Non-BPS Branes of Supersymmetric Brane Worlds

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Abstract

We consider five-dimensional brane worlds with $N = 2$ gauged supergravity in the bulk coupled supersymmetrically to two boundary branes at the fixed points of a Z_2 symmetry. We analyse two mechanisms that break supersymmetry either by choosing flipped fermionic boundary conditions on the boundary branes or by modifying the gravitino variation to include both Z_2-odd and Z_2-even operators. In all cases we find the corresponding background. Including an even part in the gravitino variation leads to tilted branes. Choosing the flipped boundary conditions leads to AdS_4 branes and stabilized radion in the detuned case, when the expectation value of the even variation is nonzero. Another solution has the interpretation of moving AdS_4 branes separated by a horizon. The solution with moving branes separated by a horizon can be extended to the tuned case. In the presence of a horizon, temperature mediation communicates supersymmetry breakdown to the branes.

1 Introduction

It has been realized recently that there exist locally supersymmetric theories in five dimensions that include nontrivial physics localized on four dimensional branes [1, 2, 3, 4, 5]. The brane sectors may contain arbitrary four dimensional gauge theories as well as localized interactions between bulk fields. In the bulk one has a gauged 5d supergravity, which can be further coupled to five dimensional gauge sectors. Such five dimensional models with branes are likely to lead to interesting extensions of the Standard Model, pertaining to novel approaches to the hierarchy problem. The class of brane–bulk supergravities that has been studied so far in some detail, [6], is the one that allows for BPS solutions in five dimensions, thus resulting in a class of four-dimensional supergravities. There exists however a possibility, that Lagrangians that are locally supersymmetric in the whole 5d space, including branes, do not allow for a supersymmetric, BPS, vacuum. Such scenarios, where five-dimensional supersymmetry is broken down to nothing by the vacuum configurations were considered in the local case by Fabinger and Horava, [7], and their phenomenology at the level of *globally* (non)supersymmetric models has been studied by Barbieri et. al. and in the subsequent papers [8] (see also [9] for related discussion). However, eventually warped gravity and moduli fields have to be consistently included into the game. In this note we make some steps in this direction. We study explicitly gauged supergravity coupled supersymmetrically to four-dimensional branes. However, we generalize the construction of [3, 4] in two ways. Firstly, we extend supersymmetry transformations, and the prepotential, by Z_2-even terms. These terms introduce supersymmetric detuning between brane tensions and bulk potential energy. Secondly, we allow the Z_2-projections to be different on different branes. These new ingredients give rise to interesting new families of vacuum solutions, which are in fact examples of non-BPS branes. We give the explicit form of vacuum configurations, and discuss their properties.

The supergravities we consider in this paper form a background for interesting particle physics. After inserting supersymmetric gauge sectors on the branes, supersymmetry breakdown will be transmitted to the chiral models living there, but the supersymmetric pattern of couplings will be preserved. In the case of a charged bulk sector, the mass splits in the multiplets will be caused directly by different boundary conditions for bosons and fermions. The 'flipped' models are five–dimensional representatives of Dp–$D\bar{p}$ brane–antibrane systems. The work we present here should help understanding the low–energy physics of these interesting theories.

2 Supersymmetry Breaking

Let us recall the basic features of pure 5d N=2 gauged supergravity. The gravity multiplet $(e_\alpha^m, \psi_\alpha^A, \mathcal{A}_\alpha)$ consists of the metric (here we use a vielbein), a pair of symplectic Majorana gravitini, and a vector field called the graviphoton. There is a global SU(2) R-symmetry which rotates the two supercharges into each other. Making use of the graviphoton we can gauge a U(1) subgroup of the R-symmetry group. Such gauging can be described by an SU(2) algebra valued matrix $\mathcal{P} = \vec{P} \cdot i\vec{\sigma}$ (prepotential). We do not give the complete form of the action and supersymmetry transformation laws in gauged supergravity (see [10] for details), but only the relevant terms. The gravitino transformation law gets the following correction due to gauging (we use the normalization of [11])

$$\delta\psi^A_\alpha = -i\frac{\sqrt{2}}{3}\gamma_\alpha g\mathcal{P}^A_B\epsilon^B \tag{1}$$

where g is the U(1) gauge charge. Gauging introduces also the potential term into the action

$$V = \frac{8}{3}g^2 Tr(\mathcal{P}^2). \tag{2}$$

Without bulk matter fields the prepotential is just a constant matrix so the potential term corresponds to a (negative) cosmological constant.

The distinguishing feature of the Randall-Sundrum scenario is that the fifth dimension is an orbifold S_1/Z_2. It is equivalent (and technically easier) to work on a smooth circle S_1 with the fifth coordinate ranging from $-\pi\rho$ to $\pi\rho$ and impose Z_2 symmetry on the fields of our Lagrangian. The Z_2 symmetry acts by $x^5 \to -x^5$. Under its action the bosonic fields $g_{\mu\nu}$, g_{55} and $\mathcal{A}_5$ are even, while $g_{\mu 5}$ and $\mathcal{A}_\mu$ are odd. The Z_2 action on the gravitino is defined as follows

$$\psi^A_\mu(-x^5) = \gamma_5\mathcal{Q}^A_B\psi^B_\mu(x^5) \quad \psi^A_5(-x^5) = -\gamma_5\mathcal{Q}^A_B\psi^B_5(x^5) \tag{3}$$

where $\mathcal{Q} = \vec{Q}\cdot\vec{\sigma}$ and $\vec{Q}^2 = 1$. In [5] it was shown that this is the most general form of the Z_2 action consistent with the symplectic properties of the gravitino. The Z_2 action on the supersymmetry generating parameter ϵ must be the same as that on the 4d components of the gravitino.

Further we need to define the Z_2 symmetry under reflection around the second fixed point at $x^5 = \pi\rho$

$$\psi^A_\mu(\pi\rho - x^5) = \alpha\gamma_5\mathcal{Q}^A_B\psi^B_\mu(\pi\rho + x^5) \quad \psi^A_5(\pi\rho - x^5) = -\alpha\gamma_5\mathcal{Q}^A_B\psi^B_5(\pi\rho + x^5). \tag{4}$$

Apart from the conventional case $\alpha = 1$, in this letter we also consider the 'flipped' supersymmetry with $\alpha = -1$. In the latter case supersymmetry is always broken globally, as different spinors survive the orbifold projection on each wall. Note also, that in the flipped case we have $\psi^A_\alpha(x^5 + 2\pi\rho) = -\psi^A_\alpha(x^5)$.

It is straightforward to check that the 5d *ungauged* supergravity action is invariant under transformations (3) and (4) but the *gauged* supergravity action is not invariant if the prepotential $\mathcal{P}$ is general. In reference [5] the constraint stating that $\mathcal{P}$ must either commute or anticommute with $\mathcal{Q}$ was derived. We find that there exists a less restrictive possibility. The action and the supersymmetry transformation laws are still Z_2 invariant if we choose the prepotential of the form:

$$g\mathcal{P} = g_1\epsilon(x^5)\mathcal{R} + g_2\mathcal{S} \tag{5}$$

where $\mathcal{R} = \vec{R}\cdot i\vec{\sigma}$ commutes with $\mathcal{Q}$ and $\mathcal{S} = \vec{S}\cdot i\vec{\sigma}$ anticommutes with $\mathcal{Q}$. Equivalently, $\mathcal{R} = i\sqrt{\vec{R}^2}\mathcal{Q}$ and $\mathcal{S} = (\vec{Q}\times\vec{U})\cdot i\vec{\sigma}$ with some arbitrary vector $\vec{U}$. Note that the cosmological constant does not contain the step function and is given by

$$\Lambda_5 = -\frac{16}{3}(g_1^2\vec{R}^2 + g_2^2\vec{S}^2). \tag{6}$$

The supergravity action with prepotential (5) contains both symmetric and antisymmetric (multiplied by $\epsilon(x^5)$) gravitino masses. The presence of the Z_2-symmetric piece $\mathcal{S}$ in the prepotential results in the supersymmetric detuning between the brane tensions and the bulk cosmological term. The part of the gravitino transformation law due to gauging is now:

$$\delta\psi^A_\alpha = -i\frac{\sqrt{2}}{3}\gamma_\alpha(g_1\epsilon(x^5)\mathcal{R}^A_B + g_2\mathcal{S}^A_B)\epsilon^B. \tag{7}$$

The presence of the step function in the above transformation law implies that the 5d action is not supersymmetric. The fifth derivative in the gravitino kinetic term acts on the step function producing an expression multiplied by a delta function. The uncancelled variation is:

$$\delta\mathcal{L} = -2i\sqrt{2}g_1(\delta(x^5) - \delta(x^5 - \pi\rho))e_4\mathcal{R}^A_B\overline{\psi_\mu}_A\gamma^\mu\gamma^5\epsilon^B. \tag{8}$$

Using the fact that the matrix $\mathcal{R}$ is proportional to $\mathcal{Q}$ we have the following relations:

$$\begin{aligned}\gamma_5\mathcal{R}^A_B\epsilon^B(0) &= i\sqrt{\vec{R}^2}\epsilon^A(0)\\ \gamma_5\mathcal{R}^A_B\epsilon^B(\pi\rho) &= i\alpha\sqrt{\vec{R}^2}\epsilon^A(\pi\rho).\end{aligned} \tag{9}$$

Thus:

$$\delta\mathcal{L} = 2\sqrt{2}g_1\sqrt{\vec{R}^2}e_4\overline{\psi_\mu}_A\gamma^\mu\epsilon^A(\delta(x^5) - \alpha\delta(x^5 - \pi\rho)). \tag{10}$$

The variation (10) can be cancelled by the variation of the determinant in the brane tension term:

$$\mathcal{L}_T = -4\sqrt{2}g_1\sqrt{\vec{R}^2}e_4(\delta(x^5) - \alpha\delta(x^5 - \pi\rho)). \tag{11}$$

Summarizing (and changing the normalization to that used by Randall and Sundrum), we constructed a *locally supersymmetric Lagrangian*, which has the following bosonic gravity part:

$$M^{-3}S = \int d^5x\sqrt{-g_5}(\frac{1}{2}R + 6k^2) - 6\int d^5x\sqrt{-g_4}kT(\delta(x^5) - \alpha\delta(x^5 - \pi\rho)) \tag{12}$$

where we have defined

$$k = \sqrt{\frac{8}{9}(g_1^2R^2 + g_2^2S^2)} \tag{13}$$

and

$$T = \frac{g_1\sqrt{\vec{R}^2}}{\sqrt{(g_1^2R^2 + g_2^2S^2)}}. \tag{14}$$

The BPS relation between the bulk cosmological constant and the brane tensions ('the Randall-Sundrum fine-tuning') corresponds to $T = 1$, which holds only when $\langle\mathcal{S}\rangle = 0$. In such case the vacuum solution is AdS_5 in the bulk with flat Minkowski branes [12]. This vacuum preserves one half of the supercharges corresponding to unbroken N=1 supersymmetry in four dimensions [3]. As soon as we switch on non-zero $\mathcal{S}$, we get $T < 1$, the BPS relation is destroyed and the vacuum necessarily breaks all supersymmetries. This is not surprising, since only the case $\langle\mathcal{S}\rangle = 0$ (with the 'odd gauge charge' and the 'antisymmetric gravitino masses') corresponds to a thin wall limit of smooth supersymmetric domain wall solutions [13].

It is clear, that supersymmetry is broken when $\alpha = \pm 1$ and $\langle\mathcal{S}\rangle \neq 0$. If $\alpha = -1$, then supersymmetry is always broken globally, independently of the expectation value of $\mathcal{S}$. To see this explicitly, let us take $\langle\mathcal{S}\rangle = 0$ and note that the fermions which are allowed to propagate on the left and right branes have to obey the conditions $W_0{}^A{}_B\psi^B = 0$ and $W_\pi{}^A{}_B\psi^B = 0$ respectively, where $W_{0,\,\pi}$ are given by (4). The projection operators $\Pi_\pm{}^A{}_B = \frac{1}{2}(\mathbf{1}\delta^A_B \pm \gamma_5\mathcal{Q}^A_B)$ split each spinor into two components, one of which is annihilated by W_0: $W_0\epsilon_+ = W_0\Pi_+\epsilon = 0$. The second component, $\Pi_-\epsilon$, is annihilated by W_π if $\alpha = -1$; for $\alpha = +1$ $W_0 = W_\pi$. The BPS conditions imply (we take here $ds^2 = a^2(x^5)dx^2 + (dx^5)^2$)

$$\frac{a'}{a}\gamma_5\epsilon^A + \frac{2\sqrt{2}}{3}g_1\epsilon(x^5)\sqrt{R^2}\mathcal{Q}^A_B\epsilon^B = 0 \tag{15}$$

(this holds for AdS_4 and Minkowski foliations). When we apply the operator Π_+ to (15), we obtain the conditions $\frac{a'}{a}\frac{2\sqrt{2}}{3}g_1\epsilon(x^5)\sqrt{R^2} = 0$ or $\epsilon_+ \equiv 0$. The first possibility leads to discontinuities of the warp factor at the fixed points: $[\frac{a'}{a}]_0 = -2kT$, $[\frac{a'}{a}]_{\pi\rho} = +2kT$. However, the matching conditions in the equations of motion give $[\frac{a'}{a}]_0 = -2kT$, $[\frac{a'}{a}]_{\pi\rho} = +2\alpha kT$, which are in contradiction with the BPS condition for $\alpha = -1$, unless $\epsilon_+ \equiv 0$. Applying to (15) the second projector, Π_-, one finds out immediately that boundary conditions and BPS conditions agree on both branes only for $\epsilon_- \equiv 0$. Thus there exists no globally defined Killing spinor in the setup with flipped Z_2 acting on fermions (bosons are acted on as in the unflipped case), and all supersymmetries are broken, in particular the 4d effective theory is explicitly nonsupersymmetric.

3 Non-BPS Branes

In the remainder of this letter we construct static vacuum solutions in the detuned case ($T < 1$ and/or $\alpha = -1$). We start with the case $\alpha = 1$, i.e. opposite brane tensions. In [14, 15] a method applicable to branes violating the BPS relation was outlined. The idea was to write the BPS solution in time-boosted coordinates, so that it satisfied the matching conditions for $T > 1$. Here, we perform a similar trick which works for $T < 1$.

We write the bulk solution in the conformal gauge

$$ds^2 = a^2_{BPS}(y)(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2). \tag{16}$$

In the case at hand

$$a_{BPS}(y) = 1/(ky). \tag{17}$$

Then we apply a rotation to the bulk defining the new coordinates parametrized by the three-vector $\vec{h}$ ($i = 1, 2, 3$)

$$\tilde{x^i} = \frac{x^i - h^i y}{\sqrt{1+h^2}} \tag{18}$$

$$\tilde{y} = \frac{y + h^i x^i}{\sqrt{1+h^2}} \tag{19}$$

and $\tilde{x}_0 = x_0$, with h being the norm of $\vec{h}$. Under this change of coordinates $\tilde{g}_{\alpha\beta}(\tilde{x}) = g_{\alpha\beta}(x)$ so we can rewrite (16) as:

$$ds^2 = a^2_{BPS}(\frac{\tilde{y} - h^i \tilde{x}^i}{\sqrt{1+h^2}})(\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu + d\tilde{y}^2). \tag{20}$$

This is the same solution as (16) written in another coordinate system so this is also a legitimate solution in the bulk. The matching conditions are

$$\frac{\partial_{\tilde{y}} a_{BPS}}{a^2_{BPS}} = -kT. \tag{21}$$

Note that

$$\frac{\partial_{\tilde{y}} a_{BPS}}{a^2_{BPS}} = -k(1+h^2)^{-1/2}, \tag{22}$$

hence we find

$$\sqrt{1+h^2}T = 1. \tag{23}$$

Thus, we have to perform a rotation parametrized by a vector $\vec{h}$ whose norm is

$$h = \frac{\sqrt{1-T^2}}{T}. \tag{24}$$

Alternatively, one can view this solution as AdS_5 bulk with 'tilted' branes, i.e. the transverse position of the branes depends on x^μ and is given by

$$y_1 = -h_1^i x_1^i + \sqrt{1+h^2}\ \pi\rho \tag{25}$$

for the second brane. To match boundary conditions on the first brane one can imagine playing the same trick, i.e. rotating the brane by a vector h_0^i, $\ i = 1,2,3$, independent of $\vec{h_1}$. Again, locally near the brane the bulk solution is AdS_5 in the conformal frame, and the equation defining the brane is

$$y_0 = -h_0^i x_0^i. \tag{26}$$

Since in the bulk the two local AdS_5 solutions match smoothly onto each other, the branes given by (25) and (26) plus the AdS_5 metric in the bulk (in conformal coordinates) form a complete solution to the equations of motion. Let us specialize to the case where both vectors point in the x_3 direction. The branes are rotated with respect to the x_3–axis by angles given by

$$\tan\theta_1 = -h_1^3, \quad \tan\theta_0 = -h_0^3. \tag{27}$$

Since $h_0^3 = \pm h_1^3$ one obtains $\tan\theta_k = \pm\frac{\sqrt{1-T^2}}{T}$, $k = 0, 1$. It follows that the rotation angles of the tilted branes are proportional to the supersymmetry breaking expectation value of the Z_2–even part of the gravitino supersymmetry transformations, $\langle\mathcal{S}\rangle$,

$$\tan\theta_k = \pm\frac{g_2\sqrt{S^2}}{g_1\sqrt{R^2}}. \tag{28}$$

One can formulate this observation in a different way: the rotation of a brane induces $N = 2 \rightarrow N = 0$ supersymmetry breakdown whose magnitude is proportional to the rotation angle. The second brane also needs to be rotated to match the boundary conditions properly.

Now we move on to the 'flipped susy' case $\alpha = -1$. A vacuum solution in the warped product form can be found

$$ds^2 = a^2(x^5) g_{\mu\nu} dx^\mu dx^\nu + R_0^2 (dx^5)^2, \tag{29}$$

where $g_{\mu\nu}$ is the AdS_4 metric with cosmological constant $\bar{\Lambda}$

$$g_{\mu\nu} dx^\mu dx^\nu = e^{-2\sqrt{-\bar{\Lambda}} x_3} (-dt^2 + dx_1^2 + dx_2^2) + dx_3^2 \tag{30}$$

and the third coordinate x_3 has been singled out. As long as $T < 1$ the static vacuum solution is AdS_5 in the bulk and the warp factor can be parametrized as [16, 17]:

$$a(x^5) = \frac{\sqrt{-\bar{\Lambda}}}{k} \cosh(kR_0|x_5| - C). \tag{31}$$

This metric is isometric to the warped metric of AdS_5:

$$ds^2 = e^{-2kR_0 y}(-d\tilde{t}^2 + d\tilde{x}_1^2 + d\tilde{x}_2^2 + d\tilde{x}_3^2) + R_0^2 dy^2, \tag{32}$$

where

$$\tilde{x}_3 = (\sqrt{-\bar{\Lambda}})^{-1} \tanh(C - kR_0 x_5)\, e^{\sqrt{-\bar{\Lambda}} x_3}, \;\; y = \sqrt{-\bar{\Lambda}} \frac{x_3}{kR_0} - \frac{\ln(\frac{\sqrt{-\bar{\Lambda}}}{k} \cosh(C - kR_0 x_5))}{kR_0} \tag{33}$$

and $\tilde{t} = t,\; \tilde{x}_1 = x_1,\; \tilde{x}_2 = x_2$. This coordinate transformation will be useful in the following section where we discuss the existence of a geodesically complete submanifold of AdS_5 with a horizon where the two branes can be embedded.

The matching conditions for AdS_4 branes embedded in AdS_5 read

$$\tanh(C) \;=\; T \tag{34}$$

$$\tanh(kR_0 \pi\rho - C) \;=\; -\alpha T \;=\; T. \tag{35}$$

The first condition sets the integration constant C and the second fixes the size of the fifth dimension. The radion is stabilized at the value

$$\pi\rho k R_0 = \ln(\frac{1+T}{1-T}). \tag{36}$$

Moreover, the magnitude of the brane cosmological constant is fixed by the normalization $a(0) = 1$. This leads to

$$\bar{\Lambda} = (T^2 - 1)k^2 < 0. \tag{37}$$

The cosmological constant on the brane depends directly on the scale of supersymmetry breaking on the brane. The same is true for the expectation value of the radion. Notice, that $\langle R_0 \rangle$ can be expressed solely in terms of k and $\bar{\Lambda}$. The formula equivalent to (36) in the case $\alpha = +1$

gives $R_0 = 0$. To summarize, the nonzero expectation value of $\mathcal{S}$ gives rise to detuning between brane and bulk tensions and as a consequence to stabilization of the radion. The $\langle\mathcal{S}\rangle$ contributes also to supersymmetry breakdown, but in the case of $\alpha = -1$ supersymmetry is broken even if $\langle\mathcal{S}\rangle = 0$.

One finds that (31) is not a valid solution in the case $T = 1$. Indeed, this implies that the brane cosmological constant vanishes and the second brane is sent to infinity. In that case we expect a global mismatch due to the boundary condition on one of the branes. This is the subject of the following section.

4 Flipped Supersymmetry and Moving Branes

In this section we deal with the flipped case $\alpha = -1$ corresponding to opposite chiralities of the boundary branes and focus on the case where the brane tensions on both branes are equal. In order to find non-singular solutions to the equations of motion it is sufficient to use locally AdS spaces in the bulk. Consider the following bulk metric with $l = 1/k$

$$ds^2 = -(\frac{r^2}{l^2} - 1)dt^2 + \frac{1}{\frac{r^2}{l^2} - 1}dr^2 + r^2 d\Sigma^2 \tag{38}$$

where $d\Sigma^2$ is the metric on a compact hyperbolic three-space Σ of constant curvature -1. Such spaces can be obtained by quotienting the usual hyperbolic three-space by a discrete group. The metric is the usual AdS-black hole metric with a hyperbolic horizon where the mass of the black hole has been set to zero. Locally one can define, [18],

$$u = -\xi\sqrt{1 - \frac{l^2}{r^2}}e^{-t/l}, \;\; v = \xi\sqrt{1 - \frac{l^2}{r^2}}e^{t/l} \tag{39}$$

and

$$\tilde{r} = l\frac{r}{\xi}, \tag{40}$$

where the metric on the hyperbolic three-space is

$$d\Sigma^2 = \frac{d\xi^2 + dx_i dx^i}{\xi^2}. \tag{41}$$

This leads to the AdS_5 metric in Poincare coordinates

$$ds^2 = \frac{\tilde{r}^2}{l^2}(-dudv + d\tilde{x}_i d\tilde{x}^i) + \frac{l^2}{\tilde{r}^2}d\tilde{r}^2. \tag{42}$$

We have used the obvious relation between the light cone coordinates and orthogonal coordinates

$$u = \tilde{t} - \tilde{x}_3 \;\; v = \tilde{t} + \tilde{x}_3 \tag{43}$$

and identified $\tilde{x}_i = x_i$. Globally one must restrict the range of r to $r \geq l$ where the horizon sits at $r = l$. The Carter-Penrose diagram can be seen in figure 1 where the maximal analytic

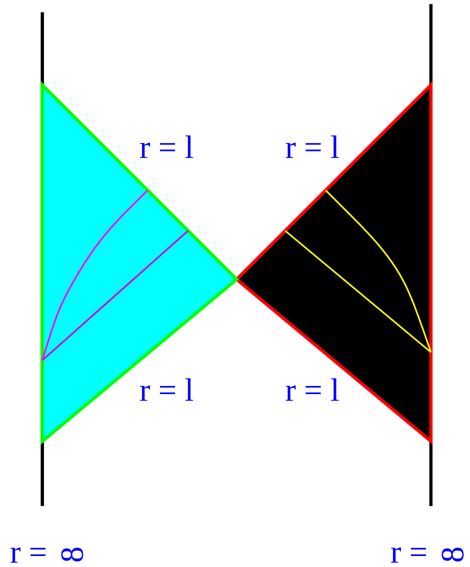

Figure 1: The Carter-Penrose diagram of the geodesically complete manifold $\mathcal{M}$ embedded in the AdS_5 strip. Both sides of the horizon $r = l$ correspond to $r \geq l$. Two light cones have been depicted. The moving branes stay within the respective light cones reaching the speed of light at the horizon.

extension of the manifold $\mathcal{M}$ (defined by (38)–(42)) has been drawn. It consists of two copies of the same $r \geq l$ space glued at $r = l$ by a wormhole called the Einstein-Rosen bridge whose topology is the one of Σ. Notice that the horizon sits in the middle of the Carter-Penrose diagram of AdS_5.

Let us come back to the flipped supersymmetric brane model where two branes of the same tension are present. It is relevant to locate the two branes with respect to the horizon. Notice that the parametrization (39) of $\mathcal{M}$ implies that $\tilde{x}_3 \geq 0$. However, the two AdS_4 branes of the previous section are such that $\tanh(kR_0x_5 - C)$ have opposite signs. Given that we are interested in solutions related to the static AdS_4 solutions, this leads to the assignment of the opposite sign $\tilde{x}_3$ coordinates for the two branes in the figure 1. Indeed, the static branes are located at

$$x_5 = 0,\ x_5 = \frac{2C}{kR_0}. \tag{44}$$

The horizon at $r = l$ corresponds to $\tilde{x}_3 = 0$ and that through (33) corresponds to $x_5 = \frac{C}{kR_0}$, the point lying just in the middle between the static branes. We thus find that the two branes are on both sides of the horizon. As $T \to 1$ the second brane goes to infinity and the static solution is not valid anymore. We will find that, by embedding the two branes in $\mathcal{M}$, this problem can be alleviated.

One can find a solution to this model by embedding each brane in one of the two copies of the local AdS space-time $r \geq l$. The two branes are separated by the hyperbolic Einstein-Rosen bridge located at the horizon. In each half space the motion of the branes is determined by the

the boundary condition for moving branes

$$(\frac{r^2}{l^2} - 1 + \dot{r}^2)^{1/2} = T\frac{r}{l} \tag{45}$$

in proper time τ defined by

$$-(\frac{r^2}{l^2} - 1)\dot{t}^2 + \frac{1}{\frac{r^2}{l^2} - 1}\dot{r}^2 = -1. \tag{46}$$

The boundary condition is equivalent to the Friedmann equation

$$H^2 = \frac{T^2 - 1}{l^2} + \frac{1}{r^2} \tag{47}$$

where $H = \dot{r}/r$. Now the induced metric on the brane reads

$$ds_B^2 = -d\tau^2 + r^2(\tau)d\Sigma^2. \tag{48}$$

The solution to Friedmann's equation is

$$r(\tau) = \pm\frac{l}{\sqrt{1-T^2}}\sin(\frac{\sqrt{1-T^2}}{l}\tau) \tag{49}$$

leading to the identification of the moving brane with an AdS_4 brane with cosmological constant $\bar{\Lambda}$. Notice that $r \geq l$ implies that τ does not cover the whole circle. This implies that the moving branes are only a slice of AdS_4 branes, with an infinite period oscillation in the t coordinate. In conclusion the moving brane is nothing but another way of embedding AdS_4 branes in AdS_5 which is only locally isometric to the static branes of section 3.

We can now treat the degenerate case $T = 1$ for which the above brane cosmological constant $\bar{\Lambda} \sim (T^2 - 1)$ vanishes. Now the brane has the topology of an open FRW space-time with curvature $k = -1$. The motion is given by

$$r(\tau) = \pm\tau + r_0. \tag{50}$$

We can now deduce the evolution equation

$$\frac{r^2(t)}{l^2} - 1 = e^{\pm 2t/l}. \tag{51}$$

This implies that the brane either recedes away from the horizon or converges towards the horizon. Moreover it takes an infinite amount of time to reach the horizon. This is to be contrasted with the finite amount of proper time that it takes to reach the horizon.

It convenient to use tortoise coordinates defined by

$$u = \frac{l}{2}\ln(\frac{\frac{r}{l} - 1}{\frac{r}{l} + 1}) \tag{52}$$

leading to

$$ds^2 = -(\text{cotanh}^2\frac{u}{l} - 1)(dt^2 - du^2) + l^2\text{cotanh}^2\frac{u}{l}d\Sigma^2. \tag{53}$$

The horizon is now at $u = -\infty$ and the spatial infinity at $u = 0$. In the vicinity of the horizon the metric becomes

$$ds^2 = 4e^{2u/l}(-dt^2 + du^2) + l^2 d\Sigma^2. \tag{54}$$

Putting $x = 2le^{u/l}$ we find that the horizon has the structure of a Rindler space times the hyperbolic space Σ. In particular the temperature of the horizon is

$$T_H = \frac{1}{2\pi l}. \tag{55}$$

It can be interpreted as the emission of gravitons and gravitini coming out through the wormhole. One may wonder how an observer bound to a brane would interpret the supersymmetry breakdown seen on his/her brane, with no classical communication with the second brane from which he/she is separated by the horizon. We note that there is a nonzero temperature of the horizon, of order k, which is observable, and can be seen locally on a given side of the horizon as the reason for the mass splittings in the bulk multiplets. One could describe this phenomenon as temperature mediated supersymmetry breakdown.

In the system of coordinates that we adopted the light-like geodesics follow

$$u = r_0 \pm t \tag{56}$$

showing that particles take an infinite amount of time to reach the horizon. Notice that the trajectory of the moving brane is given by

$$u(t) = \frac{l}{2}\ln(\frac{\frac{r(t)}{l} - 1}{\frac{r(t)}{l} + 1}) \tag{57}$$

corresponding to a speed

$$\dot{u} = \pm\frac{l}{r}. \tag{58}$$

The brane reaches the speed of light at the horizon.

Let us make a comment on trapping of gravity on the brane. When the brane is far away from the horizon, $r(t) \gg l$, the metric (38) takes the form of the AdS_5 metric in conformal gauge, and the brane is moving very slowly with respect to it. Hence, locally around the brane gravity should be indistinguishable from that around the positive tension Randall–Sundrum brane. However, when the brane comes close to the horizon, the specific shape of the metric near horizon becomes important, and also the Hubble length on the brane, $d_H = H^{-1} = r(t)$, becomes comparable to l, hence gravity on the brane becomes nonstandard.

5 Summary

In this paper we have presented generalized supersymmetric brane worlds. We have enhanced supersymmetry transformations by including both Z_2–odd and Z_2–even components in the 5d prepotential. Further, we have allowed the Z_2 symmetry of the Lagrangian to act on fermions locally, differently in the vicinity of each wall (this is the case of flipped boundary conditions for fermionic degrees of freedom). In models with flipped boundary conditions or with nonzero vacuum expectation value for Z_2–even parts of supersymmetry transformations ($\sim \langle \mathcal{S} \rangle$) the $N = 2$ supersymmetry is broken down to nothing by vacuum configurations. These configurations, which are non–BPS domain walls in 5d, have the form of rotated branes if $\langle \mathcal{S} \rangle \neq 0$, with supersymmetry breakdown proportional to the rotation angle. There are no singularities, but 4d Lorentz invariance is broken in these vacua.

Models with flipped boundary conditions correspond to Dp–$D\bar{p}$ brane–antibrane systems in 5d. There supersymmetry is broken, which can be understood as a mismatch between possible Killing spinors in the bulk and boundary conditions on the branes. There exist static AdS_4 branes with a stabilized radion. Other non–BPS branes that we have found are moving, and in addition there is a horizon (but no essential singularity) separating the branes. One may wonder how an observer bound to a brane would interpret supersymmetry breakdown seen on his brane, without classical communication with the second brane. The relevant observation is that there is a nonzero temperature of the horizon, of the order of k, which is observable, and can be seen locally on a given side of the horizon as the reason for the mass splittings in the bulk multiplets. One could describe this phenomenon as temperature mediated supersymmetry breakdown. An interesting problem is to add brane matter to the backgrounds we have discussed, and to derive low–energy phenomenology of such models. We leave this issue to a future publication.

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