

GUT Breaking on the Lattice

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July 27, 2001

Abstract

We construct a supersymmetric grand unified model in the framework of a latticized extra dimension. The $SU(5)$ symmetries on the lattice are broken by the vacuum expectation values of the link fields connecting adjacent $SU(5)$ sites, leaving just the MSSM at low energies. Below the $SU(5)$ breaking scale, the theory gives rise to a similar spectrum as in orbifold breaking of $SU(5)$ symmetry in 5 dimensions, and shares many features with the latter scenario. We discuss gauge coupling unification and proton decay emphasizing the differences with respect to the usual grand unified theories. Our model may be viewed as an effective four dimensional description of the orbifold symmetry breaking in higher dimensions.

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1 Introduction

Recently, a new approach to gauge theories in extra dimensions has been introduced by considering extra dimensions on a transverse lattice [1, 2]. This provides an “ultraviolet complete” gauge invariant description of the higher dimensional gauge theory. On the other hand, from a purely 4-dimensional point of view, the extra dimensions are “generated” through a series of gauge groups and link fields among them. This latticizing or deconstructing approach to the extra dimensions provides a great tool to understand higher dimensional gauge theories, and to obtain new models both in pure 4 dimensions and higher dimensions [3, 4, 5, 6].

In this note, we examine the orbifold breaking of the grand unified (GUT) gauge symmetry [7, 8, 9, 10] from the point of view of the deconstructed extra dimensions. In the case of $SU(5)$ GUT breaking, a reflection around an orbifold fixed point $y = 0$ with the parity transformation $A_\mu(-y) = P^{-1}A_\mu(y)P$, $P = \text{diag}(-1, -1, -1, +1, +1)$ projects out the zero modes of the X, Y gauge bosons and breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. In field theory, the orbifold should be viewed as a theory defined on a finite interval with suitable boundary conditions. In this case, the boundary condition is such that the gauge symmetry at the boundary point $y = 0$ is only $SU(3) \times SU(2) \times U(1)$, while the $SU(5)$ gauge symmetry is preserved in the bulk. The usual gauge coupling unification can be preserved because the gauge couplings are dominated by the $SU(5)$ symmetric bulk contributions which are enhanced by the volume factor relative to the contributions from the boundary. There are several nice features of this GUT breaking mechanism. It is easy to obtain doublet-triplet splitting in the Higgs sector, at the same time avoiding proton decay mediated by the colored triplet Higgs fields, which may already pose a problem with the current experimental bounds in the usual 4-dimensional GUT. The gaugino mediated supersymmetry (SUSY) breaking [11] can naturally be incorporated in this framework to solve the SUSY flavor problem.

In the following, we consider this orbifold GUT breaking on latticized extra dimensions. It becomes a 4-dimensional theory with a series of gauge groups, broken down to the diagonal subgroup by the link field vacuum expectation values (VEV's). The simplest realization is to have only $SU(3) \times SU(2) \times U(1)$ gauge symmetry on one lattice point at the end, and $SU(5)$'s on all other lattice points. However, we prefer to start with $SU(5)$'s on all lattice points, and break them down to $SU(3) \times SU(2) \times U(1)$ with the VEV's of the link fields. Since the link fields are identified with the A_5 component of the gauge field

in the continuum limit, this is equivalent to the “Wilson line breaking,” which has been shown to be equivalent to the orbifold breaking in the continuum theory [12, 13, 14]. In this model, the $SU(3)$, $SU(2)$, and $U(1)$ gauge couplings are truly unified at some high scale. We will find that the spectrum of the continuum theory is reproduced in the limit of large number of lattice points. We will also discuss related issues such as doublet-triplet splitting and gauge coupling unification in this 4-dimensional picture. While finishing this work, we learned that a similar idea is being pursued by C. Csaki, G. D. Kribs, and J. Terning [15].

2 Formalism

2.1 SUSY $SU(5)$ on the orbifold lattice

We will begin with a supersymmetric $SU(5)$ theory on a latticized extra dimension. We assume that we have $N + 1$ $SU(5)$ gauge groups with common gauge coupling g , with $N + 1$ $N = 1$ vector multiplets V_i ($i = 0, \dots, N$), one for each $SU(5)$. There are also two sets of N chiral multiplets Φ_i and $\bar{\Phi}_i$, Φ_i forms $(\bar{5}_{i-1}, 5_i)$ under the two nearest $SU(5)$'s, while $\bar{\Phi}_i$ has the opposite charges. The Lagrangian for the vector multiplets and the chiral fields is the following:

$$\mathcal{L} = \int d^4x \left[\int d^4\theta \sum_{i=1}^N \left(\Phi_i^\dagger e^{(V_{i-1}-V_i)} \Phi_i + \bar{\Phi}_i^\dagger e^{(-V_{i-1}+V_i)} \bar{\Phi}_i \right) + \int d^2\theta \sum_{i=0}^N W_i^\alpha W_{i,\alpha} \right]. \quad (2.1)$$

As shown in [5], if the diagonal components of the link fields, Φ_i and $\bar{\Phi}_i$, acquire universal vacuum expectation values $v/\sqrt{2}$ which preserve the $N = 1$ supersymmetry,

$$\langle \Phi_i \rangle = \langle \bar{\Phi}_i \rangle = \frac{v}{\sqrt{2}} \text{diag}(1, 1, 1, 1, 1), \quad (2.2)$$

then $SU(5)^{N+1}$ is spontaneously broken down to a diagonal $SU(5)$.¹ The vector multiplets have the mass spectrum $M_{V,n} = 2gv \sin \frac{n\pi}{2(N+1)}$, $n = 0 \dots N$, while certain linear combinations of some components in the link fields Φ and $\bar{\Phi}$, which become part of the massive $N = 1$ vector multiplets, receive D term contributions and acquire the mass spectrum $M_{\Phi, \bar{\Phi}, n} = 2gv \sin \frac{n\pi}{2(N+1)}$, $n = 1 \dots N$. The other components of Φ and $\bar{\Phi}$ acquire masses $\sim v$ or higher and thus decouple from the low energy effective theory. Therefore, one recovers an $N = 1$ $SU(5)$ theory at the zero mode level.

¹For simplicity and ease of comparison with the result of orbifold breaking in 5D, we assume that the gauge couplings and the link VEV's are the same for all lattice points.

In addition, we have four sets of chiral fields: $H_{5,i} = \{H_{C,i}, H_{U,i}\}$ and its conjugate $H_{5,i}^c = \{H_{C,i}^c, H_{U,i}^c\}$; as well as $H_{\bar{5},i} = \{H_{\bar{C},i}, H_{D,i}\}$ and its conjugate $H_{\bar{5},i}^c = \{H_{\bar{C},i}^c, H_{D,i}^c\}$, where the subscripts show the charges of the fields under each $SU(5)$. We assume that on the zeroth brane, one only has $H_{5,0}$ and $H_{\bar{5},0}$, but not their conjugate partners. The superpotential for these fields is the following,

$$\mathcal{L} \sim \int d^4x \int d^2\theta \sum_{i=1}^N (M_H H_{5,i} H_{5,i}^c - \lambda \Phi_i H_{5,i-1} H_{5,i}^c + M_H H_{\bar{5},i} H_{\bar{5},i}^c - \lambda \bar{\Phi}_i H_{\bar{5},i-1} H_{\bar{5},i}^c) \dots \quad (2.3)$$

When Φ_i and $\bar{\Phi}_i$ acquire VEV's, and assuming $M_H = \lambda v / \sqrt{2}$, the mass spectra for the H fields arising from the superpotential are such that H_5 and $H_{\bar{5}}$ have massless zero modes which preserve $N = 1$ SUSY, while all conjugate fields become massive. The massive H and H^c fields have the spectra $M_{H,H^c,n} = 2M_H \sin \frac{n\pi}{2(N+1)}$, $n = 1, \dots, N$, which is the same as the massive vector multiplets and the massive chiral link fields, given the choice $M_H = gv$ ($\lambda = \sqrt{2}g$).

The results map onto a continuum five-dimensional theory with $N = 1$ supersymmetry compactified on a Z_2 orbifold of size $L = (N+1)/gv$. Orbifolding breaks the $N = 1$ SUSY in five dimensions (which is equivalent to $N = 2$ SUSY in four dimensions) down to $N = 1$ SUSY in four dimensions. The Higgs fields H_5 and $H_{\bar{5}}$ are complete hypermultiplets in the 5D theory, while in 4D $N = 1$ language each of them includes two chiral multiplets that are conjugate of each other.

2.2 $SU(5)$ breaking

To generate $SU(5)$ breaking, we assume that the first set of link fields takes on a different form. We assume that there are four link fields, $\Phi_1, \bar{\Phi}_1, \Phi'_1, \bar{\Phi}'_1$ that are charged under $SU(5)_0$ and $SU(5)_1$. Φ_1 and Φ'_1 form $(\bar{5}_0, 5_1)$ representation, and $\bar{\Phi}_1$ and $\bar{\Phi}'_1$ form $(5_0, \bar{5}_1)$. Their VEV's have the following structure,

$$\begin{aligned} \langle \Phi_1 \rangle = \langle \bar{\Phi}_1 \rangle &= \frac{v}{\sqrt{2}} \text{diag}(1, 1, 1, 0, 0); \\ \langle \Phi'_1 \rangle = \langle \bar{\Phi}'_1 \rangle &= \frac{v}{\sqrt{2}} \text{diag}(0, 0, 0, 1, 1); \end{aligned} \quad (2.4)$$

These VEV's can be obtained with suitable superpotential interaction [16, 17]. All other link fields have the same structure and VEV's as previously discussed. The unbroken gauge group is then $SU(3) \times SU(2) \times U(1)$, which is easily seen from the mass spectrum of the gauge bosons. For the $SU(3) \times SU(2) \times U(1)$ gauge bosons, the mass matrix

remains the same as in the case considered previously in Sec. 2.1,

$$M_{3-2-1}^2 = \frac{1}{2}g^2v^2 \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ & & & \cdots & \\ 0 & 0 & \cdots & -1 & 1 \end{pmatrix}. \quad (2.5)$$

Hence, there is a zero mode for each of the gauge groups. The X, Y gauge bosons, however, acquire a different mass spectrum, due to the fact that the VEV's of Φ_1 and Φ'_1 do not generate off-diagonal mass terms between the gauge bosons of $SU(5)_0$ and $SU(5)_1$,

$$M_{X,Y}^2 = \frac{1}{2}g^2v^2 \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ & & & \cdots & \\ 0 & 0 & \cdots & -1 & 1 \end{pmatrix}. \quad (2.6)$$

As a result, the X, Y gauge bosons on the 0th brane are decoupled from the rest of the lattice, and have masses $\overline{M}_0 = gv$. The other X, Y bosons on branes 1.. N acquire the mass spectrum $M_{X,Y,n} = 2gv \sin \frac{(n-1/2)\pi}{(2N+1)}$, $n = 1, \dots, N$.

Since the model preserves $N = 1$ SUSY, we expect it to contain the full vector multiplets of $SU(3) \times SU(2) \times U(1)$, and the X, Y vector multiplets to exhibit the same mass spectrum as their scalar components. The corresponding components in the Φ and $\overline{\Phi}$ fields also split in a similar fashion.

In the Higgs sector, we modify the couplings between the Higgs fields on the 0th and 1st brane and the corresponding link fields, while keeping the couplings on all other branes the same. The superpotential takes the following form,

$$W \sim \lambda' \frac{H_{5,0}\Phi_1\overline{\Phi}_1H_{\overline{5},0}}{M} - \lambda H_{5,0}\Phi'_1H_{5,1}^c + M_H H_{5,1}H_{5,1}^c - \lambda H_{\overline{5},0}\Phi'_1H_{\overline{5},1}^c + M_H H_{\overline{5},1}H_{\overline{5},1}^c + \dots \quad (2.7)$$

where the ... include the couplings of the H, H^c fields present in Eq. (2.3).

Since Φ'_1 and $\overline{\Phi}'_1$ have non-zero VEV's only in their last two diagonal components, the Higgs doublets $H_{U,i}$ and $H_{D,i}$ from $H_{5,i}$ and $H_{\overline{5},i}$ acquire the same mass spectrum as the $SU(3) \times SU(2) \times U(1)$ vector multiplets, as we previously discussed. Namely,

$M_{H_U, H_D, n} = 2gv \sin \frac{n\pi}{2(N+1)}$, $n = 0 \cdots N$. However, the structure for the colored Higgs components is changed. Φ'_1 and $\bar{\Phi}'_1$ do not generate off-diagonal mass terms between the 0th and 1st colored Higgs field, hence, the colored triplets on the 0th brane $H_{C,0}$ and $H_{\bar{C},0}$ are decoupled from the rest of the lattice. The $N \times N$ mass matrix for the colored Higgses on the $n = 1, \cdots N$ branes takes the following form,

$$M_{H_C, H_{\bar{C}}}^2 = M_H^2 \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ & & & \cdots & \\ 0 & 0 & \cdots & -1 & 1 \end{pmatrix}. \quad (2.8)$$

Therefore, there are N massive modes, with the spectrum $M_{H_C, H_{\bar{C}}, n} = 2gv \sin \frac{(n-1/2)\pi}{(2N+1)}$, $n = 1, \cdots N$. Finally, the colored components in $H_{5,0}$ and $H_{\bar{5},0}$ acquire masses from the higher dimensional coupling that is localized on the first brane, as shown in eqn.(2.7). Their masses are $\lambda' v^2/2M$. One can tune the parameter λ' , assuming that v is comparable to M , such that $\lambda' v^2/2M = gv$. Hence, the complete colored Higgs spectrum matches onto that of the X, Y vector multiplets.

It is easy to verify that the H^c fields also exhibit the same splitting between their colored components and their doublet components, due to the vacuum structure of the first set of link fields. The doublet components of the H^c fields have the spectrum $M_{H_{U,D}^c} = 2gv \sin \frac{n\pi}{2(N+1)}$, $n = 1 \cdots N$, while the triplets have the spectrum $M_{H_C^c} = 2gv \sin \frac{(n-1/2)\pi}{(2N+1)}$.

In summary, the massless modes in our model include $N = 1$ $SU(3) \times SU(2) \times U(1)$ vector multiplets and two Higgs chiral multiplets H_U and H_D . The massive modes fall into two types according to their spectrum.

- $M_{1n} = 2gv \sin(\frac{n\pi}{2(N+1)})$, $n = 1, \cdots, N$. The fields that have this type of mass spectrum are the KK modes of the $SU(3) \times SU(2) \times U(1)$ gauge supermultiplets, which include components coming from the link fields Φ and $\bar{\Phi}$, and the KK towers of Higgs doublets including $H_{U,D}$ and $H_{U,D}^c$.
- $\bar{M}_0 = gv$ and $M_{2n} = 2gv \sin \frac{(n-1/2)\pi}{(2N+1)}$, $n = 1, \cdots N$. This category includes the massive X, Y vector multiplets, which contain components from the link fields, and the KK towers of Higgs triplets which include $H_{C,\bar{C}}$. At the same time, the massive colored Higgs modes belonging to the $H_{5,\bar{5}}^c$ (there is a total of N of those) do not include \bar{M}_0 .

There are other components of the link fields (and other possible fields required to generate the link VEV's) acquiring masses of order v or higher from minimizing the potential.

At $n \ll N$, $M_{1n} \approx gv \frac{n\pi}{N}$, while $M_{2n} \approx gv \frac{(n-1/2)\pi}{N}$. Hence, the masses of the two sets of KK modes have a relative shift of $\frac{gv\pi}{2N}$. The low energy spectrum is the same as that of the KK modes in [9], in which a SUSY $SU(5)$ model in five dimensions is compactified on a $Z_2 \times Z_2$ orbifold.

One complete family of quarks and leptons comes from a $\bar{\mathbf{5}}$ and a $\mathbf{10}$ of the $SU(5)$. We can assume that these matter fields are localized on a single lattice point (i.e., transforming under a single gauge group). Having matter fields localized on the boundary which preserves (breaks) the $SU(5)$ gauge symmetry in the continuum theory corresponds to having them transforming under the N th (0th) gauge group. Alternatively, they can have wavefunctions distributed in the latticized bulk if one adds $\mathbf{10}$, $\bar{\mathbf{10}}$ and $\bar{\mathbf{5}}$, $\mathbf{5}$ on several lattice points, linked by the Φ , $\bar{\Phi}$ fields as in the Higgs sector. Because the zero modes of the Higgs doublets are equal linear combinations of $H_{U,i}$ and $H_{D,i}$ on all lattice points, they couple to fermions localized on different branes through Yukawa couplings and generate masses and mixings for the standard model fermions after the electroweak symmetry is broken.

3 Discussion

Given the spectrum presented in the previous section, the running of the gauge couplings at the 1-loop level including the threshold corrections from all massive modes can be easily calculated as follows,

$$\alpha_a^{-1}(M_Z) = \alpha_G^{-1}(M_*) + \frac{1}{2\pi} \left[\beta_a \ln\left(\frac{M_*}{M_Z}\right) + \gamma_a \sum_{n=1}^N \ln\left(\frac{M_*}{M_{1n}}\right) + \delta_a \sum_{n=1}^N \ln\left(\frac{M_*}{M_{2n}}\right) + \delta'_a \ln\left(\frac{M_*}{M_0}\right) + \Delta_a \right]. \quad (3.9)$$

Here $\alpha_G = g^2/(4\pi(N+1))$, and the numerical coefficients are determined only by the group structure of the fields. β_a ($a = 1, 2, 3$ refers to $U(1)$, $SU(2)$ and $SU(3)$) includes the contribution from the zero modes, γ_a includes the contribution from the modes which have a Type I mass spectrum, δ_a accounts for the the modes with a Type II mass spectrum for $n = 1, \dots, N$. These coefficients have been calculated in [9], where a model with a similar spectrum has been constructed from a $Z_2 \times Z_2$ compactification of a supersymmetric 5D theory: $\beta_a = (\frac{33}{5}, 1, -3)$, $\gamma_a = (\frac{6}{5}, -2, -6)$, $\delta_a = (-\frac{46}{5}, -6, -2)$. δ'_a counts the contributions

from the X and Y gauge bosons and $H_{C,\bar{C}}$, both with mass \overline{M}_0 . It is easy to show that $\delta'_a = (-\frac{48}{5}, -6, -3)$. Δ_a includes the threshold corrections from heavy link field components, which are near or above the $SU(5)$ breaking scale.

As discussed in Refs. [9, 18], gauge coupling unification is not ruined by the presence of the Kaluza-Klein spectrum. We now examine this in our model in more detail.

Let us define $M_G = 2 \times 10^{16}$ GeV as the scale where α_1 and α_2 meet in the MSSM. Previous studies [19, 20] have shown that with the central values for the gauge couplings at the weak scale, and a SUSY spectrum which is not unnaturally heavy, the gauge couplings miss each other at the scale M_G by

$$\varepsilon_3 \equiv \frac{g_3 - g_1}{g_1} \sim -(1 - 2)\% . \quad (3.10)$$

This mismatch should be accounted for by the GUT-scale threshold corrections within any specific grand unified model. We now proceed to calculate the prediction for ε_3 in our model.

We choose to match the MSSM onto the full GUT theory at the scale $M_* = M_G$. The condition $\alpha_1(M_G) = \alpha_2(M_G)$ implies that the threshold corrections to α_1 and α_2 at the scale M_G should be equal. This allows us to compute the value of $\overline{M}_0 = gv$ for any given fixed N :

$$\ln \frac{M_G}{\overline{M}_0} = -\frac{8}{9}(G_N - D_N), \quad (3.11)$$

where the numerical factors G_N and D_N are defined as follows,

$$\begin{aligned} G_N &\equiv \sum_{n=1}^N \ln \left[2 \sin \frac{n\pi}{2(N+1)} \right] = \frac{1}{2} \ln(N+1), \\ D_N &\equiv \sum_{n=1}^N \ln \left[2 \sin \frac{(n-1/2)\pi}{2N+1} \right] = 0 . \end{aligned} \quad (3.12)$$

(In what follows, we ignore the model-dependent effects from Δ_a .)

Having determined $\overline{M}_0 = gv$, there are no free parameters left, and for any given N we get a prediction for ε_3 at the unification scale M_G :

$$\varepsilon_3 = -\frac{\alpha_G}{3\pi}(G_N - D_N). \quad (3.13)$$

In Fig. 1 we show the prediction for ε_3 and \overline{M}_0 for several different values of N . For $N \gtrsim 20$ the proton decay rate from the dimension 6 operator exceeds the experimental bound, as discussed below. The points which are consistent with (marginally consistent

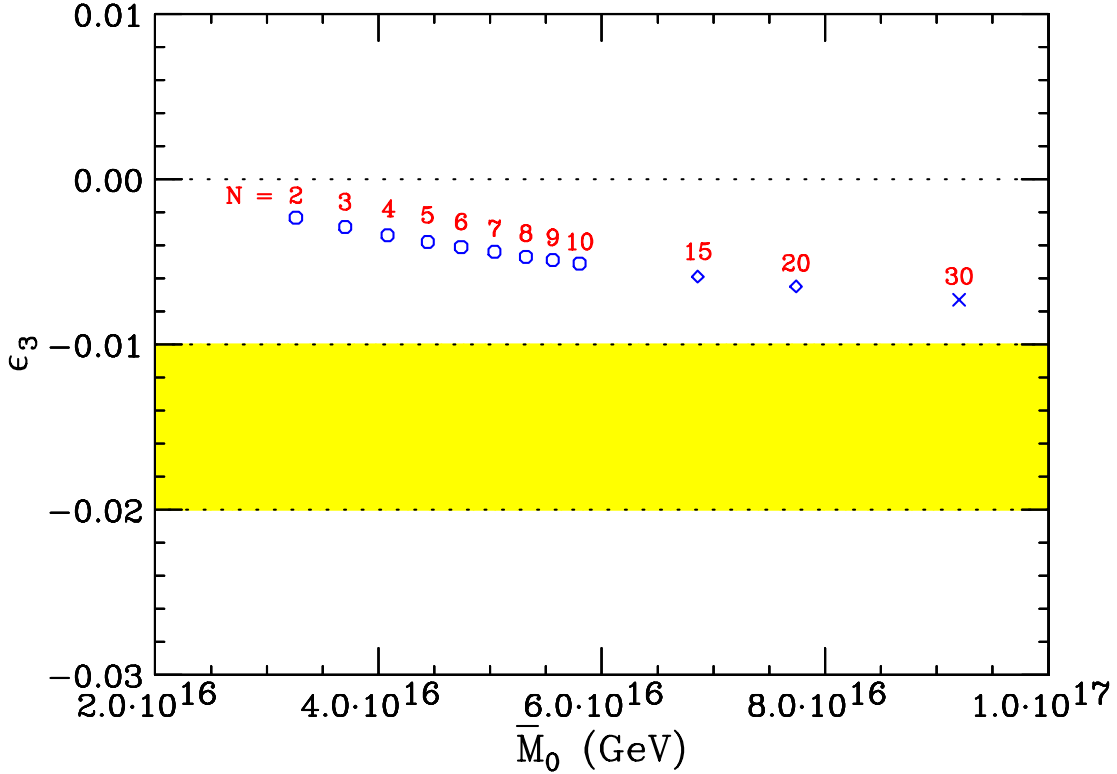


Figure 1: Predictions for ε_3 and \bar{M}_0 for several different values of N . The circles (diamonds, crosses) are consistent (marginally consistent, excluded) with the proton decay limits from dimension 6 operators. The shaded region is the range of ε_3 preferred by low energy data (see Eq. (3.10)).

with, excluded by) proton decay, are denoted by circles (diamonds, crosses). We see that the predicted threshold correction ε_3 is *negative*, i.e. goes in the right direction. However, its magnitude is not large enough to completely fix gauge coupling unification. One might hope that the additional threshold effects Δ_a due to the heavy components of the link fields will ameliorate the situation. Alternatively, gauge coupling unification can be further improved by reducing λ' , hence lowering the mass of the colored triplet Higgs on lattice point 0, which results in an additional negative contribution to ε_3 .

From Fig. 1 we also see that the $SU(5)$ breaking scale, defined as $2gv = 2\bar{M}_0$, is a few times higher than the usual M_G , and it grows for larger N . The mass of the lowest KK mode, namely, the effective compactification scale, is between $0.4 \sim 0.8 \times M_G$.

The colored triplet Higgs mediated proton decay is absent if the matter fields are localized away from the zeroth lattice point (i.e., do not transform under $SU(5)_0$), because the two sets of Higgs fields containing H_U and H_D do not couple to each other away from

lattice point 0. Although the triplets on lattice point 0, $H_{C,0}$ and $H_{\bar{C},0}$, couple through the non-renormalizable interaction λ' , they decouple from the triplets on the other lattice points. As a result, the proton decay process mediated by $H_{C,0}$ and $H_{\bar{C},0}$ can only take place if the quarks and the leptons are on the 0th brane.²

If the matter fields are localized on branes away from the 0th brane, the dimension 6 proton decay operators from the X, Y gauge boson exchange will be enhanced compared to the usual SUSY GUT, because there are many X, Y gauge bosons contributing to the process and the lightest ones are lighter than those in the traditional 4D SUSY GUT. The experimental value of the proton lifetime thus imposes constraints on the scales in our construction. The decay mode $p \rightarrow e^+ \pi^0$ through exchanging of X and Y gauge bosons requires that the lightest X and Y gauge bosons both should have mass $gv\pi/(2N+1) \geq 5 \times 10^{15}$ GeV. On the other hand, as we discussed earlier, the X, Y gauge bosons on lattice point 0 are decoupled from the other X, Y gauge bosons, and have mass gv , which is not suppressed by the volume factor N and somewhat larger than the usual SUSY GUT scale. Therefore, if the matter fields are localized on the lattice 0, the dimension 6 proton decay operators will be suppressed compared to the case when matter is localized away from the 0th brane.

As mentioned in the Introduction, gaugino mediated SUSY breaking can be easily incorporated in the orbifold GUT breaking scenario. In our case, similar superpartner spectrum can be obtained if SUSY breaking only couples to the gauge group on the lattice point away from where matter fields are localized [5, 6].

In summary, we have constructed a 4D SUSY GUT theory with many $SU(5)$ gauge groups. The gauge symmetry breaking scale is somewhat higher than the GUT scale in the usual 4D theory. However, gauge coupling unification is achieved due to the threshold corrections from the ‘‘Kaluza-Klein’’ modes lighter than the symmetry breaking scale. It shares many features with the 5D orbifold GUT breaking models, and may be viewed as an effective 4D description of these higher dimensional mechanisms.

²In Ref. [9], a $U(1)_R$ symmetry is imposed to completely forbid the dimension 5 proton decay operators. This $U(1)_R$ symmetry is not respected by the non-renormalizable interaction λ' in our model. However, the size of the dimension 5 proton decay operators depends on the flavor structure [21] and hence is difficult to estimate without a flavor theory.

Acknowledgements

We wish to thank Y. Nomura for useful discussions. We also wish to acknowledge the kind hospitality of the Aspen Center for Physics where this work was finished. H.-C. Cheng is supported by the Robert R. McCormick Fellowship and by DOE grant DE-FG02-90ER-40560. Research by JW was supported by the U.S. Department of Energy Grant DE-AC02-76CHO3000.

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