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A Bridge between CP violation at Low Energies and Leptogenesis

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Abstract

We discuss the possibility of relating the size and sign of the observed baryon asymmetry of the universe to CP violation observable at low energies, in a framework where the observed baryon asymmetry is produced by leptogenesis through the out of the equilibrium decay of heavy Majorana neutrinos. We identify the CP violating phases entering in leptogenesis as well as those relevant for CP violation at low energies in the minimal seesaw model. We show that although in general there is no relation between these two sets of phases, there are specific frameworks in which such a connection may be established and we give a specific grand unification inspired example where such a connection does exist. We construct weak-basis invariants related to CP violation responsible for leptogenesis, as well as those relevant for CP violation at low energies.

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1 Introduction

In the Standard Model (SM) neutrinos are massless and there is no mixing or CP violation in the leptonic sector. However, in any extension of the SM which incorporates neutrino masses and mixing, CP violation naturally arises also in the leptonic sector and can in principle be measured through neutrino oscillations [1]. CP violation in the leptonic sector can have profound cosmological implications, playing a crucial rôle in the generation of the observed baryon number asymmetry of the universe (BAU). Indeed, one of the most plausible scenarios for the generation of BAU is through the out-of- equilibrium decay of heavy Majorana neutrinos [2]. This mechanism has been studied in detail by several groups [3] and it has been shown that the observed BAU, $n_B/s \sim 10^{-10}$, can be obtained in the above scenario, without any fine-tuning of parameters. In this paper, we address the question of whether it is possible to establish a connection between CP breaking necessary to generate leptogenesis and CP violation at low energies. More specifically, assuming that baryogenesis is achieved through leptogenesis, can one infer the strength of CP violation at low energies from the size and sign of the observed BAU? We show that without any further assumption about the flavour structure of the leptonic mass matrices and/or the mechanism of CP violation, in general one cannot establish a direct connection,between the strength of CP violation at high energies (required for leptogenesis) and that observed at low energies. However, we shall point out that there are special scenarios where such a connection may be established. In particular, we describe a scenario inspired on grand unified theories (GUTs), where such a connection arises. Mixing and CP violation with Majorana neutrinos is often described in the decoupling limit via the 3×3 Maki-Nakagawa-Sakata matrix [4]. Since for leptogenesis, we have to analyze decays of heavy neutrinos, we need to consider the full 3×6 mixing matrix appearing in the weak charged current, connecting the charged leptons to the three light neutrinos and the three heavy neutrinos. The number of independent CP violating phases was identified for the general case in Ref.[5] and for a special case in Ref.[6]. In this paper, we identify the CP violating phases both in an appropriately chosen weak-basis (WB) and in the mass eigenstate basis. Furthermore, we construct WB invariants which must vanish if CP invariance holds. Non-vanishing of any of these WB invariants signals CP violation. We identify the WB invariants relevant to leptogenesis and those associated to CP violation at low energies. The interest of these WB invariants stems from the fact that they can be evaluated in any WB and are thus particularly suited to the analysis of specific ansätze for charged lepton and neutrino mass matrices. For any given ansatz, in order to analyze its potential for leptogenesis or whether it leads to CP violation in neutrino oscillations, one can simply compute the appropriate WB invariant. Most of our analysis does not depend on the origin of CP violation, namely on whether CP is explicitly broken at the Lagrangian level or spontaneously broken. However, we present a simple extension of the SM where CP is spontaneously broken by a single phase of the vacuum expectation value of a complex scalar, singlet under $SU(3)_c \times SU(2)_L \times U(1)$. It is shown that this phase is sufficient to produce both the CP violation necessary for baryogenesis and the CP violation observable at low energies.

2 A minimal extension of the SM

Let us consider a minimal extension of the SM which consists of adding to the standard spectrum one right-handed neutrino per generation. After spontaneous gauge symmetry breaking, the following leptonic mass terms can be written:

$$\begin{aligned}\mathcal{L}_m &= -[\overline{\nu_L^0} m_D \nu_R^0 + \frac{1}{2}\nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0] + h.c. = \\ &= -[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0] + h.c. \end{aligned} \tag{2.1}$$

where m_D, M_R and m_l denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, and $n_L = (\nu_L^0, (\nu_R^0)^c)$. The right-handed neutrino Majorana mass term is SU(2) $\times$ U(1) invariant, consequently it can have a value much above the scale v of the electroweak symmetry breaking, thus leading, through the seesaw mechanism [7], to naturally small left-handed Majorana neutrino masses, of order $\frac{{m_D}^2}{M_R}$. It is convenient to determine the nature of the various CP violating phases, both in a weak-basis (WB), where all gauge currents are real and flavour diagonal, and in a mass-eigenstate basis, where fermion mass terms are real, diagonal but there is non-trivial flavour mixing in the charged currents.

2.1 CP violating phases in a weak-basis

In order to study CP violation in a WB, let us first note that the most general CP transformation which leaves the gauge interaction invariant is:

$$\begin{aligned}\mathrm{CP} l_L (\mathrm{CP})^\dagger &= U \gamma^0 \mathrm{C} \overline{l_L}^T \quad \mathrm{CP} l_R (\mathrm{CP})^\dagger = V \gamma^0 \mathrm{C} \overline{l_R}^T \\ \mathrm{CP} \nu_L (\mathrm{CP})^\dagger &= U \gamma^0 \mathrm{C} \overline{\nu_L}^T \quad \mathrm{CP} \nu_R (\mathrm{CP})^\dagger = W \gamma^0 \mathrm{C} \overline{\nu_R}^T\end{aligned} \tag{2.2}$$

where U, V, W are unitary matrices acting in flavour space and where for notation simplicity we have dropped here the superscript 0 in the fermion fields. . Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied:

$$W^T M_R W = -M_R^* \tag{2.3}$$
$$U^\dagger m_D W = {m_D}^* \tag{2.4}$$
$$U^\dagger m_l V = {m_l}^* \tag{2.5}$$

In order to analyze the implications of the above conditions, it is convenient to choose the WB where both m_l, M_R, are real diagonal. In this basis, W is then constrained by Eq. (2.3) to be of the form

$$W = \mathrm{diag.}\,(\exp(i\alpha_1), \exp(i\alpha_2), ... \exp(i\alpha_n)) \tag{2.6}$$

where n denotes the number of generations and the α_i have to satisfy:

$$\alpha_i = (2p_i + 1)\frac{\pi}{2} \tag{2.7}$$

with p_i integer numbers. Multiplying Eq. (2.5) by its Hermitian conjugate, and taking into account that we are working on a WB where m_l is real diagonal, one concludes that U has to be of the form:

$$U = \text{diag.}\left(\exp(i\beta_1), \exp(i\beta_2), ... \exp(i\beta_n)\right) \tag{2.8}$$

where β_i are arbitrary phases. From Eqs. (2.4), (2.6), (2.8) it follows then that CP invariance constrains the matrix m_D to satisfy :

$$\arg(m_D)_{ij} = \frac{1}{2}(\beta_i - \alpha_j) \tag{2.9}$$

Note that the α_i are fixed by Eq. (2.7), up to discrete ambiguities. Therefore CP invariance constrains the matrix m_D to have only n free phases β_i. Since m_D is in an arbitrary matrix, with n^2 independent phases, it is clear that there are $n^2 - n$ independent CP restrictions. This number equals, of course, the number of independent CP violating phases which appear in general in this model. In the WB which we are considering, these phases appear as $n(n-1)$ phases which cannot be removed from m_D. It is worth counting also the remaining physical parameters in this WB in the case of three generations. The matrices m_l, M_R are diagonal and real and therefore they contain six real parameters. The matrix m_D , apart from the six CP violating phases, has nine real parameters. Thus one has a total of fifteen real parameters and six CP violating phases. We shall see that this counting of degrees of freedom agrees with the one made in the physical basis, in terms of charged lepton and neutrino masses together with the mixing angles and CP violating phases entering in the leptonic mixing matrix.

2.2 CP violating phases in the leptonic mixing matrix

For definiteness, we shall consider the case of three generations (three light neutrinos), where the full neutrino mass matrix, $\mathcal{M}$ in Eq. (2.1), is 6×6, and has the following form:

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \tag{2.10}$$

For notation simplicity, we have dropped the subscript in m_D and M_R. The neutrino mass matrix is diagonalized by the transformation:

$$V^T \mathcal{M}^* V = \mathcal{D} \tag{2.11}$$

where $\mathcal{D} = \text{diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$, with m_{ν_i} and M_{ν_i} denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write V and $\mathcal{D}$ in the following form:

$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}; \tag{2.12}$$

$$\mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \tag{2.13}$$

From Eqs. (2.10), (2.11), (2.12), (2.13) one obtains:

$$S^\dagger m^T K^* + K^\dagger m S^* + S^\dagger M S^* = d \tag{2.14}$$

$$S^\dagger m^T R^* + K^\dagger m T^* + S^\dagger M T^* = 0 \tag{2.15}$$

$$T^\dagger m^T R^* + R^\dagger m T^* + T^\dagger M T^* = D \tag{2.16}$$

From Eq. (2.15) and taking into account that both S and R are of order $\frac{m}{M}$, one obtains, to an excellent approximation:

$$S^\dagger = -K^\dagger m M^{-1} \tag{2.17}$$

From Eqs. (2.14), (2.17), it also follows to an excellent approximation that:

$$-K^\dagger m \frac{1}{M} m^T K^* = d \tag{2.18}$$

which is the usual seesaw formula. The neutrino weak-eigenstates are related to the mass eigenstates by:

$$\nu^0_{i\,L} = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, ...6 \end{pmatrix} \tag{2.19}$$

and thus the leptonic charged current interactions are given by:

$$-\frac{g}{\sqrt{2}} \left(\overline{l_{iL}} \gamma_\mu K_{ij} \nu_{j\,L} + \overline{l_{iL}} \gamma_\mu R_{ij} N_{j\,L} \right) W^\mu + h.c. \tag{2.20}$$

From Eqs. (2.19), (2.20) it follows that K and R give the charged current couplings of charged leptons to the light neutrinos ν_j and to the heavy neutrinos N_j, respectively. In the exact decoupling limit, R can be neglected and only K is relevant. However, since we want to study the connection between CP violation relevant to leptogenesis and that detectable at low energies (e.g., in neutrino oscillations) we have to keep both K and R. Now, from the relation $\mathcal{M}^* V = V^* \mathcal{D}$ and taking into account the zero entry in Eq. (2.10), one derives the following exact relation:

$$R = m T^* D^{-1} \tag{2.21}$$

From Eq. (2.16), and keeping in mind that we are working in a WB where the right-handed Majorana neutrino mass M is diagonal, one concludes that $T = 1$ up to corrections of order $\frac{m^2}{M^2}$. Therefore, one has, to an excellent approximation:

$$R = m D^{-1} \tag{2.22}$$

From Eqs. (2.18), (2.22), it is clear how the six physical phases of the Dirac mass matrix m, enter in the two blocks K, R of the 3×6 leptonic mixing matrix. It is useful to parametrize the Dirac neutrino mass matrix by [8]:

$$m = UY_{\triangle} \tag{2.23}$$

where U is a unitary matrix and $Y_{\triangle}$ has a triangular form:

$$Y_{\triangle} = \begin{pmatrix} y_1 & 0 & 0 \\ |y_{21}| \exp(i\phi_{21}) & y_2 & 0 \\ |y_{31}| \exp(i\phi_{31}) & |y_{32}| \exp(i\phi_{32}) & y_3 \end{pmatrix} \tag{2.24}$$

where the y_i are real. Since U is unitary, it contains in general six phases. However, three of these phases can be rephased away through the transformation:

$$m \to P_{\xi} m \tag{2.25}$$

where $P_{\xi} = \mathrm{diag.}\,(\exp(i\xi_1), \exp(i\xi_2), \exp(i\xi_3))$. In a WB, this corresponds to a simultaneous phase transformation of the left-handed charged lepton fields and the left-handed neutrino fields. Furthermore, $Y_{\triangle}$ defined by Eq. (2.24) can be written as:

$$Y_{\triangle} = P_{\beta}^{\dagger}\, \hat{Y}_{\triangle}\, P_{\beta} \tag{2.26}$$

where $P_{\beta} = diag.(1, \exp(i\beta_1), \exp(i\beta_2))$ and

$$\hat{Y}_{\triangle} = \begin{pmatrix} y_1 & 0 & 0 \\ |y_{21}| & y_2 & 0 \\ |y_{31}| & |y_{32}| \exp(i\sigma) & y_3 \end{pmatrix} \tag{2.27}$$

with $\sigma = \phi_{32} - \phi_{31} + \phi_{21}$. It follows from Eqs. (2.23), (2.26) that the matrix m can then be written as:

$$m = \hat{U}_{\rho} P_{\alpha} \hat{Y}_{\triangle} P_{\beta} \tag{2.28}$$

where $P_{\alpha} = diag.(1, \exp(i\alpha_1), \exp(i\alpha_2))$ and $\hat{U}_{\rho}$ contains only one phase ρ and can be written, for example, through a “standard” parametrization, as used in the case of the CKM matrix [9]. Therefore, in this WB, where m_l and M_R are diagonal and real, the phases ρ, α_1, α_2, σ, β_1, β_2 are the only physical phases and can be used to characterize CP violation in this model. We shall see in the sequel that leptogenesis is controlled by the phases σ, β_1, β_2. Taking into consideration Eq. (2.22), which is valid to an excellent approximation, one obtains:

$$R = \hat{U}_{\rho} P_{\alpha} \hat{Y}_{\triangle} D^{-1} P_{\beta} \tag{2.29}$$

Note that the phases β_1 and β_2 are of Majorana type in the sense that they could be removed from R through a phase redefinition of the heavy Majorana neutrinos N_2, N_3. Of course, since the N_i are Majorana particles, this redefinition would simply shift these phases to the mass terms of N_2, N_3. At this point, the following comment is in order. We are considering a simple extension of the SM, where there are Dirac, as well as right-handed neutrino mass terms, but no left-handed Majorana mass terms. This is, of course, due to the absence of Higgs triplets. The fact that there are no left-handed Majorana neutrino mass terms leads to special constraints. Indeed from Eqs. (2.10), (2.11), (2.12), (2.13), one readily obtains the following exact relation:

$$K^* d K^\dagger + R^* D R^\dagger = 0. \tag{2.30}$$

In order to analyse the meaning of these constraints, let us for the moment consider the most general case, i.e. the case in which a left-handed Majorana mass term is also present. The general 3×6 leptonic mixing matrix can then be exactly parametrized by the first three rows of the 6×6 unitary matrix V which diagonalizes the full neutrino mass matrix $\mathcal{M}^*$, provided that V is chosen in such a way that a minimal number of phases appears in these first three rows. Such is the case with the following explicit parametrization for V [10]:

$$V = \hat{V} P \tag{2.31}$$

where $P = \text{diag.}\,(1, \exp(i\sigma_1), \exp(i\sigma_2), ..., \exp(i\sigma_5))$ and $\hat{V}$ is given by:

$$\hat{V} = O_{56} I_6(\delta_{10}) O_{45} O_{46} I_5(\delta_9) I_6(\delta_8) ... O_{26} I_3(\delta_4) ... I_6(\delta_1) O_{12} ... O_{16} \tag{2.32}$$

where O_{ij} are orthogonal matrices mixing the ith and jth generation and $I_j(\delta_k)$ are unitary diagonal matrices of the form:

$$I_j(\delta_k) = \begin{pmatrix} 1 & & & & & & \\ & \cdot & & & & & \\ & & 1 & & & & \\ & & & e^{i\delta_k} & & & \\ & & & & 1 & & \\ & & & & & \cdot & \\ & & & & & & 1 \end{pmatrix} \begin{array}{c} \\ \\ \\ \leftarrow j \\ \\ \\ \\ \end{array} \tag{2.33}$$

It can be readily verified that the first three rows of $\hat{V}$, contain seven phases. This parametrization is particularly useful, for instance, in models with vectorial quarks [11]. Together with the five phases contained in P, one has, in the general case, twelve phases characterizing the 3×6 leptonic mixing matrix. The previous counting shows that the constraint of Eq. (2.30) leads to the decrease of the number of independent phases from twelve to six. This is to be expected, since in the general case, the left-hand side of Eq. (2.30) equals the left-handed Majorana matrices mass m_ν, a 3×3 complex symmetric matrix, which in general contains six phases. Therefore, putting $m_\nu = 0$ implies the loss of six independent phases. Of course, one can still use the parametrization defined by Eqs. (2.31), (2.32), (2.33) but the angles and phases introduced are not independent parameters, since they are constrained by Eq. (2.30).

3 Weak-basis invariants and CP violation

In section 2.1, we have analysed CP violation in a particular WB. We shall derive now WB invariants which have to vanish in order for CP invariance to hold. The non-vanishing of any of these invariants signals CP violation [5]. We are specially interested in WB invariants sensitive to the CP violating phases which appear in leptogenesis. From Eqs. (2.4), (2.3), one obtains:

$$\begin{aligned} W^\dagger h W &= h^* \\ W^\dagger H W &= H^* \end{aligned} \tag{3.1}$$

where $h = m^\dagger m$, $H = M^\dagger M$. It can be then readily derived, from Eqs. (2.3), (3.1), that CP invariance requires:

$$I_1 \equiv \mathrm{ImTr}[hHM^*h^*M] = 0 \tag{3.2}$$

Since I_1 is a WB invariant, it may be evaluated in any convenient WB. In the WB where the right-handed neutrino mass M is diagonal, one obtains:

$$\begin{aligned} I_1 &= M_1M_2({M_2}^2 - {M_1}^2)\mathrm{Im}({h_{12}}^2) + M_1M_3({M_3}^2 - {M_1}^2)\mathrm{Im}({h_{13}}^2) + \\ &+ M_2M_3({M_3}^2 - {M_2}^2)\mathrm{Im}({h_{23}}^2) = 0 \end{aligned} \tag{3.3}$$

The appearance of the quadratic term ${h_{ij}}^2$ was to be expected since it reflects the well known fact that phases of $\frac{\pi}{2}$ in h_{ij} do not imply CP violation. The interest of I_1 stems from the fact that, as will be seen in section 4, the strength of leptogenesis crucially depends on $\mathrm{Im}({h_{ij}}^2)$ $(j \neq i)$. Similarly, one can readily show that CP invariance also implies the vanishing of the WB invariants I_2, I_3:

$$\begin{aligned} I_2 &\equiv \mathrm{ImTr}[hH^2M^*h^*M] = \\ &= M_1M_2({M_2}^4 - {M_1}^4)\mathrm{Im}({h_{12}}^2) + M_1M_3({M_3}^4 - {M_1}^4)\mathrm{Im}({h_{13}}^2) + \\ &+ M_2M_3({M_3}^4 - {M_2}^4)\mathrm{Im}({h_{23}}^2) = 0 \qquad (3.4) \\ I_3 &\equiv \mathrm{ImTr}[hH^2M^*h^*MH] = \\ &= {M_1}^3{M_2}^3({M_2}^2 - {M_1}^2)\mathrm{Im}({h_{12}}^2) + {M_1}^3{M_3}^3({M_3}^2 - {M_1}^2)\mathrm{Im}({h_{13}}^2) + \\ &+ {M_2}^3{M_3}^3({M_3}^2 - {M_2}^2)\mathrm{Im}({h_{23}}^2) = 0 \qquad (3.5) \end{aligned}$$

where again we have given the explicit expressions for I_2, I_3, in the basis where M is real, diagonal. Note that the three Eqs. (3.3), (3.4), (3.5) constitute a set of linear equations in terms of the variables $\mathrm{Im}({h_{ij}}^2)$, where the coefficients are functions of the right-handed neutrino masses M_i. The determinant of the coefficients of this set of equations can be readily evaluated and one obtains:

$$Det. = {M_1}^2{M_2}^2{M_3}^2\Delta^2{}_{21}\Delta^2{}_{31}\Delta^2{}_{32} \tag{3.6}$$

where $\Delta_{ij} = ({M_i}^2 - {M_j}^2)$. From Eq. (3.6), it follows that if none of the M_i vanish and furthermore there is no degeneracy, the vanishing of I_1, I_2, I_3 implies the vanishing of $\mathrm{Im}({h_{12}}^2)$, $\mathrm{Im}({h_{13}}^2)$, $\mathrm{Im}({h_{23}}^2)$. Since there are six independent CP violating phases, one may wonder whether one can construct other three independent WB invariants, apart from I_i, which would describe CP violation in the leptonic sector. This is indeed possible, a simple choice are the WB invariants $\bar{I}_i (i = 1, 2, 3)$, obtained from I_i, through the substitution of h by $\bar{h} = m^\dagger h_l m$, where $h_l = m_l {m_l}^\dagger$. For example one has:

$$\bar{I}_1 = \mathrm{ImTr}(m^\dagger h_l m H M^* m^T {h_l}^* m^* M) \tag{3.7}$$

and similarly for $\bar{I}_2, \bar{I}_3$. As it was the case for I_i, CP invariance requires that $\bar{I}_i = 0$. And likewise, the vanishing of $\bar{I}_i$ implies (barring either vanishing or degenerate M_i) that $\mathrm{Im}(\bar{h_{ij}}^2)$ $(i \neq j)$ vanish.

4 CP violating phases relevant for leptogenesis

In this section, we identify the CP violating phases relevant for leptogenesis, obtained through the out of equilibrium decay of heavy Majorana neutrinos. In the previous section, we have seen that there are six independent CP violation sources in the minimal seesaw model by constructing WB invariants. Next we show which of these six independent CP violating sources contribute to lepton number asymmetry.

4.1 Lepton number asymmetry

Leptogenesis gives rise to the BAU through the out of the equilibrium decay of heavy Majorana neutrinos in the symmetric phase, with the generation of $L \neq 0$ while $B = 0$ is still maintained. Later on, sphaleron processes [12] restore $B + L = 0$ in the universe leaving $B - L$ invariant, thus creating a non vanishing B. Because the lepton number is generated at a very high temperature in this scenario, the lepton number asymmetry is generated in the symmetric phase, i.e., $v = 0$, where v is the Higgs vacuum expectation value. In the previous sections, we started with the Lagrangian in the broken phase and identified the CP violating quantities. In order to see the connection between the lepton number asymmetry generated in the symmetric phase and the CP violating quantities defined in the broken phase, i.e., the phases in the 3×6 mixing matrix and the WB invariant defined in terms of the Dirac mass matrix m and the Majorana mass matrix M, we use the Lagrangian in the broken phase for the computation of the asymmetry in the symmetric phase. This is possible by simply taking the symmetric limit, i.e., $v \to 0$ in the broken phase computation. By following this procedure, not only do we recover the known result in the symmetric phase, but we also clarify the relation between CP violation generating lepton asymmetry at a high energy and CP violation in the broken phase. To be definite, we compute the decay of a heavy Majorana neutrino N_j into charged leptons $l_i^{\pm}$. The light flavour indices $i = 1, 2$ and 3 correspond to e, μ and τ respectively. We can define the lepton family number asymmetry as $\Delta A^j{}_i = N^j{}_i - \overline{N}^j{}_i$ and the lepton number asymmetry is obtained by summing over the three flavours:

$$\Delta A^j = \sum_i \Delta N^j{}_i. \tag{4.1}$$

In the symmetric phase, the heavy Majorana neutrinos decay into charged leptons and charged Higgs. In the broken phase, the charged Higgs boson is absorbed into the W boson and the decay of heavy Majorana neutrinos into charged leptons and charged Higgs has no physical significance. In the broken phase, the decay into W bosons and charged leptons is the process generating lepton number asymmetry. Of all possible polarizations of the W boson as final states, the longitudinal gauge boson gives the dominant contribution. Because the asymmetry comes from the interference between the tree amplitude and the absorptive part of the one-loop amplitudes, we can write the amplitude as follows:

$$\begin{aligned}
& M(N_j \to {l_i}^+ W^-) = \langle {l_i}^+ W^- | T | N_j \rangle \\
+ \quad & i\pi \sum \langle {l_i}^+ W^- | T | {l_n}^- W^+ \rangle \langle {l_n}^- W^+ | T | N_j \rangle \delta(E_n + E_W - M_j) \\
+ \quad & i\pi \sum \langle {l_i}^+ W^- | T | \nu_n Z \rangle \langle \nu_n Z | T | N_j \rangle \delta(E_n + E_Z - M_j) \\
+ \quad & i\pi \sum \langle {l_i}^+ W^- | T | \nu_n H \rangle \langle \nu_n H | T | N_j \rangle \delta(E_n + E_H - M_j),
\end{aligned} \tag{4.2}$$

where $\langle T \rangle$ denotes the tree amplitudes. We compute the absorptive part by summing over possible two body on-shell states such as $l^- W^+$, νZ and νH, so that $\sum$ stands for the sum over flavour indices n, three momentum q, polarization, and spin. For example, for a W and a charged lepton state, $\sum$ means $\int \sum_{pol,spin,n} \frac{d^3q}{2E_n 2E_W (2\pi)^3}$. The following analysis shows that of all the polarizations, the longitudinal W and Z boson dominate in the sum. In the rest frame of the heavy Majorana neutrino, the gauge bosons which appear in the matrix elements carry half of the energy and the momentum of the decaying heavy Majorana neutrino, i.e., $E_W \simeq P_W \simeq \frac{M}{2}$. Therefore, in leading order of the $\frac{v}{M}$ expansion, the polarization of the longitudinal W (W_L) is written as:

$${\varepsilon_L}^\mu \qquad \simeq \frac{{P_W}^\mu}{M_W}. \tag{4.3}$$

By substituting it into charged current interaction, we obtain for the tree level matrix element of N_j to ${l_i}^-\ {W_L}^+$:

$$\begin{aligned}
M^{\text{tree}}(N_j \to {W_L}^+ {l_i}^-) &= -R_{ij} \frac{g}{\sqrt{2}} (\varepsilon_L)^{\mu *} \bar{u_i} \gamma_\mu L U_N, \\
&= -R_{ij} M_j \frac{g}{\sqrt{2} M_W} \bar{u_i} R U_N,
\end{aligned} \tag{4.4}$$

where we use $P_W = P_j - P_i$ and neglect the lepton mass. We note that only the interaction to the longitudinal polarization remains in the small $\frac{v}{M}$ limit:

$$R_{ij} M_j \frac{g}{\sqrt{2} M_W} \to \frac{\sqrt{2} m_{ij}}{v} = y_{Dij}, \tag{4.5}$$

with y_{Dij} the coefficients of the neutrino Yukawa couplings. In the following discussion, we only keep the longitudinal polarization for W and Z. Including the Higgs boson interaction, the relevant part of the Lagrangian needed for the computation of the matrix elements in Eq. (4.2), is given by:

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{g}{\sqrt{2}}(R^\dagger)_{ji}(\bar{N}_j\gamma_\mu l_{iL})W^{\mu+} - \frac{g}{2\cos\theta_W}(R^\dagger K)_{ji}(\bar{N}_j\gamma_\mu\nu_{iL})Z^\mu \\
& -\frac{g}{2M_W}M_j(R^\dagger K)_{ji}(\bar{N}_j\nu_{iL})H \\
& -\frac{g}{\sqrt{2}}R_{ij}(\overline{l_{iL}}\gamma_\mu N_j)W^{\mu-} - \frac{g}{2\cos\theta_W}(K^\dagger R)_{ij}(\overline{\nu_{iL}}\gamma_\mu N_j)Z^\mu \\
& -\frac{g}{2M_W}(K^\dagger R)_{ij}M_j(\overline{\nu_{iL}}N_j)H. \qquad (4.6)
\end{aligned}
$$

By neglecting the masses of the Higgs, W, Z, charged leptons, and light neutrinos compared to the heavy Majorana neutrinos masses, we obtain the following result:

$$
\begin{aligned}
& M(N_j \to l_i{}^+W_L{}^-) \\
= \;& (\bar{u}_i L U_N)\frac{gM_j}{\sqrt{2}M_W}\Big[(R^\dagger)_{ji} - i(R^\dagger)_{kn}(R^\dagger)_{ki}R_{nj}\left(\frac{gM_k}{\sqrt{2}M_W}\right)^2\frac{1}{16\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{2(1-x_k)}\right) \\
& -i(R^\dagger K)_{kn}(R^\dagger)_{ki}(K^\dagger R)_{nj}\left(\frac{gM_k}{\sqrt{2}M_W}\right)^2\frac{1}{16\pi}\left(\frac{\sqrt{x_k}}{2(1-x_k)}\right)\Big] + ..., \\
= \;& (\bar{u}_i L U_N)\frac{gM_j}{\sqrt{2}M_W}\Big[(R^\dagger)_{ji} - i(R^\dagger)_{kn}(R^\dagger)_{ki}R_{nj}\left(\frac{gM_k}{\sqrt{2}M_W}\right)^2\frac{1}{16\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{(1-x_k)}\right)\Big] \\
+ \;& ig^3\left((R^\dagger)_{ji}... + (R^\dagger R)_{jk}(R^\dagger)_{ki}...\right), \qquad (4.7)
\end{aligned}
$$

where $x_k = \frac{M_k{}^2}{M_j{}^2}$ and $I(x_k) = \sqrt{x_k}\left(1 + (1+x_k)\log(\frac{x_k}{1+x_k})\right)$. We have used $K^\dagger K = 1$ which holds up to $O(\frac{v^2}{M^2})$. M_k denotes the mass of the k th heavy Majorana neutrino which contributes to s channel and t channel scattering amplitudes denoted by $\langle lW|T|lW\rangle$, $\langle lZ|T|lW\rangle$ and $\langle lH|T|lW\rangle$. In Eq. (4.7), the first term comes from the tree level amplitude the second term comes from the absorptive part of W and charged lepton and the third term comes from sum of the absorptive part of Z and neutrino, and Higgs and neutrino. The terms indicated by ... do not contribute to the total lepton asymmetry, to leading order, because one has $\mathrm{Im}R_{ij}(R^\dagger)_{ji} = 0$ and $\mathrm{Im}(R^\dagger R)_{jk}(R^\dagger R)_{kj} = 0$. A similar formula can be obtained for the l^-W^+ final state:

$$
\begin{aligned}
& M(N_j \to l_i{}^-W_L{}^+) = \\
& -(\bar{u}_i R U_N)\frac{gM_j}{\sqrt{2}M_W}\left[R_{ij} - iR_{nk}R_{ik}(R^\dagger)_{jn}\left(\frac{gM_k}{\sqrt{2}M_W}\right)^2\frac{1}{16\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{1-x_k}\right)\right]. \qquad (4.8)
\end{aligned}
$$

The lepton number asymmetry from j th heavy Majorana particle is then given by:

$$
\begin{aligned}
A^j \;=\; & \frac{\sum_i \Delta A^j{}_i}{\sum_i\left(N^j{}_i + \overline{N^j}_i\right)} \\
= \;& \frac{g^2}{M_W{}^2}\sum_{k\neq j}\left[(M_k)^2\mathrm{Im}\left((R^\dagger R)_{jk}(R^\dagger R)_{jk}\right)\frac{1}{16\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{1-x_k}\right)\right]\frac{1}{(R^\dagger R)_{jj}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{g^2}{M_W{}^2}\sum_{k\neq j}\left[\mathrm{Im}\left((m^\dagger m)_{jk}(m^\dagger m)_{jk}\right)\frac{1}{16\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{1-x_k}\right)\right]\frac{1}{(m^\dagger m)_{jj}}, \\
&= \sum_{k\neq j}\left[\mathrm{Im}\left(({y_D}^\dagger y_D)_{jk}({y_D}^\dagger y_D)_{jk}\right)\frac{1}{8\pi}\left(I(x_k)+\frac{\sqrt{x_k}}{1-x_k}\right)\right]\frac{1}{({y_D}^\dagger y_D)_{jj}}, \qquad (4.9)
\end{aligned}
$$

where in the third equality, we used the approximate formulae: $R_{ij}M_j = m_{ij}$ and in the final expression, we substituted $m_{ij} = y_{Dij}\frac{v}{\sqrt{2}}$. This expression for A^j agrees with the one computed in the symmetric phase [13]. The expression in terms of the mixing matrix R was first derived, in a different way, in [6] without using the unitary gauge as we have done in this paper. To summarize our results, starting with the Lagrangian in the broken phase, we have computed the lepton number asymmetry and in the limit $v \to 0$, we have exactly recovered the lepton number asymmetry computed in the symmetric phase. The following argument shows that this result was to be expected. In the symmetric phase, the Nambu-Goldstone boson becomes physical and the gauge bosons become massless. The heavy Majorana neutrino decays through the Yukawa coupling to the would-be Nambu-Goldstone boson. In the broken phase, the gauge bosons become massive and the Nambu-Goldstone boson is absorbed as their longitudinal component. The heavy Majorana neutrino can decay into both transverse and longitudinal gauge bosons. The interaction to the transverse polarization is suppressed by the mixing R and can be neglected for small $\frac{v}{M}$, while the coupling to the longitudinal polarization remains finite in the limit of $\frac{v}{M} \to 0$ and this is given by the original Yukawa coupling (See Eq. (4.5)). This is the essential reason why we obtain the same result as the one found in the symmetric phase although we started with the broken phase.

4.2 CP violation in lepton number asymmetry

From Eq. (4.9) together with the parametrization chosen in Eq. (2.28) it can be seen that the lepton number asymmetry is only sensitive to the CP violating phases appearing in Y_Δ, since

$$
\mathrm{Im}\{(m^\dagger m)_{jk}\}^2 \;=\; \mathrm{Im}\{({Y_\Delta}^\dagger Y_\Delta)_{jk}\}^2, \quad (j\neq k). \qquad (4.10)
$$

Explicitly, one has:

$$
\begin{aligned}
(m^\dagger m)_{12} &= |y_{21}|y_2\exp(i\beta_1) + |y_{31}y_{32}|\exp\left(i(\beta_1+\sigma)\right) \\
(m^\dagger m)_{23} &= |y_{32}|y_3\exp(-i(\beta_2-\beta_1-\sigma)) \\
(m^\dagger m)_{31} &= |y_{31}|y_3\exp(-i\beta_2). \qquad (4.11)
\end{aligned}
$$

Thus the three phases σ, β_1 and β_2 appearing in Y_Δ generate the lepton number asymmetry [6]. From our analysis in section 3, it is clear that the lepton asymmetry can be related to WB invariants. We have seen that there are six independent WB invariants which signal CP violation in the minimal seesaw model. The combinations which generate lepton number asymmetry can be written in terms of $h = m^\dagger m$:

$$
\mathrm{Im}\{(m^\dagger m)_{jk}\}^2 = \mathrm{Im}(h_{jk})^2. \qquad (4.12)
$$

where $j \neq k$. By going to the WB, in which both charged lepton and heavy Majorana mass matrices are real diagonal, we have shown that the three WB invariants $I_1 \sim I_3$ are functions of $\mathrm{Im}({h_{12}}^2)$, $\mathrm{Im}({h_{23}}^2)$ and $\mathrm{Im}({h_{31}}^2)$. Therefore A^j can be written in terms of WB invariants I_1, I_2 and I_3 and is independent of the other three WB invariants $\bar{I}_i$ $(i = 1, 2, 3)$.

5 Relating CP violation in leptogenesis with CP violation at low energies

5.1 Model independent analysis

One of the most fascinating questions one may ask is whether there is some connection between CP violation responsible for the generation of BAU through leptogenesis and the one measurable at low energies. More specifically, assuming that BAU is indeed achieved through leptogenesis, one may ask what can one infer about the size of CP violation at low energies, from the size and sign of the observed BAU. In order to address the above question, one should keep in mind that in the WB where both the right-handed neutrino mass matrix M and the charged lepton mass matrix m_l are diagonal, real, all information about the leptonic mixing and CP violation is contained in the Dirac neutrino mass matrix m. We have seen that in the minimal seesaw model we are considering, the matrix m contains six phases. For definiteness, let us consider the parametrization of m given by Eq. (2.28), where the six phases are ρ, α_1, α_2, σ, β_1, β_2. In general and without further assumptions about the structure of the leptonic mass matrices, these six phases are independent from each other. Furthermore, we have seen in section 4 that leptogenesis is controlled by the phases σ, β_1 and β_2. In order to see what are the phases relevant to CP violation at low energies, we have to consider the effective left-handed neutrino mass matrix, given by:

$$m_{ef} = -m\frac{1}{D}m^T = -\hat{U}_\rho P_\alpha \hat{Y}_\triangle {P_\beta}^2 \frac{1}{D} \hat{Y}_\triangle^T P_\alpha \hat{U}_\rho^T \tag{5.1}$$

The strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy WB invariant:

$$Tr[h_{ef}, h_l]^3 = 6i\Delta_{21}\Delta_{32}\Delta_{31}\mathrm{Im}\{(h_{ef})_{12}(h_{ef})_{23}(h_{ef})_{31}\} \tag{5.2}$$

where $h_{ef} = m_{ef}{m_{ef}}^\dagger$, $h_l = m_l {m_l}^\dagger$ and $\Delta_{21} = ({m_\mu}^2 - {m_e}^2)$ with analogous expressions for Δ_{31}, Δ_{32}. From Eqs. (5.1), (5.2), it is clear that all the six phases contained in m affect the strength of CP violation at low energies. In the decoupling limit, leptonic mixing is characterized, of course, by a 3×3 unitary matrix, containing one CP violating Dirac phase δ and two Majorana phases, which have an interesting geometrical interpretation in terms of unitarity triangles [14]. CP violation in neutrino oscillations is only affected by the phase δ. The important point is that the phase δ is, in general, a function of all the six phases ρ, α_1, α_2, σ, β_1, β_2 as can be seen from Eqs. (2.18), (2.28). Since leptogenesis only depends on σ, β_1 and β_2, it is clear that, in general, one cannot directly relate the size of CP violation responsible for leptogenesis with the strength of CP violation at low energies. More specifically, one cannot derive the size of the phase δ from the knowledge of the amount of CP violation required to generate the observed baryon asymmetry, through leptogenesis. A related important question is the following. Throughout the paper, we have assumed completely generic leptonic mass matrices. Let us now consider a scheme where the only non-vanishing phases are those responsible for leptogenesis, namely σ, β_1 and β_2. Can one in such a scheme, generate CP violation at low energies? In order to answer this question, one has to compute the WB invariant $Tr[h_{ef}, h_l]^3$ given in Eq. (5.2), and, in particular, Im$Q = \mathrm{Im}\{(h_{ef})_{12}(h_{ef})_{23}(h_{ef})_{31}\}$. Through a tedious but straightforward computation one can verify that ImQ does not vanish in general. This is true even if only one of the phases σ, β_1 or β_2 is non-vanishing. One concludes then that in a model where the leptonic mass matrices are constrained (e. g. by flavour symmetries) so that only one of the phases (for example σ) is non-vanishing, one can establish a direct connection between the size of the observed BAU and the strength of CP violation at low energies, observable for example in neutrino oscillations. In section 5.2 we shall describe a framework where the only non-vanishing phases are those responsible for leptogenesis.

5.2 A special GUT inspired scenario

In the previous subsection, our analysis, done in the framework of the minimal seesaw, has been model independent and we have discussed, with all generality, the phases which can be seen by leptogenesis, as well as those appearing at low energy CP violating processes. Let us now assume that the Hermitian matrices $m_l m_l^\dagger$, $mm^\dagger$ are diagonalized by the same left-handed unitary transformation, a situation which occurs in some GUTs. In this case, in the WB where m_l and the rigt-handed Majorana neutrino matrix M are both real and diagonal, the neutrino Dirac mass matrix has the form:

$$m = dU_R \tag{5.3}$$

where d is diagonal and U_R is a generic unitary matrix. It can be easily seen that in this case there are only three independent phases. In fact the phases of U_R corresponding to P_ξ in Eq. (2.25) can be eliminated in the same way, due to commutativity with the diagonal matrix d. Two of the remaining three phases in U_R can be factored out and we may write:

$$m = d\hat{U}(\delta)P_\alpha, \quad P_\alpha = diag(1, \exp(i\alpha_1), \exp(i\alpha_2)) \tag{5.4}$$

We conclude from Eq. (2.22) that in the limit $T = 1$ the phases α_1 and α_2 are seen, in the heavy neutrino sector, as Majorana type phases in analogy to the phases β_1 and β_2 in Eq. (2.29). On the other hand $\hat{U}(\delta)$ can be parametrized as the CKM matrix with only one phase δ. As a result, the matrix m will contain only three phases, namely the phases δ, α_1 and α_2. In section 4 we pointed out that leptogenesis is only sensitive to the phases appearing in $m^\dagger m$ and so it is clear that these three phases are exactly those appearing in leptogenesis. Furthermore it can be readily verified that these phases do lead to CP violation at low energies. This is due to the fact that from Eq. (5.3), one obtains for the effective lefthanded neutrino mass matrix:

$$m_{ef} = -dU_R \frac{1}{D} U_R^T d \tag{5.5}$$

where D is the matrix M in the WB where it is diagonal. The crucial point is that the phases in U_R do not cancel out in Eq. (5.5). It is well known [15], [16] that in such a framework the large neutrino mixing required to account for the atmospheric neutrino anomaly can be generated even for small mixing in $\hat{U}(\delta)$, with hierarchical entries in D. Specific examples can be constructed and even in the case of a single nonvanishing phase, one still generates CP violation at low energies - this is the case for only $\delta \neq 0$ or in alternative, for instance, only $\alpha_2 \neq 0$. These features are particularly striking since, as was pointed out above, these phases appear in the heavy sector with a different nature (Dirac versus Majorana type).

There is an alternative WB for this scenario where m and m_l are diagonalized simultaneously and M becomes:

$$M = U_R^* D U_R^\dagger \tag{5.6}$$

As a result, one might view all CP violating effects as related to the phases δ, α_1 and α_2 generated at high energies. It is common in the literature to assume hierarchical entries in D. One might wonder whether this choice plays a crucial rôle in the connection between CP violation at high energies and CP violation at low energies. In order to answer this question it is worthwhile computing $\mathrm{Im}\{(h_{ef})_{12}(h_{ef})_{23}(h_{ef})_{31}\}$ in the case of exact degeneracy of M. In the WB where $m = d$ and m_l is diagonal we have:

$$M = U_R^* D U_R^\dagger = \mu U_R^* U_R^\dagger = \mu Z_0 \tag{5.7}$$

with μ the common degenerate mass and Z_0 a symmetric unitary matrix. It was shown in Ref. [17] that one may have a nontrivial M, even in the limit of exact degeneracy, which can be parametrized by:

$$M = \mu Z_0 = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{pmatrix} \begin{pmatrix} c\theta & s\theta & 0 \\ s\theta & -c\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{pmatrix} \tag{5.8}$$

where c, s stand for cosine and sine. In the case of full degeneracy, there are only two independent angles ϕ and θ and one CP violating phase, α. It can be easily checked that:

$$\begin{aligned} \mathrm{Im}\{(h_{ef})_{12}(h_{ef})_{23}(h_{ef})_{31}\} &= d_1^2 d_2^2 d_3^2 (d_1^2 - d_2^2)(d_2^2 - d_3^2)(d_1^2 - d_3^2) \tfrac{1}{\mu^6} s\alpha c\theta s^2\theta s^2\phi c^2\phi = \\ &= d_1^2 d_2^2 d_3^2 (d_1^2 - d_2^2)(d_2^2 - d_3^2)(d_1^2 - d_3^2) \tfrac{1}{\mu^6} \left(\tfrac{1}{\mu^4} \mathrm{Im}(M_{11} M_{12}^* M_{21}^* M_{22}) \right) \end{aligned} \tag{5.9}$$

where the d_i's are the diagonal elements of $m = d$ and the imaginary part of a quartet of the unitary matrix M appears. This result means that even in this very special limit of exact degeneracy in M, CP violation at high energies can lead to CP violation at low energies. Note that in a framework where the CP violating phases appear only in M, CP is only softly broken by the Lagrangian.

5.3 A model with spontaneous CP violation at a high energy scale

So far, in our analysis, we have not dealt with the origin of the CP violating phases. These phases can appear as explicit CP violating terms in the Lagrangian, either through complex Yukawa couplings and/or complex entries in the right-handed neutrino Majorana mass M, or, in alternative, as the result of spontaneous CP violation. In this subsection, we describe a minimal extension of the SM, where CP is a good symmetry of the Lagrangian, only broken by the vacuum. We show that the model has the interesting feature that the breaking of CP originates at a high energy scale, through a single phase of a complex scalar field which is sufficient to generate CP violation necessary for leptogenesis and at the same time generate CP violation at low energies. Let us consider an extension of the SM where one adds one right-handed neutrino per generation, as well as a complex scalar S, invariant under $SU(3)_c \times SU(2)_L \times U(1)$. The most general Yukawa and mass terms invariant under $SU(3)_c \times SU(2)_L \times U(1)$ can be written:

$$\begin{aligned} \mathcal{L}_Y &= \left(\overline{\nu^0}, \overline{e^0}\right)_L Y_l \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} e^0_R + \left(\overline{\nu^0}, \overline{e^0}\right)_L Y_\nu \begin{bmatrix} \phi^{0*} \\ -\phi^- \end{bmatrix} \nu^0_R + \\ &+ \frac{1}{2}\nu_R^{0T} \overline{M} \nu_R^0 + \frac{1}{2} Y_S \nu_R^{0T} C \nu_R^0 S + \frac{1}{2} Y'_S \nu_R^{0T} C \nu_R^0 S^* + h.c. \end{aligned} \tag{5.10}$$

Since we impose CP invariance on the Lagrangian, all the Yukawa couplings Y_l, Y_ν, Y_S, Y'_S, as well as the mass term $\overline{M}$ are real. Due to the presence in the Higgs potential of terms like S^2, S^{*2}, S^4, S^{*4}, it can be readily seen that there is a region of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi^0 \rangle = v; \quad \langle S \rangle = V e^{i\theta} \tag{5.11}$$

The following mass terms are thus generated:

$$m_l = vY_l; \quad m = vY_\nu; \quad M = \overline{M} + Y_S V e^{i\theta} + Y'_S V e^{-i\theta} \tag{5.12}$$

Since the Yukawa terms Y_S, Y'_S are real, but arbitrary, M is a general complex symmetric matrix, while m_l, m_D are real. It is clear that one can go to a WB where m_l, M are real, diagonal while the neutrino Dirac mass matrix is of the form:

$$m = {O_L}^T d U_R \tag{5.13}$$

with O_L an orthogonal matrix, while U_R is a generic unitary matrix. The CP violating phases contained in U_R arise from the diagonalization of M and therefore these phases have their origin in the vacuum phase θ of Eq. (5.11). Of course, U_R can be written as $P'_R \hat{U}_R(\phi) P_R$, with P'_R, P_R diagonal unitary matrices. However, it should be noted that, due to the presence of the non-trivial orthogonal matrix O_L, it is not possible to eliminate the three phases contained in P'_R.

Although CP violation arises in this model from the single vacuum phase θ, it is clear that in general one has CP violation necessary to obtain leptogenesis (with the relevant phases being the three phases appearing in $m^\dagger m = {U_R}^\dagger d^2 U_R$) as well as CP violation at low energies. So far, we have not dealt with the quark sector. Since in the above scenario we have assumed CP invariance at the Lagrangian level, the quark Yukawa couplings will also be real. It has been shown [18] that the phase θ of $\langle S \rangle$ can still generate unsuppressed CP violation in the quark sector at low energies, provided one introduces at least one isosinglet vector-like quark. These vector-like quarks play a rôle analogous to the one played by right-handed neutrinos in the leptonic sector, in the sense that they allow the phase θ to be seen by the charged W interactions connecting standard quarks.

6 Conclusions

We have studied the various sources of CP violation in the minimal seesaw model, identifying both the CP violating phases and the weak-basis invariants which are associated to leptogenesis as well as those relevant for CP violation at low energies. We have addressed the question of whether it is possible to establish a connection between CP violation responsible for leptogenesis and CP violation observable at low energies, for example through neutrino oscillations. It was shown that, in general, such a connection between the two phenomena cannot be established. However, we have described a class of models where the phases which are responsible for leptogenesis are also the ones that generate CP violation at low energies. This situation naturally occurs in models with the feature of having, in a weak-basis, the left-handed component of charged leptons aligned in flavour space with the left-handed neutrinos. In this class of models a direct connection may in principle be established between the size and sign of the observed baryon asymmetry obtained through leptogenesis and CP violation observed at low energies.

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