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Doublet–Triplet Splitting in Realistic Heterotic String Derived models

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Abstract

There has been recently a surge of interest in Grand Unified Theories on orbifolds of higher dimensional spaces. In particular, the higher dimensional doublet–triplet splitting mechanism has been of much interest. I revisit the superstring doublet–triplet splitting mechanism in which the color triplets are projected out by the GSO projections, while leaving the electroweak doublets in the physical spectrum. The connection with the higher dimensional theories is elucidated. It is shown that the doublet–triplet splitting depends crucially on the assignment of boundary conditions in the compactified directions. The possibility of reducing the number of Higgs multiplets by using the GSO projections is also discussed.

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1 Introduction

Superstring theory provides a consistent framework for perturbative unification of gravity and gauge theories. Among the five perturbative string limits the heterotic string is the only one that also admits the standard grand unification structures, for example that of $SO(10)$. The specific implications of this is that the heterotic string can in principle preserve the embedding of the standard model generations in the 16 representation of $SO(10)$ as well as the canonical normalization of the weak hypercharge. However, as is well known supersymmetric grand unified theories [1] give rise to proton decay from dimension four, five and six operators [2].

Superstring theory offers resolutions to the proton decay from all of these sources. The issue of proton decay in realistic string constructions has been amply discussed in the past [3, 4, 5, 6, 7] and therefore I will be brief in respect to the details that are given in the earlier literature. The purpose of this note, prompted by the recent interest in grand unified higher dimensional theories [8], is to elucidate the connection between the superstring doublet–triplet splitting mechanism, which was derived in the free fermionic formulation and the higher dimensional theories.

2 Realistic free fermionic models

The class of models under consideration are constructed in the free fermionic formulation [9]. The notation and details of the construction of these models are given elsewhere [10, 11, 12, 13, 14, 15, 16, 17]. In the free fermionic formulation of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. In the light–cone gauge the world–sheet field content consists of two transverse left– and right–moving space–time coordinate bosons, $X^\mu_{1,2}$ and $\bar{X}^\mu_{1,2}$, and their left–moving fermionic superpartners $\psi^\mu_{1,2}$, and additional 62 purely internal Majorana–Weyl fermions, of which 18 are left–moving, χ^I, and 44 are right–moving, ϕ^a. In the supersymmetric sector the world–sheet supersymmetry is realized non–linearly and the world–sheet supercurrent is given by $T_F = \psi^\mu \partial X_\mu + i\chi^I y^I \omega^I$, $(I = 1, \cdots, 6)$. The $\{\chi^I, y^I, \omega^I\}$ $(I = 1, \cdots, 6)$ are 18 real free fermions transforming as the adjoint representation of $SU(2)^6$. Under parallel transport around a noncontractible loop on the toroidal world–sheet the fermionic fields pick up a phase

$$f \;\rightarrow\; -\,\mathrm{e}^{i\pi\alpha(f)} f \;, \quad \alpha(f) \in (-1, +1]. \qquad (2.1)$$

Each set of specified phases for all world–sheet fermions, around all the non–contractible loops is called the spin structure of the model. Such spin structures are usually given is the form of 64 dimensional boundary condition vectors, with each element of the vector specifying the phase of the corresponding world–sheet fermion. The basis vectors are constrained by string consistency requirements and completely determine the vacuum structure of the model. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S–matrix elements between external states [18].

The boundary condition basis defining a typical realistic free fermionic heterotic string models is constructed in two stages. The first stage consists of the NAHE set, which is a set of five boundary condition basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [15]. The gauge group after imposing the GSO projections induced by the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ supersymmetry. The space–time vector bosons that generate the gauge group arise from the Neveu–Schwarz sector and from the sector $\mathbf{1}+b_1+b_2+b_3$. The Neveu–Schwarz sector produces the generators of $SO(10)\times SO(6)^3\times SO(16)$. The sector $\zeta \equiv \mathbf{1}+b_1+b_2+b_3$ produces the spinorial **128** of $SO(16)$ and completes the hidden gauge group to E_8. The NAHE set divides the internal world–sheet fermions in the following way: $\bar{\phi}^{1,\cdots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\cdots,5}$ generate the $SO(10)$ gauge group, and $\{\bar{y}^{3,\cdots,6}, \bar{\eta}^1\}$, $\{\bar{y}^1, \bar{y}^2, \bar{\omega}^5, \bar{\omega}^6, \bar{\eta}^2\}$, $\{\bar{\omega}^{1,\cdots,4}, \bar{\eta}^3\}$ generate the three horizontal $SO(6)^3$ symmetries. The left–moving $\{y, \omega\}$ states are divided to $\{y^{3,\cdots,6}\}$, $\{y^1, y^2, \omega^5, \omega^6\}$, $\{\omega^{1,\cdots,4}\}$ and χ^{12}, χ^{34}, χ^{56} generate the left–moving $N = 2$ world–sheet supersymmetry. At the level of the NAHE set the sectors b_1, b_2 and b_3 produce 48 multiplets, 16 from each, in the 16 representation of $SO(10)$. The states from the sectors b_j are singlets of the hidden E_8 gauge group and transform under the horizontal $SO(6)_j$ $(j = 1, 2, 3)$ symmetries. This structure is common to all the realistic free fermionic models.

The second stage of the basis construction consists of adding to the NAHE set three (or four) additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors b_1, b_2 and b_3, and simultaneously break the four dimensional gauge group. The assignment of boundary conditions to $\{\bar{\psi}^{1,\cdots,5}\}$ breaks $SO(10)$ to one of its subgroups $SU(5) \times U(1)$ [10], $SO(6) \times SO(4)$ [12], $SU(3) \times SU(2) \times U(1)^2$ [11, 13, 14, 16] or $SU(3) \times SU(2)^2 \times U(1)$ [17]. Similarly, the hidden E_8 symmetry is broken to one of its subgroups by the basis vectors which extend the NAHE set. The flavor $SO(6)^3$ symmetries in the NAHE–based models are always broken to flavor $U(1)$ symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such $U(1)_j$ symmetries are always obtained in the NAHE based free fermionic models, from the subgroup of the observable E_8, which is orthogonal to $SO(10)$. These are produced by the world–sheet currents $\bar{\eta}\bar{\eta}^*$ $(j = 1, 2, 3)$, which are part of the Cartan sub–algebra of the observable E_8. Additional unbroken $U(1)$ symmetries, denoted typically by $U(1)_j$ $(j = 4, 5, ...)$, arise by pairing two real fermions from the sets $\{\bar{y}^{3,\cdots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\bar{\omega}^{1,\cdots,4}\}$. The final observable gauge group depends on the number of such pairings.

3 Superstring Higgs doublet–triplet splitting

In the free fermionic models, representations in the 5 and $\bar{5}$ of $SU(5)$ which yield electroweak Higgs doublets and color Higgs triplets arise from the untwisted (Neveu–

Schwarz) and twisted sectors. The color triplets arising from these sectors are those that can mediate rapid proton decay from dimension five operators. Additional color triplets may arise from "Wilsonian" sectors, but their interactions with the Standard Model states may be protected by discrete symmetries [5].

For the Higgs multiplets arising from the Neveu–Schwarz sector there exists a doublet–triplet splitting mechanism which operates by the assignment of boundary conditions to the set of internal world–sheet fermions $\{y,\omega|\bar{y},\bar{\omega}\}^{1,\cdots,6}$. The Neveu–Schwarz sector gives rise to three fields in the 10 representation of $SO(10)$. These contain the Higgs electroweak doublets and color triplets. Each of those is charged with respect to one of the horizontal $U(1)$ symmetries $U(1)_{1,2,3}$. Each one of these multiplets is associated, by the horizontal symmetries, with one of the twisted sectors, b_1, b_2 and b_3. The doublet–triplet splitting results from the boundary condition basis vectors which breaks the $SO(10)$ symmetry to $SO(6)\times SO(4)$. We can define a quantity Δ_i in these basis vectors which measures the difference between the boundary conditions assigned to the internal fermions from the set $\{y,w|\bar{y},\bar{\omega}\}$ and which are periodic in the vector b_i,

$$\Delta_i = |\alpha_L(\text{internal}) - \alpha_R(\text{internal})| = 0,1 \quad (i=1,2,3) \tag{3.1}$$

If $\Delta_i = 0$ then the Higgs triplets, D_i and $\bar{D}_i$, remain in the massless spectrum while the Higgs doublets, h_i and $\bar{h}_i$ are projected out and the opposite occurs for $\Delta_i = 1$.

Thus, the rule in Eq. (3.1) is a generic rule that can be used in the construction of the free fermionic models. The model of table (3.2) illustrates this rule. In this model $\Delta_1 = \Delta_2 = 0$ while $\Delta_3 = 1$. Therefore, this model produces two pairs of color triplets and one pair of Higgs doublets from the Neveu–Schwarz sector, D_1, $\bar{D}_1$ D_2, $\bar{D}_2$ and h_3, $\bar{h}_3$.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	1	1	0	0	1 1 1 0 0	1	0	1	1 1 1 1 0 0 0 0
β	1	0	1	0	1 1 1 0 0	0	1	1	1 1 1 1 0 0 0 0
γ	1	0	0	1	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	$y^3\bar{y}^3$	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$y^6\bar{y}^6$	$y^1\bar{y}^1$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\omega^6\bar{\omega}^6$	$\omega^2\omega^3$	$\omega^1\bar{\omega}^1$	$\omega^4\bar{\omega}^4$	$\bar{\omega}^2\bar{\omega}^3$
α	1	0	0	1	0	0	1	0	0	0	1	1
β	0	0	0	1	0	1	1	0	0	1	0	1
γ	1	1	0	0	1	1	0	0	0	0	0	1

(3.2)

With the choice of generalized GSO coefficients:

$$c\binom{b_1, b_3, \alpha, \beta, \gamma}{\alpha} = -c\binom{b_2}{\alpha} = c\binom{\mathbf{1}, b_j, \gamma}{\beta} =$$
$$c\binom{\gamma}{b_3} = -c\binom{\gamma}{\mathbf{1}, b_1, b_2} = -1 \qquad (j=1,2,3),$$

with the others specified by modular invariance and space–time supersymmetry. In ref. [4] this doublet–triplet splitting mechanism is proven in terms of the world–sheet modular invariance constraints and the GSO projections. The constraint

$$|\alpha_L(b_j) - \alpha_R(b_j)| = 1 \quad , \quad (j = 1, 2, 3), \tag{3.3}$$

in the basis vectors that break $SO(10)$ to $SO(6)\times SO(4)$, guarantees that the Neveu–Schwarz color triplets, D_j and $\bar{D}_j$, are projected out and that the electroweak doublets, h_j and $\bar{h}_j$, remain in the massless spectrum. It is now possible to construct models in which all the color Higgs triplets from the Neveu–Schwarz sector are projected out by the GSO projections. Table (3.4) provides an example of such a model. It is noted that the NS Higgs–doublet triplet mechanism operates irrespective of the choice of the GSO projection coefficients.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	1

(3.4)

As the Higgs–doublet triplet mechanism operates irrespective of the GSO phases they are not displayed here explicitly. The sector $b_1 + b_2 + \alpha + \beta$ produces additional states that transform solely under the observable sector. In particular it can give rise to additional electroweak doublets and color triplets. The color triplets from this sector may cause problems with proton lifetime constraints. However, a similar doublet–triplet splitting mechanism works for this sector as well. There exist choices of boundary conditions for the set of left–right symmetric internal fermions, $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$, for which the triplets are projected out and the doublets remain in the massless spectrum. For example, in the model of ref. [13] this sector produces one pair of electroweak doublets and one pair of color triplets,

$$h_{45} \equiv [(1, 0); (2, -1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \quad D_{45} \equiv [(3, -1); (1, 0)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \tag{3.5}$$

while in the model of table (3.4) this sector produces two pairs of electroweak doublets,

$$h_{45} \equiv [(1, 0); (2, -1)]_{\frac{1}{2},\frac{1}{2},0,0,0,0} \quad h'_{45} \equiv [(1, 0); (2, -1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \tag{3.6}$$

and all the color triplets, from the Neveu–Schwarz sector and the sector $b_1+b_2+\alpha+\gamma$, are projected out from the physical spectrum by the GSO projections. The two models differ only by the assignment of boundary conditions to the set of internal fermions, $\{y,\omega|\bar{y},\bar{\omega}\}^{1,\cdots,6}$. The simplicity and elegance of the superstring doublet–triplet splitting mechanism is striking. There is no need for exotic representations of high dimensionality as in minimal $SU(5)$ extension of the Standard Model [19]. Moreover, the superstring doublet–triplet splitting mechanism does not depend on additional assumptions on Yukawa couplings as is required in all GUT doublet–triplet splitting mechanism. In the superstring doublet–triplet splitting mechanism the dangerous color triplets simply do not exist in the massless spectrum. Furthermore, due to discrete and custodial non-Abelian symmetries [5] there exists examples of models in which proton decay mediating operators are not generated.

Another relevant question with regard to the Higgs doublet–triplet splitting mechanism is whether it is possible to construct models in which both the Higgs color triplets and electroweak doublets from the Neveu–Schwarz sector are projected out by the GSO projections. This is a viable possibility as we can choose for example

$$\Delta_j^{(\alpha)} = 1 \text{ and } \Delta_j^{(\beta)} = 0,$$

where $\Delta^{(\alpha,\beta)}$ are the projections due to the basis vectors α and β respectively. This is a relevant question as the number of Higgs representations, which generically appear in the massless spectrum, is larger than what is allowed by the low energy phenomenology. Consider for example the model in table (3.7)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	1	1	0	0	1 1 1 0 0	1	0	1	1 1 1 1 0 0 0 0
β	1	0	1	0	1 1 1 0 0	0	1	1	1 1 1 1 0 0 0 0
γ	1	0	0	1	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	0	1	0	0	0	0	1	0	0
γ	0	1	0	0	0	1	0	1	1	0	0	0

(3.7)

With the choice of generalized GSO coefficients:

$$c\begin{pmatrix} b_j \\ S, b_j, \alpha, \beta, \gamma \end{pmatrix} = c\begin{pmatrix} \alpha \\ \alpha, \beta, \gamma \end{pmatrix} =$$
$$c\begin{pmatrix} \beta, \\ \beta, \gamma \end{pmatrix} = c\begin{pmatrix} \gamma \\ \mathbf{1} \end{pmatrix} = -1$$

(j=1,2,3), with the others specified by modular invariance and space–time supersymmetry. In this model $\Delta_1^{(\alpha)} = \Delta_2^{(\alpha)} = \Delta_3^{(\alpha)} = 1$, and $\Delta_1^{(\beta)} = \Delta_3^{(\beta)} = 0$, Therefore, In this model irrespective of the choice of the generalized GSO projection coefficients, both the Higgs color triplets and electroweak doublets associated with b_1 and b_3 are projected out by the GSO projections. However, it is found that the combination of these projections also results in the projection of some of the representations from the corresponding sectors b_1 and b_3 and therefore these sectors do not produce the full chiral 16 of $SO(10)$. Therefore, realization of this mutual projection of both Higgs triplets and doublets from the Neveu–Schwarz sector requires that the chiral generations be obtained from non–NAHE set basis vectors.

4 Correspondence with orbifolds

In the previous section it was shown that the superstring doublet–triplet splitting mechanism depend on the assignment of boundary conditions to the set of internal world–sheet fermions $\{y, \omega | \bar{y}, \bar{\omega}\}$. In this section I show that this set of internal world–sheet fermions in fact corresponds to six internal compactified dimensions. Consequently, in the bosonic language, *i.e.* in the language of the compactified dimensions, the boundary condition of the internal fermions translate to twisting of the internal dimensions with orbifold fixed points.

The correspondence with the orbifold construction is illustrated by extending the NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$, by one additional boundary condition basis vector [20]

$$X = (0, \cdots, 0 | \underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}}, 0, \cdots, 0) \ . \tag{4.1}$$

with a suitable choice of the GSO projection coefficients the model possess an $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge group and $N = 1$ space–time supersymmetry. The matter fields include 24 generations in 27 representations of E_6, eight from each of the sectors $b_1 \oplus b_1 + X$, $b_2 \oplus b_2 + X$ and $b_3 \oplus b_3 + X$. Three additional 27 and $\overline{27}$ pairs are obtained from the Neveu–Schwarz $\oplus$ X sector.

To construct the model in the orbifold formulation one starts with a model compactified on a flat torus with nontrivial background fields [21]. The action of the six dimensional compactified dimensions is given by

$$S = \frac{1}{8\pi} \int d^2\sigma (G_{ij} \partial X^i \partial X^j + B_{ij} \partial X^i \partial X^j) \tag{4.2}$$

where

$$G_{ij} = \frac{1}{2} \sum_{I=1}^{D} R_i e_i^I R_j e_j^I \tag{4.3}$$

is the metric of the six dimensional compactified space and $B_{ij} = -B_{ji}$ is the anti-symmetric tensor field. The $e^i = \{e_i^I\}$ are six linear independent vectors normalized to $(e_i)^2 = 2$. The subset of basis vectors

$$\{\mathbf{1}, S, X, I = \mathbf{1} + b_1 + b_2 + b_3\} \tag{4.4}$$

generates a toroidally-compactified model with $N = 4$ space–time supersymmetry and $SO(12) \times E_8 \times E_8$ gauge group. The same model is obtained in the geometric (bosonic) language by constructing the background fields which produce the $SO(12)$ lattice. Taking the metric of the six-dimensional compactified manifold to be the Cartan matrix of $SO(12)$:

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \tag{4.5}$$

and the antisymmetric tensor

$$b_{ij} = \begin{cases} g_{ij} & ; i > j, \\ 0 & ; i = j, \\ -g_{ij} & ; i < j. \end{cases} \tag{4.6}$$

When all the radii of the six-dimensional compactified manifold are fixed at $R_I = \sqrt{2}$, it is seen that the left– and right–moving momenta

$$P^I_{R,L} = [m_i - \frac{1}{2}(B_{ij} \pm G_{ij})n_j] e_i^{I^*} \tag{4.7}$$

reproduce all the massless root vectors in the lattice of $SO(12)$, where in (4.7) the $e_i^{I^*}$ are dual to the e_i, and $e_i^* \cdot e_j = \delta_{ij}$.

The orbifold models are obtained by moding out the six dimensional torus by a discrete symmetry group, P [23]. The allowed discrete symmetry groups are constrained by modular invariance. The Hilbert space is obtained by acting on the vacuum with twisted and untwisted oscillators and by projecting on states that are invariant under the space and group twists. A general left–right symmetric twist is given by $(\theta^i_j, v^i; \Theta^I_J, V^I)$ $(i = 1, \cdots, 6)$ $(I = 1, \cdots, 16)$ and $X^i(2\pi) = \theta^i_j X^j(0) + v^i$; $X^I(2\pi) = \Theta^I_J X^J(0) + V^I$. The massless spectrum contains mass states from the untwisted and twisted sectors. The untwisted sector is obtained by projecting on states that are invariant under the space and group twists. The twisted string centers around the points that are left fixed by the space twist. In the case of "standard embedding" one acts on the gauge degrees of freedom in an $SU(3) \in E_8 \times E_8$ with the same action as on the six compactified dimensions + NSR fermions. In this case the number of chiral families (27's of E_6) is given by one half the Euler characteristic,

$$\chi = \frac{1}{|P|} \sum_{g,h \in P} \chi(g, h), \tag{4.8}$$

where $\chi(g,h)$ is the number of points left fixed simultaneously by h and g. The mass formula for the right–movers in the twisted sectors is given by,

$$M_R^2 = -1 + \frac{(P+V)^2}{2} + \Delta c_\theta + N_R \tag{4.9}$$

where V^I are the shifts on the gauge sector and $\Delta c_\theta = \frac{1}{4}\sum_k \eta_k(1-\eta_k)$ is the contribution of the twisted bosonic oscillators to the zero point energy and $\eta_k = \frac{1}{2}$ for a Z_2 twist.

To translate the fermionic boundary conditions to twists and shifts in the bosonic formulation we bosonize the real fermionic degrees of freedom, $\{y,\omega|\bar{y},\bar{\omega}\}$. Defining, $\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = -ie^{iX_i}$, $\eta_i = \sqrt{\frac{1}{2}}(y_i - i\omega_i) = -ie^{-iX_i}$ with similar definitions for the right movers $\{\bar{y},\bar{\omega}\}$ and $X^I(z,\bar{z}) = X_L^I(z) + X_R^I(\bar{z})$. With these definitions the world–sheet supercurrents in the bosonic and fermionic formulations are equivalent,

$$T_F^{int} = \sum_i \chi_i y_i \omega_i = i\sum_i \chi_i \xi_i \eta_i = \sum_i \chi_i \partial X_i. \tag{4.10}$$

The momenta P^I of the compactified scalars in the bosonic formulation are identical with the $U(1)$ charges $Q(f)$ of the unbroken Cartan generators of the four dimensional gauge group,

$$Q(f) = \frac{1}{2}\alpha(f) + F(f) \tag{4.11}$$

where $\alpha(f)$ are the boundary conditions of complex fermions f, reduced to the interval $(-1,1]$ and $F(f)$ is a fermion number operator.

The boundary condition vectors b_1 and b_2 now translate into $Z_2\times Z_2$ twists on the bosons X_i and fermions χ_i and to shifts on the gauge degrees of freedom. The massless spectrum of the resulting orbifold model consist of the untwisted sector and three twisted sectors, θ, θ' and $\theta\theta'$. From the untwisted sector we obtain the generators of the $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge groups. The only roots of $SO(12)$ that are invariant under the $Z_2\times Z_2$ twist are those of the subgroup $SO(4)^3$. Thus, the $SO(12)$ symmetry is broken to $SO(4)^3$. Similarly, the shift in the gauge sector breaks one E_8 symmetry to $E_6 \times U(1)^2$. In addition to the gauge group generators the untwisted sector produces: three copies of $27 + \bar{27}$, one pair for each of the complexified NSR left–moving fermions; three copies of, $1+\bar{1}$, E_6 singlets which are charged under $U(1)^2$. These singlets are the untwisted moduli of the $Z_2 \times Z_2$ orbifold model and match the number of untwisted moduli in the free fermionic model. The $E_8 \times E_8$ singlets are obtained from the root lattice of $SO(12)$ and transform as $(1,4,4)$ under the $S0(4)^3$ symmetries, one for each of the complexified NSR left–moving fermions.

The number of fixed points in each twist is 32. The total number of fixed points is 48. The number of chiral 27's is 24, eight from each twisted sector, and matches the number of chiral 27's in the fermionic model. For every fixed point we obtain the $SO(4)^3 \times E_6 \times E_8$ singlets. These are obtained for appropriate choices of the momentum vectors, P^I, and correspond to twisted moduli. The $E_6 \times E_8$ singlets can be obtained by acting on the vacuum with twisted oscillators and from combinations of the dual of the invariant lattice, I^*, [22]. The spectrum of the orbifold model and its symmetries are seen to coincide with the spectrum and symmetries of the fermionic model [20].

It is noted that the effect of the additional basis vector X, Eq. (4.1), is to separate the gauge degrees of freedom, spanned by the world–sheet fermions $\{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,8}\}$ from the internal compactified degrees of freedom $\{y, \omega|\bar y, \bar\omega\}^{1,\cdots,6}$. in the realistic free fermionic models this is achieved by the vector 2γ [20], with

$$2\gamma = (0, \cdots, 0| \underbrace{1, \cdots, 1}_{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3} \bar\phi^{1,\cdots,4}}, 0, \cdots, 0) \ , \tag{4.12}$$

which breaks the $E_8 \times E_8$ symmetry to $SO(16) \times SO(16)$.

5 Discussion

In section (3) it was shown that the assignment of boundary conditions to the set of world–sheet fermions $\{y, \omega|\bar y, \bar\omega\}^{1,\cdots,6}$ is the one that selects between the electroweak Higgs doublet, versus the color Higgs triplets, according to the quantity Δ_j in Eq. (3.1). In section (4) on the other hand it was shown that this set of world–sheet fermions corresponds to the internal compactified dimensions in an orbifold construction. This means that the assignment of boundary conditions to this set of world–sheet fermions in fact corresponds to further Z_2 orbifold twisting of the compactified dimensions. This type of construction is precisely the one that has been recently rediscovered in [8]. It is therefore very rewarding to note that in the context of the heterotic–string construction such a mechanism operates in a framework of realistic three generation models, compatible with perturbative quantum gravity. It ought to be further remarked that elimination of color triplets by Wilson line breaking has also been noted in other orbifold compactifications [24].

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