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arXiv:hep-ph/0107025v1 3 Jul 2001

A new mass relation among the hadron vector resonances.

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Abstract

We show that the hadron vector resonances are described by fields transforming according to different inequivalent representations of the Lorentz group: $(1/2, 1/2)$ and $(1, 0) + (0, 1)$. The vector representation $(1/2, 1/2)$ is well studied and corresponds to the gauge fields. On the other hand, the chiral representations $(1, 0)$ and $(0, 1)$ are described by the second rank antisymmetric tensor fields, for which interaction theory has not yet been constructed. In the framework of the phenomenological Nambu–Jona-Lasinio approach we have introduced and used all these fields for a description of the vector resonances. A new mass relation between low-lying hadron vector and axial-vector resonances is obtained. This relation is in agreement with the present experimental data.

1 Introduction

The bound states of a quark and an antiquark are characterized by the quantum numbers J^{PC}, where J is the total angular momentum, P is the parity and C is the charge conjugation. There exist three types of different quantum numbers for the known vector mesons [1]. They are 1^{--}, 1^{++} and 1^{+-}. For instance, the first quantum numbers can be assigned to the ρ and ω, ϕ vector mesons with isospin $I = 1$ and $I = 0$. The second and the third quantum numbers are assigned to the axial-vector mesons a_1, f_1 and b_1, h_1, correspondingly. Note that we have two different types of axial-vector mesons and the key point is the difference between the last two assignments for these.

Let us consider the extended Nambu–Jona-Lasinio (NJL) [2] models. In such models the Lagrangian contains only the quark fields, while the spontaneous symmetry breaking and the hadron states are generated dynamically by the model itself. The mesonic states arise as excitations of quark–antiquark pairs and that defines their interactions with the quarks.

In the relativistic theory the symmetry group is the Lorentz group $O(3,1)$, which is isomorphic to the direct product of the two spatial rotation groups $O(3) \times O(3)$. The lowest representations of the $O(3,1)$ group, which can be used as building blocks for the construction of higher spin representations, are chiral fundamental spinors transforming according to two inequivalent representations $(1/2,0)$ and $(0,1/2)$. They correspond to quarks with different chiralities. There exists two possibilities to construct the meson fields with the spin 1.

The vector representation $(1/2,1/2)$ arises from the product between the left $(1/2,0)$ and the right $(0,1/2)$ fundamental spinors. There are the vector $\overline{\psi}\gamma_\mu\psi$ and the axial-vector $\overline{\psi}\gamma_\mu\gamma^5\psi$ bilinear forms of the quark spinor fields with quantum numbers 1^{--} and 1^{++}, which couple to the vector and axial-vector mesons respectively. They have gauge-like interactions with the quarks and can be described by the gauge vector V_μ and axial-vector A_μ fields.

To describe the mesons with the quantum numbers 1^{+-} we must use other inequivalent chiral representations $(1,0)$ and $(0,1)$, which can be constructed, if one uses only the product either of the left $(1/2,0)$ or of the right $(0,1/2)$ fundamental spinors. These mesonic states correspond to the bilinear form $\overline{\psi}\sigma_{\mu\nu}\psi$, and are described by the antisymmetric tensor field $T_{\mu\nu}$, rather than by the gauge fields. The antisymmetric tensor field $T_{\mu\nu}$ have six independent components: three-vector and three-axial-vector, and can be decomposed in the axial-vector $B_\mu = \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}\hat{\partial}_\nu T_{\alpha\beta}$ and the vector $R_\mu = \hat{\partial}_\nu T_{\nu\mu}$ [3], where $i\hat{\partial}_\mu = i\partial_\mu/\sqrt{-\partial^2}$ is a dimensionless unit vector[1]. Each of the vectors B_μ and R_μ has three independent components due to the antisymmetric property of $T_{\mu\nu}$. Besides the axial-vector mesons B_μ with quantum numbers 1^{+-}, there are additional vector mesons R_μ with quantum numbers 1^{--}, like those of the gauge mesons V_μ, but having different coupling to the quarks.

These excitations *were missed* in the NJL model and they are not considered as real particles at the present time. In this paper we shall show how we can use the newly introduced quasi-particles in the NJL framework. To demonstrate this idea we deal with the one-flavour NJL model only.

[1] We assume that repeated indices are summed in all cases.

2 The effective Lagrangian.

One of the most important symmetries of the real world and QCD, which holds in the NJL model, is chiral symmetry. Following the classical paper [4] we require that the primary fermion interaction must be invariant under γ^5- and ordinary phase transformations:

$$\psi \to \exp[i\alpha\gamma^5]\ \psi, \qquad \bar\psi \to \bar\psi\ \exp[i\alpha\gamma^5], \tag{1}$$

$$\psi \to \exp[i\alpha]\ \psi, \qquad \bar\psi \to \bar\psi\ \exp[-i\alpha], \tag{2}$$

where α is a constant and ψ is the Dirac spinor corresponding to a quark field. We restrict ourselves to the consideration of quark-antiquark bound state formations as real particles. These states are explicitly invariant under transformations (2).

Whilst the Dirac spinor has four components one can construct 16 independent bilinear forms in quark-antiquark channel: $\bar\psi\psi$, $\bar\psi\gamma^5\psi$, $\bar\psi\gamma_\mu\psi$, $\bar\psi\gamma_\mu\gamma^5\psi$ and $\bar\psi\sigma_{\mu\nu}\psi$. Under the Lorentz group they transform as scalar, pseudoscalar, vector, axial-vector and antisymmetric tensor, correspondingly. To deal with the chiral properties of these bilinear forms it is useful to define chiral currents: $\mathcal{V}_\mu = \bar\psi\gamma_\mu\psi$, $\mathcal{A}_\mu = \bar\psi\gamma_\mu\gamma^5\psi$, $\mathcal{S}^\pm = \bar\psi(1\pm\gamma^5)\psi$ and $\mathcal{T}^\pm_{\mu\nu} = \bar\psi\sigma_{\mu\nu}(1\pm\gamma^5)\psi$. The vector $\mathcal{V}_\mu$ and axial-vector $\mathcal{A}_\mu$ currents obviously satisfy the chiral invariance. The last two terms transform under (1) as follows:

$$\mathcal{S}^\pm \to \exp[\pm 2i\alpha]\ \mathcal{S}^\pm, \qquad \mathcal{T}^\pm_{\mu\nu} \to \exp[\pm 2i\alpha]\ \mathcal{T}^\pm_{\mu\nu}. \tag{3}$$

Now we can construct the chiral invariant Lagrangian choosing scalar $\mathcal{S}^\pm$ and tensor $\mathcal{T}^\pm_{\mu\nu}$ current-current interactions with opposite chiralities and quadratic forms of $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ interactions. The former is the primary interaction in the original work [4] of Nambu and Jona-Lasinio. The latter interaction is used in the extensions of the NJL model to achieve a sufficient attractive force in the axial-vector channel [5]. What about the tensor interactions? It is easy to check that its Lorentz invariant form

$$\mathcal{T}^+_{\mu\nu}\ \mathcal{T}^-_{\mu\nu} = (\bar\psi\sigma_{\mu\nu}\psi)^2 - (\bar\psi\sigma_{\mu\nu}\gamma^5\psi)^2 \equiv 0 \tag{4}$$

exactly equals zero, because $\mathcal{T}^+_{\mu\nu}$ and $\mathcal{T}^-_{\mu\nu}$ belong to different irreducible representations of the Lorentz group, namely $(1,0)$ and $(0,1)$. It is therefore impossible to include tensor excitations in a local one-flavour NJL model. Possible minimal extension has been proposed in [3]. The effective four-fermion Lagrangian has the form

$$\begin{aligned} \mathcal{L}_{int} = \ &+\ G_S\ [(\bar\psi\psi)^2 - (\bar\psi\gamma^5\psi)^2]\ -\ G_V\ (\bar\psi\gamma_\mu\psi)^2 -\ G_A\ (\bar\psi\gamma_\mu\gamma^5\psi)^2 \\ &-\ G_T\ [\hat\partial_\mu(\bar\psi\sigma_{\mu\lambda}\psi)\cdot\hat\partial_\nu(\bar\psi\sigma_{\nu\lambda}\psi) - \hat\partial_\mu(\bar\psi\sigma_{\mu\lambda}\gamma^5\psi)\cdot\hat\partial_\nu(\bar\psi\sigma_{\nu\lambda}\gamma^5\psi)]. \end{aligned} \tag{5}$$

3 The collective states.

The nonlinear current-current interactions (5) can be linearized by means of the formula

$$\exp[-\frac{i}{2}J\mathcal{K}^{-1}J] = C\int[\mathrm{d}\varphi]\exp[iJ\varphi + \frac{i}{2}\varphi\mathcal{K}\varphi], \tag{6}$$

where auxiliary fields φ will play the role of collective meson states. Then the initial Lagrangian takes the form

$$\mathcal{L}_{init} \ = \ i\ \bar\psi\partial\!\!\!/\psi + g_S\ \bar\psi\psi\ S - \frac{g_S^2}{4G_S}\ S^2 + i\ g_P\ \bar\psi\gamma^5\psi\ P - \frac{g_P^2}{4G_P}\ P^2$$

$$+ \quad g_V \; \bar\psi\gamma_\mu\psi \; V_\mu + \frac{g_V^2}{4G_V} \; V_\mu^2 + g_A \; \bar\psi\gamma_\mu\gamma^5\psi \; A_\mu + \frac{g_A^2}{4G_A} \; A_\mu^2$$
$$+ \quad g_R \; \hat\partial_\nu(\bar\psi\sigma_{\mu\nu}\psi) \; R_\mu + \frac{g_R^2}{4G_R} R_\mu^2 + i \; g_B \; \hat\partial_\nu(\bar\psi\sigma_{\mu\nu}\gamma^5\psi) \; B_\mu + \frac{g_B^2}{4G_B} B_\mu^2. \qquad (7)$$

Here we have introduced all possible low-lying collective states. They are scalar S, pseudoscalar P, two vectors V_μ, R_μ and two axial-vectors A_μ, B_μ with the following quantum numbers:

meson fields:	S	P	V_μ	A_μ	R_μ	B_μ
quantum numbers:	0^{++}	0^{-+}	1^{--}	1^{++}	1^{--}	1^{+-}

Let us note that there are two different vector mesons V_μ and R_μ with the same quantum numbers. Therefore, the physical states of such mesons can be linear combinations of them[2].

Integration over the quark field ψ leads to an effective Lagrangian for the meson fields with proper induced kinetic and mass terms. Various interactions among the meson fields also arise [3]. Here we are only interested in a mass matrix.

4 The mass matrix.

It is well known that after spontaneous symmetry breaking, when the quark field acquires mass m, we get the scalar meson with the mass $2m$ and the massless pseudoscalar meson. Due to interactions with the scalar field S the mass terms for the vector and axial-vector mesons are also modified. The final Lagrangian for the mass terms reads

$$\mathcal{L}_M = \frac{M_V^2}{2} V_\mu^2 + \sqrt{\frac{3}{2}} m V_{\mu\nu}\hat R_{\mu\nu} + \frac{M_T^2 - 6m^2}{2} R_\mu^2 + \frac{M_A^2 + 6m^2}{2} A_\mu^2 + \frac{M_T^2 + 6m^2}{2} B_\mu^2, \qquad (8)$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $\hat R_{\mu\nu} = \hat\partial_\mu R_\nu - \hat\partial_\nu R_\mu$. Here M_V, M_A, M_T and m masses can be independent. But if we believe that the effective four-fermion interactions of the quarks could originate in QCD by gluon exchange in the $1/N_c$ limit, one obtains $M_V = M_A$ [2]. This reduces the number of unknown parameters to three. Therefore, we can derive one relation for four physical masses.

As it is expected, the vector mesons with the same quantum numbers are mixed

$$\mathcal{M}^2(q^2) = \begin{pmatrix} M_V^2 & \sqrt{6m^2q^2} \\ \sqrt{6m^2q^2} & M_T^2 - 6m^2 \end{pmatrix}. \qquad (9)$$

Let us note that it is dynamical mixing, because the mass matrix contains explicitly momentum q^2. Therefore, the mixing angle depends on the momenta.

As long as the isospin triplets consist of *up* and *down* quarks with approximately the same constituent masses we can apply this one-flavour model to the charged mesons ρ, a_1, b_1 and ρ' in order to avoid mixing with s quark for $I = 0$ multiplets. In this case $m_{a_1}^2 \equiv M_A^2 + 6m^2$, $m_{b_1}^2 \equiv M_T^2 + 6m^2$, and $m_\rho^2 = \lambda_1(m_\rho^2)$, $m_{\rho'}^2 = \lambda_2(m_{\rho'}^2)$ are solutions of the quadratic equation:

$$\lambda^2 - (m_{a_1}^2 + m_{b_1}^2 - 12m^2)\;\lambda + (m_{a_1}^2 - 6m^2)(m_{b_1}^2 - 12m^2) = 0. \qquad (10)$$

[2]It means, in particular, that the ρ meson can have both gauge and anomalous tensor couplings with the quarks [6], while the axial-vector mesons with quantum numbers 1^{++} have only gauge interactions and the axial-vector mesons with quantum numbers 1^{+-} have only tensor interactions.

Using the Viète theorem we get immediately from (10) the following useful relationships:

$$\left\{ \begin{array}{lcl} m_{\rho'}^2 + m_\rho^2 & = & m_{a_1}^2 + m_{b_1}^2 - 12m^2, \\ m_{\rho'}^2 \, m_\rho^2 & = & (m_{a_1}^2 - 6m^2)(m_{b_1}^2 - 12m^2), \end{array} \right. \tag{11}$$

or

$$\boxed{m_{b_1}^2 = \frac{m_{\rho'}^4 + m_\rho^4 - m_{a_1}^4}{m_{\rho'}^2 + m_\rho^2 - m_{a_1}^2}.} \tag{12}$$

The last equation is the main result of our work, which can be compared with experiment.

As long as the masses of ρ and b_1 mesons are known with better precision than the ρ' or a_1 masses, we take them as input parameters [1]: $m_\rho^{exp} = 769.9 \pm 0.8$ MeV and $m_{b_1}^{exp} = 1229.5 \pm 3.2$ MeV. Then we can compare the predicted $m_{\rho'} - m_{a_1}$ relation (3σ allowed region between curves in Fig. 1) with the experimental data. For this purpose we have shown in Fig. 1 the combined result for the charged $\rho' \equiv \rho(1450)$ mass measurements [7] and the PDG result [1] for a_1 mass with 3σ allowed intervals. The last high-precision measurement of a_1 mass [8] is also shown.

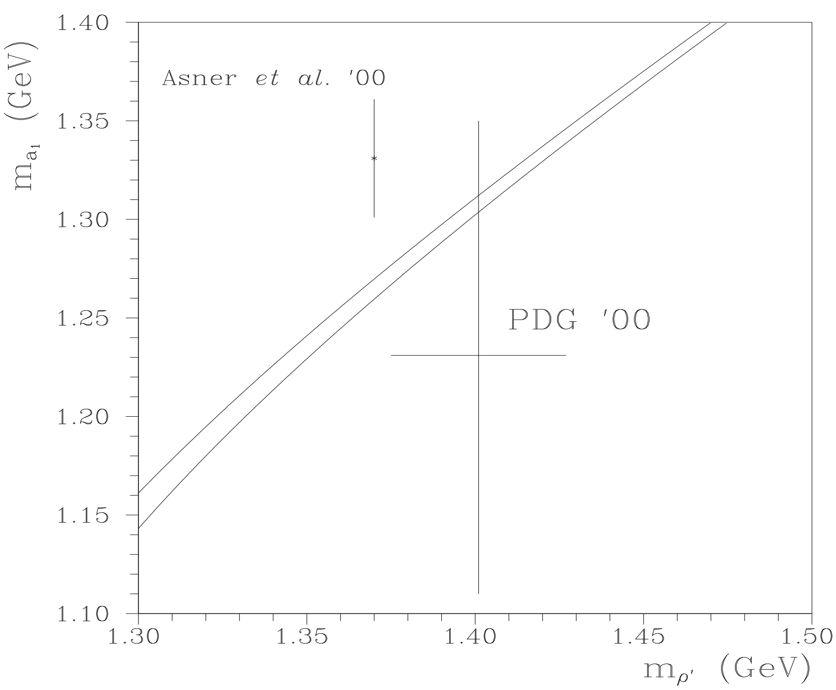

Figure 1: The 3σ allowed region for ρ' and a_1 masses versus the experimental data.

We can conclude that there is reasonable agreement between the prediction (12) and the experimental data. Quantitatively, the higher value of a_1 mass $m_{a_1} = 1307 \pm 40$ MeV is favoured (in comparison with PDG), if the following ρ' mass value $m_{\rho'} = 1401 \pm 26$ MeV [7, 9] is accepted. The latter is confirmed by the latest experimental data. The corresponding quark mass is $m = 235.5 \pm 3.4$ MeV. It leads to the prediction for the mass of the sigma meson $m_\sigma = 2m = 471 \pm 7$ MeV that is also in excellent concordance with the recent experiment [10] $m_\sigma^{exp} = 478 \pm 29$ MeV.

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