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# Wrapped fluxbranes

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## Abstract

We consider the construction of fluxbranes in certain curved geometries, generalizing the familiar construction of the Melvin fluxtube as a quotient of flat space. The resulting configurations correspond to fluxbranes wrapped on cycles in curved spaces. The non-trivial transverse geometry leads in some instances to solutions with asymptotically constant dilaton profiles. We describe explicitly several supersymmetric solutions of this kind. The solutions inherit some properties from their flat space cousins, like flux periodicity. Interestingly type IIA/0A fluxbrane duality holds near the core of these fluxbranes, but does not persist in the asymptotic region, precisely where it would contradict perturbative inequivalence of IIA/0A theories.

# 1 Introduction

The study of solitonic localized lumps of energy in string theory has turned out to provide far-reaching insights into its nature. This has been the case for BPS  $Dp$ -branes (or  $p$ -branes in general), and more recently for non-BPS D-branes. Recently, a different kind of objects, the fluxbranes, has started to receive attention [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. A flux  $p$ -brane (F $p$ -brane) is a higher dimensional generalization of a flux-tube, namely a  $(p + 1)$ -dimensional object with non-zero flux for a degree  $(9 - p)$  field-strength form  $F_{9-p}$  (or wedge product of field strength forms) in the transverse  $(9 - p)$ -dimensional space. Hence the core of the fluxbrane carries a  $F_{9-p}$  flux piercing the transverse space.

Remarkably, a non-trivial version of these objects, generalizing the Melvin flux-tube [12], can be constructed by imposing non-trivial identifications of points in (one dimension higher) flat space and performing a ‘skew’ Kaluza-Klein reduction [13]<sup>1</sup>. The simple lift to the higher dimensional theory of this fluxbrane allows to study its properties in detail. For instance, its instability and decay modes [13, 1, 2], duality between fluxbrane solutions in type IIA and 0A theories [4], and duality with other string configuration [6]. It has also allowed to use such fluxbranes as sources to trigger Myers’ dielectric effect on D-branes [14] and build new hopefully stable configurations [7, 8].

Unfortunately, the Melvin type solutions lead to blowing up dilaton values at large distance, which prevent for a more detailed understanding and interpretation of the solution from the lower dimensional perspective. Also, some directly constructed fluxbranes have non-normalizable transverse flux. In a sense these results stem from the fact that these fluxbranes live in flat space (which becomes curved after introduction of the flux). In this setup self-gravitation of the flux, although leads to the formation of a flux-brane, does not prevent it from spreading too much. One would expect that considering fluxbranes in a background gravitational field (to which self-gravity subsequently adds) would improve the above features.

The purpose of this paper is to discuss the construction of fluxbranes in certain curved spaces, by a simple generalization of the construction of the Melvin fluxbranes from flat space. In some situations, the flux is ‘confined’ by the background transverse geometry, so that asymptotically in the transverse space the corresponding solutions have constant dilaton, and vanishing flux. We find that in this situation certain prop-

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<sup>1</sup>See [6, 7, 5, 9] for discussion on the direct constructions of general fluxbrane solutions in supergravity.

erties of the flat space solution, like flux periodicity and certain dualities, remain, while for instance IIA/0A fluxbrane duality is substantially modified.

These solutions also illustrate that fluxbranes are dynamical objects, able to wrap non-trivial cycles in curved spaces. Consequently they should contain world-volume zero modes associated to deformations of the cycles, and a rich dynamics which remains unexplored.

The paper is organized as follows. In Section 2 we present a generalized version of the construction of e.g. type IIA fluxbranes in a spacetime  $\mathbf{X}_{10}$  by modding out  $\mathbf{X}_{11} = \mathbf{R} \times \mathbf{X}_{10}$  by a simultaneous shift in the additional dimension and an arbitrary  $U(1)$  isometry in  $\mathbf{X}_{10}$ . In Section 3 we review the case of rotational isometries in flat space  $\mathbf{X}_{10}$ , which leads to the Melvin fluxbrane and some simple generalizations. We point out an unnoticed connection with fluxbranes at orbifold singularities. In Sections 4,5 we use our generalized construction to build fluxbranes wrapped on cycles in curved spaces. We present explicit examples of IIA F5-branes in Taub-NUT space, and on a 3-cycle in the  $G_2$  holonomy manifold in [15]. These solutions have asymptotically constant dilaton and vanishing flux. We also show that wrapping does not guarantee the latter features, and give examples of wrapped fluxbranes with blowing up dilaton. In Section 6 we briefly discuss examples where the isometry in  $\mathbf{X}_{10}$  has no fixed points, and construct a fluxbrane in an AdS compactification. In Section 7 we discuss IIA/0A fluxbrane duality in models with constant asymptotic dilaton. Finally, Section 8 contains our final remarks.

## 2 General Strategy

In this section we provide a generalization of the familiar procedure [13, 1, 2] to obtain fluxbranes in a  $d$ -dimensional theory as quotients of simpler configurations in  $d + 1$  dimensions. For concreteness we phrase the discussion in terms of IIA/M theory language, even though it is clear how to use the results to build e.g. NS-NS fluxbranes.

We consider M-theory on a space  $\mathbf{X}_{11} = \mathbf{R} \times \mathbf{X}_{10}$ , with  $\mathbf{X}_{10}$  admitting a  $U(1)$  isometry parametrized by  $\tau$ . The general form of the metric is

$$ds_{11}^2 = dx_{10}^2 + f(x) (d\tau + f_\mu(x) dx^\mu)^2 + g_{\mu\nu} dx^\mu dx^\nu \quad (2.1)$$

where  $x$  denotes the set of  $x^\mu$ 's,  $\mu = 0, \dots, 8$ , and  $x, \tau$  parametrize  $\mathbf{X}_{10}$ , and  $x_{10}$  parametrizes  $\mathbf{R}$ . The isometry generated by the Killing vector  $\partial_\tau$ , and we normalize the period of  $\tau$  to  $2\pi$ .

Consider modding by  $x_{10} \rightarrow x_{10} + 2\pi R_{10}$ ,  $\tau \rightarrow \tau + 2\pi B$ . We are interested in performing a KK reduction along the Killing  $R_{10}\partial_{10} + B\partial_\tau$ . Defining the adapted coordinate  $\tilde{\tau} = \tau - \chi x_{10}$ , with  $\chi = B/R_{10}$  the metric becomes

$$ds_{11}^2 = dx_{10}^2 + f(x) (d\tilde{\tau} + \chi dx_{10} = f_\mu dx^\mu)^2 + g_{\mu\nu} dx^\mu dx^\nu \quad (2.2)$$

In these coordinates, the KK reduction is simply along the Killing vector  $\partial_{10}$ . In order to obtain the ten-dimensional quantities after the reduction, we compare with the KK ansatz metric

$$ds_{11}^2 = e^{4\Phi/3} (dx_{10} + R_{10}A_\mu dx^\mu)^2 + e^{-2\Phi/3} ds_{10}^2 \quad (2.3)$$

where  $\Phi$  and  $A$  are the ten-dimensional dilaton and RR 1-form fields, respectively. After some algebra, one obtains

$$e^{4\Phi/3} = \Lambda(x) \quad ; \quad R_{10}A = \Lambda^{-1} f(x) \chi (d\tilde{\tau} + f_\mu dx^\mu) \quad (2.4)$$

$$ds_{10}^2 = \Lambda^{-1/2} f(x) (d\tilde{\tau} + f_\mu dx^\mu)^2 + \Lambda^{1/2} (g_{\mu\nu} dx^\mu dx^\nu + d\vec{x}^2)$$

with  $\Lambda = 1 + \chi^2 f(x)$ .

The resulting ten-dimensional solution has non-zero RR 1-form field strength, related to the variations of the geometric features of the original  $U(1)$  isometry orbits in  $\mathbf{X}_{10}$ . When the resulting field strength configuration forms a lump, the solution is interpreted as a fluxbrane, generalizing the familiar concept of fluxtubes.

The prototypical case of these solutions, reviewed in next section, is given by starting with flat space  $\mathbf{X}_{11} = \mathbf{R} \times \mathbf{M}_{10}$ , and using a rotational isometry in  $\mathbf{M}_{10}$ . The size of the  $U(1)$  orbits increases steadily as one moves away from the fixed locus of the rotation, which implies the resulting solution is a fluxbrane spanning the (image after KK reduction of the) fixed plane. Unfortunately, the growing of the orbit size has no bound, and leads to a unbounded dilaton growth driving the final solution to strong coupling at large distances away from the fluxbrane.

New more general possibilities, leading to wrapped branes of different kinds, are discussed in subsequent sections.

### 3 Flat space fluxbranes

In this section we apply the strategy above to review the construction of Melvin fluxbranes (and generalizations) in flat space, essentially following [2].

### 3.1 The IIA F7-brane

Let us review the construction of the Melvin fluxbrane solution in IIA string theory starting from flat space in M-theory <sup>2</sup>. Consider M-theory in flat space, with metric

$$ds_{11}^2 = dx_{10}^2 + d\rho^2 + \rho^2 d\varphi^2 + d\vec{x}^2 \quad (3.1)$$

where  $\rho, \varphi$  are polar coordinates in an  $\mathbf{R}^2$  and  $\vec{x} \in \mathbf{M}^8$ . Consider performing the identification

$$\begin{aligned} x_{10} &\rightarrow x_{10} + 2\pi R_{10} \\ \varphi &\rightarrow \varphi + 2\pi B \end{aligned} \quad (3.2)$$

Namely, a  $2\pi$  shift identification along the Killing  $R_{10}\partial_{10} + B\partial_{\varphi}$ . This action breaks all the supersymmetries of the configuration. Comparing with (2.1) we have

$$\tau = \varphi \quad ; \quad f = \rho^2 \quad ; \quad f_{\mu} = 0 \quad ; \quad g_{\mu\nu} dx^{\mu} dx^{\nu} = d\rho^2 + d\vec{x}^2 \quad (3.3)$$

Applying those results, we define the adapted coordinate  $\tilde{\varphi} = \varphi - (B/R_{10})x_{10}$ , and replacing into (2.5) we obtain

$$e^{4\Phi/3} = 1 + \frac{B^2 \rho^2}{R_{10}^2} \quad ; \quad A_{\tilde{\varphi}} = \frac{B\rho^2}{R_{10}^2 + B^2 \rho^2} \quad (3.4)$$

$$ds_{10}^2 = \Lambda^{1/2} d\rho^2 + \Lambda^{-1/2} \rho^2 d\tilde{\varphi}^2 + \Lambda^{1/2} d\vec{x}^2 \quad (3.5)$$

with  $\Lambda = 1 + B^2 \rho^2 / R_{10}^2$ . The solution describes a 8-dimensional Poincare invariant configuration of non-vanishing RR 1-form field strength flux turned on in the  $(\rho, \tilde{\varphi})$  plane. Near the origin the metric is smooth and has small curvature, and the RR field strength is approximately constant. This configuration is a F7-brane. At large  $\rho$  the fluxbrane induces strong coupling, thus affecting the validity of the description. This property makes the interpretation of the solution difficult in certain regimes. As discussed in the introduction, the root of this problem lies in the use of isometries with growing orbit length in the base space.

The weakly coupled region is at  $\rho \ll R_{10}/B$ , while the ten-dimensional approximation holds for  $\rho \gg R_{10}$ . Hence, for both conditions to hold one needs  $B \ll 1$ , weak field strength. Below we discuss certain duality properties, which typically involve strong/weak field strength relations, and therefore imply either strong coupling or breakdown of the ten-dimensional approximation.

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<sup>2</sup>Our conventions differ slightly from other references.

### 3.2 Lower dimensional F-branes

For completeness, let us discuss the construction of other fluxbranes with non-vanishing flux for the  $2k$ -form  $F \wedge \dots \wedge F$  in their  $(10 - 2k)$ -dimensional core, denoted  $F(9 - 2k)$ -branes. Let us write the 11-dimensional flat metric as

$$ds_{11}^2 = dx_{10}^2 + \sum_{i=1}^k (d\rho_i^2 + \rho_i^2 d\varphi_i^2) + d\vec{x}^2 \quad (3.6)$$

where  $\rho, \varphi$  are polar coordinates in the  $i^{\text{th}}$   $\mathbf{R}^2$  and  $\vec{x} \in \mathbf{M}^{10-2k}$ . Let us perform the identification

$$\begin{aligned} x_{10} &\rightarrow x_{10} + 2\pi R_{10} \\ \varphi_i &\rightarrow \varphi_i + 2\pi B_i \end{aligned} \quad (3.7)$$

Equivalently, defining complex coordinates  $z_j = \rho_j e^{i\varphi_j}$ , the action on  $\mathbf{R}^{2k}$  is  $z_j \rightarrow e^{iB_j} z_j$ . For generic choices of  $B_i$  this action breaks all the supersymmetries of the configuration. However, for actions on  $\mathbf{R}^{2k}$  lying in a subgroup of  $SU(k)$  some supersymmetry is preserved.

In this case it is simpler to repeat the calculation instead of using the result in Section 2. Defining the adapted coordinates

$$\tilde{\varphi}_i = \varphi_i - \frac{B_i}{R_{10}} x_{10} \quad (3.8)$$

the metric becomes

$$ds_{11}^2 = dx_{10}^2 + \sum_i [d\rho_i^2 + \rho_i^2 (d\tilde{\varphi}_i + \frac{B_i}{R_{10}} dx_{10})^2] + d\vec{x}^2 \quad (3.9)$$

Performing the KK reduction along  $\partial_{10}$ , and comparing with (2.3). We obtain

$$\begin{aligned} e^{4\varphi/3} = \Lambda \quad ; \quad A_{\tilde{\varphi}_i} = B_i \rho_i^2 / (R_{10}^2 + \sum_{j=1}^k B_j^2 \rho_j^2) \quad (3.10) \\ ds_{10}^2 = \Lambda^{1/2} \sum_{i=1}^k (d\rho_i^2 + \rho_i^2 d\tilde{\varphi}_i^2) + \Lambda^{1/2} d\vec{x}^2 - \Lambda^{-1/2} [\sum_{i=1}^k (B_i \rho_i^2 / R_{10}) d\tilde{\varphi}_i]^2 \end{aligned}$$

with  $\Lambda = 1 + \sum_{i=1}^k B_i^2 \rho_i^2 / R_{10}^2$ . The solution describes a  $(10 - 2k)$  Poincare invariant object with nonzero  $F^k$ -flux in the transverse  $\mathbf{R}^{2k}$ , a  $F(9 - 2k)$ -brane. In analogy with the previous situation, the dilaton blows up away from the core of the fluxbrane.

### 3.3 Dualities

We conclude this section by reviewing some of the remarkable properties of these solutions. Although they are very surprising from the 10-dimensional viewpoint, they

are readily obtained from the above 11-dimensional construction, by the simple strategy of choosing KK reductions with respect to different Killing isometries.

A first such property is flux periodicity. Namely, the fluxbrane configurations for parameters  $B$  and  $B+2$  are clearly equivalent, since they have the same 11-dimensional lift (since a rotation by  $4\pi$  is trivial). Concretely, starting with flat space and the identification (3.2) reduction along  $R_{10}\partial_{10} + B\partial_\varphi$  leads to the IIA F7-brane with parameter  $B$ , while reduction along  $R_{10}\partial_{10} + (B+2)\partial_\varphi$  leads to the F7-brane with parameter  $B+2$ . The equivalence is however highly non-trivial from the viewpoint of the 10-dimensional variables.

A second duality property is derived analogously. Start again with the quotient of flat space by the identification (3.2). Reducing along the Killing  $R_{10}\partial_{10} + B\partial_\varphi$  we obtain the IIA F7-brane. On the other hand, reducing along the killing  $R_{10}\partial_{10} + (B+1)\partial_\varphi$ , fermions are antiperiodic along the corresponding  $\mathbf{S}^1$  orbit (due to the additional  $2\pi$  rotation in the  $(\rho, \varphi)$  plane). As pointed out in [4], given the IIA/0A/M-theory duality proposal in [16], the resulting F7-brane is embedded in type 0A theory. Hence one obtains a duality between the type IIA and 0A F7-branes, for different values of the flux.

Considering the same starting point, the IIA F7-brane can be related to yet another familiar configuration in cases where  $B = 1/N$ . Namely, reducing the 11-dimensional configuration along the Killing  $\partial_{10}$ , the resulting configuration is simply an orbifold flat 10-dimensional space, with orbifold action generated by

$$z \rightarrow e^{2\pi i B} z \tag{3.11}$$

where  $z = \rho e^{i\varphi}$ . This action is moreover embedded into the RR 1-form gauge degree of freedom, as in [17]. This is the F7/cone duality [6].

Analogously the  $F(9 - 2k)$ -branes can be shown to be dual, for suitable  $B_i$ , to  $\mathbf{C}^k/\mathbf{Z}_N$  orbifold singularities. In fact, an interesting connection has been proposed between the generic instabilities of non-supersymmetric fluxbranes [1, 2] and the closed string twisted tachyons in non-supersymmetric orbifold singularities [18] (see also [3]).

### 3.4 Fluxbranes at singularities

These dualities are straightforwardly extended to lower-dimensional fluxbranes. In the following we consider a particular duality not explicitly discussed in the literature, and which leads to interesting systems of fluxbranes at orbifold singularities.

In particular, consider flat 11-dimensional space modded out by the identification (3.7). Reducing along the Killing  $R_{10} \partial_{10} + \sum_{i=1}^k B_i \partial_{\varphi_i}$  leads to a  $F(9 - 2k)$ -brane in flat space. Consider the case in which a subset of the  $B_i$  are rational numbers  $a_i/N$ . We may index such entries by  $i = 1, \dots, k_1$ , and the remaining  $i' = 1, \dots, k_2$ , with  $k_1 + k_2 = k$ . Reducing along the Killing  $R_{10} \partial_{10} + \sum_{i'=1}^{k_2} B_{i'} \partial_{\varphi_{i'}}$ , we obtain a  $F(9 - 2k_2)$ -brane sitting transverse to a  $k_1$ -fold singularity, generated by

$$(z_1, \dots, z_{k_1}) \rightarrow (e^{2\pi \frac{a_i}{N}} z_1, \dots, e^{2\pi \frac{a_{k_1}}{N}} z_{k_1}) \quad (3.12)$$

with the twist embedded in the RR 1-form gauge degree of freedom i.e. a flat connection RR field turned on, as in [17]. The  $F^{k_2}$  flux is turned on in the  $\mathbf{R}^{2k_2}$  directions transverse to the fluxbrane and the orbifold  $\mathbf{C}^{k_1}/\mathbf{Z}_N$ . Interestingly, for certain choices of the  $B$ 's (which determine the fluxbrane species and the orbifold action) the system of fluxbranes at the singularity is supersymmetric, even if neither of them is supersymmetric independently. From this viewpoint, the introduction of fluxbranes at non-supersymmetric orbifold singularities provides a stabilization mechanism for the latter (and vice-versa). It would be interesting to understand this from an analysis similar to [18].

Even though the above dualities are suggestive, one cannot avoid the feeling that their usefulness (or even validity) is questioned by the blowing up behaviour in the large distance regime. It would be desirable to have a similarly simple construction of fluxbranes in a more controlled setup, with asymptotically constant dilaton and small curvature. In the following sections we provide such a construction, which leads to fluxbranes wrapped on cycles in a curved space. Intuitively, the non-trivial transverse geometry cuts off the flux at large distances, so that asymptotically the backreaction of the flux on the geometry and the dilaton gradient fall off.

## 4 Fluxbranes in Taub-NUT space

In this section we discuss the construction of a fluxbrane with constant asymptotic dilaton. It has the interpretation of a fluxbrane transverse to a Taub-NUT space.

Our starting point is the  $N$ -center Taub-NUT space [19], defined by the metric

$$\begin{aligned} ds^2 &= U^{-1} (d\tau + \vec{\omega} \cdot d\vec{r})^2 + U d\vec{r}^2 \\ U &= \frac{1}{R^2} + \sum_{i=1}^N \frac{1}{|\vec{r} - \vec{r}_i|} \quad ; \quad \vec{\nabla} \times \vec{\omega} = \vec{\nabla} U \end{aligned} \quad (4.1)$$



The coordinate  $\tau$  has periodicity  $2\pi$ . This space is an  $\mathbf{S}^1$  fibration over  $\mathbf{R}^3$ , with fiber of constant asymptotic radius and degenerating over the points  $\vec{r}_i$ , the centers. The metric is smooth as long as the latter are generic, and develops  $A_{k-1}$  orbifold singularities when  $k$  centers coincide.

The metric is hyperkahler, therefore has an  $SU(2)$  isometry. This is most manifest for coinciding centers, where it amounts to rotations on the base  $\mathbf{R}^3$ . The metric also has a  $U(1)$  isometry along the  $\mathbf{S}^1$  fiber, corresponding to shifts in  $\tau$ . The orbits of this isometry have asymptotically constant length  $2\pi R$ , and degenerate over the centers. It is this isometry that we will exploit to build our fluxbrane.

Let us consider compactifying M-theory in Taub-NUT space, a configuration which preserves sixteen supersymmetries. Denoting the additional coordinates  $x_{10}$  and  $\vec{x} \in \mathbf{R}^6$ , the metric is

$$ds_{11}^2 = U^{-1} (d\tau + \vec{\omega} \cdot d\vec{r})^2 + U d\vec{r}^2 + dx_{10}^2 + d\vec{x}^2 \quad (4.2)$$

Let us identify points related by the action  $x_{10} \rightarrow x_{10} + 2\pi R_{10}$ ,  $\tau \rightarrow \tau + 2\pi R B$ , namely points related by a  $2\pi$  shift along the Killing direction  $R_{10}\partial + RB\partial_\tau$ . This identification preserves all sixteen supersymmetries in the system.

Comparing with (2.1), we have

$$f = U^{-1} \quad ; \quad f_{\vec{r}} = \vec{\omega} \quad ; \quad f_{\vec{x}} = 0 \quad ; \quad g_{\mu\nu} dx^\mu dx^\nu = U d\vec{r}^2 + d\vec{x}^2 \quad (4.3)$$

We introduce the adapted coordinate

$$\tilde{\tau} = \tau - \chi x_{10}, \quad \text{with } \chi = \frac{RB}{R_{10}} \quad (4.4)$$

and working directly, or using (2.5), the KK reduction gives

$$\begin{aligned} e^{4\Phi/3} &= \Lambda \quad ; \quad R_{10} A = \chi \Lambda^{-1} U^{-1} (d\tilde{\tau} + \vec{\omega} \cdot d\vec{r}) \\ ds_{10}^2 &= \Lambda^{-1/2} U^{-1} (d\tilde{\tau} + \vec{\omega} \cdot d\vec{r})^2 + \Lambda^{1/2} U d\vec{r}^2 + \Lambda^{1/2} d\vec{x}^2 \end{aligned} \quad (4.5)$$

with  $\Lambda = 1 + \chi^2 U^{-1}$ .

The geometry is similar to that of Taub-NUT space, with a ‘squashing’ which is important near the centers, and dies off for large  $r$ . The system is however supersymmetric due to the presence of a non-trivial RR 1-form flux, again localized near the centers  $\vec{r}_i$ , and asymptoting to a flat connection. Finally the dilaton is well behaved everywhere, and asymptotes to a constant value. The metric asymptotically tends to a standard Taub-NUT compactification, with redefined radius due to the new proper

length along the orbit of  $\tilde{\tau}$ . The  $U(1)$  isometry in the neighbourhood of the Taub-NUT center, is locally of the form

$$(z_1, z_2) \rightarrow (e^{i\alpha} z_1, e^{-i\alpha} z_2) \quad (4.6)$$

for suitable complex coordinates  $z_1, z_2$ <sup>3</sup>. Hence, we obtain non-zero  $F^2$  flux localized at those points. At large  $r$ , the isometry becomes translational, and the KK reduction simply reproduces a flat connection, a RR 1-form Wilson line. Since the flux dies off asymptotically, one recovers a Taub-NUT space and a constant dilaton in the large  $r$  region.

The solution therefore corresponds to an F5-brane transverse to a Taub-NUT geometry. The final configuration preserves sixteen supersymmetries, the same amount as a Taub-NUT geometry with no flux. Hence in this case the fluxbrane does not break any further supersymmetries.

An advantage of the good asymptotics of this solution is that the fluxbrane can be understood as an excitation of type IIA theory on Taub-NUT space. As such, one may hope to learn about the existence of zero modes of the excitation, localized on its core, by looking at symmetries preserved by the background and broken by the excited state, namely Goldstone bosons or goldstinos. Unfortunately, the fluxbrane does not break any further symmetries, hence no such modes arise. This does not contradict the claim that fluxbranes are dynamical; even for D-branes, there exist cases (like branes trapped at singularities) where no Goldstone zero modes exist. Hence our solution describe a trapped fluxbrane in the same sense.

We refrain from discussing duality properties of this solution and variants thereof, which are easily derived from the 11-dimensional perspective by applying the philosophy in [6]. Instead, we turn to the construction of other interesting examples.

## 5 More general wrapped fluxbranes

### 5.1 Wrapped fluxbranes with asymptotically constant dilaton

In this section we would like to further explore the above ideas to construct examples of fluxbranes with constant asymptotic dilaton, and wrapped on diverse cycles in non-trivial geometries. In order to get constant asymptotic dilaton, it is crucial that the

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<sup>3</sup>Near a center the Taub-NUT metric is  $ds^2 = \frac{1}{r} d\tilde{r}^2 + r (d\tau + \vec{\omega} \cdot d\tilde{r})^2$ . Changing variables as  $q = ae^{i\tau}$ , where  $a$  is a purely imaginary quaternion, the metric becomes manifestly flat  $ds^2 = dqd\bar{q}$ , and the  $U(1)$  isometry becomes (4.6) in terms of  $q = z_1 + jz_2$ , see [20] for details.

base spaces have  $U(1)$  isometries with orbits of asymptotic constant length. Metrics for such spaces are not easy to construct in dimensions higher than four, hence in this section we use an indirect route to consider such geometries.

The key idea is that such geometries are predicted to exist by string duality. Specifically, the lift to M-theory of configurations involving D6-branes corresponds to non-trivial geometries with a  $U(1)$  isometry of constant asymptotic length (related to the asymptotic IIA dilaton). For supersymmetry preserving D6-brane configurations, a nice list of the special holonomies arising in M-theory is given in [21]. We will use this duality relation to generate spaces on which we subsequently wrap fluxbranes.

### General examples

Let us construct e.g. a F5-fluxbrane wrapped on a holomorphic 2-cycle on a Calabi-Yau threefold. We will be interested in discussing CY spaces with a  $U(1)$  isometry of constant asymptotic radius orbit. A simple way to generate such spaces is to use string duality, as follows. Consider a D6-brane wrapped on a (possibly non-compact) holomorphic 2-cycle  $\Sigma_2$  within a (possibly non-compact) four-dimensional hyperkahler space  $\mathbf{X}_4$ . Lifting the configuration results in a Calabi-Yau threefold  $\mathbf{X}_6$ , topologically a  $\mathbf{C}^*$  fibration over  $\mathbf{X}_4$ , with fibers degenerating (to two intersecting complex planes) over  $\Sigma_2$ . The lift also provides us with some metric information, namely that the  $\mathbf{S}^1$  fiber within the  $\mathbf{C}^*$  is the orbit of a  $U(1)$  isometry, has constant asymptotic radius (the radius of the M-theory circle), and degenerates over  $\Sigma_2$ . This space is a threefold generalization of the situation encountered in Section 4. In fact, the full geometry is just a fibration of the Taub-NUT space over  $\Sigma_2$ .

Let  $\tau$  parametrize the  $\mathbf{S}^1$  fiber, with period  $2\pi$ , and  $x_{10}$  parametrize one of the additional flat non-compact dimensions. Consider modding out by the symmetry  $x_{10} \rightarrow x_{10} + 2\pi R_{10}$ ,  $\tau \rightarrow \tau + 2\pi B$ , and perform a KK reduction along the Killing vector  $R_{10} \partial_{10} + B \partial_\tau$ . Since at every point in  $\Sigma_2$  the geometry is locally like in the Taub-NUT example, one obtains an F5-brane spanning four flat non-compact dimensions and wrapped on  $\Sigma_2$  within (a squashed version of)  $\mathbf{X}_6$ . The construction preserves 8 supercharges, the same amount preserved by  $\mathbf{X}_6$  alone. So the fluxbrane does not break further supersymmetries.

Notice that if  $\Sigma_2$  is not rigid the configuration contains zero modes associated to deformations of the cycle. These however exist already in the geometry of  $\mathbf{X}_6$ , and hence should not be assigned to the fluxbrane. Again, the fluxbrane is trapped at the core of the Taub-NUT fibers, and is simply dragged by the deformations of the geometry.

The construction is obvious to generalize to similar fluxbranes on diverse cycles. Sticking to supersymmetric cycles in special holonomy manifolds, many examples can be generated using string duality. Without repeating details, one may build: 1) Starting with a D6-brane on a Slag 3-cycle  $\Sigma_3$  within a Calabi-Yau threefold  $\mathbf{X}_6$ , we get a F5-brane on a 3-cycle in a (squashed version of a)  $G_2$  manifold; 2) Starting with a D6-brane on a holomorphic 4-cycle  $\Sigma_4$  in a Calabi-Yau threefold, an F5-brane on a holomorphic 4-cycle in a Calabi-Yau fourfold; 3) Starting with a D6-brane on a coassociative 4-cycle in a  $G_2$  manifold, we get an  $F^2$  fluxbrane in a 4-cycle in a  $Spin(7)$  manifold; etcetera.

It should be clear that these examples generate F5-branes due to the special nature  $U(1)$  fixed loci in supersymmetry preserving manifolds, which are most familiar. Clearly, use of other isometries can lead to e.g. wrapped F7-branes using analogous techniques (see Section 5.2 and 6 for some discussion).

### F5-brane in a 3-cycle in a $G_2$ manifold

In the following we consider an example of the above construction. Recently a  $G_2$  holonomy metric manifold with a  $U(1)$  isometry with orbits of asymptotically constant radius has been constructed in [15]. In the above language, it corresponds to the M-theory lift of a D6-brane wrapped on the 3-sphere in a deformed conifold. However, once the manifold metric is known, it can be used to build a fluxbrane without further appeal to how it was obtained, as we do below.

The metric for the compactification of M-theory in such space is

$$ds_{11}^2 = \sum_{a=1}^7 e^a \otimes e^a + d\vec{x}^2 \quad (5.1)$$

$$\begin{aligned} e^1 &= A(r) (\sigma_1 - \Sigma_1) & ; & & e^2 &= A(r) (\sigma_2 - \Sigma_2) \\ e^4 &= B(r) (\sigma_1 + \Sigma_1) & ; & & e^5 &= B(r) (\sigma_2 + \Sigma_2) \\ e^3 &= D(r) (\sigma_3 - \Sigma_3) & ; & & e^6 &= r_0 C(r) (\sigma_3 + \Sigma_3) \\ & & & & e^7 &= dr/C(r) \end{aligned} \quad (5.2)$$

where  $\vec{x} \in \mathbf{R}^3$ , and  $\sigma_i, \Sigma_i$  are  $SU(2)$  left-invariant 1-forms in  $\mathbf{S}^3$

$$\begin{aligned} \sigma_1 &= \cos \psi d\theta + \sin \psi \sin \theta d\phi & ; & & \Sigma_1 &= \cos \hat{\psi} d\hat{\theta} + \sin \hat{\psi} \sin \hat{\theta} d\hat{\phi} \\ \sigma_2 &= -\sin \psi d\theta + \cos \psi \sin \theta d\phi & ; & & \Sigma_2 &= -\sin \hat{\psi} d\hat{\theta} + \cos \hat{\psi} \sin \hat{\theta} d\hat{\phi} \\ \sigma_3 &= d\psi + \cos \theta d\phi & ; & & \Sigma_3 &= d\hat{\psi} + \cos \hat{\theta} d\hat{\phi} \end{aligned} \quad (5.3)$$

and  $A, B, C, D$  are radial functions, explicitly given in eq.(3.6) of [15]. The metric has isometry  $SU(2) \times SU(2) \times U(1)$ , with the  $U(1)$  corresponding to a simultaneous shift in  $\psi, \hat{\psi}$  (which also induces a simultaneous rotation of  $(\sigma_1, \sigma_2)$  and  $(\Sigma_1, \Sigma_2)$ ). The orbit

length is controlled by  $C(r)$ , and asymptotes to constant  $4\pi r_0$  at large  $r$ , and shrinks to zero at  $r = 9r_0/2$ , which corresponds to an associative 3-cycle with  $\mathbf{S}^3$  topology [15]. In fact, the full space can be thought of as a fibration of Taub-NUT over such cycle.

Defining  $\tau = \psi + \hat{\psi}$ , we may cast (5.2) in the form

$$ds_{11}^2 = d\vec{x}^2 + \sum_{a \neq 6} e^a \otimes e^a + r_0^2 C(r)^2 (d\tau + \cos \theta d\phi + \cos \hat{\theta} d\hat{\phi})^2 \quad (5.4)$$

analogous to (2.1). Notice that the  $\sum_a e^a e^a$  term is actually  $\psi$ -independent. Let us mod out this configuration by the isometry

$$x_{10} \rightarrow x_{10} + 2\pi R_{10} \quad ; \quad \tau = \tau + 4\pi B \quad (5.5)$$

We define  $\tilde{\tau} = \tau - \chi x_{10}$ , with  $\chi = 2B/R_{10}$  and apply (2.5) with

$$f = r_0^2 C(r)^2 \quad ; \quad f_\phi = \cos \theta \quad ; \quad f_{\hat{\phi}} = \cos \hat{\theta} \quad ; \quad g_{\mu\nu} = \sum_{a \neq 6} e^a \otimes e^a + d\vec{x}^2 \quad (5.6)$$

We get a ten-dimensional IIA configuration given by

$$\begin{aligned} e^{4\Phi/3} &= \Lambda \quad ; \quad \Lambda = 1 + \chi^2 r_0^2 C(r)^2 \\ A &= \Lambda^{-1} r_0^2 C(r)^2 (\chi/R_{10}) (d\tilde{\tau} + \cos \theta d\phi + \cos \hat{\theta} d\hat{\phi}) \\ ds_{10}^2 &= \Lambda^{1/2} d\vec{x}^2 + \Lambda^{1/2} \sum_{a=1}^6 e^a \otimes e^a + \Lambda^{-1/2} r_0^2 C(r)^2 (d\tilde{\tau} + \cos \theta d\phi + \cos \hat{\theta} d\hat{\phi})^2 \end{aligned} \quad (5.7)$$

The solution, by the above arguments, corresponds to an F5-brane wrapped on a 3-sphere in (a squashed version of) the  $G_2$  manifold in [15].

An a priori simpler realization of the same idea would be provided by starting e.g. with a D6-brane wrapped on for instance the 2-sphere in the Eguchi-Hanson space. Its lift would correspond to a Calabi-Yau three-fold  $\mathbf{X}_6$  given by a Taub-NUT fibration over such 2-cycle. Using this space to build a fluxbrane as described above would result in a supersymmetric F5-brane wrapped on a 2-cycle in (an squashed version of)  $\mathbf{X}_6$ . Even though this configuration is in principle simpler than the  $G_2$  example above, the metric for  $\mathbf{X}_6$  with asymptotically constant  $\mathbf{S}^1$  radius is not available in the literature.

We conclude by pointing out that these fluxbranes present flux periodicity by the same argument as in flat space. This supports the expectation of this being a generic feature of fluxbranes [6]. As in the flat case, our models with F5-branes do not show any connection between IIA and 0A solutions. Whether wrapped F7-branes do provide such connection or not is addressed in Section 7.

## 5.2 Wrapped fluxbranes with blowing dilaton

The remarkable property that the above fluxbranes have asymptotically constant dilaton is not generic within the class of wrapped fluxbranes. In other words, wrapping a fluxbrane does not guarantee that the dilaton is bounded at large distances. This is obvious e.g. in the above examples, by taking limits in which the asymptotic  $U(1)$  circle decompactifies. For completeness we present some examples of wrapped fluxbranes with blowing dilaton.

### More fluxbranes at singularities

Considering for instance F5-branes transverse to the  $k$ -center Taub-NUT case in Section 4. The original Taub-NUT space (4.1), in the limit  $R \rightarrow \infty$  becomes an ALE space. For coincident centers, it is simply a  $\mathbf{C}^2/\mathbf{Z}_k$ , with the orbifold action generated by  $(z_1, z_2) \rightarrow (e^{2\pi i/k} z_1, e^{-2\pi i/k} z_2)$ . Hence this construction yields a different configuration of fluxbranes at singularities, namely F5-branes transverse to  $\mathbf{C}^2/\mathbf{Z}_N$ , which is simply an orbifold of the flat space F5-brane solutions. Clearly, these solutions have large dilaton at large distances.

As an interesting aside, let us point out that using ideas similar to [6], one readily derives a duality between fluxbranes in flat space and fluxbranes at singularities. In particular, consider M-theory in flat 11-dimensional space, with identification of points related by

$$x_{10} \rightarrow x_{10} + 2\pi R_{10} \quad ; \quad \varphi_1 \rightarrow \varphi_1 + 2\pi B \quad ; \quad \varphi_2 \rightarrow \varphi_2 - 2\pi B \quad (5.8)$$

Reducing to ten dimensions along the Killing  $R_{10}\partial_{10} + B\partial_{\varphi_1} - B\partial_{\varphi_2}$  lead to a F5-brane in flat space. On the other hand, reducing along  $R_{10}\partial_{10} + (B+1/k)\partial_{\varphi_1} - (B+1/k)\partial_{\varphi_2}$ , leads to F5-branes for a different value of the flux, and transverse to a  $\mathbf{C}^2/\mathbf{Z}_k$  singularity. The latter are precisely the same branes at singularities encountered above.

### Intersecting F7-branes in Taub-NUT

To show that wrapped F7-branes can be achieved in our setup, let us build e.g. a F7-brane wrapped on a 2-cycle in multi Taub-NUT space, by using the  $SU(2)$  (rather than the  $U(1)$ ) isometry of this space. Considering the case of coincident centers, any  $SU(2)$  element amounts to an  $SO(2)$  rotation along some symmetry axis, say  $x_3$ , passing through the center location. Let us mod out by a  $2\pi R_{10}$  shift in  $x_{10}$  and a simultaneously  $2\pi B$  rotation around  $x_3$ . Such identification breaks all the supersymmetries in the configuration. The result after KK reduction yields a RR 1-form flux localized on a 2-dimensional subspace of Taub-NUT, parametrized by  $x_3$  and  $\tau$ . Hence it corresponds to an F7-brane wrapped on a 2-cycle in Taub-NUT space, which is holomorphic in a

suitable complex structure. As a complex space, Taub-NUT with coinciding centers can be described by  $xy = v^N$ . Adapting the coordinates, the rotational isometry corresponds to  $x \rightarrow e^{i\alpha}x, y \rightarrow e^{i\alpha}y, v \rightarrow e^{2i\alpha/N}v$  for real  $\alpha$ . The flux is localized on its fixed points, which corresponds to the holomorphic (reducible) cycle  $xy = 0$ . Hence we obtain two F7-branes wrapped on intersecting holomorphic cycles. The wrapping a non-supersymmetric object on a supersymmetric cycle explains the breaking of all supersymmetries.

### F5-branes in 2-cycle in the conifold

Our final example is an F5-brane wrapped on a (non-compact) 2-cycle in the resolved conifold. Consider the resolved conifold, which is the cotangent bundle space of the 3-sphere  $\mathbf{T}^*\mathbf{S}^3$ . The explicit metric can be found in [22], but for our purposes it is enough to know that it admits an  $SU(2)^2$  isometry. As a complex manifold the space can be described by the equation  $z_1^2 + z_2^2 + z_3^2 + z^4 = \epsilon$ , where epsilon is the deformation parameter, and the coordinates  $z_i$  transform in the  $(2, 2)$  representation of the  $SU(2)^2$  isometry. A generic  $U(1)$  rotation within  $SU(2)^2$  acts as

$$\begin{aligned} x &\rightarrow e^{i\alpha}x & ; & & y &\rightarrow e^{-i\alpha}y \\ z &\rightarrow e^{i\beta}z & ; & & w &\rightarrow e^{-i\beta}w \end{aligned} \tag{5.9}$$

in adapted complex coordinates in which the geometry is described by  $xy - zw = \epsilon$ . Here  $\alpha$  and  $\beta$  define the embedding of  $U(1)$  in the  $U(1)^2$  within  $SU(2)^2$ .

Since the initial  $SO(4)$  is the isometry of the 3-sphere in the resolution, the orbits of these symmetries grow with the distance. To build a fluxbrane, consider e.g. modding by the a simultaneous shift in  $x_{10}$  and a  $U(1)$  rotation with  $\beta = 0$ . After KK reduction, we obtain a supersymmetric F5-brane wrapped on  $x = y = 0$ , namely the holomorphic 2-cycle  $zw = \epsilon$ , which has the topology of a cylinder (it intersects the base  $\mathbf{S}^3$  over a maximal  $\mathbf{S}^1$ ).

Alternatively, we could have chosen an isometry with non-zero  $\alpha, \beta$ . This isometry is freely acting on the deformed conifold, a situation not encountered before. Examples of this kind are briefly discussed in Section 6. For the undeformed conifold,  $xy - zw = 0$ , this action leaves the origin as the only fixed point. The resulting fluxbrane is a supersymmetric F3-brane transverse to the conifold singularity.

Let us conclude by pointing out that these examples have interesting duality properties as well. Using by now familiar arguments [6], the last examples are related to orbifolds of the conifold considered in [23], with non-trivial embedding of the orbifold twist in the RR 1-form gauge degree of freedom.

## 6 Freely acting isometries

A possibility not explored in the literature is the construction of fluxbranes by using freely acting  $U(1)$  isometries. The previous examples led to fluxbranes with core located, so to speak, at the fixed points of the associated isometry. Hence one may incorrectly conclude that freely acting isometries would not generate fluxbranes. In fact, the construction in Section 2 did not assume the existence of fixed points, and led to non-trivial flux configurations in the lower dimensional theory, and no obstruction to have well-defined cores for the corresponding flux lumps.

Let us discuss an example of freely acting isometry, namely the Hopf isometry for  $\mathbf{S}^3$  with the round metric. Of course string or M-theory compactifications on  $\mathbf{R}^d \times \mathbf{S}^3$ , for  $d = 7, 8$  respectively, do not satisfy the equations of motion. However, the example provides a good toy model, which can be embedded in more complicated (and interesting) situations, like for instance when the  $\mathbf{S}^3$  is a cycle within say a Calabi-Yau space, or for Anti de Sitter compactifications, see below.

The round metric for  $\mathbf{S}^3$  can be written

$$ds_3^2 = \sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2 + \sigma_3 \otimes \sigma_3 \quad (6.1)$$

where  $\sigma_i$  are the  $SU(2)$  left-invariant one-forms defined in the first column in (5.3). This  $\mathbf{S}^3$  is a (Hopf) fiber bundle over  $\mathbf{S}^2$  with fiber  $\mathbf{S}^1$  of constant length. The base is parametrized by  $\theta, \phi$ , while the fiber is parametrized by  $\psi$ , which is an isometric direction with no fixed points.

Introducing an additional flat dimension, parametrized by  $x$ , the metric in the form (2.1) is

$$ds^2 = dx^2 + \sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2 + (d\psi + \cos \theta d\phi)^2 \quad (6.2)$$

Modding by the action  $x \rightarrow x + 2\pi R$ ,  $\psi \rightarrow \psi + 4\pi B$ , and KK reducing, the resulting 3-dimensional configuration is <sup>4</sup>

$$e^{4\Phi/3} = \Lambda \quad ; \quad A = \Lambda^{-1}(\chi/R) (d\tilde{\psi} + \cos \theta d\phi) \\ ds^2 = \Lambda^{1/2}(\sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2) + \Lambda^{-1/2}(d\tilde{\psi} + \cos \theta d\phi)^2 \quad (6.3)$$

with  $\chi = 2B/R$ , and  $\Lambda = 1 + \chi^2$  is a constant. The configuration is a 3-sphere, squashed on the  $\mathbf{S}^1$  fiber, and preserving the base  $\mathbf{S}^2$  with round metric. Moreover

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<sup>4</sup>Since we are not compactifying from 11 to 10 dimensions, one should modify the form of the KK ansatz. We skip this point, or if the reader wishes, we indeed look at M-theory on say  $\mathbf{X}_8 \times \mathbf{S}^3$  (with the radius of the 3-sphere a function of some time coordinate in  $\mathbf{X}_8$ , to account for the instability of the system).



there is constant non-zero field strength on the base  $\mathbf{S}^2$ . The configuration can be thought of as a fluxbrane wrapped on the  $\mathbf{S}^1$  fiber (times the space transverse to  $\mathbf{S}^3$  if included). The fact that the fluxbrane is smeared over  $\mathbf{S}^2$  is simply consequence of the constant radius of the  $U(1)$  isometry, rather than of the lack of fixed points, and is avoidable by starting with a lumpy, rather than round, metric in  $\mathbf{S}^3$ .

We conclude by discussing a simple context in which a round 3-sphere appears, namely in type IIB string compactification on  $\mathbf{AdS}_3 \times \mathbf{S}^3 \times \mathbf{X}_4$ , with RR 3-form field strength flux through  $\mathbf{S}^3$  and where  $\mathbf{X}_4$  a Ricci-flat compact four-manifold. These geometries have arisen in the context of the AdS/CFT duality as supergravity duals of the theory on D1- and D5-branes transverse (resp. wrapped) on  $\mathbf{X}_4$  [24]. Here however we look at the configuration in its own right. Considering the case  $\mathbf{X}_4 = \mathbf{T}_4$ , we may use this 10-dimensional configuration to construct a IIB NS-NS fluxbrane in a 9-dimensional compactification. Since it is not possible to add a non-compact flat  $x_{10}$  direction, we may simply use one of the flat (compact) dimensions within  $\mathbf{T}^4$ . Denoting its coordinate  $x$  with periodicity  $2\pi R$ , we build our 9-dimensional fluxbrane by modding by

$$x \rightarrow x + 2\pi R/N \quad ; \quad \psi \rightarrow \psi + 2\pi n/N \tag{6.4}$$

where  $\psi$  denotes the above Hopf coordinate, and  $n$  is an integer (so that the identification we are imposing is consistent with the identification implicit in the original torus. The resulting configuration is a 9-dimensional  $\mathbf{AdS}_3 \times \tilde{\mathbf{S}}^3 \times \mathbf{T}^3$  model, with a squashed 3-sphere with constant NS-NS 2-form flux through the base  $\mathbf{S}^2$ , and (non-constant) RR 3-form flux through  $\mathbf{S}^3$ . This can be described as a NS-NS flux 6-brane wrapping  $\mathbf{AdS}_3 \times \mathbf{S}^1 \times \mathbf{T}^3$ .

Fluxes have found many applications in the context of AdS/CFT, where they correspond in the field theory language to supersymmetry breaking terms, like scalar masses, etc. In fact, the above configuration leads to partial breaking of supersymmetry, from 8 to 4 supercharges. It would be interesting to understand the role of the above construction in the AdS/CFT context.

## 7 Type IIA/0A fluxbrane duality

In [4] the authors proposed a duality of between IIA/0A F7-branes for parameters  $B$  and  $B + 1$  (in our convention), respectively. Due to the blowing up of the dilaton and the different  $B$  parameters, a weakly coupled IIA fluxbranes is related to a

strongly coupled 0A fluxbrane, and viceversa, hence avoiding contradiction with the perturbative inequivalence of both theories.

One may worry that F7-brane solutions with asymptotically finite dilaton and vanishing flux built using our construction can lead to precisely this kind of contradiction, due to the asymptotic region where the theory is weakly coupled regardless of the  $B$  parameter controlling the flux at the origin. In this section we discuss this issue and solve the potential puzzle.

Unfortunately we are not aware of any manifold satisfying the equations of motion and admitting a  $U(1)$  isometry suitable to obtain F7-branes, and with asymptotically constant orbits length. Hence we carry out the discussion in terms of a toy model, but which is sufficient to illustrate the main point. Interestingly, we find that IIA/0A fluxbrane duality holds as in [4] near the fluxbrane core, and breaks down in the asymptotic region, precisely where problems could arise. This can be taken as mild additional evidence for the proposed duality.

To construct our F7-brane, we consider  $\mathbf{X}_{11} = \mathbf{M}_8 \times \mathbf{R}^2 \times \mathbf{R}$ , with metric

$$ds_{11}^2 = d\vec{x}^2 + d\rho^2 + g(\rho) d\varphi^2 + dx_{10}^2 \quad (7.1)$$

where  $g(\rho)$  is a function that interpolates between  $\rho^2$  at small distance and a constant  $R^2$  at large distances, for instance  $g(\rho) = \rho^2/(1+\rho^2/R^2)$ . The ansatz has the drawback that it does not satisfy the equations of motion. However, one may include time dependence (and perhaps a mild dilaton profile) to render it consistent. The main point in the following, though, would go through to such situation, so we avoid cluttering the discussion.

The  $(\rho, \varphi)$  space looks like a (half) cigar, with space becoming flat near the tip ( $\rho \ll R$ ), and  $\mathbf{R} \times \mathbf{S}^1$  away from it ( $\rho \gg R$ ). The coordinate  $\varphi$  parametrizes a  $U(1)$  isometry, which is rotational near the tip and has constant orbits of length  $2\pi R$  in the asymptotic region.

Let us construct a IIA F7-brane by modding out by the identification  $x_{10} \rightarrow x_{10} + 2\pi R_{10}$ ,  $\varphi \rightarrow \varphi + 2\pi RB$ . Reducing along  $R_{10}\partial_{10} + BR\partial_{\varphi}$ , we obtain

$$\begin{aligned} e^{4\Phi/3} = \Lambda = 1 + (BR/R_{10})^2 g(\rho) \quad ; \quad A_{\tilde{\varphi}} = \Lambda^{-1}(BR/R_{10}^2) g(\rho) \\ ds_{10}^2 = \Lambda^{1/2} d\vec{x}^2 + \Lambda^{1/2} d\rho^2 + \Lambda^{-1/2} g(\rho) d\tilde{\varphi}^2 \end{aligned} \quad (7.2)$$

Near the core, the solution looks like the Melvin F7-brane in Section 2, while for large  $\rho$  the dilaton stabilizes to a finite value, and the RR field strength drops.

The question is the nature of the solution if we reduce along  $R_{10}\partial_{10} + (B+1)R\partial_{\varphi}$ . For an observer near  $\rho = 0$ , things look like in flat space, hence the IIA F7-brane

with parameter  $(B + 1)$  should be equivalent to the 0A F7-brane with parameter  $B$ . This is indeed the case because near  $\rho = 0$  the isometry is rotational and fermions are antiperiodic along its orbit, allowing the use of [16] to derive the fluxbrane duality.

However, if fermions would also be antiperiodic along the  $U(1)$  orbit in the asymptotic region, we would get to the conclusion that the IIA and 0A configurations are equivalent in a region where the flux is negligible and the dilaton remains under control. Happily, such a contradiction is avoided because in the asymptotic region the fermions are periodic along the  $U(1)$  orbit. This follows because when the fermions are transported along the asymptotic circle, they pick up an additional phase from the holonomy of the spin connection. This is given by the exponential of the integrated scalar curvature in the interior of the loop, which by the Gauss-Bonnet theorem equals  $e^{i\pi} = -1$ . Hence in the asymptotic region both reductions lead to a IIA configuration with no relation to 0A fluxbranes.

## 8 Conclusions

In this paper we have proposed a simple extension of the construction of the Melvin fluxtube as a quotient of flat space, to build wrapped versions of these branes. In particular we have shown that in some instances the background curved space confines the flux and leads to solutions where the asymptotic fields strength dies off and the dilaton becomes constant.

We believe this is an interesting mechanism, worth further exploration, and expect these new solutions to help illuminating the still obscure properties of fluxbranes. Hopefully, the solutions presented can also lead to useful ansatze in the construction of fluxbranes for RR fields other than the 1-form (beyond the obvious ones obtained by T-dualizing our examples), perhaps with finite flux in the transverse directions (as opposed to certain flat fluxbranes of this kind [5, 6, 7]).

Another interesting direction is the study of the world-volume zero modes of these objects. In most of our examples the wrapped fluxbranes preserved the same symmetries as the background, and led to no obvious goldstone zero modes. In a sense, the branes are trapped by the geometry, and are simply dragged by the latter is deformed. This does not disagree with the fact that the objects are dynamical in better circumstances. We hope that further research leads to a more explicit display of dynamical features of fluxbranes.

Finally, it would be interesting to introduce D-brane probes in these configurations,

and learn about D-brane dynamics in new fluxbranes backgrounds. In particular, we expect dielectric D-branes to behave better in wrapped fluxbranes with asymptotically constant dilatons than in their flat space cousins. In particular, the dying off of the flux at large distance prevents the dielectric branes from running off to infinity, and may lead to stable minima hopefully within the reach of gravitational description (see [7] and [8] for discussion and criticism).

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