# Single gluino production in the R-parity lepton number violating MSSM at the LHC * 

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#### Abstract

We examine the $R_{p}$-violating signal of single gluino production associated with a charged lepton or neutrino at the large hadron collider (LHC), in the model of R-parity relaxed supersymmetric model. If the parameters in the $R_{p}$ supersymmetric interactions are not too small, and the mass of gluino is considered in the range from several GeV (as the Lightest Supersymmetric Particle) to 800 GeV , the cross section of the single gluino production via Drell-Yan processes can be in the order of $10^{2} \sim 10^{3}$ femto barn, and that via gluon fusion in the order of $10^{-1} \sim 10^{3}$ femto barn. If the gluino decay can be well detected in the CERN LHC, this process provides a prospective way to probe supersymmetry and $R_{p}$ violation.


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## I. Introduction

The new physics beyond the standard model (SM) has been intensively studied in the past years[1]. The supersymmetric models (SUSY) are the most attractive ones among the general extended models of the SM. As we know that electroweak gauge invariance requires absence of the terms in the lagrangian that change either baryon number or lepton number. Usually these terms may lead to unacceptable short proton lifetime. One way to solve the proton-decay problem is to impose a discrete symmetry conservation called R-parity $\left(R_{p}\right)$ conservation[2]. Actually this conservation is put into the MSSM for the purpose to retain the symmetries of the SM. But the most general SUSY extension of the SM should contain such terms.

If the R-parity is conserved, all supersymmetric partner particles must be produced in pair, thus the lightest of superparticles must be stable. If R-parity is violated, the feature of the SUSY particles changes a lot. Until now we have been lacking in credible theoretical argument and experimental tests for $R_{p}$ conservation, we can say that the $R_{p}$ violation ( $R_{p}$ ) would be equally well motivated in the supersymmetric extension of the SM. Finding the signature of $R_{p}$ violation has recently motivated some investigation[3] because of experimentally observed discrepancies.

Experimentally searching for the effects of $R_{p}$ interactions has been done with many efforts in the last few years. Unfortunately, up to now we have only some upper limits on $R_{p}$ parameters. It is necessary to continue these works on finding $R_{p}$ signal or getting further stringent constraints on the $R_{p}$ parameters in future experiments. Detecting signals of the supersymmetric particle is a prospective way in searching for R-parity violation. The process of single chargino/neutralino production with $R_{p}$ has been intensively studied in former works[4]. The
strongly interacting supersymmetric paticles, squark $\tilde{q}$ and gluino $\tilde{g}$, can be produced with the largest cross sections at hadron colliders. From the evolution of parameters, $\tilde{g}$ should be the heaviest gauginos at low scale, since the ratios of gaugino masses to coupling constants do not change with scale in one-loop approximation[5][6]. However, in some other models gluino may exist as the Lightest Supersymmetric Particle (LSP)[7]. The production and decay of gluino with or without R-parity conservation have been investigated in [8] [9]. It is showed in Ref.[10] that if gluino is lighter than $\sim 1 \mathrm{TeV}$, the signal of $\tilde{g}$ decay can be detected by initial searches for sparticles at the LHC even if R-parity is not conserved. In Ref.[11] single gluino production via Drell-Yan process with R-parity baryon number violation has been studied. It is showed that the production of gluino can be well detected when the $R_{p}$ parameters are not too small, e.g. $\lambda^{\prime \prime}=0.01 \sim 0.1$.

In this paper we studied the single gluino production associated with a charged lepton or neutrino in the framework of the MSSM with R-parity lepton number violation for both the Drell-Yan process and the one-loop gluon fusion process. In section II we present the model and calculation of the process $p p \rightarrow \tilde{g}+X$. Numerical results and discussion are given in section III. In section IV we give a short summary. The details of expressions in the calculation can be found in the Appendix.

## II. Calculation

The general $R_{p}$ superpotential can be written as

$$
\begin{equation*}
W_{k_{p}}=\epsilon_{i j}\left(\lambda_{I J K} \tilde{L}_{i}^{I} \tilde{L}_{j}^{J} \tilde{R}^{K}+\lambda_{I J K}^{\prime} \tilde{L}_{i}^{I} \tilde{Q}_{j}^{J} \tilde{D}^{K}+\epsilon_{I} H_{i}^{2} \tilde{L}_{j}^{I}\right)+\lambda_{I J K}^{\prime \prime} \tilde{U}^{I} \tilde{D}^{J} \tilde{D}^{K} \tag{1}
\end{equation*}
$$

$L^{I}, Q^{I}, H^{I}$ denotes the $\mathrm{SU}(2)$ doublets of lepton, quark and Higgs superfields respectively,
while $R^{I}, U^{I}, D^{I}$ are the singlets lepton and quark superfields. The bilinear $R_{p}$ term $\epsilon_{i j} \epsilon_{I} H_{i}^{2} \tilde{L}_{j}^{I}$ will lead to the mixture of mass eigenstates and give the neutrino masses. However, their effects are assumed to be negligible in our process. The constraints on the couplings[9],

$$
\begin{equation*}
\mid\left(\lambda \text { or } \lambda^{\prime}\right) \lambda^{\prime \prime} \left\lvert\,<10^{-10}\left(\frac{\tilde{m}}{100 G e V}\right)^{2}\right. \tag{2}
\end{equation*}
$$

is usually taken to indicate that only lepton or baryon number violation exist. We consider only the lepton number violation, i.e., $\lambda$ and $\lambda^{\prime}$ are assumed to be non-zero while $\lambda^{\prime \prime}$ terms are neglected.

In this paper we denote the Drell-Yan tree-level processes

$$
\begin{gather*}
p p \rightarrow u \bar{d}(\bar{u} d) \rightarrow \tilde{g} \tau^{+}\left(\tilde{g} \tau^{-}\right)  \tag{3}\\
p p \rightarrow d \bar{d} \rightarrow \tilde{g} \nu_{\tau} \tag{4}
\end{gather*}
$$

and the one-loop process through gluon fusion

$$
\begin{equation*}
p p \rightarrow g g \rightarrow \tilde{g} \nu_{\tau} \tag{5}
\end{equation*}
$$

Because of charge conjugation invariance, the cross section of subprocess $u \bar{d} \rightarrow \tilde{g} \tau^{+}$coincides with $\bar{u} d \rightarrow \tilde{g} \tau^{-}$. Then we give only the calculations for the subprocess $u \bar{d} \rightarrow \tilde{g} \tau^{+}$. We should mention that there in no $R_{p}$ coupling of $u \overline{\tilde{u}} \nu$ in the lowest order, so we don't consider the $u \bar{u} \rightarrow \tilde{g} \nu$ channel. The cross section of the Drell-Yan process are determined by $R_{p}$ parameters of the first generation, i.e., $\lambda_{311}^{\prime}$, while the gluon fusion process depend on $R_{p}$ parameters of all generations, especially $\lambda_{333}^{\prime}$. Although in our calculation, the contribution of the one-loop gluon fusion process appears to be relatively small, they are not negligible due to the facts that
there is large gluon luminosity in protons and the $R_{p}$ couplings of the third generation could be large.

The Feynman diagrams for the tree-level process $u \bar{d} \rightarrow \tilde{g} \tau$ and $d \bar{d} \rightarrow \tilde{g} \nu_{\tau}$ are given in Fig. 1 and Fig. 2 respectively. The $g g \rightarrow \tilde{g} \nu_{\tau}$ one-loop diagrams at the lowest order are plotted in Fig.3. In our calculation the ultraviolet divergence in subprocess $g g \rightarrow \tilde{g} \nu_{\tau}$ should be cancelled automatically and it's not necessary to consider the renormalization at one-loop level. Fig.3(a) contains the s-channel diagrams, Fig.3(b) the box diagrams, and Fig.3(c) the quartic diagrams. The relevant Feynman rules without $R_{p}$ interactions can be found in references[12]. The related Feynman rules with $R_{p}$ interactions can be read out from Eq.(1), which are listed in the Appendix.

We define the Mandelstam variables as usual

$$
\begin{align*}
& \hat{s}=\left(p_{1}+p_{2}\right)^{2}=\left(k_{1}+k_{2}\right)^{2} \\
& \hat{t}=\left(p_{1}-k_{1}\right)^{2}=\left(p_{2}-k_{2}\right)^{2}  \tag{6}\\
& \hat{u}=\left(p_{1}-k_{2}\right)^{2}=\left(p_{2}-k_{1}\right)^{2}
\end{align*}
$$

The amplitude of $u \bar{d} \rightarrow \tilde{g} \tau^{+}$(Feynman diagrams in Fig.1) is given by

$$
\begin{equation*}
M_{u \bar{d}}=M_{u \bar{d}}^{\hat{t}}+M_{u \bar{d}}^{\hat{u}} \tag{7}
\end{equation*}
$$

where

$$
\begin{align*}
M_{u \bar{d}}^{\hat{t}} & =\bar{u}\left(k_{1}\right)\left[i \sqrt{2} g_{s} T_{\beta \alpha}^{\gamma}\left(-Z_{U}^{* 1 i} P_{L}+Z_{U}^{* 2 i} P_{R}\right)\right] u\left(p_{1}\right) \frac{i}{\bar{t}-m_{\bar{u}_{i}}^{2}} \bar{v}\left(p_{2}\right)\left(i \lambda_{311}^{\prime} Z_{U}^{1 i} P_{R}\right) v\left(k_{2}\right) \\
M_{u \bar{d}}^{\hat{u}} & =-\bar{v}\left(p_{2}\right)\left[i \sqrt{2} g_{s} T_{\beta \alpha}^{\gamma}\left(-Z_{D}^{* 1 i} P_{R}+Z_{D}^{* 2 i} P_{L}\right)\right] v\left(k_{1} \frac{i}{\hat{u}-m_{\tilde{d}_{i}}^{2}} \bar{u}\left(k_{2}\right)\left(i \lambda_{311}^{\prime} Z_{D}^{* 2 i} P_{L}\right) u\left(p_{1}\right)\right. \tag{8}
\end{align*}
$$

$Z_{D}^{i j}$ and $Z_{U}^{i j}$ denote the matrices used to diagonalize the down-type squark and up-type squark mass matrices, respectively. $\alpha, \beta$ and $\gamma$ denote the color indices of the initial up-quark, downquark and final gluino respectively. Similarly the amplitude of $u \bar{d} \rightarrow \tilde{g} \nu_{\tau}$ (Feynman diagrams
in Fig.1) is given by

$$
\begin{equation*}
M_{d \bar{d}}=M_{d \bar{d}}^{\hat{t}}+M_{d \bar{d}}^{\hat{u}}, \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
M_{d \bar{d}}^{\hat{t}}= & \bar{u}\left(k_{1}\right)\left[i \sqrt{2} g_{s} T_{\alpha \beta}^{\gamma}\left(-Z_{D}^{1 i} P_{L}+Z_{D}^{2 i} P_{R}\right)\right] u\left(p_{1}\right) \frac{i}{\bar{t}-m_{\tilde{d}_{i}}^{2}} \\
& \times \bar{v}\left(p_{2}\right)\left[i \lambda_{311}^{\prime}\left(Z_{D}^{2 i} P_{R}+Z_{D}^{* 1 i} P_{L}\right)\right] v\left(k_{2}\right), \\
M_{d \bar{d}}^{\hat{u}}= & -\bar{v}\left(p_{2}\right)\left[i \sqrt{2} g_{s} T_{\beta \alpha}^{\gamma}\left(-Z_{D}^{* 1 i} P_{R}+Z_{D}^{* 2 i} P_{L}\right)\right] v\left(k_{1}\right) \frac{i}{\frac{u}{u}-m_{\tilde{d}_{i}}^{2}}  \tag{10}\\
& \times \bar{u}\left(k_{2}\right)\left[i \lambda_{311}^{\prime}\left(Z_{D}^{* 2 i} P_{L}+Z_{D}^{1 i} P_{R}\right)\right] u\left(p_{1}\right),
\end{align*}
$$

$\alpha, \beta$ and $\gamma$ denote the initial down-quarks and final gluino respectively.
The amplitudes squared summed over the spins and colors can be written explicitly as follows, where we assume that $Z_{D}^{i j}$ and $Z_{U}^{i j}$ are real.

$$
\begin{align*}
\sum\left|M_{u d}\right|^{2} & =8 g_{s}^{2}\left(\lambda_{311}^{\prime}\right)^{2}\left\{\frac{\left(Z_{U}^{1 i}\right)^{2}}{\left(t-m_{\tilde{u}_{i}}^{2}\right)^{2}}\left(\hat{t}-m_{\tau}^{2}\right)\left(\hat{t}-m_{\tilde{g}}^{2}\right)+\frac{\left(Z_{D}^{2 i}\right)^{2}}{\left(\hat{u}-m_{\tilde{d}_{i}}^{2}\right)^{2}}\left(\hat{u}-m_{\tau}^{2}\right)\left(\hat{u}-m_{\tilde{g}}^{2}\right)\right. \\
& \left.-\frac{\left(Z_{U}^{1 i}\right)^{2} Z_{D}^{1 j} Z_{D}^{2 j}}{\left(\hat{t}-m_{\tilde{u}_{i}}^{2}\left(\hat{u}-m_{\tilde{d}_{j}}^{2}\right)\right.}\left[\left(\hat{t}-m_{\tau}^{2}\right)\left(\hat{t}-m_{\tilde{g}}^{2}\right)+\left(\hat{u}-m_{\tau}^{2}\right)\left(\hat{u}-m_{\tilde{g}}^{2}\right)-\hat{s}\left(\hat{s}-m_{\tilde{g}}^{2}-m_{\tau}^{2}\right)\right]\right\} \\
\sum\left|M_{d d}\right|^{2} & =8 g_{s}^{2}\left(\lambda_{311}^{\prime}\right)^{2}\left\{\frac{1}{\left(\hat{t}-m_{\tilde{d}_{i}}^{2}\right)^{2}} \hat{t}\left(\hat{t}-m_{\tilde{g}}^{2}\right)+\frac{1}{\left(\hat{u}-m_{\tilde{d}_{i}}^{2}\right)^{2}} \hat{u}\left(\hat{u}-m_{\tilde{g}}^{2}\right)\right. \\
& \left.-\frac{2 Z_{D}^{1 i} Z_{D}^{1 j} Z_{\hat{D}}^{2 i} Z_{D}^{2 j}}{\left(\hat{t}-m_{\tilde{d}_{i}}^{2}\right)\left(\hat{u}-m_{\tilde{d}_{j}}^{2}\right)}\left[\hat{t}\left(\hat{t}-m_{\tilde{g}}^{2}\right)+\hat{u}\left(\hat{u}-m_{\tilde{g}}^{2}\right)-\hat{s}\left(\hat{s}-m_{\tilde{g}}^{2}\right)\right]\right\} \tag{11}
\end{align*}
$$

The corresponding amplitude of $g\left(p_{1}, a, \mu\right) g\left(p_{2}, b, \nu\right) \rightarrow \tilde{g}\left(k_{1}, c\right) \nu_{\tau}\left(k_{2}\right)$ (Feynman diagrams in Fig.3) can be written as

$$
\begin{align*}
\mathcal{M} & =\mathcal{M}^{b}+\mathcal{M}^{q}+\mathcal{M}^{t r} \\
& =\epsilon^{\mu}\left(p_{1}\right) \epsilon^{\nu}\left(p_{2}\right) \bar{u}\left(k_{1}\right)\left\{f_{1} g_{\mu \nu}+f_{2} k_{1 \mu} k_{1 \nu}+f_{3} g_{\mu \nu} \gamma_{5}+f_{4} k_{1 \mu} k_{1 \nu} \gamma_{5}+f_{5} k_{1 \nu} \gamma_{\mu}+f_{6} k_{1 \mu} \gamma_{\nu}\right. \\
& +f_{7} g_{\mu \nu} \not p_{1}+f_{8} k_{1 \mu} k_{1 \nu} \not_{1}+f_{9} g_{\mu \nu} \not p_{2}+f_{10} k_{1 \mu} k_{1 \nu} \not p_{2}+f_{11} k_{1 \nu} \gamma_{5} \gamma_{\mu}+f_{12} k_{1 \mu} \gamma_{5} \gamma_{\nu} \\
& +f_{13} g_{\mu \nu} \gamma_{5} \not p_{1}+f_{14} k_{1 \mu} k_{1 \nu} \gamma_{5} \not p_{1}+f_{15} g_{\mu \nu} \gamma_{5} \not p_{2}+f_{16} k_{1 \mu} k_{1 \nu} \gamma_{5} \not p_{2}+f_{17} \gamma_{\mu} \gamma_{\nu} \\
& +f_{18} k_{1 \nu} \gamma_{\mu} \not b_{1}+f_{19} k_{1 \mu} \gamma_{\nu} \not p_{2}+f_{20} \gamma_{5} \gamma_{\mu} \gamma_{\nu}+f_{21} k_{1 \nu} \gamma_{5} \gamma_{\mu} \not b_{1}+f_{22} k_{1 \mu} \gamma_{5} \gamma_{\nu} \not 2_{2} \\
& +f_{23} \gamma_{\mu} \gamma_{\nu} p_{1}+f_{24} \gamma_{\mu} \gamma_{\nu} \not 2_{2}+f_{25} k_{1 \nu} \gamma_{\mu} b_{1} \not p_{2}+f_{26} k_{1 \mu} \gamma_{\nu} \not_{1} \not b_{2}+f_{27} \gamma_{5} \gamma_{\mu} \gamma_{\nu} \not p_{1} \\
& +f_{28} \gamma_{5} \gamma_{\mu} \gamma_{\nu} \not p_{2}+f_{29} k_{1 \nu} \gamma_{5} \gamma_{\mu} p_{1} p_{2}+f_{30} k_{1 \mu} \gamma_{5} \gamma_{\nu} p_{1} \not p_{2}+f_{31} \gamma_{\mu} \gamma_{\nu} p_{1} p_{2} \\
& \left.+f_{32} \gamma_{5} \gamma_{\mu} \gamma_{\nu} p_{1} \not p_{2}+f_{33} g_{\mu \nu} \not b_{1} \not p_{2}+f_{34} g_{\mu \nu} \gamma_{5} \not p_{1} \not p_{2}\right\} v\left(k_{2}\right) \tag{12}
\end{align*}
$$

where $\mathcal{M}^{b}, \mathcal{M}^{q}$ and $\mathcal{M}^{t r}$ are the matrix elements contributed by box, quartic and triangle interaction diagrams, respectively.

The cross sections for the subprocesses $u \bar{d}(\bar{u} d, d \bar{d}) \rightarrow \tilde{g} \tau^{+}\left(\tilde{g} \tau^{-}, \tilde{g} \nu_{\tau}\right)$ and $g g \rightarrow \tilde{g} \nu_{\tau}$ can be obtained by using the following equation

$$
\begin{equation*}
\hat{\sigma}(\hat{s})=\frac{1}{16 \pi \hat{s}^{2}} \int_{\hat{t}^{-}}^{\hat{t}^{+}} d \hat{t} \sum^{-}|\mathcal{M}|^{2} \tag{13}
\end{equation*}
$$

where the bar over sum means average over the initial spin and color. In above equations, $\hat{t}$ is the momentum transfer squared from one of the incoming particle to the gluino in the final state. For the subprocesses $g g(d \bar{d}) \rightarrow \tilde{g} \nu_{\tau}$ we have

$$
\hat{t}^{ \pm}=\frac{1}{2}\left[\left(m_{\tilde{g}}^{2}+m_{\nu_{\tau}}^{2}-\hat{s}\right) \pm \sqrt{\left(m_{\tilde{g}}^{2}+m_{\nu_{\tau}}^{2}-\hat{s}\right)^{2}-4 m_{\tilde{g}}^{2} m_{\nu_{\tau}}^{2}}\right]
$$

and for the subprocesses $u \bar{d}(\bar{u} d) \rightarrow \tilde{g} \tau^{+}\left(\tilde{g} \tau^{-}\right)$,

$$
\hat{t}^{ \pm}=\frac{1}{2}\left[\left(m_{\tilde{g}}^{2}+m_{\tau}^{2}-\hat{s}\right) \pm \sqrt{\left(m_{\tilde{g}}^{2}+m_{\tau}^{2}-\hat{s}\right)^{2}-4 m_{\tilde{g}}^{2} m_{\tau}^{2}}\right] .
$$

With the results from Eq.(13), we can easily obtain the total cross section at pp collider by folding the cross section of subprocess with the quark and gluon luminosity correspondingly.

$$
\begin{equation*}
\sigma(s)=\int_{\left(m_{\tilde{g}}+m_{\left.\tau, \nu_{\tau}\right)^{2} / s}^{1}\right.}^{1} d \tau \frac{d \mathcal{L}_{i j}}{d \tau} \hat{\sigma}_{i j}(\hat{s}=\tau s) \tag{14}
\end{equation*}
$$

where $\sqrt{s}$ and $\sqrt{\hat{s}}$ are the $p p$ collision and subprocess c.m.s. energies respectively and $d \mathcal{L}_{i j} / d \tau$ is the distribution function of parton luminosity, which is defined as

$$
\begin{equation*}
\frac{d \mathcal{L}_{i j}}{d \tau}=\frac{1}{1+\delta_{i j}} \int_{\tau}^{1} \frac{d x_{1}}{x_{1}}\left\{\left[f_{i}\left(x_{1}, Q^{2}\right) f_{j}\left(\frac{\tau}{x_{1}}, Q^{2}\right)\right]+\left[f_{j}\left(x_{1}, Q^{2}\right) f_{i}\left(\frac{\tau}{x_{1}}, Q^{2}\right)\right]\right\}, \quad(i \geq j) \tag{15}
\end{equation*}
$$

here $\tau=x_{1} x_{2}$, the definition of $x_{1}$ and $x_{2}$ are from Ref.[13], and in our calculation we adopt the MRS set G parton distribution function [14]. $f_{i, j}\left(x_{n}, Q^{2}\right)$ are the coreesponding quark and
gluon distribution functions of protons. The factorization scale Q was chosen as the average of the final particles masses $\frac{1}{2}\left(m_{\tilde{g}}+m_{\tau, \nu_{\tau}}\right)$.

## III. Numerical Calculations and Discussions

In our numerical calculation to get the low energy scenario from the MSUGRA [5], the renormalization group equations(RGE's)[15] are run from the weak scale $m_{Z}$ up to the GUT scale, taking all thresholds into account. We use two loop RGE's only for the gauge couplings and the one-loop RGE's for the other supersymmetric parameters. The GUT scale boundary conditions are imposed and the RGE's are run back to $m_{Z}$, again taking threshold into account. The effects of $R_{p}$ to RGE's are assumed to be small, and $R_{p}$ parameters in the weak scale are directly taken under the experimental upper bounds.

Since we consider this process via lepton number violation terms in the $R_{p}$ model, and the $R_{p}$ couplings $\lambda^{\prime}$ in the terms of Eq.(1) inducing heavy lepton can be very large from present upper limits[16], we choose $\lambda_{311}^{\prime}=0.05, \lambda_{322}^{\prime}=0.18$ and $\lambda_{333}^{\prime}=0.39$, and the other trilinear parameters, i.e., $\lambda, \lambda_{1 i j}^{\prime}, \lambda_{2 i j}^{\prime}$ and $\lambda^{\prime \prime}$ have no contribution to our process. The effect of bilinear breaking terms are assumed to be very small and can be negligible. The SM input parameters[17] are chosen as: $m_{t}=173.8 \mathrm{GeV}, m_{Z}=91.187 \mathrm{GeV}, m_{b}=4.5 \mathrm{GeV}, \sin ^{2} \theta_{W}=$ 0.2315 , and $\alpha_{E W}=1 / 128$. We take a simple one-loop formula for the running strong coupling constant $\alpha_{s}$

$$
\begin{equation*}
\alpha_{s}(\mu)=\frac{\alpha_{s}\left(m_{Z}\right)}{1+\frac{33-2 n_{f}}{6 \pi} \alpha_{s}\left(m_{Z}\right) \ln \frac{\mu}{m_{Z}}} \tag{16}
\end{equation*}
$$

where $\alpha_{s}\left(m_{Z}\right)=0.117$ and $n_{f}$ is the number of active flavors at the energy scale $\mu$.
If the single gluino is produced in association with a charged lepton, here assumed to be $\tau$,
both final particles can be well detectable. When a neutrino is produced instead of a charged lepton, it would lead to an energy missing. If the gluino decay could be well detected by the detector, it provides a prospective way to observe the $R_{p}$ process. For a heavy gluino, the decay channel

$$
\begin{equation*}
\tilde{g} \rightarrow q \tilde{q} \tag{17}
\end{equation*}
$$

will dominate if kinematically allowed. Since the $R_{p}$ violation parameters are strongly constrained by experimental results, then the dominant subsequent decays of squark are

$$
\begin{equation*}
\tilde{q} \rightarrow q \tilde{\chi}_{i}^{0}, q^{\prime} \tilde{\chi}_{i}^{ \pm} \tag{18}
\end{equation*}
$$

where $\tilde{\chi}_{i}^{0}(\mathrm{i}=1-4)$ denote the neutralinos and $\tilde{\chi}_{j}^{ \pm}(\mathrm{j}=1,2)$ the charginos. The charginos and neutralinos may decay further as

$$
\begin{equation*}
\tilde{\chi}_{j}^{ \pm} \rightarrow \tilde{\chi}_{i}^{0} q \bar{q}^{\prime},(i=1-4, j=1,2), \quad \tilde{\chi}_{k}^{0} \rightarrow \tilde{\chi}_{1}^{0} q \bar{q},(k=2,3,4) \tag{19}
\end{equation*}
$$

Therefore, like the R-parity conservation consequence, the typical signatures for gluinos would mainly be two, four or six jets and missing energy, carried away by the possible LSP $\tilde{\chi}_{1}^{0}$. If squarks are heavier than gluino, the following decays are possible:

$$
\begin{equation*}
\tilde{g} \rightarrow q \bar{q} \tilde{\chi}_{i}^{0}, \quad q \bar{q}^{\prime} \tilde{\chi}_{i}^{ \pm}, \quad g \tilde{\chi}_{i}^{0} \tag{20}
\end{equation*}
$$

The following decays may be important for large $R_{p}$ parameters.

$$
\begin{equation*}
\tilde{g} \rightarrow q \bar{q} \nu, \quad q \bar{q}^{\prime} l, \quad g \nu \tag{21}
\end{equation*}
$$

If gluino is the LSP and $\lambda^{\prime}$ is not extremely small, the above $R_{p}$ processes could be the only possible decay channels and the gluino will decay inside the detector. If the couplings are
very small, the gluino can form bound states with gluon or quarks before decaying, known as R-hardrons[7][18].

In Fig. 4 we depict the dependence of the cross section for the process $p p \rightarrow \tilde{g} \tau^{+}\left(\tilde{g} \nu_{\tau}\right)+X$ on mass of gluino. All the cross sections of the single gluino production via Drell-Yan and gluon fusion subprocesses are plotted. In order to obtain a wide variety of gluino mass, we abolish the MSUGRA model and choose $m_{\tilde{g}}$ and $m_{\tilde{q}_{L, R}}$ arbitrarily. We consider a gluino with its mass varying from 5 GeV , when gluino may be the LSP, up to 800 GeV . We assume that there is no mass mixing between $\tilde{q}_{L}$ and $\tilde{q}_{R}$ for all the up-type- and down-type-squarks of the first two generations, since in general the mixing size is proportional to the mass of the related ordinary quark[19]. As a representation example of the parameter space, we choose the squark masses as below,

$$
\begin{gathered}
m_{\tilde{u}_{L}}=m_{\tilde{c}_{L}}=392.9 \mathrm{GeV}, m_{\tilde{u}_{R}}=m_{\tilde{c}_{R}}=384.3 \mathrm{GeV} \\
m_{\tilde{d}_{L}}=m_{\tilde{s}_{L}}=400.0 \mathrm{GeV}, m_{\tilde{d}_{R}}=m_{\tilde{s}_{R}}=385.1 \mathrm{GeV} \\
m_{\tilde{b}_{1}}=358.5 \mathrm{GeV}, m_{\tilde{b}_{2}}=385.0 \mathrm{GeV}, m_{\tilde{t}_{1}}=312.5 \mathrm{GeV}, m_{\tilde{t}_{2}}=404.4 \mathrm{GeV} .
\end{gathered}
$$

The total cross section of single gluino associated production drops from 153 fb to 4 fb with the increment of $m_{\tilde{g}}$. The figure shows that the single gluino production rate via one-loop process $p p \rightarrow g g \rightarrow \tilde{g}+X$ is comparable with those from other production mechanisms. We can see when the gluino mass is less than 30 GeV , the cross section of $p p \rightarrow g g \rightarrow \tilde{g}+X$ can be over two times larger than the cross section of $p p \rightarrow u \bar{d}(\bar{u} d, d \bar{d}) \rightarrow \tilde{g}+X$ quantitatively. When the gluino mass is greater than 200 GeV , the contribution of gluon fusion to the gluino production process drops to small values. If the integrated luminosity at the LHC is $30 \mathrm{fb}^{-1}$, typically $10^{2} \sim 10^{3}$ raw events can be produced when we take the values of the $R_{p}$ parameters are close
to the present upper bounds. With the parameters taken in Fig.4, the signatures of gluinos can be detected in following ways:
(1) If $m_{\tilde{g}}$ is heavier than 400 GeV , the signals of gluinos can be two, four or six jets together with energy missing, which are induced by the decay of (17) and the subsequent cascade decays of squarks (shown in Eqs.(18)and (19)). The number of jets depends on the kinematical phase space of the decays (17), (18) and (19)
(2) If the mass of gluino is less than about 400 GeV , the gluinos have the possible decays shown in Eq.(20) and charginos and neutralinos subsequent decays as in Eqs.(19). The gluino signals are two or four jets with missing energy, and one or three jets with a energetic lepton.
(3)If the gluinos are the LSP or gluinos are almost degenerate with the lightest neutralino and chargino, $R_{p}$ decays of gluinos shown in Eq.(21)will be significant, their signatures will be one or two jets with missing energy, or two jets with an energetic lepton.

In Fig. 5 we plotted the cross section in the MSUGRA scenario, with $m_{0}$ varying from 100 GeV to 800 GeV . The other input parameters are chosen as: $m_{\frac{1}{2}}=150 \mathrm{GeV}, A_{0}=$ $300 \mathrm{GeV}, \tan \beta=4$ and set $\operatorname{sign}(\mu)=+$. With above MSUGRA input parameters, we get the mass of gluino ranging from 395 GeV to 439 GeV , the lighter scalar top and bottom quarks $\left(\tilde{t}_{1}, \tilde{b}_{1}\right)$ have the masses between 270 GeV to 493 GeV and 332 GeV to 700 GeV respectively, the first two generation squarks are ranged from 346 GeV to 865 GeV . The calculation shows that the gluino mass is not sensitive to $m_{0}$ with those input parameters, then the production cross sections decrease gently with the increment of $m_{0}$. In this case the contribution from the one-loop process of $p p \rightarrow g g \rightarrow \tilde{g}+X$ reaches $1 \%$ of the total cross section of single gluino associated production at the LHC. As we have mentioned, the cross section of the gluon fusion
process is still not negligible since it depends on different $R_{p}$ parameters with the Drell-Yan process. With these MSUGRA parameters, the gluino decays $\tilde{g} \rightarrow q \tilde{q},(q=u, d, c, s)$ are allowed kinematically when $m_{0}$ is about 100 GeV , but the decays of Eq. (20) may be used as gluino signatures when $m_{0}$ approaches 800 GeV .

## IV.Summary

As shown in reference [10][11], the gluino decay can be detectable at the LHC even when R-parity is not conserved. In this work we have studied the single gluino production associated with a charged lepton or neutrino in the $R_{p}$ MSSM at the LHC through the process $p p \rightarrow$ $\tilde{g} \tau^{+}\left(\tilde{g} \nu_{\tau}\right)+X$. We investigated contributions from both the tree-level Drell-Yan process and the one-loop gluon fusion process. In the subprocesses $d \bar{d} \rightarrow \tilde{g} \nu$ and $g g \rightarrow \tilde{g} \nu$, a neutrino will be produced, which leads to an energy missing by neutrino. We studied the dependence of the cross section on $m_{\tilde{g}}$ and $m_{0}$. The results show that when the $R_{p}$-violating coupling parameters are not too small and the $m_{\tilde{g}} \sim 200 \mathrm{GeV}$, the production rate of the single gluino associated with a charged lepton or neutrino can reach 30 femto barn. That means $10^{3}$ raw events can be obtained at the LHC with integrated luminosity $30 \mathrm{fb}^{-1}$. We conclude that the single gluino production associated with a charged lepton or neutrino could be observable at the LHC, when the $R_{p}$-violating parameters are close to the upper bounds. In this case we see the single gluino production associated with a charged lepton or neutrino can provide a quite prospective way to probe supersymmetry and $R_{p}$ violation. Even if we couldn't find the signal of single gluino production in the experiment, we can get more stringent constraints on $R_{p}$ couplings.

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## Appendix

The Feynman rules for the R-parity violating couplings we used are listed below:

$$
\begin{aligned}
& u_{I}-l_{K}-\tilde{D}_{J, m}^{c}:-V_{u_{I} l_{K} \tilde{D}_{J, m}}^{(1)} C^{-1} P_{L} \\
& d_{I}^{c}-l_{K}-\tilde{U}_{J, m}: V_{d_{I} l_{K} \tilde{U}_{J, m}}^{(1)} P_{L} \\
& d_{I}-\nu_{K}-\tilde{D}_{J, m}^{c}: C^{-1}\left[V_{d_{I} \nu_{K} \tilde{D}_{J, m}^{(2) *}}^{\left(P_{L}+V_{d_{I} \nu_{K} \tilde{D}_{J, m}}^{(1) *} P_{R}\right]}\right. \\
& d_{I}^{c}-\nu_{K}-\tilde{D}_{J, m}: V_{d_{I} \nu_{K} \tilde{D}_{J, m}^{(1)}} P_{L}+V_{d_{I} \nu_{K} \tilde{D}_{J, m}}^{(2)} P_{R}
\end{aligned}
$$

where C is the charge conjugation operator, $P_{L, R}=\frac{1}{2}\left(1 \mp \gamma_{5}\right)$. The coefficients of vertices can be written as:

$$
\begin{aligned}
& V_{u_{I} l_{K} \tilde{D}_{J, m}}^{(1)}=i \lambda_{K I J}^{\prime} Z_{D_{J}}^{2 m *}, V_{d_{l_{K}} \tilde{U}_{J, m}}^{(1)}=i \lambda_{K I J}^{\prime} Z_{U_{J}}^{1 m *} \\
& V_{d_{I} \nu_{K} \tilde{D}_{J, m}}^{(1)}=-i \lambda_{K J I}^{\prime} Z_{D_{J}}^{1 m *}, V_{d_{I} \nu_{K} \tilde{D}_{J, m}}^{(2)}=-i \lambda_{K I J}^{\prime} Z_{D_{J}}^{2 m}
\end{aligned}
$$

$Z_{D}^{i j}$ and $Z_{U}^{i j}$ are the matrices to diagonalize the down-squark and up-squark mass matrices, respectively. We write down form factors for one-loop diagrams of the subprocess $g\left(p_{1}, a, \mu\right) g\left(p_{2}, b, \nu\right) \rightarrow$ $\tilde{g}\left(k_{1}, c\right) \nu_{\tau}\left(k_{2}\right)$. The amplitude parts for the $\hat{u}$-channel box diagrams can be obtained from the $\hat{t}$-channel's by doing the exchanges as below:

$$
\mathcal{M}^{\hat{u}}=\mathcal{M}^{\hat{t}}\left(\hat{t} \rightarrow \hat{u}, k_{1} \leftrightarrow k_{2}, \mu \leftrightarrow \nu, a \leftrightarrow b\right)
$$

Then we present only the t-channel form factors for box diagrams. In this appendix, we use the notifications defined below for abbreviation:

$$
\begin{aligned}
& C_{0}^{1, k}, C_{i j}^{1, k}=C_{0}, C_{i j}\left[k_{1},-p_{1}-p_{2}, m_{\tilde{d}_{k}}, m_{d}, m_{d}\right] \\
& C_{0}^{2, k}, C_{i j}^{2, k}=C_{0}, C_{i j}\left[k_{1},-p_{1}-p_{2}, m_{d}, m_{\tilde{d}_{k}}, m_{\tilde{d}_{k}}\right] \\
& C_{0}^{3, k}, C_{i j}^{3, k}=C_{0}, C_{i j}\left[k_{2}, k_{1}, m_{\tilde{d}_{k}}, m_{d}, m_{\tilde{d}_{k}}\right] \\
& C_{0}^{4}=C_{0}\left[-p_{1},-p_{2}, m_{d}, m_{d}, m_{d}\right] \\
& D_{0}^{1, k}, D_{i j}^{1, k}, D_{i j l}^{1, k}=D_{0}, D_{i j}, D_{i j l}\left[k_{1},-p_{1},-p_{2}, m_{\tilde{d}_{k}}, m_{d}, m_{d}, m_{d}\right] \\
& D_{0}^{2, k}, D_{i j}^{2, k}, D_{i j l}^{2, k}=D_{0}, D_{i j}, D_{i j l}\left[k_{1},-p_{2},-p_{1}, m_{d}, m_{\tilde{d}_{k}}, m_{\tilde{d}_{k}}, m_{\tilde{d}_{k}}\right] \\
& D_{0}^{3, k}, D_{i j}^{3, k}, D_{i j l}^{3, k}=D_{0}, D_{i j}, D_{i j l}\left[-p_{2}, k_{1},-p_{1}, m_{\tilde{d}_{k}}, m_{\tilde{d}_{k}}, m_{d}, m_{d}\right] \\
& F_{1, i}^{ \pm}=Z_{D}^{1 i *} V_{D_{i} d \nu}^{(1)} \pm Z_{D}^{2 i *} V_{D_{i} d \nu}^{(2)} \\
& F_{2, i}^{ \pm}=Z_{D}^{1 i *} V_{D_{i} d \nu}^{(2)} \pm Z_{D}^{2 i *} V_{D_{i} d \nu}^{(1)}
\end{aligned}
$$

The form factors in the amplitude of the quartic interaction diagrams Fig.3(b) are expressed as

$$
\begin{aligned}
f_{1}^{q} & =-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}} \sum_{i=1}^{2}\left(C_{0}^{3, i} F_{2, i}^{-} m_{d}-C_{12}^{3, i} m_{\tilde{g}} F_{1, i}^{-}\right) d_{a b c}-h . c . \\
f_{3}^{q} & =-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}} \sum_{i=1}^{2}\left(C_{0}^{3, i} F_{2, i}^{+} m_{d}+C_{12}^{3, i} m_{\tilde{g}} F_{1, i}^{+}\right) d_{a b c}+h . c .
\end{aligned}
$$

For the other form factors of the quartic interaction diagrams, $f_{i}^{q}=0$. The none-zero form factors in the amplitude from the triangle diagrams depicted in Fig.3(c) are listed below:

$$
\begin{aligned}
f_{1}^{t r} & =\frac{i g_{s}^{3} f_{a b c}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2}\left\{C_{21}^{1, i} m_{\tilde{g}}(\hat{t}-\hat{u}) F_{1, i}^{-}-C_{0}^{1, i}(\hat{s}-\hat{t}+\hat{u})\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)-C_{11}^{1, i}\left[m_{\tilde{g}}(\hat{s}\right.\right. \\
& \left.-2 \hat{t}+2 \hat{u}) F_{1, i}^{-}+(-\hat{t}+\hat{u}) F_{2, i}^{-} m_{d}\right]+(\hat{t}-\hat{u})\left[C_{21}^{2, i} m_{\tilde{g}} F_{1, i}^{-}-C_{0}^{2, i} F_{2, i}^{-} m_{d}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\left.+C_{11}^{2, i}\left(m_{\tilde{g}} F_{1, i}^{-}-m_{d} F_{2, i}^{-}\right)\right]\right\}-h . c . \\
f_{3}^{t r} & =-\frac{i g_{s}^{3} f_{a b c}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2}\left\{C_{21}^{1, i} m_{\tilde{g}}(\hat{t}-\hat{u}) F_{1, i}^{+}-C_{0}^{1, i}(\hat{s}-\hat{t}+\hat{u})\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)-C_{11}^{1, i}\left[m_{\tilde{g}}(\hat{s}\right.\right. \\
& \left.-2 \hat{t}+2 \hat{u}) F_{1, i}^{+}+(\hat{t}-\hat{u}) F_{2, i}^{+} m_{d}\right]+(\hat{t}-\hat{u})\left[C_{21}^{2, i} m_{\tilde{g}} F_{1, i}^{+}+C_{0}^{2, i} F_{2, i}^{+} m_{d}\right. \\
& \left.\left.+C_{11}^{2, i}\left(m_{\tilde{g}} F_{1, i}^{+}+m_{d} F_{2, i}^{+}\right)\right]\right\}+h . c . \\
f_{7}^{t r} & =\frac{i g_{s}^{3} f_{a b c}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2} F_{1, i}^{-}\left\{C_{22}^{1, i} \hat{s}+C_{21}^{1, i} m_{\tilde{g}}^{2}+C_{12}^{1, i}\left[\hat{s}-2\left(m_{\tilde{g}}^{2}-\hat{u}\right)\right]-2 C_{24}^{1, i}+2 C_{11}^{1, i} m_{\tilde{g}}^{2}\right. \\
& \left.-2 C_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+C_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)+2 C_{24}^{2, i}-\left(C_{12}^{2, i}+C_{23}^{2, i}\right)(\hat{t}-\hat{u})\right\}-h . c . \\
f_{9}^{t r} & =-\frac{i g_{s}^{3} f_{a b c}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2} F_{1, i}^{-}\left\{C_{22}^{1, i} \hat{s}+C_{21}^{1, i} m_{\tilde{g}}^{2}+C_{12}^{1, i}\left[\hat{s}-2\left(m_{\tilde{g}}^{2}-\hat{t}\right)\right]-2 C_{24}^{1, i}+2 C_{11}^{1, i} m_{\tilde{g}}^{2}\right. \\
& \left.-2 C_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+C_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)+2 C_{24}^{2, i}+\left(C_{12}^{2, i}+C_{23}^{2, i}\right)(\hat{t}-\hat{u})\right\}-h . c . \\
f_{13}^{t r} & =\frac{i g_{s}^{3} f_{a b c}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2} F_{1, i}^{+}\left\{C_{22}^{1, i} \hat{s}+C_{21}^{1, i} m_{\tilde{g}}^{2}+C_{12}^{1, i}\left[\hat{s}-2\left(m_{\tilde{g}}^{2}-\hat{u}\right)\right]-2 C_{24}^{1, i}+2 C_{11}^{1, i} m_{\tilde{g}}^{2}\right. \\
& \left.-2 C_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+C_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)+2 C_{24}^{2, i}-\left(C_{12}^{2, i}+C_{23}^{2, i}\right)(\hat{t}-\hat{u})\right\}+h . c . \\
f_{15}^{t r} & =-\frac{i g_{s}^{3} f_{a b c}^{32}}{32 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2} F_{1, i}^{+}\left\{C_{22}^{1, i} \hat{s}+C_{21}^{1, i} m_{\tilde{g}}^{2}+C_{12}^{1, i}\left[\hat{s}-2\left(m_{\tilde{g}}^{2}-\hat{t}\right)\right]-2 C_{24}^{1, i}+2 C_{11}^{1, i} m_{\tilde{g}}^{2}\right. \\
& \left.-2 C_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+C_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)+2 C_{24}^{2, i}+\left(C_{12}^{2, i}+C_{23}^{2, i}\right)(\hat{t}-\hat{u})\right\}+h . c . \\
f_{33}^{t r} & =\frac{i g_{s}^{3} f_{a b c}^{2}}{16 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2}\left[C_{11}^{1, i} m_{\tilde{g}} F_{1, i}^{-}+C_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+m_{d} F_{2, i}^{-}\right)\right]-h . c . \\
f_{34}^{t r} & =-\frac{i g_{s}^{3} f_{a b c}^{16 \sqrt{2} \pi^{2} \hat{s}} \sum_{i=1}^{2}\left[C_{11}^{1, i} m_{\tilde{g}} F_{1, i}^{+}+C_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-m_{d} F_{2, i}^{+}\right)\right]+h . c .}{}
\end{aligned}
$$

The none-zero form factors of the amplitude part form t-channel box diagrams, Fig.3(a) are written as

$$
\begin{aligned}
f_{1}^{b, t} & =-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{311}^{2, i} m_{\tilde{g}} F_{1, i}^{-}-D_{27}^{2, i} F_{2, i}^{-} m_{d}-D_{312}^{3, i} m_{\tilde{g}} F_{1, i}^{-}\right. \\
& \left.-D_{27}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)-D_{311}^{1, i} m_{\tilde{g}} F_{1, i}^{-}-D_{27}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+m_{d} F_{2, i}^{-}\right)\right]-h . c . \\
f_{2}^{b, t} & =-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[-D_{31}^{2, i} m_{\tilde{g}} F_{1, i}^{-}+D_{0}^{2, i} F_{2, i}^{-} m_{d}-D_{11}^{2, i}\left(m_{\tilde{g}} F_{1, i}^{-}-2 F_{2, i}^{-} m_{d}\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
& -\quad D_{12}^{2, i}\left(2 m_{\tilde{g}} F_{1, i}^{-}-F_{2, i}^{-} m_{d}\right)+D_{32}^{3, i} m_{\tilde{g}} F_{1, i}^{-}+D_{12}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right) \\
& +\quad D_{22}^{3, i}\left(2 m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)+D_{31}^{1, i} m_{\tilde{g}} F_{1, i}^{-}+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right) \\
& \left.+D_{21}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)+D_{11}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{-}+2 F_{2, i}^{-} m_{d}\right)\right]-h . c . \\
& f_{3}^{b, t}=\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{311}^{2, i} m_{\tilde{g}} F_{1, i}^{+}+D_{27}^{2, i} F_{2, i}^{+} m_{d}-D_{312}^{3, i} m_{\tilde{g}} F_{1, i}^{+}\right. \\
& \left.-D_{27}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)-D_{311}^{1, i} m_{\tilde{g}} F_{1, i}^{+}-D_{27}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\right]+h . c . \\
& f_{4}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{31}^{2, i} m_{\tilde{g}} F_{1, i}^{+}+D_{0}^{2, i} F_{2, i}^{+} m_{d}+D_{12}^{2, i}\left(2 m_{\tilde{g}} F_{1, i}^{+}+F_{2, i}^{+} m_{d}\right)\right. \\
& +D_{11}^{2, i}\left(m_{\tilde{g}} F_{1, i}^{+}+2 F_{2, i}^{+} m_{d}\right)-D_{32}^{3, i} m_{\tilde{g}} F_{1, i}^{+}-D_{12}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right) \\
& -\quad D_{22}^{3, i}\left(2 m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)-D_{31}^{1, i} m_{\tilde{g}} F_{1, i}^{+}-D_{11}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{+}-2 F_{2, i}^{+} m_{d}\right) \\
& \left.-D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)-D_{21}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\right]+ \text { h.c. } \\
& f_{5}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left[2\left(D_{27}^{2, i}+D_{311}^{2, i}\right)+2 D_{27}^{3, i}+4 D_{312}^{3, i}-D_{32}^{3, i} m_{\tilde{g}}^{2}-D_{310}^{3, i} \hat{s}\right. \\
& +D_{38}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{26}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}\right)+D_{36}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{24}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right) \\
& -D_{22}^{3, i}\left(m_{\tilde{g}}^{2}+\hat{u}\right)-D_{12}^{3, i}\left(\hat{s}+\hat{u}-m_{d}^{2}\right)+4\left(D_{27}^{1, i}+D_{311}^{1, i}\right)+\left(-3 D_{21}^{1, i}-D_{31}^{1, i}\right) m_{\tilde{g}}^{2}-\left(D_{26}^{1, i}+D_{310}^{1, i}\right) \hat{s} \\
& +D_{12}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+2 D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{34}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{25}^{1, i}\left(2 m_{\tilde{g}}^{2}-\hat{s}-2 \hat{u}\right) \\
& \left.+D_{35}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)-D_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)-D_{11}^{1, i}\left(3 m_{\tilde{g}}^{2}-m_{d}^{2}\right)\right]-h . c . \\
& f_{6}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left[2 D_{27}^{2, i}+2 D_{311}^{2, i}-2 D_{27}^{3, i}-2 D_{312}^{3, i}-C_{0}^{4}+2 D_{27}^{1, i}\right. \\
& \left.-2 D_{311}^{1, i}-D_{21}^{1, i} m_{\tilde{g}}^{2}-D_{26}^{1, i} \hat{s}+D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{25}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{0}^{1, i} m_{\tilde{d}_{i}}^{2}\right]-h . c . \\
& f_{7}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(-D_{313}^{2, i}+D_{27}^{3, i}+D_{313}^{3, i}+D_{27}^{1, i}+D_{312}^{1, i}\right)-h . c . \\
& f_{8}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(2 D_{25}^{2, i}+D_{13}^{2, i}+D_{35}^{2, i}-D_{26}^{3, i}-D_{38}^{3, i}-2 D_{24}^{1, i}-D_{12}^{1, i}-D_{34}^{1, i}\right)-h . c . \\
& f_{9}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(-D_{312}^{2, i}+D_{27}^{3, i}+D_{311}^{3, i}+D_{313}^{1, i}\right)-h . c .
\end{aligned}
$$

$$
\begin{aligned}
& f_{10}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(2 D_{24}^{2, i}+D_{12}^{2, i}+D_{34}^{2, i}-D_{12}^{3, i}-D_{22}^{3, i}-D_{24}^{3, i}-D_{36}^{3, i}-2 D_{25}^{1, i}\right. \\
& \left.-D_{13}^{1, i}-D_{35}^{1, i}\right)-h . c . \\
& f_{11}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left[2\left(D_{27}^{2, i}+D_{311}^{2, i}\right)+2 D_{27}^{3, i}+4 D_{312}^{3, i}-D_{32}^{3, i} m_{\tilde{g}}^{2}-D_{310}^{3, i} \hat{s}\right. \\
& +D_{38}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{26}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}\right)+D_{36}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{24}^{3, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right) \\
& -D_{22}^{3, i}\left(m_{\tilde{g}}^{2}+\hat{u}\right)-D_{12}^{3, i}\left(\hat{s}+\hat{u}-m_{d}^{2}\right)+4\left(D_{27}^{1, i}+D_{311}^{1, i}\right)+\left(-3 D_{21}^{1, i}-D_{31}^{1, i}\right) m_{\tilde{g}}^{2} \\
& -\left(D_{26}^{1, i}+D_{310}^{1, i}\right) \hat{s}+D_{12}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+2 D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{34}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right) \\
& +D_{25}^{1, i}\left(2 m_{\tilde{g}}^{2}-\hat{s}-2 \hat{u}\right)+D_{35}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)-D_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right) \\
& \left.-D_{11}^{1, i}\left(3 m_{\tilde{g}}^{2}-m_{d}^{2}\right)\right]+h . c . \\
& f_{12}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left[2 D_{27}^{2, i}+2 D_{311}^{2, i}-2 D_{27}^{3, i}-2 D_{312}^{3, i}-C_{0}^{4}+2 D_{27}^{1, i}\right. \\
& \left.-2 D_{311}^{1, i}-D_{21}^{1, i} m_{\tilde{g}}^{2}-D_{26}^{1, i} \hat{s}+D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{25}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)+D_{0}^{1, i} m_{\tilde{d}_{i}}^{2}\right]+h . c . \\
& f_{13}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(-D_{313}^{2, i}+D_{27}^{3, i}+D_{313}^{3, i}+D_{27}^{1, i}+D_{312}^{1, i}\right)+h . c . \\
& f_{14}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(2 D_{25}^{2, i}+D_{13}^{2, i}+D_{35}^{2, i}-D_{26}^{3, i}-D_{38}^{3, i}-2 D_{24}^{1, i}-D_{12}^{1, i}-D_{34}^{1, i}\right)+h . c . \\
& f_{15}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(-D_{312}^{2, i}+D_{27}^{3, i}+D_{311}^{3, i}+D_{313}^{1, i}\right)+h . c . \\
& f_{16}^{b, t}=-\frac{g_{s}^{3}}{16 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(2 D_{24}^{2, i}+D_{12}^{2, i}+D_{34}^{2, i}-D_{12}^{3, i}-D_{22}^{3, i}-D_{24}^{3, i}-D_{36}^{3, i}-2 D_{25}^{1, i}\right. \\
& \left.-D_{13}^{1, i}-D_{35}^{1, i}\right)+h . c . \\
& f_{17}^{b, t}=-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left\{\left[-D_{31}^{1, i} m_{\tilde{g}}^{2}+6 D_{311}^{1, i}-D_{310}^{1, i} \hat{s}+D_{34}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{35}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)\right] m_{\tilde{g}} F_{1, i}^{-}\right. \\
& +2 D_{27}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{-}+2 F_{2, i}^{-} m_{d}\right)+D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)\left(2 m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right) \\
& -D_{21}^{1, i} m_{\tilde{g}}\left[\left(2 m_{\tilde{g}}^{2}+\hat{t}\right) F_{1, i}^{-}+m_{\tilde{g}} F_{2, i}^{-} m_{d}\right]+D_{25}^{1, i}\left[m_{\tilde{g}}\left(2 m_{\tilde{g}}^{2}-\hat{s}-2 \hat{u}\right) F_{1, i}^{-}+\left(m_{\tilde{g}}^{2}-\hat{u}\right) F_{2, i}^{-} m_{d}\right] \\
& -D_{11}^{1, i}\left[\left(m_{\tilde{g}}^{2}+\hat{t}\right) F_{2, i}^{-} m_{d}+m_{\tilde{g}} F_{1, i}^{-}\left(m_{\tilde{g}}^{2}+2 \hat{t}-m_{d}^{2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& \left.+\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)\left[-D_{26}^{1, i} \hat{s}+D_{12}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)+D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{0}^{1, i}\left(-\hat{t}+m_{d}^{2}\right)\right]\right\}-h . c . \\
& f_{18}^{b, t}=\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{22}^{3, i} m_{\tilde{g}} F_{1, i}^{-}+D_{12}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)+D_{21}^{1, i} m_{\tilde{g}} F_{1, i}^{-}\right. \\
& \left.+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)+D_{11}^{1, i}\left(2 m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)\right]-h . c . \\
& f_{19}^{b, t}=\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{21}^{1, i} m_{\tilde{g}} F_{1, i}^{-}+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)+D_{11}^{1, i}\left(2 m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)\right]-h . c . \\
& f_{20}^{b, t}=-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left\{\left[D_{31}^{1, i} m_{\tilde{g}}^{2}-6 D_{311}^{1, i}+D_{310}^{1, i} \hat{s}-D_{34}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)-D_{35}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)\right] m_{\tilde{g}} F_{1, i}^{+}\right. \\
& -2 D_{27}^{1, i}\left(3 m_{\tilde{g}} F_{1, i}^{+}-2 F_{2, i}^{+} m_{d}\right)-D_{24}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)\left(2 m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right) \\
& +D_{21}^{1, i} m_{\tilde{g}}\left[\left(2 m_{\tilde{g}}^{2}+\hat{t}\right) F_{1, i}^{+}-m_{\tilde{g}} F_{2, i}^{+} m_{d}\right]-D_{25}^{1, i}\left[m_{\tilde{g}}\left(2 m_{\tilde{g}}^{2}-\hat{s}-2 \hat{u}\right) F_{1, i}^{+}+\left(-m_{\tilde{g}}^{2}+\hat{u}\right) F_{2, i}^{+} m_{d}\right] \\
& +\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\left[D_{26}^{1, i} \hat{s}-D_{12}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)-D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{0}^{1, i}\left(\hat{t}-m_{d}^{2}\right)\right] \\
& \left.+\quad D_{11}^{1, i}\left[-\left(m_{\tilde{g}}^{2}+\hat{t}\right) F_{2, i}^{+} m_{d}+m_{\tilde{g}} F_{1, i}^{+}\left(m_{\tilde{g}}^{2}+2 \hat{t}-m_{d}^{2}\right)\right]\right\}+ \text { h.c. } \\
& f_{21}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{22}^{3, i} m_{\tilde{g}} F_{1, i}^{+}+D_{12}^{3, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)+D_{21}^{1, i} m_{\tilde{g}} F_{1, i}^{+}\right. \\
& \left.+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)+D_{11}^{1, i}\left(2 m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\right]+h . c . \\
& f_{22}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{21}^{1, i} m_{\tilde{g}} F_{1, i}^{+}+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)+D_{11}^{1, i}\left(2 m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\right]+h . c . \\
& f_{23}^{b, t}=-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left[-2 D_{27}^{3, i}-4 D_{27}^{1, i}-6 D_{312}^{1, i}+D_{34}^{1, i} m_{\tilde{g}}^{2}+D_{38}^{1, i} \hat{s}+\left(D_{22}^{1, i}+D_{36}^{1, i}\right)\left(-m_{\tilde{g}}^{2}+\hat{t}\right)\right. \\
& \left.+\quad D_{24}^{1, i}\left(m_{\tilde{g}}^{2}+\hat{t}\right)-D_{310}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)-D_{26}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{12}^{1, i}\left(\hat{t}-m_{d}^{2}\right)\right]-h . c . \\
& f_{24}^{b, t}=-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left[-2 D_{27}^{1, i}-6 D_{313}^{1, i}+\left(2 D_{11}^{1, i}+D_{21}^{1, i}+D_{35}^{1, i}\right) m_{\tilde{g}}^{2}\right. \\
& +D_{39}^{1, i} \hat{s}-D_{310}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)-D_{26}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}\right)+\left(D_{12}^{1, i}+D_{24}^{1, i}\right)\left(-m_{\tilde{g}}^{2}+\hat{t}\right)-D_{37}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right) \\
& \left.-D_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{25}^{1, i}(\hat{t}+\hat{u})+D_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)-D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}-\hat{u}+m_{d}^{2}\right)\right]-h . c . \\
& f_{25}^{b, t}=-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(D_{12}^{3, i}+D_{24}^{3, i}+D_{13}^{1, i}+D_{25}^{1, i}\right)-h . c . \\
& f_{26}^{b, t}=\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{-}\left(D_{12}^{1, i}+D_{24}^{1, i}\right)-h . c .
\end{aligned}
$$

$$
\begin{aligned}
f_{27}^{b, t} & =-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left[-2 D_{27}^{3, i}-4 D_{27}^{1, i}-6 D_{312}^{1, i}+D_{34}^{1, i} m_{\tilde{g}}^{2}+D_{38}^{1, i} \hat{s}+\left(D_{22}^{1, i}+D_{36}^{1, i}\right)\left(-m_{\tilde{g}}^{2}+\hat{t}\right)\right. \\
& \left.+D_{24}^{1, i}\left(m_{\tilde{g}}^{2}+\hat{t}\right)-D_{310}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right)-D_{26}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{12}^{1, i}\left(\hat{t}-m_{d}^{2}\right)\right]+h . c . \\
f_{28}^{b, t} & =-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left[-2 D_{27}^{1, i}-6 D_{313}^{1, i}+\left(2 D_{11}^{1, i}+D_{21}^{1, i}+D_{35}^{1, i}\right) m_{\tilde{g}}^{2}\right. \\
& +D_{39}^{1, i} \hat{s}-D_{310}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{t}\right)-D_{26}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}\right)+\left(D_{12}^{1, i}+D_{24}^{1, i}\right)\left(-m_{\tilde{g}}^{2}+\hat{t}\right)-D_{37}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{u}\right) \\
& \left.-D_{23}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{u}\right)+D_{25}^{1, i}(\hat{t}+\hat{u})+D_{0}^{1, i}\left(m_{\tilde{g}}^{2}-m_{d}^{2}\right)-D_{13}^{1, i}\left(m_{\tilde{g}}^{2}-\hat{s}-\hat{t}-\hat{u}+m_{d}^{2}\right)\right]+h . c . \\
f_{29}^{b, t} & =-\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(D_{12}^{3, i}+D_{24}^{3, i}+D_{13}^{1, i}+D_{25}^{1, i}\right)+h . c . \\
f_{30}^{b, t} & =\frac{g_{s}^{3}}{32 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right) F_{1, i}^{+}\left(D_{12}^{1, i}+D_{24}^{1, i}\right)+h . c . \\
f_{31}^{b, t} & =\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{11}^{1, i} m_{\tilde{g}} F_{1, i}^{-}+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{-}+F_{2, i}^{-} m_{d}\right)\right]-h . c . \\
f_{32}^{b, t} & =-\frac{g_{s}^{3}}{64 \sqrt{2} \pi^{2}}\left(d_{a b c}-i f_{a b c}\right)\left[D_{11}^{1, i} m_{\tilde{g}} F_{1, i}^{+}+D_{0}^{1, i}\left(m_{\tilde{g}} F_{1, i}^{+}-F_{2, i}^{+} m_{d}\right)\right]+h . c .
\end{aligned}
$$

In this work we follow the definitions of two-, three-, and four-point one loop integral functions of Passarino-Veltman as shown in Ref.[20]. All the vector and tensor integrals can be calculated by deducing them into the forms of scalar integrals [21].

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## Figure captions

Fig. 1 The $R_{p}$ MSSM tree-level diagrams of the process $u \bar{d} \rightarrow \tilde{g} l^{+}$.
Fig. 2 The $R_{p}$ MSSM tree-level diagrams of the process $d \bar{d} \rightarrow \tilde{g} \nu$.
Fig. 3 The $R_{p}$ MSSM one-loop diagrams of the process $g g \rightarrow \tilde{g} \nu$. (a) box diagrams; (b) quartic coupling diagrams; (c) triangle diagrams.

Fig. 4 The cross sections of all the processes contributing to the production of single gluino associated with a charged lepton or neutrino at the LHC are depicted as functions of the gluino mass, with the c.m. collision energy $\sqrt{s}$ at 14 TeV and $m_{\tilde{g}}$ varying from 5 GeV to 800 GeV . The full-line is for total cross section of single gluino production associated with a charged lepton or neutrino $p p \rightarrow \tilde{g}+X$, The dotted-line for $p p \rightarrow \bar{u} d \rightarrow \tilde{g}+X$. The dashed-line for $p p \rightarrow u \bar{d} \rightarrow \tilde{g}+X$. The dash-dotted-line for $p p \rightarrow d \bar{d} \rightarrow \tilde{g}+X$. The long-dashed and short-dashed line for $p p \rightarrow g g \rightarrow \tilde{g}+X$.

Fig. 5 The cross sections of all the processes contributing to the production of single gluino associated with a charged lepton or neutrino at the LHC are depicted as functions of parameter $m_{0}$ in the scenario of $R_{p}$ conserved MSUGRA, with the c.m. collision energy $\sqrt{s}$ at 14 TeV . The parameter $m_{0}$ is chosen to vary from 100 GeV to 800 GeV . The full-line is for total cross section of single gluino production associated with a charged lepton or neutrino $p p \rightarrow \tilde{g}+X$, The dotted-line for $p p \rightarrow \bar{u} d \rightarrow \tilde{g}+X$. The dash-line for $p p \rightarrow u \bar{d} \rightarrow \tilde{g}+X$. The dash-dotted-line for $p p \rightarrow d \bar{d} \rightarrow \tilde{g}+X$. The long-dashed and short-dashed line for $p p \rightarrow g g \rightarrow \tilde{g}+X$.


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