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Predictions for Higgs and SUSY spectra from SO(10) Yukawa Unification with $\mu > 0$

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Abstract

We use t, b, τ Yukawa unification to constrain SUSY parameter space. We find a narrow region survives for $\mu > 0$ (suggested by $b \to s\gamma$ and the anomalous magnetic moment of the muon) with $A_0 \sim -1.9\, m_{16}$, $m_{10} \sim 1.4\, m_{16}$, $m_{16} \sim 1200-3000$ GeV and $\mu, M_{1/2} \sim 100-500$ GeV . Demanding Yukawa unification thus makes definite predictions for Higgs and sparticle masses.

Minimal supersymmetric [SUSY] SO(10) grand unified theories [GUTs] have many profound features [1]: all fermions in one family sit in one **16** dimensional spinor representation; the two Higgs doublets of the minimal supersymmetric standard model sit in one **10** dimensional fundamental representation, and gauge coupling unification at a GUT scale $M_G \sim 3 \times 10^{16}$ GeV fits well with the low energy data [2, 3]. In addition in the simplest version of SO(10) the third generation Yukawa couplings are given by a single term in the superpotential $W = \lambda$ **16 10 16** resulting in Yukawa unification $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$ and a prediction for M_t with large $\tan\beta \sim 50$ [4]. [1] This beautiful result is however marred by potentially large weak scale threshold corrections [7, 8]

$$m_b(M_Z) = \lambda_b(M_Z) \frac{v}{\sqrt{2}} \, cos\beta \, (1 + \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}^+} + \Delta m_b^{\tilde{\chi}^0} + \Delta m_b^{\log}).$$

For $\mu > 0$ the gluino term is positive and in most regions of SUSY parameter space it is the dominant contribution to Δm_b . Reasonable fits prefer $\Delta m_b < 0$; hence Yukawa unification is easy to satisfy with $\mu < 0$.

The decay $b \to s\gamma$ and the muon anomalous magnetic moment also get significant corrections proportional to $\tan\beta$[7]. These corrections come from one loop diagrams similar to those contributing to the bottom mass. The chargino term typically dominates and has opposite sign to the SM and charged Higgs contributions, thus reducing the branching ratio for $\mu > 0$. This is necessary to fit the data since the SM contribution is somewhat too big. $\mu < 0$ would on the other hand constructively add to the branching ratio and is problematic. In addition, the recent measurement of the anomalous magnetic moment of the muon $a_\mu^{NEW} = (g-2)/2 = 43\ (16) \times 10^{-10}$ also favors $\mu > 0$ [9]. Thus it is important to confirm that Yukawa unification can work consistently with $\mu > 0$.

In this paper we assume exact Yukawa unification and search, using a χ^2 analysis, for regions of SUSY parameter space with $\mu > 0$ providing good fits to the low energy data. We show that Yukawa unification dramatically constrains the Higgs and SUSY spectra. These results are sensitive to the SUSY breaking mechanism.

It is much easier to obtain EWSB with large $\tan\beta$ when the Higgs up/down masses are split ($m_{H_u}^2 < m_{H_d}^2$) [10]. In our analysis we consider two particular schemes we refer to as universal and D term splitting. In the first case the third generation squark and slepton soft masses are given by the universal mass parameter m_{16} , and only Higgs masses are split: $m_{(H_u,\,H_d)}^2 = m_{10}^2\ (1 \mp \Delta m_H^2)$. In the second case we assume D term splitting, i.e. that the D term for $U(1)_X$ is non-zero, where $U(1)_X$ is obtained in the decomposition of $SO(10) \to SU(5) \times U(1)_X$. In this second case, we have $m_{(H_u,\,H_d)}^2 = m_{10}^2 \mp 2D_X$, $m_{(Q,\,\bar{u},\,\bar{e})}^2 = m_{16}^2 + D_X$, $m_{(\bar{d},\,L)}^2 = m_{16}^2 - 3D_X$. The universal case does not at first sight appear to be similarly well motivated. It is quite clear however that in any SUSY model

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[1] Note, GUT scale threshold corrections to this Yukawa unification boundary condition are naturally small (< 1%), since they only come at one loop from the SO(10) gauge sector and the third generation - Higgs Yukawa coupling [5]. This is in contrast to GUT scale threshold corrections to gauge coupling unification which may be significant, coming from doublet/triplet splitting in the Higgs sector and, even more importantly, the SO(10) breaking sector which typically has many degrees of freedom. The data requires $\epsilon_3 = \frac{\alpha_3(M_G) - \alpha_G(M_G)}{\alpha_G(M_G)} \sim$ -4% [6].

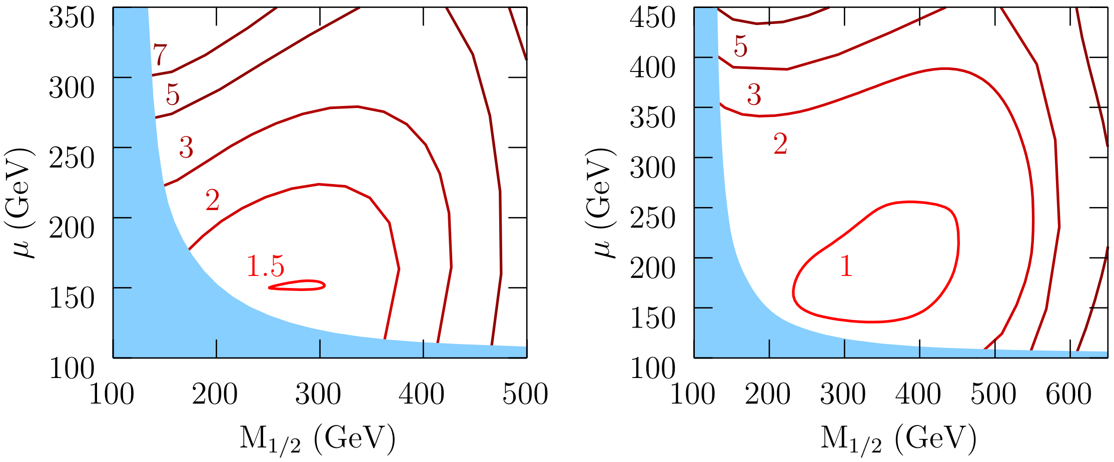

Figure 1: χ^2 contours for $m_{16} = 1500$ GeV (Left) and $m_{16} = 2000$ GeV (Right). The shaded region is excluded by the chargino mass limit $m_{\chi^+} > 103$ GeV .

the Higgs bosons are very special. R parity is used to distinguish Higgs superfields from quarks and leptons. In addition, a supersymmetric mass term μ with value of order the weak scale is needed for an acceptable low energy phenomenology. Since μ is naturally of order M_G, one needs some symmetry argument why it is suppressed. Of course, if the Higgs are special, then it is reasonable to assume splitting of Higgs, while maintaining universal squark and slepton masses. This can be achieved by GUT threshold corrections to the soft SUSY breaking scalar masses as will be discussed in [11] (see also [10]).

Our analysis is a top-down approach with 11 input parameters, defined at M_G, varied to minimize a χ^2 function composed of 9 low energy observables. The 11 input parameters are: M_G, $\alpha_G(M_G)$, ϵ_3; the Yukawa coupling λ, and the 7 soft SUSY breaking parameters μ, $M_{1/2}$, A_0, $\tan\beta$, m_{16}^2, m_{10}^2, Δm_H^2 (D_X) for universal (D term) case. We use two (one)loop renormalization group [RG] running for dimensionless (dimensionful) parameters from M_G to M_Z and complete one loop threshold corrections at M_Z [8]. We require electroweak symmetry breaking using an improved Higgs potential, including m_t^4 and m_b^4 corrections in an effective 2 Higgs doublet model below M_{stop} [12]. The χ^2 function includes the 9 observables; 6 precision electroweak data α_{EM}, G_μ, $\alpha_s(M_Z)$, M_Z, M_W, ρ_{NEW} and the 3 fermion masses M_{top}, $m_b(m_b)$, M_τ. The experimental values used for the low energy observables are given in the table.

Fig. 1 shows the constant χ^2 contours for $m_{16} = 1500$ and 2000 GeV in the case of universal squark and slepton masses. We find acceptable fits ($\chi^2 < 3$) for $A_0 \sim -1.9\ m_{16}$, $m_{10} \sim 1.4\ m_{16}$ and $1.2 \leq m_{16} \leq 3$ TeV. The best fit is for $m_{16} \sim 2000$ GeV with $\chi^2 < 1$. Note, electroweak symmetry breaking in this region of parameter space requires splitting Higgs up/down masses, $\Delta m_H^2 \sim$ O(13%). This range of soft SUSY parameters is consistent with solution (B) of Olechowski and Pokorski [10]. In the table we present the input parameters, the fits and the predicted Higgs and SUSY

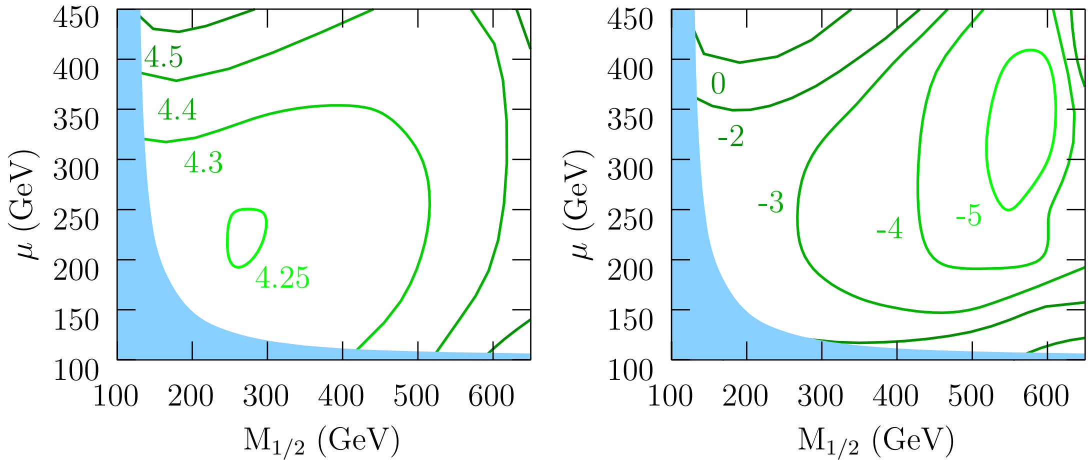

Figure 2: Contours of constant $m_b(m_b)$[GeV] (Left) and Δm_b in % (Right) for $m_{16} = 2000$ GeV .

spectra for two representative points with universal squark and slepton masses and the best fit value for D term splitting. We have not presented the contour plots for D term splitting since as can be seen from the best fit point in the table, the bottom quark mass is poorly fit in this case and $\chi^2 > 5$. Recall, since we have 11 input parameters and only 9 observables, we consider such poor fits unacceptable.

Fig. 2 gives the constant $m_b(m_b)$ and Δm_b contours for $m_{16} = 2000$ GeV . We see that the best fits, near its central value, are found with $\Delta m_b \leq -2\%$. Why does Yukawa unification only work in this narrow region of SUSY parameter space? The log corrections $\Delta m_b^{\log} \sim 4 - 6\%$ (total contribution from gluino, neutralino, chargino and electroweak loops) are positive and they must be cancelled in order to obtain $\Delta m_b \leq -2$ %. The leading mass insertion corrections proportional to $\tan\beta$ are approximately given by [7]

$$\Delta m_b^{\tilde{g}} \approx \frac{2\alpha_3}{3\pi}\,\frac{\mu m_{\tilde{g}}}{m_{\tilde{b}}^2}\, tan\beta \qquad \text{and} \qquad \Delta m_b^{\tilde{\chi}^+} \approx \frac{\lambda_t^2}{16\pi^2}\,\frac{\mu A_t}{m_{\tilde{t}}^2}\, tan\beta.$$

They can naturally be as large as 40%. The chargino contribution is typically opposite in sign to the gluino, since A_t runs to an infrared fixed point $\propto -M_{1/2}$(see for example, Carena et al. [7]). Hence in order to cancel the positive contribution of both the log and gluino contributions, a large negative chargino contribution is needed. This can be accomplished for $-A_t > m_{\tilde{g}}$ and $m_{\tilde{t}_1} << m_{\tilde{b}_1}$. The first condition can be satisfied for A_0 large and negative, which helps pull A_t away from its infrared fixed point. The second condition is also aided by large A_t. However in order to obtain a large enough splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{b}_1}$, large values of m_{16} are needed. Note, that for universal scalar masses, the lightest stop is typically lighter than the sbottom. We typically find $m_{\tilde{b}_1} \sim 3\ m_{\tilde{t}_1}$. On the other hand, D term splitting with $D_X > 0$ gives $m_{\tilde{b}_1} \leq m_{\tilde{t}_1}$. As a result in the case of universal boundary conditions excellent fits are obtained for top,

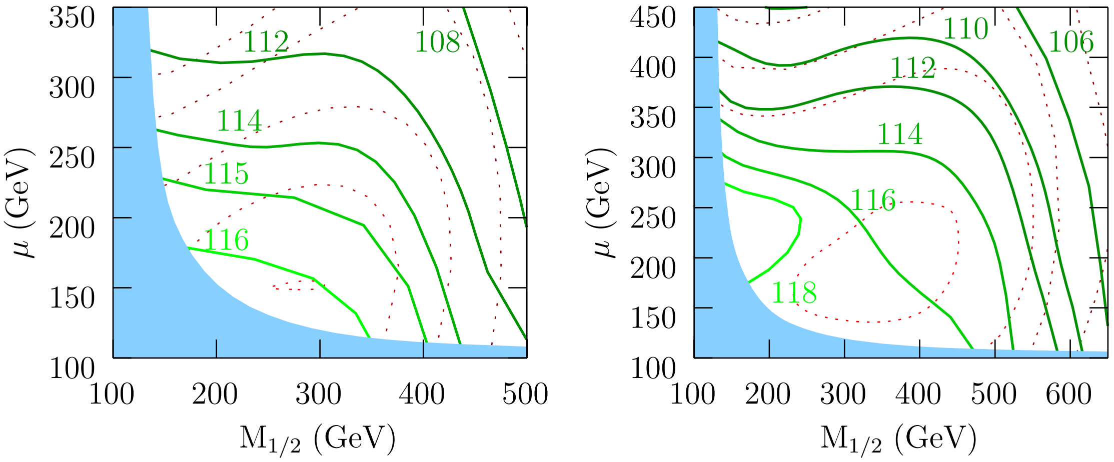

Figure 3: Contours of constant m_h [GeV] (solid lines) with χ^2 contours from Fig 1 (dotted lines) for $m_{16} = 1500$ GeV (Left) and $m_{16} = 2000$ GeV (Right).

bottom and tau masses; while for D term splitting the best fits give $m_b(m_b) \geq 4.59$ GeV .

Finally in Fig. 3 we show the constant light Higgs mass contours for $m_{16} = 1500$ and 2000 GeV (solid lines) with the constant χ^2 contours overlayed (dotted lines). Yukawa unification for $\chi^2 \leq 1$ clearly prefers light Higgs mass in a narrow range, 114 - 118 GeV . In this region the CP odd, the heavy CP even Higgs and the charged Higgs bosons are also quite light (see fit 2 in the table).[2] In addition we find the mass of $\tilde{t}_1 \sim (150-250)$ GeV, $\tilde{b}_1 \sim (450-650)$ GeV, $\tilde{\tau}_1 \sim (200-500)$ GeV, $\tilde{g} \sim (600-1200)$ GeV, $\tilde{\chi}^+ \sim (100-250)$ GeV, and $\tilde{\chi}^0 \sim (80-170)$ GeV. All first and second generation squarks and sleptons have mass of order m_{16}. The light stop and chargino may be visible at the Tevatron. With this spectrum we expect $\tilde{t}_1 \rightarrow \tilde{\chi}^+ \, b$ with $\tilde{\chi}^+ \rightarrow \tilde{\chi}^0_1 \, \bar{l} \, \nu$ to be dominant. Lastly χ^0_1 is the LSP and possibly a good dark matter candidate [13].

Note, the range of SUSY parameters with $m_{16} > 1200$ GeV and $m_{16} >> M_{1/2}$ is also preferred by nucleon decay experiments [14]. However large values of $m_{16} \geq 1200$ GeV lead to very small values for $a_\mu^{NEW} \leq 16 \times 10^{-10}$.

The region of SUSY parameter space preferred by Yukawa unification may be consistent with a supergravity mechanism for SUSY breaking at M_{Pl} with RG running from M_{Pl} to M_G (see for example Murayama et al. [10]). It however cannot be obtained with gauge mediated or gaugino mediated SUSY breaking mechanisms where $A_0 = 0$ at zeroth order. It may also be obtained in anomally mediated schemes but in this case one still has to worry about slepton masses squared and also the fact that in this case, since the

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[2] It would be interesting to see how sensitive our results, for Higgs masses, may be to alternative electroweak symmetry breaking approximations. In this paper we have used the effective 2 Higgs doublet analysis of [12] with an estimated 3 GeV uncertainty in Higgs masses. This approximation may be particularly well suited to the light Higgs spectrum we obtain in our analysis. The alternative scheme, in which the Higgs tadpoles are evaluated at a scale of order M_{stop} [8] is however more frequently used in the literature.

Table 1: **Three representative points of the fits.**
We fit the central values: $M_Z = 91.188$, $M_W = 80.419$, $G_\mu \times 10^5 = 1.1664$, $\alpha_{EM}^{-1} = 137.04$, $M_\tau = 1.7770$ with 0.1% numerical uncertainties; and the following with the experimental uncertainty in parentheses: $\alpha_s(M_Z) = 0.1180$ (0.0020), $\rho_{new} \times 10^3 = -0.200$ (1.1), $M_t = 174.3$ (5.1), $m_b(m_b) = 4.20$ (0.20). The neutral Higgs masses h, H, A_0 are pole masses; while all other sparticle masses are running masses.

Data points	1	2	3
Input parameters			
α_G^{-1}	24.46	24.66	24.73
$M_G \times 10^{-16}$	3.36	3.07	3.13
ϵ_3	−0.042	−.0397	−0.046
λ	0.70	0.67	0.80
m_{16}	1500	2000	2000
m_{10}	2027	2706	2400
Δm_H^2	0.13	0.13	0.07
$M_{1/2}$	250	350	350
μ	150	200	115
$\tan\beta$	51.2	50.5	54.3
A_0	−2748	−3748	−731
χ^2 observables			
M_Z	91.13	91.14	91.15
M_W	80.45	80.45	80.44
$G_\mu \times 10^5$	1.166	1.166	1.166
α_{EM}^{-1}	137.0	137.0	137.0
$\alpha_s(M_Z)$	0.1175	0.1176	0.1161
$\rho_{new} \times 10^3$	0.696	0.460	0.035
M_t	175.5	174.6	177.9
$m_b(m_b)$	4.28	4.27	4.59
M_τ	1.777	1.777	1.777
TOTAL χ^2	1.50	0.87	5.42
h_0	116	116	115
H_0	120	121	117
A_0	110	110	110
H^+	148	148	146
$\tilde{\chi}_1^0$	86	130	86
$\tilde{\chi}_2^0$	135	190	126
$\tilde{\chi}_1^+$	123	178	105
$\tilde{g}$	661	913	902
$\tilde{t}_1$	135	222	1020
$\tilde{b}_1$	433	588	879
$\tilde{\tau}_1$	288	420	1173
$a_\mu^{SUSY} \times 10^{10}$	9.7	5.5	6.1

gluino and chargino masses have opposite sign, it is difficult to fit both $b \rightarrow s\gamma$ and a_μ.

In a future paper [11] we present the sparticle spectrum in more detail and consequences for Tevatron searches. We discuss the sensitivity of our results to small GUT scale threshold corrections to Yukawa unification with both universal and D term Higgs up/down splitting.

In previous works Yukawa unification with $\mu > 0$ was not possible.[3] Pierce et al. [8] assume $\Delta m_H^2 = 0$ and, as a result, they are not able to enter the region of SUSY parameter space consistent with both EWSB and Yukawa unification. Baer et al. [15] also cannot obtain Yukawa unification with $\mu > 0$. This is because they use D term splitting for Higgs up/down which as discussed typically leads to sbottom lighter than stop.

While completing this article, the paper by Baer and Ferrandis [16] appeared which confirmed our results [17] on the existence of a preferred region of SUSY parameter space consistent with Yukawa unification and $\mu > 0$. Their results however require significant GUT threshold corrections to $\lambda_t = \lambda_b = \lambda_\tau$ of order 8 - 15% which helps them obtain $m_{\tilde{t}_1} < m_{\tilde{b}_1}$. They also claim that better fits are obtained with D term splitting than with the universal splitting case. We believe the latter is only true because the authors do not allow their SUSY parameters, in particular m_{16} and A_0, to explore the region of parameter space discussed in [17] and this paper.

Acknowledgements

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