# BEAM BASED MEASUREMENTS OF FIELD MULTIPOLES IN THE RHIC LOW BETA INSERTIONS AND EXTRAPOLATION OF THE METHOD TO THE LHC

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#### Abstract

The multipolar content of the dipoles and quadrupoles is known to limit the stability of the beam dynamics in superconducting machines like RHIC and even more in LHC. The low-beta quadrupoles are thus equipped with correcting coils up to the dodecapole order. The correction is planned to rely on magnetic measurements. We show that a relatively simple method allows an accurate measurement of the multipolar field aberrations using the beam. The principle is to displace the beam in the non-linear fields by local closed orbit bumps and to measure the variation of sensitive beam observables. The resolution and robustness of the method are found appropriate. Experimentation at RHIC showed clearly the presence of normal and skew sextupolar field components in addition to a skew quadrupolar component in the interaction regions. Higher-order components up to decapole order appear as well.

# **1 PRINCIPLE OF THE MEASUREMENT**

To avoid a strong perturbation of the beam dynamics, the beam is moved **locally** to large amplitudes using a closed orbit bump. The non-linearity inside the orbit bump acts by feed-down to all lower orders. Its order and magnitude are deduced from the variation of suitable observables with the bump amplitude. This method is extrapolated from [1] where it was used to measure the azimuthal dependence of a parasitic skew gradient in LEP. In this paper, we consider the feed-downs to dipole and quadrupole orders.

**Measurement of the Orbit Perturbation** We select as an observable the rms orbit perturbation measured **outside** the bump. The sign of the field perturbation is recovered by fitting an oscillation. The rms orbit  $\tilde{z}$  is given by:

$$\tilde{z} = f_z(c_n, z_{co}) = \frac{\sqrt{\beta_{z_{arc}}}}{2\sqrt{2}\sin\pi Q_z} \frac{B_N}{B\rho} \int \sqrt{\beta_z} c_n \left(\frac{z_{co}}{R}\right)^{n-1}$$

*z* stands for *x* or *y*, *R* is the reference radius,  $B_N = B_y(x=R)$ ,  $c_n$  stands for  $b_n$  or  $a_n$  the normal and skew field components,  $z_{co}$  stands for the *x* or *y* bump amplitude. The rms orbit perturbation depends both on the plane of the bump (H, V) and of the parity of the multipole order *n*:

$$\begin{split} \tilde{x}_{co}(H) &= f_x(b_n, x_{co}) \quad \tilde{y}_{co}(H) = f_y(a_n, x_{co}) \\ \tilde{x}_{co}(V, odd) &= -1^{(n-1)/2} f_x(b_n, x_{co}) \\ \tilde{y}_{co}(V, odd) &= -1^{(n-1)/2} f_y(a_n, x_{co}) \\ \tilde{x}_{co}(V, even) &= -1^{n/2} f_x(b_n, x_{co}) \\ \tilde{y}_{co}(V, even) &= -1^{(n-2)/2} f_y(a_n, x_{co}) \end{split}$$

**Measurement of the Tune Shifts** The measured tune shifts either arise from normal gradients  $\Delta Q$  or from the repelling effect of the linear coupling measured by  $\vec{c}$ 

$$\Delta Q = g(c_n, z_{co}) = \frac{n-1}{4\pi} \frac{B_N}{B\rho} \int \beta c_n \frac{z_{co}^{n-2}}{R^{n-1}} ds$$
$$\vec{c} = h(c_n, z_{co}) = -\frac{n-1}{2\pi} \frac{B_N}{B\rho} \int \sqrt{\beta_x \beta_y} a_n \frac{z_{co}^{n-2}}{R^{n-1}} e^{i(\mu_x - \mu_y)} ds$$

The selection of one or the other effect depends on the plane of the bump, on whether the multipole is erect or skew and on the parity of the multipole order n:

Bump	$b_3$	$a_3$	$b_4$	$a_4$	$b_5$	$a_5$	$b_6$
Н	$\Delta Q$	$\Delta c$	$\Delta Q$	$\Delta c$	$\Delta Q$	$\Delta c$	$\Delta Q$
V	$\Delta c$	$\Delta Q$	$\Delta Q$	$\Delta c$	$\Delta c$	$\Delta Q$	$\Delta Q$

$$\begin{aligned} \Delta Q(H, erect) &= g(b_n, x_{co}) \quad \vec{c}(H, skew) = h(a_n, x_{co}) \\ \Delta Q(V, erect, even) &= -1^{(n-2)/2} g(b_n, y_{co}) \\ \vec{c}(V, erect, odd) &= -1^{(n-1)/2} h(b_n, y_{co}) \\ \vec{c}(V, skew, even) &= -1^{n/2} h(a_n, y_{co}) \\ \Delta Q(V, skew, odd) &= -1^{(n-1)/2} g(a_n, y_{co}) \end{aligned}$$

# **2** SIMULATION IN LHC

**Orbit Response** We compute with MAD the orbit response to horizontal or vertical bumps moving the beam by up to 10 mm (9  $\sigma_x$ ) in the low- $\beta$  insertion IP5 of LHC. The linear imperfections used are 2 per mil gradient error ( $\Delta\beta/\beta \approx 20\%$ ), 1 mrad roll ( $|\vec{c}| \approx 0.04$ ). The multipole imperfections are all set to 10 units in only one quadrupole (Q2B.R5) out of the four low- $\beta$  magnet blocks. The perturbations should therefore be just on the pessimistic side.

Figure 1 shows the orbit response versus bump amplitude multipole per multipole. It decreases with the order but seems large enough for modern BPM systems. Discrepancies with the first-order approximation can be noticed (non-vanishing horizontal orbit for horizontal bumps with  $a_2$ , small asymmetry with the polarity of the bump with  $b_3$ ) and justify the use of a full numerical model.

**Tune Response** Figure 2 shows the tune response to horizontal bumps in presence of a multipole. Use of the normalized sum and difference of the tunes helps to disentangle normal tune shifts from coupling effects [3]. The signatures of the various multipoles appear sufficiently different and the magnitude of the tune shifts significant. Again some small side-effects with respect to first-order formulae show up.



Figure 1: Rms orbits versus bump amplitude

### **3** ACCURACY AND ROBUSTNESS

Measurement of the Multipoles with the orbit leakage Column 2 of the next table shows the BPM resolution at which the proper multipole order cannot be distinguished from the next order by comparing the  $\chi^2$  (20 measurement points assumed). In all cases, the accuracy

Perturbation	BPM resolution	resolution on $c_n$		
Roll=.1 mrad	$15\mu m rms$	15%		
$b_3 = 7.6  10^{-4}$	$8\mu m rms$	7%		
$b_4 = 7.2  10^{-4}$	$3.5\mu m rms$	5%		

of the measurement of the multipole strength is appropriate. A gain by a factor of two may be obtained by collecting 100 orbits rather than 20, which is technically feasible.

Measurement of the Multipoles with the tunes Assuming again 20 measurements, it is possible to decide on the multipole order up to  $b_6$  if the tune resolution is better than  $2 \ 10^{-4}$  rms based on the  $\chi^2$  per dof. In all cases, the multipole strength is evaluated to better than 10%.

**Roll of the Orbit Correctors** A roll of the bump orbit correctors in the mrad range is only relevant to the measurement of  $a_2$  and negligible otherwise. Consistency checks (H and V bumps, use of different correctors) may help reducing this uncertainty.

**Imperfections of the BPM's** The orbit leakage obtained by subtracting two orbits is immune to a dc offset of the BPM's. The use of all BPM's except those inside the bump allows a large reduction of the random imperfections. The amplitude of the orbit leakage is less than 1 mm



Figure 2:  $\Sigma Q$  and  $\Delta Q$  versus horizontal bump amplitude

in a range where the BPM's are highly linear.

**Side-effects** A parasitic  $\beta$ -beating is averaged out by the measurement principle. We notice a small change of the orbit amplification factor when a tune shift occurs, some focusing perturbations and interference between the leaking orbit and the lattice sextupoles or other non-linearities. The use of the MAD machine model and an iterative procedure is necessary to reach the ultimate accuracy.

#### **4** THE EXPERIMENT AT RHIC

During the RHIC run 2000 measurements were done in several interaction regions using local closed orbit bumps around the interaction region triplets. The measurements were inspired by the goal of evaluating the value of betatron coupling coming from the interaction region triplets, because early in the run strong sources of the coupling were observed there. The results of the analysis of the IR coupling using the orbit bumps can be found in these proceedings [4]. Besides the coupling evaluation we measured and analyzed nonlinear error harmonics. The motivation here is the fact that at the collision lattice with  $\beta^* = 1 - 2m$  the nonlinear field harmonics in the interaction region triplet quadrupoles are the main factor limiting the dynamic aperture. Extracting information about high order errors from the beam measurements with orbit bumps can provide the basis for setting the strength of the nonlinear triplet correctors and hopefully improve machine performance.

The measurements were done at 3 interaction regions (IR2,IR6,IR8) in both RHIC rings, Blue and Yellow. Tight

time constraint for data taking during a very busy commissioning RHIC run limited us to take systematically data only with horizontal orbit bumps, although a few scans with vertical bumps at selected triplets were also recorded. Bump amplitudes was varied typically from 0 to  $6\sigma$  (or until beam survival in the machine) and rms closed orbit and tune data were systematically taken. The analysis of the rms orbit data are very useful to evaluate local linear coupling but for the analysis of higher-order multipoles betatron tune shifts proved more powerful.



Figure 3: Horizontal tune shift versus horizontal orbit bump amplitude in the Yellow IR triplets.



Figure 4: Vertical tune shift versus horizontal orbit bump amplitude in the Yellow IR triplets.

Figures 3 and 4 show examples of the tune scan dependences that were measured. The plot shows horizontal and vertical betatron tune shifts as a function of orbit bump strength for the Yellow ring interaction regions. The precision of the tune measurement is about  $10^{-3}$ . A sextupole

Triplet	$b_{2,3}$	$a_{2,3}$	$b_{3,4}$	$b_{4,5}$	$a_{4,5}$	$b_{5,6}$
YO5	0.94	-0.55	0.03	-0.08	0.11	-0.01
YI6	-0.95	0.14	0.36	0.03	-0.06	-0.03
YI7	1.01	-0.22	0.81	-0.36	-0.17	-0.15
YO8	3.81		-0.47	-1.85		0.06
YO1	0.32		-0.14	0		0
YI2	1.51		0.76	-0.75		-0.21

Table 1: Measured Yellow IR harmonics in units

components is easily identifiable by the linear dependence. For higher order terms we fitted the data with a fifth degree polynomial function. Except isolated data set the fitting worked very well.

As described in section 1, the betatron tune shift resulting from the bumps beside the normal gradient part( $\Delta Q$ ) includes the coupling contribution (c). It is not quite straitforward to analyze and disentangle errors from the coupling part of the tune shift. Fortunately the coupling contributes with opposite sign to the horizontal and vertical tune shift. Thus we eliminated it by adding measured horizontal and vertical tune shifts together. However, as follows from the table in the section 1, the  $a_{3,4}$  error harmonic can only be extracted from the coupling contribution. Thus we could not extract this particular error harmonic in this analysis. Likewise, some of the skew error harmonics can be only extracted from the vertical bump dependence, and we did not have data at all IR triplets.

The error harmonic analysis has been completed for the Yellow RHIC ring. The resulting values, expressed in  $10^{-4}$  units, are listed in table 1. For error harmonic normalization, the field  $B_N$  was taken as the field at the Q2 quadrupole of the IR triplet at the R = 10mm radius.

# **5** CONCLUSION

The measurement of multipoles using beam bumps seems well adapted to the correction of super-conducting machines. Its advantages are the accuracy (bump to aperture limit, non-linearity  $\sqrt{2}$  more effective than for a free oscillation), the small perturbation to the beam dynamics which may be further reduced by a simultaneous correction of the tunes, chromaticities, ..., an optimal use of the instrumentation (measurement of variations in the small amplitude range, use of a statistical averaging of its imperfections). The first attempt on RHIC delivered some interesting information and will be pursued.

#### **6 REFERENCES**

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