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(2,0) superconformal OPEs in $D = 6$, selection rules and non-renormalization theorems

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ABSTRACT: We analyse the OPE of any two 1/2 BPS operators of (2,0) $SCFT_6$ by constructing all possible three-point functions that they can form with another, in general long operator. Such three-point functions are uniquely determined by superconformal symmetry. Selection rules are derived, which allow us to infer "non-renormalization theorems" for an abstract superconformal field theory. The latter is supposedly related to the strong-coupling dynamics of N_c coincident M5 branes, dual, in the large-N_c limit, to the bulk M-theory compactified on $AdS_7 \times S_4$. An interpretation of extremal and next-to-extremal correlators in terms of exchange of operators with protected conformal dimension is given.

KEYWORDS: AdS/CFT Correspondance, Conformal and W Symmetry, Extended Supersymmetry, Superspaces.

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1. Introduction

In recent time, many tests (for reviews of the different approaches see, e.g. [1, 2, 3]) of the AdS/CFT duality (see, e.g. [4]) between bulk supergravity on AdS_{p+2} and the boundary conformal field theory of the world volume p-brane dynamics, have relied on the particular case of $p = 3$ branes of IIB strings on AdS_5. This is because the $N = 4$ superconformal field theory can be defined in this case for arbitrary values of the gauge coupling, in view of the exceptional ultraviolet properties of four-dimensional $N = 4$ superconformal field theory (vanishing beta function).

Unlike this case, the superconformal field theories of M2 and M5 branes, which are dual to M-theory on $\mathrm{AdS}_{4(7)} \times \mathrm{S}_{7(4)}$, are only understood as strongly coupled conformal field theories, where the conformal fixed point is only defined in a formal way. However, the powerful constraints of superconformal invariance allow one, even in these cases, to extract some general information, which is supposed to remain valid in the fully-fledged non-perturbative theory.

The most popular example is the comparison [5] of the spectrum of the 1/2 BPS operators (sometimes called, by an abuse of language, chiral primary operators) of the superconformal algebra with the so-called Kaluza-Klein states [6] of $D = 11$ supergravity on $AdS_{4(7)} \times S_{7(4)}$ [7]. The 1/2 BPS operators are the simplest short UIRs of superconformal algebras, since they correspond to superfields with maximal shortening (1/2 of the θs missing). In the case of the superalgebra $OSp(8^*/4)$, which is the subject of the present paper, 1/2 BPS operators in superspace have been considered in [8, 9]. These UIRs have a simple description [6, 10] in terms of the oscillator method developed in the 1980s in the pioneering papers of ref. [11]. However, many more short UIRs exist for generic superconformal algebras, even in interacting field theories (such as $N = 4$ super-Yang-Mills (SYM) theory in $D = 4$) and these have been systematically classified by using superfields of different kinds [12, 13]. In particular, BPS multiplets are described by Grassmann (G-)analytic superfields, a generalization of the familiar notion of chiral superfields of $N = 1$, $D = 4$ superconformal algebra $SU(2, 2/1)$.

For conformal field theories in $D = 6$, non-perturbative information on their superconformal regime can be extracted by superconformal OPEs, which encode many of the non-perturbative definitions of a generic superconformal field theory.

Actually, such an approach is a revival of the so-called "bootstrap program" of the 1970s, when conformal techniques were popular in connection to the study of the short-distance behaviour of scale-invariant field theories (for a review see [14]).

The main new fact, in the case where conformal symmetry is merged with supersymmetry, are the so-called "non-renormalization theorems" of superconformal field theories, which have a simple explanation in terms of the existence, in these theories, of a wide class of operators with protected "conformal dimension" due to shortening conditions [15, 16, 17] of the Harish-Chandra modules [18, 19] of the UIRs.

In the present paper we focus the analysis on the M5 $(2, 0)$ conformal theory, based on the $OSp(8^*/4)$ superconformal algebra, and we analyse the OPE of two superconformal 1/2 BPS primary operators. Following [20, 21], this is done by resolving the UIR spectrum of operators, which have a non-vanishing three-point function with the two 1/2 BPS operators. As we explain later on, superconformal symmetry uniquely fixes such three-point functions, and further implies selection rules for the third operator. Partial results on $D = 6$ $(2, 0)$ three-point functions appear in the literature [22], also in the case of 1/2 BPS operators [23].

From standard properties of superconformal field theories this technique allows one to analyse n-point correlator functions by multiple OPEs and, in some cases, to extract remarkable non-perturbative information, which has a counterpart on the

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supergravity side, such as the extremal, next-to-extremal and near-extremal correlators [24]–[27].

Many analogies and differences with the four-dimensional case emerge from this analysis. In $D = 4$ the superconformal algebra $PSU(2, 2/4)$ admits three series of UIRs [16], one series (A) with a continuous spectrum of the conformal dimension and two isolated series (B and C) with fixed ("quantized") dimension. On the other hand, the UIRs of the $D = 6$ superconformal algebra $OSp(8^*/4)$ fall into four distinct series [17], one continuous series (A) and three isolated series (B, C and D) [9, 12].

Let us denote by

$$\mathcal{D}(\ell; J_1, J_2, J_3; a_1, a_2) \tag{1.1}$$

the quantum numbers of a generic supermodule of $OSp(8^*/4)$. Here ℓ is the conformal dimension, J_1, J_2, J_3 are the Dynkin labels of the $D = 6$ Lorentz group $SU^*(4) \equiv$ Spin $SO(1, 5)$ and a_1, a_2 are the Dynkin labels of the R symmetry group $USp(4)$ $\equiv$ Spin $SO(5)$. The four unitary series correspond to:

$$\begin{aligned}
&A) J_1, J_2, J_3 \text{ unrestricted}\,, \qquad l \geq 6 + \frac{1}{2}(J_1 + 2J_2 + 3J_3) + 2(a_1 + a_2)\,,\\
&B) J_3 = 0\,, \qquad l = 4 + \frac{1}{2}(J_1 + 2J_2) + 2(a_1 + a_2)\,,\\
&C) J_2 = J_3 = 0\,, \qquad l = 2 + \frac{1}{2}J_1 + 2(a_1 + a_2)\,,\\
&D) J_1 = J_2 = J_3 = 0\,, \qquad l = 2(a_1 + a_2)\,.
\end{aligned} \tag{1.2}$$

As we see, the three isolated series B, C, D occur for $J_1 J_2 J_3 = 0$, while the continuous series A exists for all values of J_1, J_2, J_3. Operators from series A saturating the unitarity bound, as well as all the operators in the isolated series B and C correspond to "semishort" superfields with some missing powers of θs in their expansion [12]. The isolated series D contains 1/2 and 1/4 BPS states realized in terms of G-analytic superfields independent of two or one θ, respectively. The 1/2 BPS states correspond to $a_1 = 0$, i.e. to the symmetric traceless representations of $SO(5)$. Massless conformal fields ("supersingletons" [28]) belong both to series D with $a_1 + a_2 = 1$ and to series C with $a_1 = a_2 = 0$ [10, 29].

The main result of the present paper consists in selection rules for the three-point function of two 1/2 BPS operators $\mathcal{D}(2m; 0, 0, 0; 0, m)$ and $\mathcal{D}(2n; 0, 0, 0; 0, n)$ with a third operator $\mathcal{D}(\ell; 0, s, 0; a_1, a_2)$.[2] What we find is summarized below.

The allowed values of a_1, a_2 are

$$a_1 = 2j\,, \qquad a_2 = m + n - 2k - 2j\,, \qquad \text{with} \qquad \begin{aligned} &0 \leq k \leq \min(m, n)\,,\\ &0 \leq j \leq \min(m, n) - k\,. \end{aligned}$$

[2]Note that the $SU^*(4)$ representations $[0, s, 0]$ or, equivalently, the symmetric traceless rank s tensors of $SO(1, 5)$, are the only Lorentz irreps allowed to appear in the OPE of two scalar operators.

Depending on the value of k, there are three distinct cases:

$$\textit{(i)}\ k=0\,,\ s=0\,,\ \ell=2(m+n)\,;\ \text{series D, BPS states}\,,\ \frac{1}{2}\ \text{BPS if}\ j=0\,,$$

$$\textit{(ii)}\ k=1\,,\ s=0\,,\ \ell=2(m+n-2)\,;\ \text{series D, BPS states}\,,\ \frac{1}{2}\ \text{BPS if}\ j=0\,,$$
$$s\geq 0\,,\ \ell=4+s+2(m+n-2)\,;\ \text{series B with}\ J_1=0\,,$$
$$\text{semishort multiplets}\,,$$

(iii) $k \geq 2$; no restrictions on the UIR (continuous spectrum possible).

The important fact is that in cases *(i)* and *(ii)* the operators have "protected dimension" because they are either BPS or "semishort representations".

A similar phenomenon has recently been observed in the case of $N = 4$, $D = 4$ SYM theory [21]. However, there the semishort operators are at the threshold of the unitarity bound of the continuous series rather than at an isolated point. Operators at the threshold of continuous series or at isolated points can be realized as products of "supersingleton" [12]. The surprising fact is that for $D = 6$ the bilinear supersingleton composite operators belong to the isolated series (*protected dimension*) rather than to the continuous series (*unprotected dimension*) as is the case in $D = 4$. Remarkably, the continuous unitary series starts at the three-singleton threshold and this is one of the mysteries of $D = 6$ superconformal field theory.

The above selection rules have some dramatic consequences for extremal n-point correlators of 1/2 BPS states (i.e. those for which $a_1^k = 0$, $k = 1,\ldots,n$ and $a_2^1 = \sum_{k=2}^n a_2^k$). By multiple superconformal OPE we show that extremal correlators correspond to the exchange of only 1/2 BPS states, which confirms the non-renormalization conjecture of ref. [27]. One may say that the extremal correlators of 1/2 BPS operators correspond to a sub-field theory solely built in terms of 1/2 BPS states. Incidentally, we remark that BPS operators form a "ring" under multiplication. A possible rôle of the algebra of BPS states was also put forward some time ago in conjunction with other aspects of string theory and M-theory [30].

This paper is organized as follows:
In section 2 we recall $D = 6$ $(2,0)$ superconformal fields, harmonic superspace and short representations constructed in terms of the 1/2 BPS supersingleton (the tensor multiplet). We also explain how semishort superfields are constructed in terms of supersingletons. In section 3 we study superconformal three-point functions of two 1/2 BPS operators and a third, *a priori* general operator. The crucial property of such three-point functions is that they are uniquely determined by conformal supersymmetry. Imposing the shortness conditions at two points, we derive selection rules for the operator at the third point. In this way we establish the OPE spectrum of 1/2 BPS operators. In section 4 we apply these results to n-point extremal and to four-point next-to-extremal correlators of 1/2 BPS operators and compare with the AdS supergravity non-renormalization predictions.

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2. Representations of OSp(8*/4) and $D = 6$ (2,0) harmonic superspace

A simple method of constructing the series of UIRs of superconformal algebras was proposed in [12]. The idea is to start with the massless supermultiplets ("supersingletons"). Then, by multiplying them in all possible ways, we are able to construct the four series of UIRs of OSp(8*/4) found in [17]. An essential ingredient in this construction is harmonic superspace [31].[3] In this section we give a brief summary, paying special attention to the UIRs that have fixed conformal dimension, namely the BPS and the semishort multiplets. They will play an important rôle in our OPE analysis in section 3.

2.1 Supersingletons of OSp(8*/4)

There exist three types of massless multiplets in six dimensions corresponding to ultrashort UIRs (supersingletons) of OSp(8*/4) (see, e.g. [10]). All of them can be formulated in terms of constrained superfields as follows [9].

(i) The first type comes in two species. The first one is described by a superfield $W^i(x,\theta)$, $i = 1,\ldots,4$ in the fundamental irrep $[1,0]$ of the R symmetry group USp(4). It satisfies the on-shell constraint

$$D_\alpha^{(k}W^{i)} = 0 \Longrightarrow \mathcal{D}(2;0,0,0;1,0)\,, \tag{2.1}$$

which reduces the superfield to the OSp(8*/4) UIR indicated in (2.1). The spinor covariant derivatives D^i_α obey the supersymmetry algebra

$$\left\{D^i_\alpha, D^j_\beta\right\} = -2i\Omega^{ij}\gamma^\mu_{\alpha\beta}\partial_\mu\,. \tag{2.2}$$

It is convenient to make the non-standard choice of the symplectic matrix $\Omega^{ij} = -\Omega^{ji}$ with non-vanishing entries $\Omega^{14} = \Omega^{23} = -1$. The chiral SU*(4) spinors satisfy a pseudo-reality condition of the type $\overline{D^i_\alpha} = \Omega^{ij}D^\beta_j c_{\beta\alpha}$, where c is a 4×4 unitary "charge conjugation" matrix.

The second species, the so-called (2,0) tensor multiplet [33, 34], is of special interest in $D = 6$ SCFT, since it is related to the basic degrees of freedom of the M5 brane world-volume theory [4]. It is described by an antisymmetric traceless real superfield $W^{ij} = -W^{ji} = \overline{W_{ij}}$, $\Omega_{ij}W^{ij} = 0$ (USp(4) irrep $[0,1]$) subject to the on-shell constraint

$$D_\alpha^{(k}W^{i)j} = 0 \Longrightarrow \mathcal{D}(2;0,0,0;0,1)\,. \tag{2.3}$$

(ii) The second type of supersingletons is described by a (real) Lorentz scalar and USp(4) singlet superfield, $w(x,\theta)$ obeying the constraint

$$D^{(i}_{[\alpha}D^{j)}_{\beta]}w = 0 \Longrightarrow \mathcal{D}(2;0,0,0;0,0)\,. \tag{2.4}$$

[3] The harmonic superspace formulation of the basic $D = 6$ (2,0) 1/2 BPS multiplet, the tensor multiplet, and of composite 1/2 BPS operators made out of it, was first proposed in [8] (see also [32]).

(iii) Finally, there exists an infinite series of multiplets described by USp(4) singlet superfields with n totally symmetrized external Lorentz spinor indices, $w_{(\alpha_1\cdots\alpha_n)}(x,\theta)$ (they can be made real in the case of even n) obeying the constraint

$$D^i_{[\beta}w_{(\alpha_1]\cdots\alpha_n)} = 0 \Longrightarrow \mathcal{D}(2+n/2; n,0,0;0,\dots,0)\,. \tag{2.5}$$

The constraints (2.1), (2.3), (2.4) and (2.5) restrict the θ expansion of the above superfields to just a few massless fields. In this sense the supersingletons are "ultra-short" superfields.

2.2 Harmonic superspace, Grassmann analyticity and BPS multiplets

The massless multiplets *(i)* admit an alternative formulation in harmonic superspace. The advantage of this formulation is that the constraints (2.1) and (2.3) become conditions for Grassmann analyticity which simply mean that the superfield does not depend on one or more of the Grassmann variables θ^α.

We introduce harmonic variables describing the coset USp(4)/ U(1) × U(1):

$$u \in \mathrm{USp}(4) : u^I_i u^i_J = \delta^I_J\,, \qquad u^I_i\Omega^{ij}u^J_j = \Omega^{IJ}\,, \qquad u^I_i = \left(u^i_I\right)^*\,. \tag{2.6}$$

Here the indices i,j belong to the fundamental representation of USp(4) and I,J are labels corresponding to the U(1) × U(1) projections. The harmonic derivatives

$$D^{IJ} = \Omega^{K(I}u^{J)}_i\frac{\partial}{\partial u^K_i} \tag{2.7}$$

are compatible with the definition (2.6) form the algebra of USp(4) realized on the indices I,J of the harmonics.

Let us now project the defining constraint (2.1) with the harmonics $u^1_k u^1_i$:

$$D^1_\alpha W^1 = 0\,, \tag{2.8}$$

where $D^1_\alpha = D^k_\alpha u^1_k$ and $W^1 = W^i u^1_i$. Indeed, the constraint (2.1) now takes the form of a *G-analyticity condition.*[4] It is integrable because $\{D^1_\alpha, D^1_\beta\} = 0$, as follows from (2.2). This allows us to find an *analytic basis* in superspace:

$$x^{\alpha\beta}_A = x^{\alpha\beta} - i\left(\theta^{1[\alpha}\theta^{4\beta]} + \theta^{2[\alpha}\theta^{3\beta]}\right)\,, \qquad \theta^{I\alpha} = \theta^{i\alpha}u^I_i \tag{2.9}$$

in which

$$D^1_\alpha = \frac{\partial}{\partial\theta^{4\alpha}}\,. \tag{2.10}$$

[4]Grassmann analyticity [35] is a generalization of the concept of chiral superfields familiar from $N=1$, $D=4$ supersymmetry. In six dimensions all superfields are chiral, so G-analyticity remains the only non-trivial notion.

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Then the solution to (2.8) is a *Grassmann analytic* superfield, which does not depend on one of the four odd (spinor) coordinates, $\theta^{4\alpha}$ (hence the name *1/4 BPS short superfield*):

$$W^1\left(x_A,\theta^1,\theta^2,\theta^3,u\right), \tag{2.11}$$

but it depends on the harmonic variables. Note that the *analytic superspace* with coordinates $x_A,\theta^1,\theta^2,\theta^3,u$ is closed under the Poincaré (Q) supersymmetry transformations,

$$\delta_Q x_A^{\alpha\beta} = -2i\left(\theta^{1[\alpha}\epsilon^{4\beta]} + \theta^{2[\alpha}\epsilon^{3\beta]} + \theta^{3[\alpha}\epsilon^{2\beta]}\right), \qquad \delta_Q\theta^{I\alpha} = \epsilon^{I\alpha} \equiv \epsilon^{i\alpha}u_i^I, \quad I = 1,2,3, \tag{2.12}$$

as well as under the conformal (S) supersymmetry transformations.

The G-analytic superfield (2.11) is infinitely reducible under USp(4) because of its harmonic dependence. In order to make it irreducible (or harmonic short), we impose the harmonic differential conditions

$$D^{11}W^1 = D^{12}W^1 = D^{13}W^1 = D^{22}W^1 = 0. \tag{2.13}$$

The harmonic derivatives (2.7) appearing in (2.13) correspond to the raising operators (positive roots) of USp(4), thus (2.13) can be interpreted as the defining condition of the HWS of the irrep $[1,0]$. Alternatively, if a complex parametrization of the harmonic coset $\mathrm{USp}(4)/\,\mathrm{U}(1)\times\mathrm{U}(1)$ is used, eqs. (2.13) take the form of harmonic (H-)analyticity (Cauchy-Riemann) conditions.

The tensor multiplet (2.3) can be reformulated in a similar way. Projecting the defining constraint with the harmonics $u_k^K u_i^1 u_j^2$, $K=1,2$, we obtain:

$$D^1_\alpha W^{12} = D^2_\alpha W^{12} = 0 \tag{2.14}$$

whose solution, in an appropriate modification of the analytic basis (2.9), is a harmonic superfield that does not depend on half of the Grassmann variables:

$$W^{12}\left(x_A,\theta^1,\theta^2,u\right). \tag{2.15}$$

It is subject to the same USp(4) irreducibility conditions (2.13) as W^1 above, but now they define the HWS of the irrep $[0,1]$.

The G-analytic superfields $W^1(\theta^1,\theta^2,\theta^3)$ and $W^{12}(\theta^1,\theta^2)$ are the simplest examples of the two kinds of *BPS short multiplets* of OSp($8^*/4$). The latter are described by superfields which do not depend on a fraction of the total number of Grassmann variables. Thus, W^1 is 1/4 BPS short and W^{12} is 1/2 BPS short.

Further BPS short superfields can be obtained by multiplying the above two types of supersingletons and imposing the USp(4) irreducibility conditions (2.13):

$$W^{[a_1,a_2]} = \left[W^1\left(\theta^1,\theta^2,\theta^3\right)\right]^{a_1}\left[W^{12}\left(\theta^1,\theta^2\right)\right]^{a_2} \Longrightarrow \mathcal{D}(2(a_1+a_2);0,0,0;a_1,a_2). \tag{2.16}$$

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Note that the powers of each type of supersingleton correspond to the Dynkin labels of the USp(4) irrep. This can be seen by looking at the leading component ($\theta = 0$) of the superfield. The differential conditions (2.13) reduce it to a harmonic monomial:

$$W^{[a_1,a_2]}(\theta = 0) = C^{i_1\cdots i_{a_1+a_2}\, j_1\cdots j_{a_2}}\, u^1_{i_1}\cdots u^1_{i_{a_1+a_2}} u^2_{j_1}\cdots u^2_{j_{a_2}}\,, \tag{2.17}$$

where the indices of the coefficient tensor C form the (anti)symmetrized traceless combinations of the Young tableau $(a_1 + a_2, a_2)$. In general, $W^{[a_1,a_2]}$ is 1/4 BPS short unless $a_1 = 0$ when it becomes 1/2 BPS short. In the next subsection we show that (2.16) realizes the most general series of BPS short multiplets of OSp(8*/4). The case of 1/2 BPS operators is of particular importance for us, and we therefore introduce the following simplified notation for them:

$$\mathcal{W}^m \equiv W^{[0,m]} = \left[W^{12}\left(\theta^1,\theta^2\right)\right]^m \Longrightarrow\ \mathcal{D}(2m;0,0,0;0,m)\,. \tag{2.18}$$

Concluding this subsection we point out that there exists an alternative way of constructing a subclass of 1/4 BPS short multiplets that is relevant to our discussion of OPEs of 1/2 BPS operators in section 3. To obtain it we first reformulate the tensor multiplet in an equivalent form by projecting the defining constraint (2.3) with the harmonics $u^K_k u^1_i u^3_j$, $K = 1,3$:

$$D^1_\alpha W^{13} = D^3_\alpha W^{13} = 0 \Longrightarrow W^{13} = W^{13}(\theta^1,\theta^3)\,. \tag{2.19}$$

Thus we obtain a new 1/2 BPS superfield depending on a different half of the odd variables. However, this time it is not a HWS of a USp(4) irrep since the raising operator D^{22} does not annihilate it:

$$D^{22}W^{13} = W^{12}\,. \tag{2.20}$$

Further, the product of superfields $W^{12}W^{13}$ has exactly the same content as $[W^1]^2$ because they depend on the same θs and at $\theta = 0$ one finds $C^{ijkl}u^1_i u^1_{[j} u^2_k u^3_{l]} = C^{ij}u^1_i u^1_j$. In general,

$$W^{[2j,p]} = \left[W^{12}\left(\theta^1,\theta^2\right)\right]^{p+j}\left[W^{13}\left(\theta^1,\theta^3\right)\right]^j \Longrightarrow\ \mathcal{D}(2(2j+p);0,0,0;2j,p)\,. \tag{2.21}$$

Once again, this is a 1/4 BPS superfield unless $j = 0$ when it becomes 1/2 BPS. The only difference from the general BPS series (2.16) is that in (2.21) the first USp(4) Dynkin label is even.

2.3 Series of UIRs of OSp(8*/4) and semishort multiplets

A study of the most general UIRs of OSp(8*/4) (similar to the one of ref. [16] for the case of $SU(2,2/N)$) is presented in ref. [17]. We can construct these UIRs by multiplying the three types of supersingletons above:

$$w_{\alpha_1\cdots\alpha_{p_1}} w'_{\beta_1\cdots\beta_{p_2}} w''_{\gamma_1\cdots\gamma_{p_3}}\ w^k\, W^{[a_1,a_2]}\,. \tag{2.22}$$

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Here $p_1 \geq p_2 \geq p_3$ and we have used three different copies of the supersingleton with spin, so that the spinor indices can be arranged to form an $SU^*(4)$ UIR with Young tableau (p_1, p_2, p_3) or Dynkin labels $J_1 = p_1 - p_2, J_2 = p_2 - p_3, J_3 = p_3$. We thus obtain the four distinct series in (1.2).[5] The reason why the conformal dimension in series A is continuous, while it is "quantized" in the other three series, has to do with the unitarity of the corresponding irreps.[6]

Series D contains the 1/4 or 1/2 BPS short multiplets (obtained by dropping all the w factors in (2.22)). Series A generically contains "long" multiplets, unless the unitarity bound is saturated [17] (this corresponds to setting $k = 0$ in (2.22)). In series B and C some "semishortening" takes place. In the realization (2.22) this is easily seen by using the on-shell constraints on the elementary supersingletons. The full identification of such "semishort" multiplets is given in [12]. Here we restrict ourselves to the case of series B with $J_1 = 0$, the only one relevant to the OPE analysis in section 3. Let us first set $a_1 = a_2 = 0$, i.e. no BPS factor appears in (2.22). Then we have two possibilities, $J_2 \neq 0$ and $J_2 = 0$.

If $J_2 \neq 0$ we take (2.22) with only two supersingletons with equal spin ($p_1 = p_2 = J_2 \neq 0$),

$$\mathcal{O}_{\alpha_1 \cdots \alpha_{J_2}\, \beta_1 \cdots \beta_{J_2}} = w_{\alpha_1 \cdots \alpha_{J_2}}\, w'_{\beta_1 \cdots \beta_{J_2}} \quad \longrightarrow \quad \ell = 4 + J_2 \,. \tag{2.23}$$

With the help of the on-shell constraint (2.5) we obtain the following "conservation law"

$$\epsilon^{\delta\gamma\alpha_1\beta_1} D^i_\gamma\, \mathcal{O}_{\alpha_1 \cdots \alpha_{J_2}\, \beta_1 \cdots \beta_{J_2}} = 0\,. \tag{2.24}$$

If $J_2 = 0$ we keep only two scalar supersingletons in (2.22),

$$\mathcal{O} = w^2 \quad \longrightarrow \quad \ell = 4\,. \tag{2.25}$$

Using the on-shell constraint (2.4) we obtain the following "conservation law"

$$\epsilon^{\delta\gamma\beta\alpha} D^{(i}_\gamma D^j_\beta D^{k)}_\alpha\, \mathcal{O} = 0\,. \tag{2.26}$$

In both cases above, the semishort superfield is a bilinear in the supersingletons, just like the conserved current $J_\mu(x) = \bar\phi(x)\partial_\mu\phi(x) - \phi(x)\partial_\mu\bar\phi(x)$, $\partial^\mu J_\mu = 0$, constructed out of two massless scalar fields. Indeed, the conservation laws (2.24) and (2.26) imply that some of the components of the semishort superfield are conserved (spin-)tensors. However, we can relax these conservation conditions by assigning $USp(4)$ quantum numbers to the above "currents". This is done by multiplying

[5]Comparing the UIRs of $OSp(8^*/4)$ to those of the $N = 4$ superconformal algebra in four dimensions $PSU(2, 2/4)$ [16], we remark that both include one continuous series and that the number of discrete series corresponds to the rank of the Lorentz group.

[6]The author of [17] conjectures the existence of a "window" of continuous dimensions $2 + \frac{1}{2}J_1 + 2(a_1 + a_2) \leq \ell \leq 4 + \frac{1}{2}J_1 + 2(a_1 + a_2)$ if $J_2 = J_3 = 0$, but this has not been proved.

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them by a BPS short superfield (2.16) (or (2.21)):

$$\begin{aligned} \mathcal{O}^{[a_1,a_2]}_{\alpha_1\cdots\alpha_{J_2}\,\beta_1\cdots\beta_{J_2}} &= w_{\alpha_1\cdots\alpha_{J_2}}\, w'_{\beta_1\cdots\beta_{J_2}}\, W^{[a_1,a_2]} \;\longrightarrow\; \ell = 4 + J_2 + 2(a_1+a_2)\,, \\ \mathcal{O}^{[a_1,a_2]} &= w^2\, W^{[a_1,a_2]} \;\longrightarrow\; \ell = 4 + 2(a_1+a_2)\,. \end{aligned} \tag{2.27}$$

These new semishort superfields satisfy the corresponding USp(4) projections of the conservation laws (2.24) and (2.26), for example

$$\epsilon^{\delta\gamma\alpha_1\beta_1} D^1_\gamma\, \mathcal{O}^{[a_1,a_2]}_{\alpha_1\cdots\alpha_{J_2}\,\beta_1\cdots\beta_{J_2}} = 0\,, \tag{2.28}$$

$$\epsilon^{\delta\gamma\beta\alpha} D^1_\gamma D^1_\beta D^1_\alpha\, \mathcal{O}^{[a_1,a_2]} = 0 \tag{2.29}$$

in the case of a 1/4 BPS factor in (2.27).

The new weaker constraints do not imply the presence of conserved (spin-)tensors among the components of the semishort superfields, they just eliminate some of these components. In other words, some powers of θs are missing in the expansion, but not entire θ^αs as in the case of a G-analytic superfield, hence the distinction between BPS short and semishort multiplets.

A very important point is that the semishort superfields, like the BPS ones, have fixed quantized dimension. However, unlike the BPS superfields, the "conservation" conditions on the semishort ones may be broken by dynamical effects in an interacting field theory, so their quantized dimension can in principle be affected by renormalization.[7] At the same time we should stress that there is a "dimension gap" between the semishort multiplets from series B and the continuous series A. Therefore it seems impossible to change the status of these semishort operators from "protected" to "unprotected" by small radiative corrections. In this sense the six-dimensional case is quite different from the four dimensional.

3. Three-point functions involving two 1/2 BPS operators

In this section we investigate the OPE of two 1/2 BPS operators (2.18), $\mathcal{W}^m\, \mathcal{W}^n$. To this end we construct all possible superconformal three-point functions:

$$\left\langle \mathcal{W}^m(x,\theta,1)\, \mathcal{W}^n(y,\zeta,2)\, \mathcal{O}^{\mathcal{D}}(z,\lambda,3) \right\rangle\,, \tag{3.1}$$

where $\mathcal{O}^{\mathcal{D}}$ is an *a priori* arbitrary operator carrying an OSp($8^*/4$) UIR labeled by the quantum numbers $\mathcal{D}$ from eq. (1.1). The three sets of space-time, Grassmann and harmonic variables are denoted by x, y, z; θ, ζ, λ; $1^I_i, 2^I_i, 3^I_i$, respectively. It is understood that the appropriate G-analytic basis (cf. (2.9)) is used at points 1 and 2.

Conformal supersymmetry uniquely fixes such three-point functions. Indeed, the superfunction (3.1) depends on half of the Grassmann variables at points 1 and 2 and on a full set of such variables at point 3. Thus, the total number of odd variables

[7] An interesting discussion of this point is given in the recent paper [36] in the context of $N = 4$ SYM in four dimensions.

exactly matches the number of supersymmetries (Poincaré Q plus special conformal S). Therefore there exist no nilpotent superconformal invariants and the complete θ, ζ, λ expansion of (3.1) is determined from the leading ($\theta = \zeta = \lambda = 0$) component. The latter is the three-point function of two scalars and one tensor field, and is fixed by conformal invariance.

Apart from G-analyticity, the three-point function (3.1) should also satisfy the requirement of USp(4) irreducibility (H-analyticity) at points 1 and 2. This leads to selection rules for the third UIR for the following reason. The coefficients in, for instance, the θ expansion at point 1 depend on the harmonics in a way that matches the harmonic $\mathrm{U}(1) \times \mathrm{U}(1)$ charges of $\theta^{1,2}_\alpha$. The crucial point is that some of these coefficients may be harmonic singular, thus violating the requirement of H-analyticity. Demanding that such singularities be absent excludes some UIRs at point 3.

3.1 Two-point functions

Before discussing the three-point function (3.1) itself, consider first the two-point function of two tensor multiplets:

$$\left\langle W^{12}\left(x, \theta^{1,2}, 1\right) W^{12}\left(y, \zeta^{1,2}, 2\right)\right\rangle . \tag{3.2}$$

It has the leading component

$$\left\langle W^{12} W^{12}\right\rangle_{\theta=\zeta=0} = \frac{(1^{12}2^{12})}{(x-y)^4} , \tag{3.3}$$

where, for example

$$\left(1^{12}2^{12}\right) \equiv \epsilon^{ijkl} 1^1_i 1^2_j 2^1_k 2^2_l \tag{3.4}$$

is the only possible USp(4) invariant combination of the harmonics at points 1 and 2 carrying the required $\mathrm{U}(1) \times \mathrm{U}(1)$ charges.

The irreducibility (H-analyticity) conditions at point 1 are easily checked: ∂^{22}_1 trivially annihilates the numerator and ∂^{13}_1 does so because of the antisymmetrization; similarly at point 2. The space-time dependence in (3.3) follows from the fact that W has dimension 2 and spin 0.

The G-analytic superfunction (3.2) depends on $2+2$ spinor coordinates, as many as the Q supersymmetry parameters. So, it is sufficient to use only Q supersymmetry to restore the odd variable dependence starting from (3.3). Also, we will restrict our attention to harmonic singularities at point 1 (point 2 is similar), whence it is sufficient to restore the θ dependence only. Thus, we stay in a coordinate frame in which $\zeta = 0$. Since this condition does not involve the harmonics at point 1, it cannot introduce singularities with respect to them. In such a frame there is a residual Q supersymmetry given by the condition

$$\delta'_Q \zeta^{1,2} = 0 \Longrightarrow \epsilon'^i = \left(2^i_3 2^3_j + 2^i_4 2^4_j\right) \epsilon^j . \tag{3.5}$$

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In deriving (3.5) we used the definition (2.6) of the harmonics, from which it follows that $u_4 = -u^1$, $u_3 = -u^2$ (the raising and lowering of the I and i indices is independent).

Now, in the analytic basis, x transforms as follows:

$$\delta_Q x^{\alpha\beta} = -2i\left(\theta^{1[\alpha}\epsilon^{4\beta]} + \theta^{2[\alpha}\epsilon^{3\beta]}\right) \tag{3.6}$$

(compare to (2.9) and (2.12)). Replacing the parameters in (3.6) by the residual ones from (3.5) we can find $\delta'_Q x$. Then the combination

$$x^{\alpha\beta} + i\left(a_{11}\,\theta^{1[\alpha}\theta^{1\beta]} - 2\,a_{12}\,\theta^{1[\alpha}\theta^{2\beta]} + a_{22}\,\theta^{2[\alpha}\theta^{2\beta]}\right) \tag{3.7}$$

with

$$a_{11} = \frac{(1^{24}2^{12})}{(1^{12}2^{12})}\,, \qquad a_{12} = \frac{(1^{14}2^{12})}{(1^{12}2^{12})} = -\frac{(1^{23}2^{12})}{(1^{12}2^{12})}\,, \qquad a_{22} = -\frac{(1^{13}2^{12})}{(1^{12}2^{12})} \tag{3.8}$$

is invariant under the residual Q supersymmetry. Noting that $\delta'_Q y = 0$, we can write the two-point function (3.2) in the frame $\zeta = 0$ as a coordinate shift of its leading component (3.3):

$$\begin{aligned}\left\langle W^{12}(\theta) W^{12}(\zeta = 0)\right\rangle = \exp\Big\{-\frac{i}{4}\,\big(a_{11}\left(\theta^1\partial_x\theta^1\right) - 2a_{12}\left(\theta^1\partial_x\theta^2\right) + \\ + a_{22}\left(\theta^2\partial_x\theta^2\right)\big)\Big\}\,\frac{(1^{12}2^{12})}{(x-y)^4}\,.\end{aligned} \tag{3.9}$$

The coefficients a_{11}, a_{12}, a_{22} in (3.9) introduce the harmonic singularity

$$U = \frac{1}{(1^{12}2^{12})} \tag{3.10}$$

since $(1^{12}2^{12}) = 0$ when points 1 and 2 coincide. The identity

$$a_{11}\,a_{22} - a_{12}^2 = \frac{(1^{34}2^{12})}{(1^{12}2^{12})} \tag{3.11}$$

may be used to simplify the expansion of the exponential. While the product of, say, a_{11} and a_{22} contains two such denominators, the r.h.s. of (3.11) has only one power of the singularity. We find in this way that no higher singularity than U^2 occurs in the exponential, and that all terms with U^2 contain at least one operator $\Box_x$. Since $\Box(x^2)^{-3} \sim \delta^6(x)$, the harmonic-singular terms in the expansion of (3.9) are space-time contact terms. We conclude that the two-point function is regular as long as $x \neq y$, owing to its harmonic numerator. This will not automatically be so for the three-point functions.

In the following we will also need the two-point function

$$\left\langle W^{12}(1) W^{13}(2)\right\rangle \tag{3.12}$$

for the two alternative realizations (2.15) and (2.19) of the tensor multiplet. In the frame $\zeta = 0$ this is obtained from (3.9) by replacing the harmonic 2^2 by 2^3 everywhere.

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3.2 Three-point functions $\langle \mathcal{W}^m(1)\,\mathcal{W}^n(2)\,\mathcal{O}^{\mathcal{D}}(3)\rangle$

In close analogy with the two-point functions above, here we investigate the three-point functions (3.1) starting with their leading component, then restoring the dependence on θ and finally imposing H-analyticity at point 1.

The USp(4) irrep carried by $\mathcal{O}^{\mathcal{D}}(3)$ should be in the decomposition of the tensor product of the two irreps at points 1 and 2:

$$[0,m]\otimes[0,n] = \bigoplus_{k=0}^{n}\bigoplus_{j=0}^{n-k}[2j, m+n-2k-2j]\,, \tag{3.13}$$

where we have assumed that $m \geq n$. The first Dynkin label being even, the irrep $[2j,p]$ can be realized as a product of W^{12}s and W^{13}s, recall (2.21). This suggests to construct the USp(4) structure of the function (3.1) as a product of two-point functions of Ws.

Apart from the USp(4) quantum numbers the operator $\mathcal{O}(3)$ also carries spin and dimension. Since the leading components at points 1 and 2 are scalars, the Lorentz irrep at point 3 must be a symmetric traceless tensor of rank s or, equivalently, an SU*(4) irrep $[0,s,0]$. The corresponding conformal tensor structure is built out of the vector

$$Y^\mu = \frac{(x-z)^\mu}{(x-z)^2} - \frac{(y-z)^\mu}{(y-z)^2}\,. \tag{3.14}$$

All in all, the leading term is:

$$\begin{aligned}
&\big\langle \mathcal{W}^m(1)\,\mathcal{W}^n(2)\,\mathcal{O}^{(\ell;\,0,s,0;\,2j,m+n-2k-2j)}(3)\big\rangle_0 = \\
&= \left[\frac{(1^{12}2^{12})}{(x-y)^4}\right]^k \left[\frac{(1^{12}3^{12})}{(x-z)^4}\right]^{m-j-k} \left[\frac{(2^{12}3^{12})}{(y-z)^4}\right]^{n-j-k} \times \\
&\quad\times \left\{\left[\frac{(1^{12}3^{12})}{(x-z)^4}\right]\left[\frac{(2^{12}3^{13})}{(y-z)^4}\right] - \left[\frac{(1^{12}3^{13})}{(x-z)^4}\right]\left[\frac{(2^{12}3^{12})}{(y-z)^4}\right]\right\}^j \times \\
&\quad\times\ (Y^2)^{\frac{\ell-s}{2}-m-n+2k}\ Y^{\{\mu_1}\cdots Y^{\mu_s\}}\,,
\end{aligned} \tag{3.15}$$

where $\{\mu_1\cdots\mu_s\}$ denotes traceless symmetrization. The $3^2, 3^3$ antisymmetrization in the factor $\{\cdots\}^j$ reflects the properties of the USp(4) Young tableau $(m+n-2k, 2j)$ or, equivalently, the harmonic irreducibility constraints at point 3.

To study the harmonic singularities at point 1 we restore the dependence on θ, keeping $\zeta=\lambda=0$. We need both Q and S supersymmetry to reach this new frame. The harmonics 1_i^I do not participate in the frame fixing, so that there is no danger of creating harmonic singularities at point 1. Next, under conformal boosts the vector Y^μ (3.14) transforms homogeneously with parameters involving only z. We need to find a superextension of Y^μ with the same properties: It should be invariant under $Q+S$ supersymmetry at points 1 and 2 (and covariant at point 3, but we do not see this in the present frame). Remarkably, the combination (3.7) that was Q invariant in the two-point case turns out to be $Q+S$ invariant in the new frame. Thus, performing the shift (3.7) of the variable x in the vectors Y in (3.15) we obtain the desired superextension. In addition, the two-point factor $(1^{12}2^{12})/(x-y)^4$ in (3.15) undergoes the same shift. The factors involving $(x-z)^2$ are supersymmetrized by a similar shift, which, as explained in the preceding subsection, does not induce a harmonic singularity when $1\to 3$, at least up to space-time contact terms.

The harmonic factor $\{\cdots\}^j$ in (3.15) vanishes for $1\to 2$, but cannot compensate the singularities of the type U (3.10) coming from the exponential shift (3.9). To show this, we must identify the four complex coordinates on the harmonic coset $\mathrm{USp}(4)/\mathrm{U}(1)\times\mathrm{U}(1)$ in terms of USp(4) invariant combinations of the harmonics. Then it becomes clear that the singularity in U and the "zero" in the factor $\{\cdots\}^j$ correspond to different directions on the coset. We do not present the details here.

So, we can concentrate on the exponential shift (3.9). This involves the terms

$$\exp\left\{-\frac{i}{4}\left(a_{11}(\theta^1\partial_x\theta^1)-2a_{12}(\theta^1\partial_x\theta^2)+a_{22}(\theta^2\partial_x\theta^2)\right)\right\}\times$$
$$\times\left[\frac{(1^{12}2^{12})}{(x-y)^4}\right]^k\left(Y^2\right)^{\frac{\ell-s}{2}-m-n+2k}\ Y^{\{\mu_1}\cdots Y^{\mu_s\}}\,. \tag{3.16}$$

The factor $(1^{12}2^{12})^k$ can suppress singularities. But here, as opposed to the two-point case, the presence of the Y terms will not always allow this. We distinguish three cases:

(i) If $k=0$ a singularity occurs already in the $\theta\theta$ term. In order to remove it we require:

$$\partial^\nu_x\left\{\left(Y^2\right)^{\frac{\ell-s}{2}-m-n}\ Y^{\{\mu_1}\cdots Y^{\mu_s\}}\right\}=0\,, \tag{3.17}$$

which implies

$$s=0\,,\qquad \ell=2(m+n)=2(a_1+a_2)\,, \tag{3.18}$$

where $[a_1,a_2]$ is the USp(4) irrep at point 3. This constraint sends the expression (3.16) to unity, reducing the three-point function to a product of two-point functions:

$$\begin{aligned}&\left\langle\mathcal{W}^m(1)\,\mathcal{W}^n(2)\,\mathcal{O}^{(2(m+n);\,0,0,0;\,2j,m+n-2j)}(3)\right\rangle=\\&=\left\langle W^{12}(1)W^{12}(3)\right\rangle^{m-j}\ \left\langle W^{12}(2)W^{12}(3)\right\rangle^{n-j}\times\\&\times\left\{\left\langle W^{12}(1)W^{12}(3)\right\rangle\ \left\langle W^{12}(2)W^{13}(3)\right\rangle-\right.\\&\left.-\ \left\langle W^{12}(1)W^{13}(3)\right\rangle\ \left\langle W^{12}(2)W^{12}(3)\right\rangle\right\}^j\,.\end{aligned} \tag{3.19}$$

The operator at the third point is seen to belong to series D from (1.2), i.e. it is 1/4 BPS short if $j\neq 0$ or 1/2 BPS short if $j=0$.

(ii) If $k=1$ the singularity is in the U^2 terms, all of which involve at least one operator $\Box_x$. In order to remove it we demand:

$$\Box_x\left\{(x-y)^{-4}\ \left(Y^2\right)^{\frac{\ell-s}{2}-m-n+2}\ Y^{\{\mu_1}\cdots Y^{\mu_s\}}\right\}=0\,. \tag{3.20}$$

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This equation is identically satisfied in two cases:

(ii.a) We can have $\ell = -s + 2(m+n-2) = -J_2 + 2(a_1+a_2)$. Looking at (1.2) we see that this is only compatible with series D, whence $s = J_2 = 0$. So, the first solution is

$$s = 0\,, \qquad \ell = 2(a_1+a_2)\,, \tag{3.21}$$

which again corresponds to a BPS short operator at point 3.

(ii.b) We may put $\ell = s + 2(m+n-2) + 4$, i.e.

$$\ell = 4 + J_2 + 2(a_1+a_2)\,, \tag{3.22}$$

and we recognize the B series from (1.2) with $J_1 = 0$. It is therefore expected that the operator at point 3 is "semishort", i.e. that it satisfies the constraints (2.28) when $s \neq 0$ or (2.29) when $s = 0$. Indeed, in the next subsection we shall prove this.

(iii) If $k \geq 2$, the expression (3.16) is completely regular, so we obtain no selection rules from harmonic analyticity.

3.3 Semishortening at the third point

In case *(ii.b)*, the operators $\mathcal{O}(3)$ belong to series B and should thus obey the semishortness constraints (2.28) or (2.29). If we use the analytic basis (2.9) at point 3, the spinor derivative D^1_α becomes a partial derivative, see (2.10). Then the semishortness conditions on the three-point function constructed above take the following form:

$$\begin{aligned} s \neq 0 :\ & \epsilon^{\delta\gamma\alpha_1\beta_1}\frac{\partial}{\partial\lambda^{4\gamma}}\langle \mathcal{W}^m(1)\mathcal{W}^n(2)\mathcal{O}_{\alpha_1\cdots\alpha_s\ \beta_1\cdots\beta_s}(z_A,\lambda,3)\rangle = 0\,,\\ s = 0 :\ & \epsilon^{\delta\gamma\alpha\beta}\frac{\partial}{\partial\lambda^{4\gamma}}\frac{\partial}{\partial\lambda^{4\alpha}}\frac{\partial}{\partial\lambda^{4\beta}}\ \langle \mathcal{W}^m(1)\mathcal{W}^n(2)\mathcal{O}(z_A,\lambda,3)\rangle = 0\,. \end{aligned} \tag{3.23}$$

We will now verify that these conditions are indeed satisfied.

As we explained earlier, the complete θ,ζ,λ dependence of the three-point function can be restored starting from the leading component (3.15). The factors $[\cdots]^{\cdots}$ can easily be upgraded to two-point functions of the type (3.2) and (3.12). From (2.21) we know that any product of $W^{12}(3)$ and $W^{13}(3)$ is annihilated by $\partial_{4\alpha}$, hence it trivially satisfies (3.23). Thus, we only need to impose these conditions on the supersymmetrization of the factor $(Y^2)^{\frac{\ell-s}{2}-m-n+2}\ Y^{\{\mu_1}\ldots Y^{\mu_s\}}$ (recall that $k=1$ in case *(ii.b)*). There exists a standard method [22, 38] for constructing the supercovariant version of Y, but the resulting expressions are rather complicated. Fortunately, we are only interested in the dependence of the Y factor on $\lambda^{4\alpha}$ at point 3 which is very easy to reconstruct.

Using Q and S supersymmetry, translations and conformal boosts we can choose a frame where only the coordinates $x^{\alpha\beta}$, $\lambda^{4\alpha}$ and the three sets of harmonics remain:

$$y \longrightarrow \infty\,, \qquad z = 0\,, \qquad \theta^{1,2} = 0\,, \qquad \zeta^{1,2} = 0\,, \qquad \lambda^{1,2,3} = 0\,. \tag{3.24}$$

This frame is harmonic singular, but now we are not interested in harmonic analyticity at point 3. The residual transformation preserving the frame is

$$\delta'_{Q+S} x^{\alpha\beta} = 0\,, \qquad \delta'_{Q+S} \lambda^{4\alpha} = \left(x^{\alpha\beta} + 2i A_{44} \lambda^{4\alpha}\lambda^{4\beta}\right) \xi^4_\beta\,, \\ \delta'_{Q+S}\left(1^I_i, 2^I_i, 3^I_i\right) = 0\,. \tag{3.25}$$

Here ξ^4_β is the transformation parameter and

$$A_{44} = \frac{(3^1 2_{[3]})(1^{[1} 2_{4]})(1^{2]} 3_4)}{(1^{12} 2^{12})}\,, \tag{3.26}$$

where, for instance, $(3^1 2_3) \equiv 3^1_i 2^i_3$, etc.

It is then clear that the combination

$$\hat{x}^{\alpha\beta} = x^{\alpha\beta} + 2i A_{44} \lambda^{4\alpha}\lambda^{4\beta} \tag{3.27}$$

has a homogeneous transformation law,

$$\delta'_{Q+S}\, \hat{x}^{\alpha\beta} = \Lambda\, \hat{x}^{\alpha\beta} + \Sigma^{[\alpha}{}_\gamma\, \hat{x}^{\beta]\gamma}\,, \qquad \Lambda = -i A_{44} \lambda^{4\alpha} \xi^4_\alpha\,, \\ \Sigma^\alpha{}_\gamma = 4i A_{44} \lambda^{4\alpha} \xi^4_\gamma - \text{trace}\,. \tag{3.28}$$

Here Λ and Σ are the coordinate-dependent parameters of a dilatation and of a Lorentz transformation, correspondingly.

Next, in the frame (3.24) the vector Y (3.14) becomes $Y^\mu = x^\mu / x^2$. Replacing x by $\hat{x}$ from (3.27) in Y, we obtain a vector with a covariant transformation law. Thus, the $\lambda^{4\alpha}$ dependence of the Y factor in (3.15) is restored by a simple coordinate shift, the result of which is

$$\left(Y^2\right)^{\frac{\ell-s}{2} - m - n + 2}\; Y^{\{\mu_1} \cdots Y^{\mu_s\}} \;\longrightarrow \\ \longrightarrow \left\{1 - \frac{i}{2} A_{44} \lambda^4 \partial_x \lambda^4 + A^2_{44} \left(\lambda^4\right)^4 \Box_x\right\}\; \left(x^2\right)^{-\frac{\ell+s}{2} + m + n - 2}\; x^{\{\mu_1} \cdots x^{\mu_s\}}\,. \tag{3.29}$$

Now we can easily impose conditions (3.23). The first one amounts to the requirement that the rank $s+1$ tensor $(s \neq 0)$

$$\partial^\nu_x \left[\left(x^2\right)^{-\frac{\ell+s}{2} + m + n - 2}\; x^{\{\mu_1} \cdots x^{\mu_s\}}\right] \tag{3.30}$$

is symmetric and traceless. It is verified when $\ell = s + 2(m+n)$, just as expected, recall (3.22) (note that the $(\lambda^4)^4$ term in (3.29) automatically vanishes in this case). The second condition only concerns the $(\lambda^4)^4$ term in (3.29):

$$\Box_x \left(x^2\right)^{-\frac{\ell}{2} + m + n - 2} = 0 \tag{3.31}$$

which holds when $\ell = 2(m+n) - 2$ (case $(ii.a)$) or when $\ell = 2(m+n)$ (case $(ii.b)$).

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We conclude that when the operator at point 3 has the right quantum numbers to belong to series B, the three-point function automatically satisfies the corresponding semishortening condition. This is reminiscent of the situation in bosonic D-dimensional CFT. There the three-point function of two scalars of equal dimension and a vector of dimension ℓ, $\langle\phi(1)\phi(2)j^\mu(3)\rangle$ is automatically conserved, $\langle\phi(1)\phi(2)j^\mu(3)\rangle\overleftarrow{\partial}_{3\mu} = 0$, when ℓ takes the appropriate value $\ell = D-1$.

4. Extremal and next-to-extremal correlators

In this section we discuss certain classes of n-point correlation functions of 1/2 BPS operators $\mathcal{W}^m \equiv [W^{12}]^m$:

$$\langle \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2)\cdots\mathcal{W}^{m_n}(n)\rangle\,. \tag{4.1}$$

According to the terminology introduced in [24] they are called

$$\begin{aligned}
\text{"extremal" if} \qquad & m_1 = \sum_{i=2}^n m_i\,, \\
\text{"next-to-extremal" if} \qquad & m_1 = \sum_{i=2}^n m_i - 2\,, \\
\text{"near-extremal" if} \qquad & m_1 = \sum_{i=2}^n m_i - 2k\,, \qquad k\geq 2\,.
\end{aligned} \tag{4.2}$$

Using AdS supergravity arguments, it was conjectured in [27] that the extremal and next-to-extremal correlators are not renormalized and factorize into products of two-point functions, whereas the near-extremal ones are renormalized but still factorize into correlators with fewer numbers of points. With the help of the OPE results from section 3, we prove here the non renormalization and factorization of the n-point extremal correlators as well as the non renormalization of the next-to-extremal four-point correlator. We also speculate about a possible way to understand the factorization of near-extremal correlators.

4.1 The extremal case

We begin by the simplest case, which is a four-point extremal correlator. It can be represented as the convolution of two OPEs:

$$\begin{aligned}
&\langle \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2)\mathcal{W}^{m_3}(3)\mathcal{W}^{m_4}(4)\rangle = \\
&\quad = \sum\!\!\!\!\!\!\int_{5,5'} \langle \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2)\mathcal{O}(5)\rangle\ \langle\mathcal{O}(5)\mathcal{O}(5')\rangle^{-1}\ \langle\mathcal{O}(5')\mathcal{W}^{m_3}(3)\mathcal{W}^{m_4}(4)\rangle\,,
\end{aligned} \tag{4.3}$$

where the sum goes over all possible operators that appear in the intersection of the two OPEs. Owing to the orthogonality of different operators the inverse two-point

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function $\langle \mathcal{O}(5)\mathcal{O}(5')\rangle^{-1}$ only exists if $\mathcal{O}(5)$ and $\mathcal{O}(5')$ are identical.[8] To find out their spectrum, we first examine the USp(4) quantum numbers. From (3.13) we see that

$$\begin{aligned} \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2) \ &\longrightarrow \ \mathcal{O}(5) \ : \ \bigoplus_{k=0}^{m_2}\bigoplus_{j=0}^{m_2-k}[2j, m_1+m_2-2j-2k]\,, \\ \mathcal{W}^{m_3}(3)\mathcal{W}^{m_4}(4) \ &\longrightarrow \ \mathcal{O}(5') \ : \ \bigoplus_{k'=0}^{m_4}\bigoplus_{j'=0}^{m_4-k'}[2j', m_3+m_4-2j'-2k']\,, \end{aligned} \tag{4.4}$$

where we have assumed $m_3 \geq m_4$. Since in the extremal case $m_1 = m_2 + m_3 + m_4$ (recall (4.2)), the intersection is given by the following conditions:

$$j = j'\,, \qquad 0 \leq k' = k - m_2 \leq 0\,, \tag{4.5}$$

whose only solution is

$$k = m_2 \implies j = j' = 0\,, \qquad k' = 0\,. \tag{4.6}$$

Further, we deduce from (3.19) that $k' = 0$ and $j' = 0$ imply that $\mathcal{O}(5')$, and by orthogonality, $\mathcal{O}(5)$ must be identical 1/2 BPS operators,

$$\mathcal{O} = \mathcal{W}^{m_3+m_4}\,. \tag{4.7}$$

Finally, in this particular case the three-point functions in (4.3) degenerate into products of two two-point functions (recall (3.19)), so (4.3) becomes

$$\begin{aligned} &\langle \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2)\mathcal{W}^{m_3}(3)\mathcal{W}^{m_4}(4)\rangle = \\ &\quad = \int_{5'}\langle W(1)W(2)\rangle^{m_2} \int_5 \langle W(1)W(5)\rangle^{m_3+m_4} \, \langle W(5)W(5')\rangle^{-(m_3+m_4)} \times \\ &\qquad\qquad\qquad \times \langle W(5')W(3)\rangle^{m_3} \, \langle W(5')W(4)\rangle^{m_4} \\ &\quad = \langle W(1)W(2)\rangle^{m_2} \, \langle W(1)W(3)\rangle^{m_3} \, \langle W(1)W(4)\rangle^{m_4}\,. \end{aligned} \tag{4.8}$$

This clearly shows that the extremal four-point correlator factorizes into a product of two-point functions. In other words, it always takes its free (Born approximation) form, so it stays *non renormalized.*

The generalization of the above result to an arbitrary number of points is straightforward. We explain it on the example of a five-point extremal correlator. This time we have to perform three consecutive OPEs, so the analog of (4.3) is

$$\begin{aligned} \langle \mathcal{W}^{m_1}(1)\cdots\mathcal{W}^{m_5}(5)\rangle &= \langle \mathcal{W}^{m_1}(1)\mathcal{W}^{m_2}(2)\mathcal{O}(6)\rangle \bullet \langle \mathcal{O}(6)\mathcal{W}^{m_3}(3)\mathcal{O}(7)\rangle \bullet \\ &\quad \bullet \langle \mathcal{O}(7)\mathcal{W}^{m_4}(4)\mathcal{W}^{m_5}(5)\rangle\,, \end{aligned} \tag{4.9}$$

[8]In CFT every operator $\mathcal{O}$ has the so-called "shadow" operator $\tilde{\mathcal{O}}$ such that the two can form a non-diagonal two-point function of the type $\langle \mathcal{O}(1)\tilde{\mathcal{O}}(2)\rangle = \delta(1-2)$. However, these "shadows" only exceptionally have physical dimension (i.e. do not violate the unitarity bound), so they usually need not be considered in an OPE. It is easy to show that this is the case here.

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where $\bullet$ denotes the convolution with the inverse two-point functions at the internal points 6 and 7. As before, we start by examining the USp(4) quantum numbers. The sum of the Dynkin labels of an irrep is a U(1) charge. In the tensor product of two irreps the charge of the product ranges from the sum to the difference of the two charges, e.g. in $[0,m_1]\otimes[0,m_2]$ we obtain values between m_1+m_2 and $m_1-m_2=m_3+m_4+m_5$. Moving along the chain (4.9) from left to right, and each time choosing the minimal value, when we arrive at the last pair of points, we are just able to match the maximal value m_4+m_5 coming from the tensor product of the last two irreps. Thus the only possible chain of irreps is as follows:

$$\begin{aligned}[0,m_1]\otimes[0,m_2] \;\longrightarrow\; &[0,m_3+m_4+m_5]\otimes[0,m_3] \;\longrightarrow \\ \longrightarrow [0,m_4+m_5] \;\longleftarrow\; &[0,m_4]\otimes[0,m_5]\,. \end{aligned} \tag{4.10}$$

Note that at each step the first Dynkin label is 0.

Now, let us start moving from right to left. Using (3.13) we see that the first step corresponds to $k=j=0$, so it produces a single 1/2 BPS operator in the OPE, $\mathcal{O}(7)=\mathcal{W}^{m_4+m_5}(7)$. Consequently, at the second step we again have the OPE of two 1/2 BPS operators producing yet another 1/2 BPS operator $\mathcal{O}(6)=\mathcal{W}^{m_3+m_4+m_5}(6)$. If $n>5$ this process goes on until we reach the first pair of points. We conclude that the n-point extremal correlators of 1/2 BPS operators are based on exchanges of $(n-2)$ 1/2 BPS operators only. We may call this a "field theory of 1/2 BPS operators".

Finally, just as in the four-point case (4.8) above, the three-point functions in (4.9) become degenerate (products of two two-point functions) and we achieve the expected factorization of the extremal correlator:

$$\begin{aligned}&\langle\mathcal{W}^{m_1}(1)\cdots\mathcal{W}^{m_5}(5)\rangle = \\ &\quad=\int_{6',7'}\langle W(1)W(2)\rangle^{m_2}\times \\ &\quad\times\int_6\langle W(1)W(6)\rangle^{m_3+m_4+m_5}\,\langle W(6)W(6')\rangle^{-(m_3+m_4+m_5)}\langle W(6')W(3)\rangle^{m_3}\times \\ &\quad\times\int_7\langle W(6')W(7)\rangle^{m_4+m_5}\,\langle W(7)W(7')\rangle^{-(m_4+m_5)}\times \\ &\qquad\times\,\langle W(7')W(4)\rangle^{m_4}\,\langle W(7')W(5)\rangle^{m_5} \\ &\quad=\langle W(1)W(2)\rangle^{m_2}\,\langle W(1)W(3)\rangle^{m_3}\,\langle W(1)W(4)\rangle^{m_4}\,\langle W(1)W(5)\rangle^{m_5}\,.\end{aligned} \tag{4.11}$$

4.2 The next-to-extremal and near-extremal cases

The situation is considerably more complicated in the next-to-extremal case, even with just four points, $m_1=m_2+m_3+m_4-2$. Repeating the steps leading to (4.5), this time we find the conditions

$$j=j'\,,\qquad 0\le k'-1=k-m_2\le 0\,, \tag{4.12}$$

which admit two solutions:

$$k = m_2 - 1\,, \qquad k' = 0\,, \qquad \begin{cases} j = 0 \longrightarrow \mathcal{O}^{[0,m_3+m_4]}(5') & \text{is } \frac{1}{2} \text{ BPS} \\ j = 1 \longrightarrow \mathcal{O}^{[2,m_3+m_4-2]}(5') & \text{is } \frac{1}{4} \text{ BPS} \end{cases}, \qquad (4.13)$$

or

$$\begin{aligned} & k = m_2\,, \qquad k' = 1\,, \\ & j = 0 \;\longrightarrow\; \mathcal{O}^{[0,m_3+m_4-2]}(5') \qquad \text{is } \frac{1}{2} \text{ BPS or semishort}\,. \end{aligned} \qquad (4.14)$$

We see that unlike the extremal case, where only a finite number of 1/2 BPS operators are exchanged, here one encounters 1/4 BPS and semishort ones. The latter form an infinite series, so the OPE content is much richer. Still, there is an important restriction: all the operators in the OPE have protected integer dimension.[9] To put it differently, we have shown that no operators of anomalous dimension can occur in the expansion of the next-to-extremal four-point correlator. Since renormalization in ultraviolet-finite CFT is associated with the appearance of anomalous dimensions, we can conclude that the correlator is non renormalized.

However, showing that the amplitude factorizes into a product of two-point functions is not so easy now. The reason is that the three-point functions $\langle \mathcal{W}\mathcal{W}\mathcal{O} \rangle$ themselves no longer factorize, so evaluating expression (4.3) implies doing conformal four-star integrals [37]. Yet, the calculation may turn out to be rather trivial, once again because we are only dealing with integer dimensions. Indeed, in the case at hand the three-point functions $\langle \mathcal{W}\mathcal{W}\mathcal{O} \rangle$ involve singular distributions of the type $1/(x^2)^k$, $k \geq 3$ with delta-function type singularities. After properly regularizing the integrals, this is expected to result in the factorization of the amplitude.

Finally, we could try to apply our arguments to the near-extremal correlators. In this case tensoring the USp(4) irreps at, for example, points 1 and 2, and then going along the chain leaves room for irreps $[2j, m_1 + m_2 - 2k - 2j]$ with $k \geq 2$. In other words, operators with unprotected dimension are allowed to appear, so the correlator can be renormalized. One might speculate that, for instance, the near-extremal six-point condition $m_1 = \sum_{i=2}^{6} m_i - 4$ will restrict the occurrence of a $k = 2$ exchange (and hence of anomalous dimensions) to only one of the OPEs, the rest still involving operators of protected dimension. As we just explained, the latter are associated with singular distributions and thus with trivial integrations, whereas the former will give rise to a non-trivial four-point function. This is a possible scenario of the factorization conjectured in [26, 27], and it certainly deserves a careful investigation.

We remark that in [25] a different approach was used to prove the non renormalization of extremal and next-to-extremal correlators in four-dimensional SCFT. It consists in constructing directly the n-point superconformal invariant in harmonic

[9] In fact, this case resembles the conformal partial wave expansion (or double OPE) of the *free* four-point function of physical scalars of canonical dimension [14]. There one finds an infinite spectrum of operators of integer dimension (conserved tensors).

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superspace and then imposing the harmonic analyticity conditions. In the extremal case this method leads to the conclusion that the corresponding invariant is unique and coincides with its free value.[10] However, the constraints obtained in this way for next-to-extremal correlators are weaker and do not allow us to decide whether they are renormalized or not. In [25] additional dynamical information was used coming from the insertion of the SYM action into the correlator. In six dimensions there is no known dynamical principle, therefore this procedure cannot be applied. Consequently, we can say that the method based on OPE described in this paper is more powerful, at least where next-to-extremal correlators are concerned.

5. Concluding remarks

The analysis carried out in this paper actually applies to any $D = 6$ $(N, 0)$ superconformal algebra $\mathrm{OSp}(8^*/2N)$. We note that, unlike $D = 4$, these algebras have only one kind of 1/2 BPS states in the $[0, \ldots, 0, N]$ of the R symmetry group $\mathrm{USp}(2N)$. We expect to find similar selection rules in all of these cases.

The same method can also be applied to the $D = 3$ $N = 8$ superconformal field theories based on the superalgebra $\mathrm{OSp}(8/4, \mathbb{R})$.

An extension of our result, which could be relevant to the more detailed examination of next-to-extremal and near-extremal correlators, is to construct three-point functions where only at one point there is a 1/2 BPS operator. In this case superconformal invariance does not uniquely fix the three-point functions, but one might still hope to find some selection rules.

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