of Ref.[12] gives  $\Delta = (37.7 \pm (15.0)_{exp} \pm (15.6)_{th})$  where  $\Delta = (a_\mu^{exp} - a_\mu^{SM}) \times 10^{10}$ . One finds that the difference  $(a_\mu^{exp} - a_\mu^{SM})$  in these analyses is somewhat smaller and the error somewhat larger compared to the result of Eq.(9). Similar trends are reported in the analyses of Refs.[10, 13]. An interesting assessment of the hadronic contribution and the possibilities for improvement in the future is given in Ref.[14]. For the discussion of the rest of this paper we will assume the validity of Eq.(9).

One may ask what is the nature of new physics in view of Eq.(9). Some possibilities that present themselves are supersymmetry, compact extra dimensions, muon compositeness, technicolor, anomalous W couplings, new gauge bosons, lepto-quarks and radiative muon masses. We shall focus here mostly on supersymmetry as the possible origin of the difference observed by the BNL experiment. Supersymmetry has many attractive features. It helps to stabilize the hierarchy problem with fundamental Higgs, and it leads to the unification of the gauge coupling constants consistent with the LEP data. To extract meaningful results from SUSY models, however, one needs a mechanism of supersymmetry breaking. There are several mechanisms proposed for the breaking of supersymmetry such as gravity mediated, gauge mediated, anomaly mediated etc. We focus in this paper mainly on the gravity mediated models, i.e., the supergravity (SUGRA) unified models[15]. In the minimal version of this model based on a flat Kähler potential, i.e., mSUGRA, the SUSY breaking sector is described by the parameters  $m_0$ ,  $m_{\frac{1}{2}}$ ,  $A_0$ ,  $\tan\beta$  and  $sign(\mu)$  where  $m_0$  is the universal scalar mass,  $m_{\frac{1}{2}}$  is the universal gaugino mass,  $A_0$  is the universal trilinear coupling,  $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$  where  $H_2$  gives mass to the up quark and  $H_1$  gives mass to the down quark and the lepton, and  $\mu$  is the Higgs mixing parameter. The use of the curved Kähler potential results in a SUGRA model with non-universalities consisting of the minimal set of soft SUSY parameters and additional parameters which, for example, describe deviations from universality in the Higgs sector and in the third generation sector.

Some of the interesting features of SUGRA models include the fact that the radiative breaking constraints of the electro-weak symmetry leads to the lightest neutralino being the lightest supersymmetric particle (LSP) and thus under the constraint of R parity the lightest neutralino is a possible candidate for dark matter over most of the parameter space of the model. Also, analyses in SUGRA models show that the lightest Higgs must have a mass  $m_h \leq 130$  GeV under the usual assumptions of naturalness, i.e.,  $m_0, m_{\tilde{g}} < 1$  TeV. Finally, SUGRA models bring in new sources of CP violation which in any case are needed for baryogenesis. Thus, mSUGRA has two soft CP violating phases while, many more soft CP violating phases arise in non-universal SUGRA models and in the minimal supersymmetric standard model (MSSM). Regarding some of the other alternatives of SUSY breaking, one finds that the gauge mediated breaking (GMSB) does not produce a candidate for cold dark matter, while the anomaly mediated supersymme-

try breaking (AMSB) scenario now appears very stringently constrained by the BNL data when combined with its specially characteristic  $b \rightarrow s + \gamma$  constraint.

Our analysis in mSUGRA includes two loop renormalization group evolutions (RGE) for the couplings as well as soft parameters with the Higgs potential at the complete one-loop level [16] minimized at the scale  $Q \sim \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$  for radiative electroweak symmetry breaking. We have also included the SUSY QCD corrections [17] to the top quark (with  $M_t = 175$  GeV) and the bottom quark masses and we have used the code *FeynHiggs-Fast* [18] for the mass of the light Higgs boson.

## 2 SUSY contribution to $a_\mu$ at one loop

It is well known that  $a_\mu$  vanishes in the exact supersymmetric limit[19] and is non-vanishing only in the presence of supersymmetry breaking. Not surprisingly then  $a_\mu^{SUSY}$  (where  $a_\mu = a_\mu^{SM} + a_\mu^{SUSY}$ ) is sensitive to the nature of new physics[20]. Thus the analysis of  $a_\mu^{SUSY}$  requires a realistic model of supersymmetry breaking. The first such analysis within the well motivated SUGRA model was given in Refs.[21, 22]. We reproduce here partially the result of Ref.[22]

$$a_\mu^{SUSY} = a_\mu^{\tilde{W}} + a_\mu^{\tilde{Z}} \quad (10)$$

where  $a_\mu^{\tilde{W}}$  is the chargino contribution and  $a_\mu^{\tilde{Z}}$  is the neutralino contribution. The chargino contribution is typically the larger contribution over most of the parameter space and is

$$a_\mu^{\tilde{W}} = \frac{m_\mu^2}{48\pi^2} \frac{A_R^{(a)2}}{m_{\tilde{W}_a}^2} F_1 \left( \left( \frac{m_{\tilde{\nu}_\mu}}{m_{\tilde{W}_a}} \right)^2 \right) + \frac{m_\mu}{8\pi^2} \frac{A_R^{(a)} A_L^{(a)}}{m_{\tilde{W}_a}} F_2 \left( \left( \frac{m_{\tilde{\nu}_\mu}}{m_{\tilde{W}_a}} \right)^2 \right) \quad (11)$$

Here  $A_L(A_R)$  are the left(right) chiral amplitudes

$$A_R^{(1)} = -\frac{e}{\sqrt{2} \sin \theta_W} \cos \gamma_1; \quad A_L^{(1)} = (-1)^\theta \frac{em_\mu \cos \gamma_2}{2M_W \sin \theta_W \cos \beta} \quad (12)$$

$$A_R^{(2)} = -\frac{e}{\sqrt{2} \sin \theta_W} \sin \gamma_1; \quad A_L^{(2)} = -\frac{em_\mu \sin \gamma_2}{2M_W \sin \theta_W \cos \beta} \quad (13)$$

where  $\theta = 0(1)$  if the light chargino eigenvalue  $\lambda_1$  is positive (negative), and  $\gamma_{1,2}$  are mixing angles. We wish to point out that the most dominant contribution to  $a_\mu^{SUSY}$  comes from

the chirality non-diagonal lighter chargino part of  $a_\mu^{\tilde{W}}$ . First we note that for the most contributing term in the chargino part the coupling is proportional to  $1/\cos\beta$  ( $\sim \tan\beta$ ) and thus  $a_\mu$  increases almost linearly with  $\tan\beta$  [23, 24]; second due to the same dominant term the sign of  $a_\mu^{SUSY}$  is correlated strongly with the sign of  $\mu$  (we use here the  $\mu$  sign convention of Ref.[25]). It is easy to exhibit this by considering the eigenvalues  $\lambda_i$  ( $i=1,2$ ) of the chargino mass matrix (where we define  $\lambda_1$  as the eigenvalue corresponding to the lighter chargino) that  $\lambda_1 < 0$  for  $\mu > 0$  and  $\lambda_1 > 0$  for  $\mu < 0$  except for  $\tan\beta \sim 1$ , which leads to [23, 24]  $a_\mu^{SUSY} > 0, \mu > 0$  and  $a_\mu^{SUSY} < 0, \mu < 0$ .

### 3 Implications of Precise BNL Data

In the following analysis we assume CP conservation. Under this constraint and setting  $a_\mu^{SUSY} = a_\mu^{exp} - a_\mu^{SM}$ , we immediately find that the BNL data determines [26, 27, 28, 29]  $sign(\mu) = +1$ . In imposing the BNL constraint we use a  $2\sigma$  corridor

$$10.6 \times 10^{-10} < a_\mu^{SUSY} < 76.2 \times 10^{-10} \quad (14)$$

We utilize Eq.(14) in determining the allowed parameter space of mSUGRA using the one loop formula for which the chargino part is given by Eq.(11)[22]. [The leading order correction to one loop as computed in Ref.[30] gives a fractional contribution of  $-(4\alpha/\pi)\ln(M_S/m_\mu)$  where  $M_S$  is an average sparticle mass. This is typically less than 10% and is ignored in the analysis here.] In Fig.1 we give an analysis of this constraint in the  $m_0 - m_{\frac{1}{2}}$  plane for the case of  $\tan\beta = 10$ . One finds that there is now an upper limit on  $m_0$  and  $m_{\frac{1}{2}}$ . Interestingly, we find that the allowed region of the parameter space which is below the  $a_\mu^{SUSY} = \delta a_\mu^{SMALL} = 10.6 \times 10^{-10}$  line allows for a light Higgs consistent with the lower limit of about 115 GeV as given by the possible signal at LEP[31]. The white region close to  $m_{\frac{1}{2}}$  axis in Fig.1 is excluded for stau turning to be the LSP. The left side white region near the  $m_0$  axis is excluded by the constraints from chargino mass lower limit or radiative electro-weak symmetry breaking.

Next we discuss the case of a large  $\tan\beta$ , i.e.,  $\tan\beta = 55$ . This is the largest  $\tan\beta$  before one gets into a non-perturbative domain for most of the parameter space. The results of the analysis on the allowed parameter space in the  $m_0 - m_{\frac{1}{2}}$  plane are given

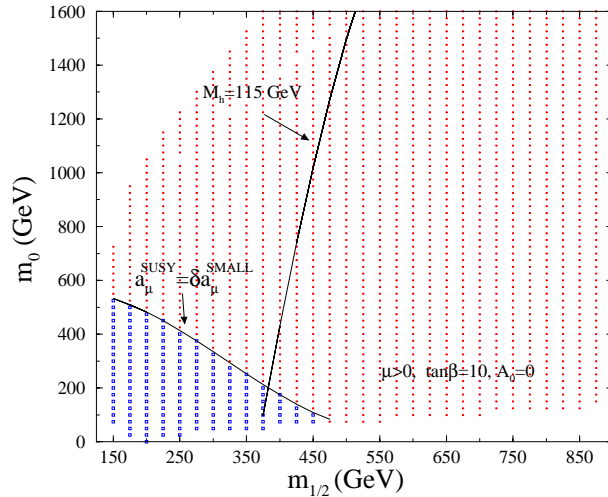


Figure 1: Upper limit in the  $m_0 - m_{\frac{1}{2}}$  plane implied by the BNL  $g - 2$  constraint for  $\tan \beta = 10$  indicated by the line  $a_{\mu}^{SUSY} = \delta a_{\mu}^{SMALL} = 10.6 \times 10^{-10}$ . The allowed region in the parameter consistent with constraint of Eq.(14) lies below this line. 115 GeV Higgs signal[31] is also indicated (from Ref.[26]).

in Fig.2 consistent with the constraints of Eq.(14). One finds that in this case there is both a lower limit and an upper limit and the allowed parameter space is the shaded area contained between the lines. The white region near  $m_0$  axis for larger  $m_0$  and smaller  $m_{\frac{1}{2}}$  values is excluded because of the chargino mass lower limit or the radiative electro-weak symmetry breaking constraints. The white region near  $m_{\frac{1}{2}}$  axis having smaller  $m_0$  values is excluded via the tachyonic stau constraint and the region just above this, corresponding to moderately large  $m_0$  values is excluded because of the CP-odd Higgs boson turning tachyonic at the tree level which is a large  $\tan \beta$  effect. Again the Higgs signal corresponding to the LEP lower limit is indicated by the solid near vertical line and is seen to lie in the allowed region of the parameter space.

A full analysis was carried out including also values of  $\tan \beta = 5, 30$  and  $45$  in Ref.[26]. We discuss the results of the full analysis from the point of view of sparticle spectra. In Fig.3 the upper limits in sneutrino-light chargino plane are given for  $\tan \beta = 5$  and  $10$ . A similar analysis is given for  $\tan \beta = 30, 45$ , and  $55$  in Fig.4. From Fig.3 and Fig.4 one finds, as expected, that there are strong correlations between the upper limits and  $\tan \beta$ . Using the entire data set in Fig.3 and Fig.4 one finds,

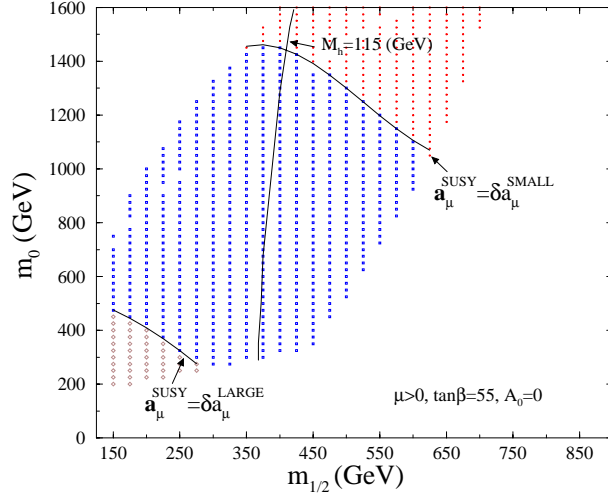


Figure 2: Upper and lower limits in the  $m_0 - m_{\frac{1}{2}}$  plane implied by the BNL  $g-2$  constraint for  $\tan\beta$  indicated by lines  $a_\mu^{SUSY} = \delta a_\mu^{SMALL} = 10.6 \times 10^{-10}$  and  $a_\mu^{SUSY} = \delta a_\mu^{LARGE} = 76.2 \times 10^{-10}$ . The allowed region in the parameter consistent with constraint of Eq.(14) lies between the lines. The 115 GeV Higgs signal is also indicated (from Ref.[26]).

$$m_{\tilde{\chi}_1^\pm} \leq 650 \text{ GeV}, m_{\tilde{\nu}_\mu} \leq 1.5 \text{ TeV} (\tan\beta \leq 55) \quad (15)$$

The corresponding limits in the  $m_0 - m_{\frac{1}{2}}$  plane are

$$m_{1/2} \leq 800 \text{ GeV}, m_0 \leq 1.5 \text{ TeV} (\tan\beta \leq 55) \quad (16)$$

The upper limits that arise in mSUGRA from the analysis of Ref.[26] are consistent with the fine tuning criteria (see, e.g., Ref.[32]), and are very encouraging from the point of view of discovery of superparticles at colliders. Thus LHC can discover squarks and gluinos up to 2 TeV[33, 34]. This means that essentially all of the squark and gluino mass spectrum allowed within mSUGRA by the Brookhaven  $g-2$  constraint will become visible at LHC[26, 34]. A comparison of the upper limits in the  $m_0 - m_{\frac{1}{2}}$  plane allowed by the  $g-2$  constraint vs the discovery potential of the LHC is given in the work of Baer et.al. in Ref.[34] and we reproduce one of the figures from that analysis here(see Fig.5).

Many further investigations of the implications of the BNL result have been carried out over the recent months[35, 36, 37, 38, 39] exploring the effects of the  $g-2$  constraint



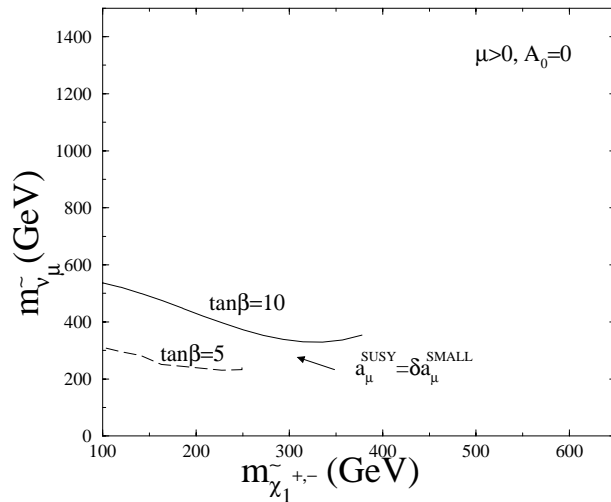


Figure 3: Upper limits in the  $m_{\tilde{\nu}_\mu} - m_{\tilde{\chi}_1^{+,-}}$  plane implied by the BNL  $g - 2$  constraint for  $\tan\beta = 5$  and 10, are indicated by the lines  $a_{\mu}^{SUSY} = \delta a_{\mu}^{SMALL} = 10.6 \times 10^{-10}$ . The allowed region in the parameter consistent with constraint of Eq.(14) lies below the lines (from Ref.[26]).

on a variety of low energy phenomena such as on  $b \rightarrow s + \gamma$ , dark matter, lepton flavor violation, trileptonic signal[41] and on other low energy SUSY signals. We briefly discuss two of these:  $b \rightarrow s + \gamma$  and dark matter. Regarding  $b \rightarrow s + \gamma$ , the Standard Model branching ratio for this process is estimated to be[42]  $B(b \rightarrow s + \gamma) = (3.29 \pm 0.33) \times 10^{-4}$ . Recent experiment gives[43]  $B(b \rightarrow s + \gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$  where the first error is statistical, and there are two types of systematic errors. Now it is well known that the imposition of the  $b \rightarrow s + \gamma$  constraint puts severe limits on the mSUGRA parameter space when  $\mu < 0$  eliminating most of the parameter space in this case[44, 45]. Thus had the sign of  $\mu$  from the BNL experiment turned out to be negative it would have eliminated most of the parameter space of the minimal model. On the other hand for the  $\mu > 0$  case one finds that the constraint  $b \rightarrow s + \gamma$  is much less severe. Thus most of the parameter space of mSUGRA in this case at least for small and moderately large values of  $\tan\beta$  is left unconstrained. For large values of  $\tan\beta$  nearing 50 the  $b \rightarrow s + \gamma$  constraint does become more stringent but a significant part of the parameter space is still allowed[37]. However, it has been emphasized in Ref.[39] that  $B(b \rightarrow s + \gamma)$  is not a pure observable and requires hard cuts for its extraction experimentally. This provides a note of caution

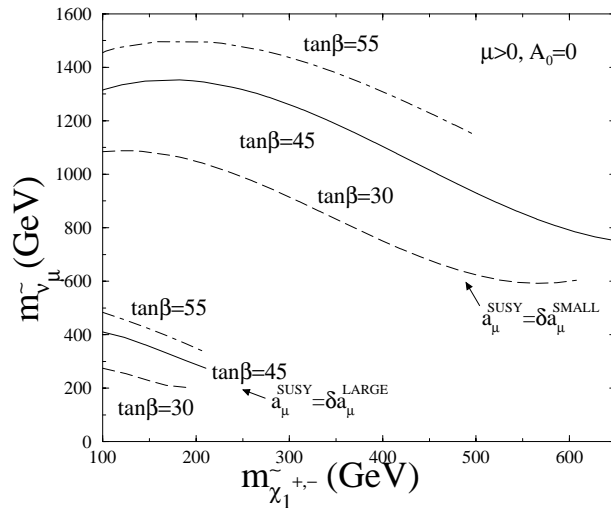


Figure 4: Upper limits and lower limits in the  $m_{\tilde{\nu}_\mu} - m_{\tilde{\chi}_{1}^{+,-}}$  plane implied by the BNL  $g-2$  constraint for  $\tan\beta = 30, 45$  and  $55$  are indicated by lines  $a_{\mu}^{SUSY} = \delta a_{\mu}^{SMALL} = 10.6 \times 10^{-10}$  and  $a_{\mu}^{SUSY} = \delta a_{\mu}^{LARGE} = 76.2 \times 10^{-10}$ . The allowed region in the parameter consistent with constraint of Eq.(14) lies between the lines (from Ref.[26]).

on imposing the  $B(b \rightarrow s + \gamma)$  constraint too stringently.

A closely related phenomenon that is sensitive to the sign of  $\mu$  is the analysis of dark matter. It was shown in the early days when the first measurement of  $b \rightarrow s + \gamma$  was made that the  $b \rightarrow s + \gamma$  branching ratio has a strong correlation with the neutralino-proton cross-sections in the direct detection of dark matter[44] in regard to the sign of  $\mu$ . This happens due to the fact that the neutralino-proton cross-sections are smaller for the case of  $\mu < 0$  than for the case of  $\mu > 0$ . Additionally, with the  $b \rightarrow s + \gamma$  constraint which eliminates most of the parameter space for  $\mu < 0$ , one finds that the neutralino-proton cross sections to be very small for the available region of parameters for this sign of  $\mu$ . Consequently, direct detection of neutralino dark matter is strongly disfavored for  $\mu < 0$  as opposed to what one finds for  $\mu > 0$ . Thus, the fact that the BNL experiment determines the  $\mu$  sign to be positive is indeed good news for the direct detection of dark matter[26, 29, 37, 38].

We now turn to a brief discussion of models other than mSUGRA. One such model is AMSB. The details of this model and procedure for its implementation can be found

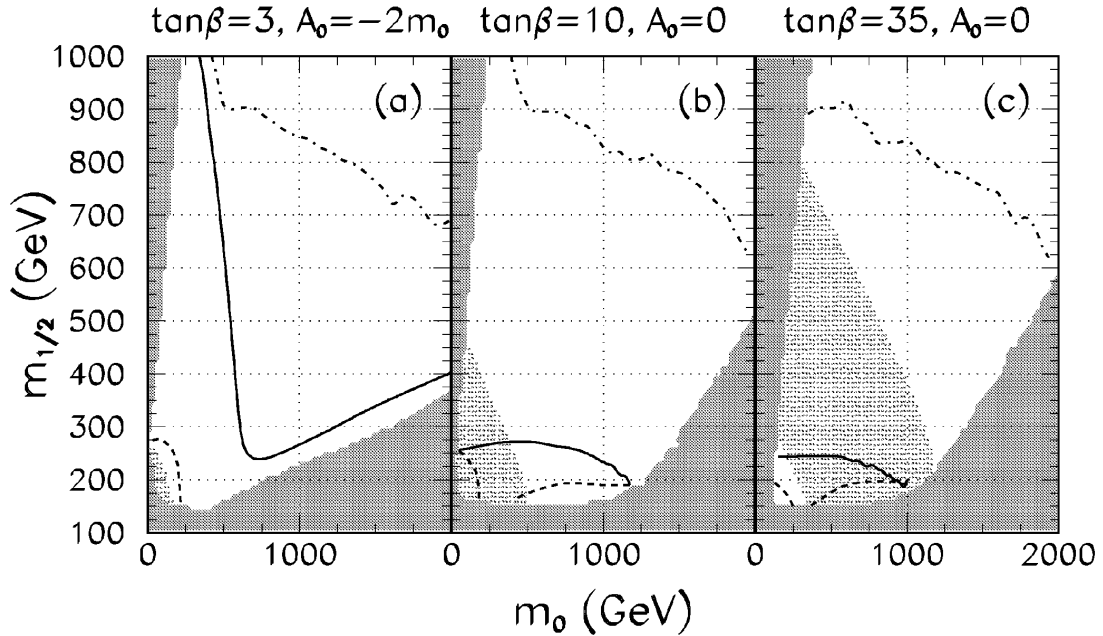


Figure 5: A plot of  $m_0$  vs.  $m_{1/2}$  parameter space in the mSUGRA model for  $\mu > 0$  and a)  $A_0 = -2m_0$  and  $\tan\beta = 3$ , b)  $A_0 = 0$  and  $\tan\beta = 10$  and c)  $A_0 = 0$  and  $\tan\beta = 35$ . The  $2\sigma$  region favored by the E821 measurement is shaded with dots. The region below the solid contour has  $m_h < 113.5$  GeV. The region below the dashed contour is accessible to Tevatron searches with  $25 \text{ fb}^{-1}$  of integrated luminosity, while the region below the dot-dashed contour is accessible via LHC sparticle searches with  $10 \text{ fb}^{-1}$  of integrated luminosity. (Taken from Ref.[34]).

in Ref.[46]. The analysis for this case is given in Ref.[26] where the upper limits in the sneutrino-chargino plane corresponding to three values of  $\tan\beta$ , i.e.,  $\tan\beta = 10, 30$ , and 40 (the maximum allowed) were analyzed which produced upper limits of  $m_{\tilde{\nu}_\mu} \leq 1.1$  TeV and  $m_{\chi_1^+} \leq 300$  GeV. These limits are lower than those of Eq.(15). Further, for  $\mu > 0$ , one finds that the constraint from  $b \rightarrow s + \gamma$  in this case excludes a significant amount of parameter space when the BNL  $g - 2$  constraint is imposed[28]. Further, analyses within the framework of the unconstrained supersymmetric standard model, and analyses within more general scenarios and their implications for colliders are given in works of Refs.[36, 40].

One possibility which must be discussed along with supersymmetry is that of contri-

butions from extra space time dimensions to  $g - 2$ . In Ref.[47] a class of realistic models with extra spacetime dimensions were considered (For reviews see Refs.[48]). It was shown that for the case of one extra dimension compactified on  $S^1/Z_2$  with matter and Higgs fields residing on the orbifolds and the gauge fields propagating in the bulk, the massless spectrum of the model coincides with the massless spectrum of MSSM. The Kaluza-Klein modes for W contribute to the Fermi constant and the current good agreement between the Standard Model determination of  $G_F$  and its experimental value leaves only a small error corridor in which the contributions from extra dimensions can reside. This constraint leads to a lower limit of about 3 TeV on the inverse compactified dimension and severely constrains the contribution of extra dimensions to the muon anomalous magnetic moment. One finds that for the case of one extra dimension, the contribution of Kaluza-Klein states is smaller than the supersymmetric contribution by more than two orders of magnitude. For the case of more than one extra dimension, the contribution to  $a_\mu$  is larger than for the case of one extra dimension but still significantly smaller than the one arising from supersymmetry. Thus we conclude that models with extra dimensions of the type considered in Ref.[47] do not create a strong background relative to the supersymmetric effects (see, however, the analysis of Ref.[49]).

## 4 Conclusions

In this review we have given a brief summary of the developments on the analyses of the muon anomaly. Implications of the difference  $a_\mu^{exp} - a_\mu^{SM}$  seen at BNL for supersymmetric models and specifically for mSUGRA were explored. An effect of the size seen at Brookhaven for  $a_\mu^{exp} - a_\mu^{SM}$  was already predicted within the SUGRA model in 1984 where it was found that the supersymmetric correction could be as large or larger than the Standard Model electro-weak correction[22]. Furthermore, we have also explored the implications of the BNL result for the direct detection of supersymmetry at accelerators and in dark matter searches. Thus a detailed analysis within mSUGRA of the BNL result using a  $2\sigma$  error corridor on the difference  $a_\mu^{exp} - a_\mu^{SM}$  leads to upper limits on sparticle masses which all lie below 2 TeV. Since the LHC can discover squarks and the gluino up to 2 TeV, most if not all of the sparticles should become visible at LHC. Further, it was pointed out that the BNL data determines the sign of  $\mu$  to be positive within the minimal

model which is very encouraging for direct dark matter searches. It was also pointed out that there is little chance of confusing the supersymmetric contribution to  $a_\mu$  with effects from extra dimensions. This is so at least in models where the Standard Model is obtained by a direct compactification of a five dimensional model on  $S^1/Z_2$  which gives a contribution to  $a_\mu$  from Kaluza-Klein excitations, significantly smaller than a typical supersymmetric contribution. The BNL data also imposes impressive constraints on CP phases. It was shown in Ref.[50] that the BNL constraint eliminates up to 60-90% of the parameter space in the  $\theta_\mu$  and  $\xi_2$  (phase of  $\tilde{m}_2$ ) plane. In the presence of phases the relationship between the sign of  $a_\mu^{SU5Y}$  and the phase of  $\mu$  may also be modified.

There is a significant amount of data from the run of 2000 which would be analyzed in the near future and BNL eventually hopes to measure  $a_\mu$  to an accuracy of  $4 \times 10^{-10}$ . Analyses including data from Beijing[51], from Novosibirsk[52] and additional  $\tau$  data from CLEO[53] should delineate the hadronic error more reliably. Further if deviation between theory and experiment persists at the current level after the analysis of the new data currently underway is carried out, and if also the error corridor shrinks then a signal for new physics will be undeniable. Such a signal interpreted as arising from supersymmetry then has dramatic new predictions for the direct observation of sparticles at accelerators. Further, if supersymmetry is the right explanation for such an effect, and there is a great bulk of theoretical reasoning in justification of this expectation, then the search for a fundamental Higgs boson becomes all the more urgent. Thus, the Brookhaven  $g - 2$  result further heightens the expectation for the observation of a light supersymmetric Higgs boson at RUNII of the Tevatron. Finally, we point out that the BNL constraint, specifically the positivity of  $\mu$  for a class of models, has an important implication for Yukawa unification in grand unified models[54] and this area is likely to be explored further in the future.

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