Three-Family Supersymmetric Standard-like Models
from Intersecting Brane Worlds

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We construct the first three family $N = 1$ supersymmetric string model with Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ from an orientifold of type IIA theory on $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ and D6-branes intersecting at angles. In addition to the minimal supersymmetric Standard Model particles, the model contains right-handed neutrinos, a chiral (but anomaly-free) set of exotic multiplets, and extra vector-like multiplets. We discuss some phenomenological features of this model.

The space of classical string vacua is highly degenerate, and at present we are unable to make definitive statements about how the string vacuum describing our universe is selected. Nonetheless, one can use phenomenological constraints as guidelines to construct semi-realistic string models and explore, with judicious assumptions, the resulting phenomenology. The purpose of such explorations is of course not to find *the* model which would fully describe our world, but to examine the generic features of these string derived solutions.

Until a few years ago, such explorations were carried out mainly in the framework of weakly coupled heterotic string theory. Indeed, a number of semi-realistic string models have been constructed and analyzed [2]. However, an important lesson from string duality is that these models represent only a corner of M-theory – the string vacuum describing our world may well be in a completely different regime in which the perturbative description of heterotic string theory breaks down [3]. Fortunately, the advent of D-branes allows for the construction of semi-realistic string models in another calculable regime, as illustrated by the various four-dimensional $N = 1$ supersymmetric Type II orientifolds ([4–13] and references therein) constructed using conformal field theory techniques. However, the constraints on supersymmetric four-dimensional models are rather restrictive, leading to not fully realistic gauge sectors and matter contents. Motivated by the search for Standard Model-like solutions, several discrete or continuous deformations of this class of models have been explored. They include: (i) blowing-up of orientifold singularities [14,15], which cannot be described by a free world-sheet conformal field theory, hence the spacetime spectrum can only be computed using field theory techniques, (ii) locating the branes at different points in the internal space (see e.g. [10,13,16]) which in a T-dual picture corresponds to turning on continuous or discrete Wilson lines, (iii) introduction of discrete values for the NS-NS B field [8,17] which in the T-dual picture corresponds to tilting the compactification tori, (iv) introduction of gauge fluxes in the D-brane world-volumes (see [19,25] for supersymmetric $D = 6$ models), which in the T-dual version corresponds to D-branes intersecting at angles (hence closely related to models in [20,27]).

An appealing feature of (iv) is that generically, there exists chiral fermions where D-branes intersect [31]. Their multiplicity is hence determined by a topological quantity, *i.e.* the intersection number of the branes. Models with D-branes intersecting at arbitrary angles are non-supersymmetric. For non-supersymmetric models, Ramond-Ramond (RR) tadpole cancellation conditions are less constraining [21], a fact exploited to construct semi-realistic models in [22,23], and more recently in [24–28] in the context of intersecting branes. However, non-supersymmetric models suffer from a less understood, complicated dynamics. At the quantum level, flat directions are lifted, leading to involved stabilization problems. In addition uncancelled NS-NS tadpoles require redefining the background geometry [29]. These difficulties are the main reasons that we focus on the construction of string models with $N = 1$ supersymmetry.

The purpose of this letter is to present the first example of a four-dimensional $N = 1$ supersymmetric type IIA orientifold with D6-branes intersecting at angles, leading to Standard Model gauge group (as part of the gauge group structure) and three quark-lepton families. Beyond the structure of the minimal supersymmetric Standard Model (MSSM), the model contains some additional gauge factors, right-handed neutrinos, a chiral set of fields with exotic standard model gauge quantum numbers, and diverse vector-like multiplets. Despite its lack of fully realistic features, it provides the first construction of supersymmetric Standard Model-like string models in the setup of (non-trivial) intersecting brane worlds. Interestingly, since only D6-branes and O6-planes are involved, the M-theory lift of this general class of supersymmetric orientifold models corresponds to purely geometrical backgrounds admitting a G_2 holonomy metric and leading to chiral four-dimensional fermions.

We shall provide the key features of the construction, the gauge group structure and the massless spectrum. The details of the construction, consistency condition,s as well as a broader class of models (including examples of grand unified theories) will be presented in a longer

companion paper [1].

The construction of the model is based on an orientifold of type IIA on $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ (related to [5] by T-duality), with D6-branes not parallel to the orientifold 6-planes (O6-planes). The generators θ, ω act as $\theta : (z_1, z_2, z_3) \to (-z_1, -z_2, z_3)$, and $\omega : (z_1, z_2, z_3) \to (z_1, -z_2, -z_3)$ on the complex coordinates z_i of $\mathbf{T}^6$, which we moreover choose to be factorizable. The orientifold action is ΩR, where Ω is world-sheet parity, and R acts by $R : (z_1, z_2, z_3) \to (\bar{z}_1, \bar{z}_2, \bar{z}_3)$. The model contains four kinds of O6-planes, associated to the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, $\Omega R\theta\omega$. We will focus on the open string (charged) spectrum. The closed string sector contains gravitational supermultiplets as well as orbifold moduli and is straightforward to determine. The cancellation of the RR crosscap tadpoles requires an introduction of K stacks of N_a D6-branes ($a = 1, \ldots, K$) wrapped on three-cycles (taken to be the product of 1-cycles (n_a^i, m_a^i) in the ith two-torus), and their images under ΩR, wrapped on cycles $(n_a^i, -m_a^i)$.

The rules to compute the spectrum are analogous to those in [25]. Consequently models with all tori orthogonal lead to even number of families. Hence we consider moels with one tilted T^2, where the tilting parameter is discrete and has a unique non-trivial value [32]. This mildly modifies the closed string sector, but has an important impact on the open string sector. Namely, a D-brane 1-cycle (n_a^i, m_a^i) along a tilted torus is mapped to $(n^i, -m^i - n^i)$. It is convenient to define $\tilde{m}^i = m^i - \frac{1}{2}n^i$, and label the cycles as $(n^i, \tilde{m}^i)$.

The orbifold actions on the Chan-Paton indices of the branes, for each stack of D6$_a$-branes, and their ΩR images, denoted by D6$_{a'}$-branes, are as follows:

$$\gamma_{\theta,a} = \mathrm{diag}\,(i\mathbf{1}_{N_a/2}, -i\mathbf{1}_{N_a/2}\,;\, -i\mathbf{1}_{N_a/2}, i\mathbf{1}_{N_a/2})$$

$$\gamma_{\omega,a} = \mathrm{diag}\left[\begin{pmatrix} 0 & \mathbf{1}_{N_a/2} \\ -\mathbf{1}_{N_a/2} & 0 \end{pmatrix} ; \begin{pmatrix} 0 & \mathbf{1}_{N_a/2} \\ -\mathbf{1}_{N_a/2} & 0 \end{pmatrix}\right]$$

$$\gamma_{\Omega R,a} = \begin{pmatrix} & & \mathbf{1}_{N_a/2} & 0 \\ & & 0 & \mathbf{1}_{N_a/2} \\ \mathbf{1}_{N_a/2} & 0 & & \\ 0 & \mathbf{1}_{N_a/2} & & \end{pmatrix} \quad (1)$$

The model is constrained by RR tadpole cancellation conditions. In ΩR orientifolds twisted tadpoles vanish automatically [20,27], whereas untwisted RR tadpoles require cancellation of D6-brane and O6-plane 7-form charges. For models with tilted third two-torus, they read

$$\sum_a N_a n_a^1 n_a^2 n_a^3 - 16 = 0$$

$$\sum_a N_a n_a^1 m_a^2 \tilde{m}_a^3 + 8 = 0$$

$$\sum_a N_a m_a^1 n_a^2 \tilde{m}_a^3 + 8 = 0$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 + 16 = 0 \quad (2)$$

The solutions of the above equations define a consistent model; the construction of the resulting spectrum is discussed in detail in [1]. Here we only summarize the results for D6-branes not parallel to O6-planes (for zero angles, the spectrum follows from [5]). The aa sector (strings stretched within a single stack of D6$_a$-branes) is invariant under θ, ω, and is exchanged with $a'a'$ by the action of ΩR. For the gauge group, the θ projection breaks $U(N_a)$ to $U(N_a/2) \times U(N_a/2)$, and ω identifies both factors, leaving $U(N_a/2)$. Concerning the matter multiplets, we obtain three adjoint $N = 1$ chiral multiplets.

The $ab + ba$ sector, strings streched between D6$_a$- and D6$_b$-branes, is invariant, as a whole, under the orbifold projections, and is mapped to the $b'a' + a'b'$ sector by ΩR. The matter content before any projection would be given by I_{ab} chiral fermions in the bifundamental $(N_a, \overline{N}_b)$ of $U(N_a) \times U(N_b)$, where

$$I_{ab} = \left(n_a^1 m_b^1 - n_b^1 m_a^1\right)\left(n_a^2 m_b^2 - n_b^2 m_a^2\right)\left(n_a^3 \tilde{m}_b^3 - n_b^3 \tilde{m}_a^3\right)$$

is the intersection number of the wrapped cycles, and the sign of I_{ab} denotes the chirality of the corresponding fermion ($I < 0$ giving left-handed fermions in our convention). For supersymmetric intersections, additional massless scalars complete the corresponding chiral supermultiplet. In principle, one needs to take into account the orbifold action on the intersection point. However the final result turns out to be insensitive to this subtlety and is still given by I_{ab} chiral multiplets in the $(N_a/2, \overline{N_b/2})$ of $U(N_a/2) \times U(N_b/2)$. A similar effect takes place in $ab' + b'a$ sector, for $a \neq b$, where the final matter content is given by $I_{ab'}$ chiral multiplets in the bifundamental $(N_a/2, N_b/2)$.

For the $aa' + a'a$ sector the orbifold action on the intersection points turns out to be crucial. For intersection points invariant under the orbifold, the orientifold projection leads to a two-index antisymmetric representation of $U(N_a/2)$, except for states with θ and ω eigenvalue $+1$, where it yields a two-index symmetric representation. For points not fixed under some orbifold element, say two points fixed under ω and exchanged by θ, one simply keeps one point, and does not impose the ω projection. Equivalently, one considers all possible eigenvalues for ω, and applies the above rule to read off whether the symmetric or the antisymmetric survives. A closed formula for the chiral piece in this sector is given in [1].

The condition that the system of branes preserve the $N = 1$ supersymmetry unbroken requires [31] that each stack of D6-branes is related to the O6-planes by a rotation in $SU(3)$: denoting by θ_i the angles the D6-brane forms with the horizontal direction in the ith two-torus, supersymmetry preserving configurations must satisfy $\theta_1 + \theta_2 + \theta_3 = 0$. In order to simplify the supersymmetry conditions, and our search for realistic models, we

will consider a particular ansatz: $(\theta_1, \theta_2, 0)$, $(\theta_1, 0, \theta_3)$ or $(0, \theta_2, \theta_3)$.

Due to the smaller number of O6-planes in tilted configurations, the RR tadpoles conditions are very stringent for more than one tilted torus. Focusing on tilting just the third torus, the search for theories with $U(3)$ and $U(2)$ gauge factors carried by branes at angles and three left-handed quarks, turns out to be very constraining, at least within our ansatz. We have found essentially a unique solution. The D6-brane configuration with wrapping numbers $(n_a^i, \widetilde{m}_a^i)$ is given in Table I.

Type	N_a	$(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, \widetilde{m}_a^3)$
A_1	8	$(0,1) \times (0,-1) \times (2,\widetilde{0})$
A_2	2	$(1,0) \times (1,0) \times (2,\widetilde{0})$
B_1	4	$(1,0) \times (1,-1) \times (1,\widetilde{3/2})$
B_2	2	$(1,0) \times (0,1) \times (0,\widetilde{-1})$
C_1	6+2	$(1,-1) \times (1,0) \times (1,\widetilde{1/2})$
C_2	4	$(0,1) \times (1,0) \times (0,\widetilde{-1})$

TABLE I. D6-brane configuration for the three-family model.

The 8 D6-branes labeled C_1 are spit in two parallel but not overlapping stacks of 6 and 2 branes, hence lead to a gauge group $U(3) \times U(1)$. Interestingly, a linear combination of the two $U(1)$'s is actually a generator within the $SU(4)$ arising for coincident branes. This ensures that this $U(1)$ is automatically non-anomalous and massless (free of linear couplings to untwisted moduli) [26,28], and turns out to be crucial in the appearance of hypercharge in this model.

For convenience we consider the 8 D6-branes labeled A_1 to be away from the O6-planes in all three complex planes. This leads to two D6-branes that can move independently (hence give rise to a group $U(1)^2$), plus their θ, ω and ΩR images. These $U(1)$'s are also automatically non-anomalous and massless. In the effective theory, this corresponds to Higgsing of $USp(8)$ down to $U(1)^2$.

The open string spectrum is tabulated in Table II. The generators Q_3, Q_1 and Q_2 refer to the $U(1)$ factor within the corresponding $U(n)$, while Q_8, Q_8' are the $U(1)$'s arising from the $USp(8)$. The hypercharge is defined as:

$$Q_Y = \frac{1}{6}Q_3 - \frac{1}{2}Q_1 + \frac{1}{2}(Q_8 + Q_8') \tag{3}$$

From our above comments, Q_Y as defined guarantees that $U(1)_Y$ is massless. The theory contains three Standard Model families, plus one exotic chiral (but anomaly-free) set of fields, and multiplets with vector-like quantum numbers under the SM gauge group.

Even though the model is an explicit string realization of the brane world scenario, the string scale is of the order of four-dimensional Planck scale because Standard Model gauge interactions are embedded in different D6-branes. The experimental bounds on Kaluza-Klein replica of Standard Model gauge bosons imply that the internal dimensions cannot be large [24].

Quarks, leptons and Higgs fields live at different intersections, hence the Yukawa couplings among the Higgs and two fermions arise from a string worldsheet of area A_{ijk} stretching between the three intersections [26], $Y_{ijk} \sim \exp(-A_{ijk}/\alpha')$. Note that one family of quarks and leptons do not have renormalizable couplings with the Higgs, due to the uncancelled Q_2 charges, and the only chiral multiplets which carry opposite Q_2 charges are charged under the weak $SU(2)$. Therefore, this family remains massless before electroweak breaking.

This model is supersymmetric for some specific choice of complex structure moduli, which determine the angles satisfying the supersymmetry condition. The supersymmetry breaking effect when the condition is violated is reflected as a Fayet-Iliopoulos term for the $U(1)$ gauge fields. It is proportional to the deviation from the supersymmetric angle situation, and reproduces correctly the tachyonic scalar masses, and zero fermion masses. The corresponding D-term is expected to be cancelled by vevs of the tachyonic scalar fields, hence shifting the configuration to a corrected vacuum, where some intersecting

Sector	Non-Abelian Reps.	Q_3	Q_1	Q_2	Q_8	Q_8'	Q_Y
A_1B_1	$3 \times 2 \times (1,\overline{2},1,1,1)$	0	0	-1	± 1	0	$\pm\frac{1}{2}$
	$3 \times 2 \times (1,\overline{2},1,1,1)$	0	0	-1	0	± 1	$\pm\frac{1}{2}$
A_1C_1	$2 \times (\overline{3},1,1,1,1)$	-1	0	0	± 1	0	$\frac{1}{3},-\frac{2}{3}$
	$2 \times (\overline{3},1,1,1,1)$	-1	0	0	0	± 1	$\frac{1}{3},-\frac{2}{3}$
	$2 \times (1,1,1,1,1)$	0	-1	0	± 1	0	$1,0$
	$2 \times (1,1,1,1,1)$	0	-1	0	0	± 1	$1,0$
B_1C_1	$(3,\overline{2},1,1,1)$	1	0	-1	0	0	$\frac{1}{6}$
	$(1,\overline{2},1,1,1)$	0	1	-1	0	0	$-\frac{1}{2}$
B_1C_2	$(1,2,1,1,4)$	0	0	1	0	0	0
B_2C_1	$(3,1,2,1,1)$	1	0	0	0	0	$\frac{1}{6}$
	$(1,1,2,1,1)$	0	1	0	0	0	$-\frac{1}{2}$
B_1C_1'	$2 \times (3,2,1,1,1)$	1	0	1	0	0	$\frac{1}{6}$
	$2 \times (1,2,1,1,1)$	0	1	1	0	0	$-\frac{1}{2}$
B_1B_1'	$2 \times (1,1,1,1,1)$	0	0	-2	0	0	0
	$2 \times (1,3,1,1,1)$	0	0	2	0	0	0

TABLE II. Chiral Spectrum of the open string sector in the three-family model. The non-Abelian gauge group is $SU(3) \times SU(2) \times USp(2) \times USp(2) \times USp(4)$. Notice that we have not included the aa sector piece, even though it is generically present in the model. The non-chiral pieces in the ab, ab' and aa' sectors are not present for branes at generic locations, hence they are not listed here.

D6-branes are recombined, triggering gauge symmetry breaking.

It is quite remarkable that these chiral models are related to the non-chiral model of [5], by recombinations of the 3-cycles on which the D6-branes wrap. This is the T-dual of a $D = 4$ version of the small instanton transition [30]. Clearly, our search is in no sense exhaustive. There exists different variants, in the framework descibed here, obtained by, *e.g.*, (i) changing the additional branes not directly involved in the SM structure, (ii) allowing branes rotated at an angle in all three torus, and (iii) compactify Type IIA orientifold on a different orbifold which preserves $N = 1$ supersymmetry. However, within our ansatz for the angles in the case of $\mathbf{Z_2} \times \mathbf{Z_2}$ orbifold, the requirements of supersymmetry, Standard Model-like gauge group and number of chiral families are rather stringent, and the model presented here is relatively unique. Let us note that D6-branes wrapping around supersymmetric 3-cycles with three non-trivial angles, say $(n_i, m_i) = (1, 1) \times (1, 1) \times (1, -1)$, contribute to some (but not all) tadpole conditions with the same sign as that of an O6-plane – a feature which is absent when all the D6-branes are parallel to some O6-planes. It would be interesting to explore such variants to eliminate the additional vector-like matter and the extra exotics. We leave this for further investigation.

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