EARTH MATTER EFFECTS ON THE SUPERNOVA NEUTRINO SPECTRA

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We explore the earth matter effects on the energy spectra of neutrinos from a supernova. We show that the observations of the energy spectra of ν_e and $\bar{\nu}_e$ from a galactic supernova may enable us to identify the solar neutrino solution, to determine the sign of Δm^2_{32}, and to probe the mixing matrix element $|U_{e3}|^2$ to values as low as 10^{-3}. We point out scenarios in which the matter effects can even be established through the observation of the spectrum at a single detector.

1 Introduction

In the solutions of the solar and atmospheric neutrino anomalies through the oscillations between the three active neutrino species, three types of ambiguities remain to be resolved: (i) the solution of the solar neutrino problem, (ii) the mass hierarchy – normal ($m_3 > m_1, m_2$) or inverted ($m_3 < m_1, m_2$), and (iii) the value of $|U_{e3}|^2$. The energy spectra of neutrinos from a Type II supernova contain signatures that can resolve some of these ambiguities irrespective of the model of supernova dynamics [1]. Here we concentrate on the effects due to the passage of these neutrinos through the earth which may give spectacular signatures of some of the neutrino mass and mixing schemes.

The neutrinos from the supernova arrive at the earth as an effectively incoherent mixture of the mass eigenstates in vacuum. Depending on the relative position of the supernova and the neutrino detector at the time of arrival, the neutrinos have to travel different distances through the earth. The mass eigenstates in vacuum then get intermixed and the observed energy spectra of neutrinos are affected. Since the energy range of the supernova neutrinos overlaps with the energy range for the MSW resonance with $\Delta m^2_{\odot}$, the matter effects can become large. In that case, we have distinctive signatures of some of the neutrino mixing schemes. The comparison of the neutrino spectra observed at two detectors can be used to measure the earth matter effects, but in certain situations even the spectrum observed at a single detector can perform the task.

2 Neutrino Conversions in the star and the earth

The neutrino transitions between different matter eigenstates inside the supernova take place mainly in the resonance regions H and L, which are char-

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acterized by $(\Delta m^2_{atm}, 4|U_{e3}|^2)$ and $(\Delta m^2_\odot, \sin^2 2\theta_\odot)$ respectively. Due to the Δm^2-hierarchy $(\Delta m^2_{atm} \gg \Delta m^2_\odot)$, the dynamics in each of the two resonance layers can be considered independently as a 2ν transition [2]. The final neutrino fluxes can then be written in terms of *survival probabilities* p and $\bar{p}$ of ν_e and $\bar{\nu}_e$ respectively (apart from the geometrical factor of $1/R^2$):

$$\begin{bmatrix} F_e \\ F_{\bar{e}} \\ 4F_x \end{bmatrix} = \begin{bmatrix} p & 0 & 1-p \\ 0 & \bar{p} & 1-\bar{p} \\ 1-p & 1-\bar{p} & 2+p+\bar{p} \end{bmatrix} \begin{bmatrix} F_e^0 \\ F_{\bar{e}}^0 \\ F_x^0 \end{bmatrix} , \tag{1}$$

where $F_e(F_e^0), F_{\bar{e}}(F_{\bar{e}}^0)$ and $F_x(F_x^0)$ are the final (initial) fluxes of $\nu_e, \bar{\nu}_e$ and ν_x (each of the non-electron neutrino or antineutrino species) respectively.

Let $P_H(\bar{P}_H)$ and $P_L(\bar{P}_L)$ be the probabilities that the neutrinos (antineutrinos) jump to another matter eigenstate in the resonance layers H and L respectively. Adiabaticity in the resonance layer H divides the possible range of $|U_{e3}|^2$ into three regions [1,3]: (I) completely adiabatic transitions, $P_H \lesssim 0.1, |U_{e3}|^2 \gtrsim 10^{-3}$, (II) partially adiabatic transitions, $0.1 \lesssim P_H \lesssim 0.9, 10^{-5} \lesssim |U_{e3}|^2 \lesssim 10^{-3}$, (III) completely adiabatic transitions, $P_H \gtrsim 0.9, |U_{e3}|^2 \lesssim 10^{-5}$. (For inverted hierarchy, P_H needs to be replaced by $\bar{P}_H$. The ranges of $|U_{e3}|^2$ do not change much, however.)

The values of p and $\bar{p}$ are determined by these *flip probabilities* and the mixing matrix elements $|U_{ei}|^2$:

$$p = \sum a_i |U_{ei}|^2 \quad , \quad \bar{p} = \sum \bar{a} |U_{ei}|^2 \ , \tag{2}$$

where a_i and $\bar{a}_i$ are as given in Table 1.

Table 1. The values of the coefficients a_i and $\bar{a}_i$ for normal and inverted hierarchies

	Hierarchy	
	normal	inverted
a_1	$P_H P_L$	P_L
a_2	$P_H(1-P_L)$	$1-P_L$
a_3	$1-P_H$	0
$\bar{a}_1$	$1-\bar{P}_L$	$\bar{P}_H(1-\bar{P}_L)$
$\bar{a}_2$	$\bar{P}_L$	$\bar{P}_H \bar{P}_L$
$\bar{a}_3$	0	$1-\bar{P}_H$

The mass eigenstates arriving at the surface of the earth oscillate in the earth matter. Let P_{ie} be the probability that a mass eigenstate ν_i entering the earth reaches the detector as a ν_e. The flux of ν_e at the detector is

$$F_e^D = \sum_i P_{ie} F_i = F_e^0 \sum a_i P_{ie} + F_x^0 (1 - \sum_i a_i P_{ie}) \ , \tag{3}$$

where we have used the unitarity condition $\sum_i P_{ie} = 1$. Thus, the ν_e flux at the detector can be written as

$$F_e^D = p^D F_e^0 + (1-p^D) F_x^0 \quad \text{with} \quad p^D = \sum_i a_i P_{ie} \ . \tag{4}$$

Similarly, for the antineutrinos, we can write

$$F_{\bar{e}}^D = \bar{p}^D F_{\bar{e}}^0 + (1 - \bar{p}^D) F_x^0 \quad \text{with} \quad \bar{p}^D = \sum_i \bar{a}_i \bar{P}_{ie} \quad . \tag{5}$$

The difference in the ν_e fluxes at the detector due to the propagation in earth equals

$$F_e^D - F_e = (p^D - p)(F_e^0 - F_x^0) \quad , \quad F_{\bar{e}}^D - F_{\bar{e}} = (\bar{p}^D - \bar{p})(F_{\bar{e}}^0 - F_x^0) \quad . \tag{6}$$

2.1 Neutrinos

In the case of normal hierarchy, using Table 1 we get

$$p^D - p = P_H (P_{2e} - |U_{e2}|^2)(1 - 2P_L) + (P_{3e} - |U_{e3}|^2)(1 - P_H - P_H P_L) \quad . \tag{7}$$

The second term in (7) can be neglected. Indeed, inside the earth, ν_3 oscillates with a very small depth:

$$P_{3e} - |U_{e3}|^2 \lesssim \left(\frac{2EV_{earth}}{\Delta m_{atm}^2} \right) \sin^2 2\theta_{e3} \quad , \tag{8}$$

where V_{earth} is the effective potential of ν_e in the earth. For neutrino energies of $5 - 50$ MeV, we have $2EV_{earth}/\Delta m_{atm}^2 \lesssim 10^{-2}$. Moreover, $\sin^2 2\theta_{e3} \leq 0.1$, so that $P_{3e} - |U_{e3}|^2 \leq 10^{-3}$. Then (7) becomes

$$p^D - p \approx P_H (P_{2e} - |U_{e2}|^2)(1 - 2P_L) \quad . \tag{9}$$

In general, when the signals from two detectors D1 and D2 are compared, we get the difference of fluxes

$$F_e^{D1} - F_e^{D2} \approx P_H \cdot (1 - 2P_L) \cdot (P_{2e}^{(1)} - P_{2e}^{(2)}) \cdot (F_e^0 - F_x^0) \quad , \tag{10}$$

where $P_{2e}^{(1)}$ and $P_{2e}^{(2)}$ are the $\nu_e \leftrightarrow \nu_2$ oscillation probabilities for the detectors D1 and D2 correspondingly.

For inverted hierarchy, using Table 1 we get

$$p^D - p \approx (P_{2e} - |U_{e2}|^2)(1 - 2P_L) \quad , \tag{11}$$

so that

$$F_e^{D1} - F_e^{D2} \approx (1 - 2P_L) \cdot (P_{2e}^{(1)} - P_{2e}^{(2)}) \cdot (F_e^0 - F_x^0) \quad . \tag{12}$$

2.2 Antineutrinos

In the case of normal hierarchy, Table 1 leads to

$$\bar{p}^D - \bar{p} = (\bar{P}_{1e} - |U_{e1}|^2)(1 - \bar{P}_L) + (\bar{P}_{2e} - |U_{e2}|^2)\bar{P}_L \quad . \tag{13}$$

Then we obtain the difference in the fluxes at two detectors $D1$ and $D2$ as

$$F_{\bar{e}}^{D1} - F_{\bar{e}}^{D2} \approx (1 - 2\bar{P}_L) \cdot (\bar{P}_{1e}^{(1)} - \bar{P}_{1e}^{(2)}) \cdot (F_{\bar{e}}^0 - F_x^0) \quad , \tag{14}$$

where we have neglected the oscillations of $\bar{\nu}_3$ inside the earth.

For the inverted hierarchy,

$$\bar{p}^D - \bar{p} = (\bar{P}_{1e} - |U_{e1}|^2)(1 - 2\bar{P}_L)\bar{P}_H + (\bar{P}_{3e} - |U_{e3}|^2)(1 - \bar{P}_H - \bar{P}_H\bar{P}_L) \quad . \tag{15}$$

Since $\bar{\nu}_3$ oscillates inside the earth with a very small depth (inequality (8) is valid with P_{3e} replaced by $\bar{P}_{3e}$), the second term in (15) can be neglected to get

$$\bar{p}^D - \bar{p} = (\bar{P}_{1e} - |U_{e1}|^2)(1 - 2\bar{P}_L)\bar{P}_H \quad . \tag{16}$$

Therefore, finally we get

$$F_{\bar{e}}^{D1} - F_{\bar{e}}^{D2} \approx \bar{P}_H \cdot (1 - 2\bar{P}_L) \cdot (\bar{P}_{1e}^{(1)} - \bar{P}_{1e}^{(2)}) \cdot (F_{\bar{e}}^0 - F_x^0) \quad . \tag{17}$$

3 "Factorized" matter effects

Note that the equations (10), (12), (14) and (17) have significant similarities. All of these may be written in the factorized form

$$F^{D1} - F^{D2} = f_{flux} \cdot f_{star} \cdot f_{osc} \ . \tag{18}$$

In this section, we shall discuss these factors and the insights they offer on the extent of the observable earth matter effects.

3.1 f_{flux}: the difference in initial fluxes

The relevant difference in initial fluxes is

$$f_{flux} = \begin{cases} (F_e^0 - F_x^0) & \text{for} \quad \nu_e \\ (F_{\bar{e}}^0 - F_x^0) & \text{for} \quad \bar{\nu}_e \end{cases} \quad .$$

Since the ν_e and $\bar{\nu}_e$ spectra are softer than the ν_x spectrum, and the luminosities of all the spectra are similar in magnitude [4], the term f_{flux} is positive at low energies and becomes negative at higher energies where the ν_x flux overwhelms the ν_e ($\bar{\nu}_e$) flux. Therefore, the earth effect has a different sign for low and high energies. The observed effect can be significant in the energy range where $\sigma \times f_{flux}$ is maximum (here σ is the cross section of the neutrino interactions).

3.2 f_{star}: the factor characterizing conversions inside the star

The value of this factor depends only on the neutrino conversions occuring inside the star, and hence it also characterizes the different scenarios:

$$f_{star} = \begin{cases} P_H(1 - 2P_L) & \text{normal hierarchy} \quad \nu_e \\ (1 - 2P_L) & \text{inverted hierarchy} \quad \nu_e \\ (1 - 2\bar{P}_L) & \text{normal hierarchy} \quad \bar{\nu}_e \\ \bar{P}_H(1 - 2\bar{P}_L) & \text{inverted hierarchy} \quad \bar{\nu}_e \end{cases} \quad . \tag{19}$$

Consider the ν_e conversions in the scenario of normal hierarchy. In the limit of $U_{e3} \to 0$ and $P_H \to 1$, (9) reduces to the expression for the earth effects in the case of two neutrino mixing :

$$[p^D - p]_{2\nu} = (P_{2e} - |U_{e2}|^2)(1 - 2P_L) \quad , \tag{20}$$

which is equivalent to the one used in literature in the context of day-night effect for solar neutrinos. Therefore (9) can be looked upon as the earth matter effect due to the two neutrino mixing (20) suppressed by a factor of P_H. The mixing of the third neutrino thus plays a major role, making the expected earth effects in the case of supernova neutrinos smaller than those expected in the case of solar neutrinos for the same mixing scheme and in the same energy range. If the H-resonance is completely adiabatic, the earth effect vanishes: all the ν_e produced are converted to ν_3 in the star, and the earth matter effect on ν_3 is negligibly small (as we have established through (8)).

Notice that the expressions for the earth matter effects in the case of the inverted mass hierarchy (12, 17) have the same form as the expressions in the case of the normal mass hierarchy (10, 14), except for the factors of P_H and $\bar{P}_H$. Indeed, since $P_H, \bar{P}_H \leq 1$, these act as suppression factors for the earth matter effects on ν_e and $\bar{\nu}_e$, in the case of normal and inverted hierarchy respectively. Then the observation of significant earth matter effects on ν_e for normal hierarchy would imply $P_H \approx 1$, i.e. $|U_{e3}|^2 \lesssim 10^{-3}$. Similarly, significant earth matter effects on $\bar{\nu}_e$ for inverted hierarchy would imply $\bar{P}_H \approx 1$, i.e. $|U_{e3}|^2 \lesssim 10^{-3}$.

3.3 f_{osc}: the difference in earth oscillation probabilities

This factor represents the difference in the neutrino conversions inside the earth due to the different distances travelled by the neutrinos inside the earth before reaching the detectors.

$$f_{osc} = \begin{cases} P_{2e}^{(1)} - P_{2e}^{(2)} & \text{for} \quad \nu_e \\ \bar{P}_{2e}^{(1)} - \bar{P}_{2e}^{(2)} & \text{for} \quad \bar{\nu}_e \end{cases} \quad .$$

If the neutrino trajectory crosses only the mantle of the earth, one can use a constant density approximation which gives

$$P_{2e}^{(1)} - P_{2e}^{(2)} \approx \sin 2\theta_{e2}^m \ \sin(2\theta_{e2}^m - 2\theta_{e2}) \ \left[\sin^2\left(\frac{\pi d_1}{l_m}\right) - \sin^2\left(\frac{\pi d_2}{l_m}\right)\right] \quad . \tag{21}$$

Here θ_{e2}^m and l_m are the mixing angle and the oscillation length in the earth matter respectively, and d_i is the distance travelled by the neutrinos inside the earth before reaching the detector D_i. The first two terms on the right hand side of (21) are positive for the scenarios with the SMA and LMA solutions, so that the sign of f_{osc} is the same as the sign of the term inside the square bracket in (21). For the scenario with the VO solution, the earth matter effects are negligible: for $\Delta m^2 \sim 10^{-10}$ eV^2, the mixing in the earth matter is highly suppressed, since $\sin 2\theta_{e2}^m$ is very small.

In the case of antineutrinos, the constant density approximation gives

$$\bar{P}_{1e}^{(1)} - \bar{P}_{1e}^{(2)} \approx -\sin 2\bar{\theta}_{e2}^m \, \sin(2\bar{\theta}_{e2}^m - 2\theta_{e2}) \left[\sin^2\left(\frac{\pi d_1}{l_m}\right) - \sin^2\left(\frac{\pi d_2}{l_m}\right) \right] \, , \quad (22)$$

where $\bar{\theta}_{e2}^m$ is the mixing angle inside the earth for the antineutrinos. For the antineutrino channel $\bar{\theta}_{e2}^m < \theta_{e2} \ll 1$ for SMA solution and $\bar{\theta}_{e2}^m$ is strongly suppressed by matter in the VO case. Therefore the earth matter effects on the $\bar{\nu}_e$ spectrum can be significant only for the scenario with the LMA (as well as LOW) solution. In this scenario, $\sin 2\bar{\theta}_{e2}^m > 0$ and $\sin(2\bar{\theta}_{e2}^m - 2\theta_{e2}) < 0$, so that the sign of f_{osc} is the same as the sign of the oscillation term inside the square bracket in (22).

If neutrinos cross both the mantle and the core, the parametric enhancement of oscillations may occur, which leads to the appearance of parametric peaks apart from the peaks due to the MSW resonances in the core and the mantle [5]. Correspondingly the factor f_{osc} will be a more complicated function of the neutrino energy.

To summarize, the earth matter effects on the ν_e spectrum can be significant only for the scenarios with the SMA or LMA or LOW solutions. Moreover, if the hierarchy is normal, $|U_{e3}|^2 \lesssim 10^{-3}$ is needed in addition. The effect on the $\bar{\nu}_e$ spectrum can only be significant for the LMA scenario. Let us consider these cases in detail below.

In the case of SMA, the factor of f_{osc} can be as large as 0.25 in the energy range of 20 – 40 MeV, where the term f_{flux} is also significant. However, even in this scenario, and with optimistic values of P_H, P_L and θ_{e2}, the net effect is only $\lesssim 10\%$, which would be difficult to disentangle from the uncertainties in the original fluxes (see Fig. 7 in [1]).

In the case of LMA, the factor f_{osc} can be as large as 0.3 in the energy range of 20 – 50 MeV both for ν_e and $\bar{\nu}_e$. Also, the transitions in the L resonance layer are completely adiabatic [1] for both neutrinos and antineutrinos, so that the factor $(1 - 2P_L)$ as well as $(1 - 2\bar{P}_L)$ is equal to 1. Indeed, the earth matter effects can be large in both channels. In Fig. 1, we show the ν_e and $\bar{\nu}_e$ spectra for different distances travelled by the neutrinos through the earth.

Note that in the case of LMA, in some region of parameter space the final ν_e spectrum shows a spectacular dip [1,6], which cannot be mimicked by any uncertainty in the original energy spectrum. Thus, the earth matter effects can be demonstrated in a clean way, independent of any model of supernova dynamics and by the observation of signals in only one detector. We demonstrate this case further through a Monte carlo simulation of the final ν_e spectrum in LMA scheme with 300 events in Fig. 2. The dip is clearly observable in the spectrum with $d = 6000$ km.

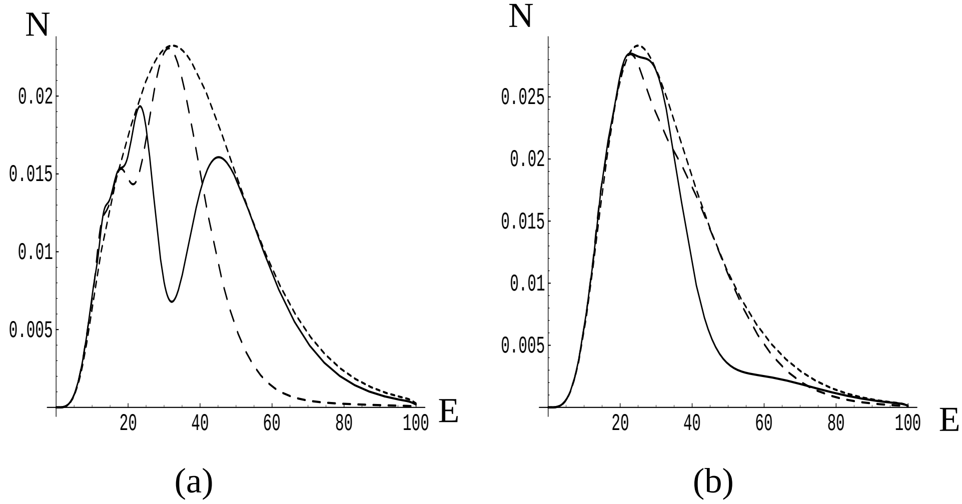

Figure 1. The earth matter effects on (a) the ν_e spectrum and (b) the $\bar{\nu}_e$ spectrum for $P_H = 1$ in the scheme with the LMA solution ($\Delta m^2 = 2 \cdot 10^{-5}$ eV^2, $\sin^2 2\theta_\odot = 0.9$) and normal hierarchy. The dotted, dashed and solid lines show the spectra of the number of $\nu - N$ charged current events when the distance travelled by the neutrinos through the earth is $d = 0$ km, $d = 4000$ km, and $d = 6000$ km respectively.

4 Conclusions

We have indicated how the observation of earth matter effects on the ν_e and $\bar{\nu}_e$ energy spectra from a supernova can help resolve ambiguties in the neutrino mass spectrum in a model independent manner.

The observation of any earth matter effects rules out the scenario with the VO solution. If the earth matter effects are observed in the neutrino channels but not in the antineutrino channels, we either have the inverted mass hierarchy, or the normal mass hierarchy with $|U_{e3}|^2 \lesssim 10^{-3}$.

The observation of earth matter effects in the antineutrino channel identifies the LMA solution. In addition, if the effects are significant in the neutrino channel also, $|U_{e3}|^2 \lesssim 10^{-3}$ may be established.

The qualitative features of the earth matter effects may be able to explain certain peculiar features of the SN1987A supernova neutrino energy spectra [7], but the number of events is too small to come to any definite conclusion. However, a galactic supernova may provide us with a sufficient number of events [8] to enable us to reconstruct the energy spectra of ν_e and $\bar{\nu}_e$.

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This talk points out the salient features of the earth matter effects on the

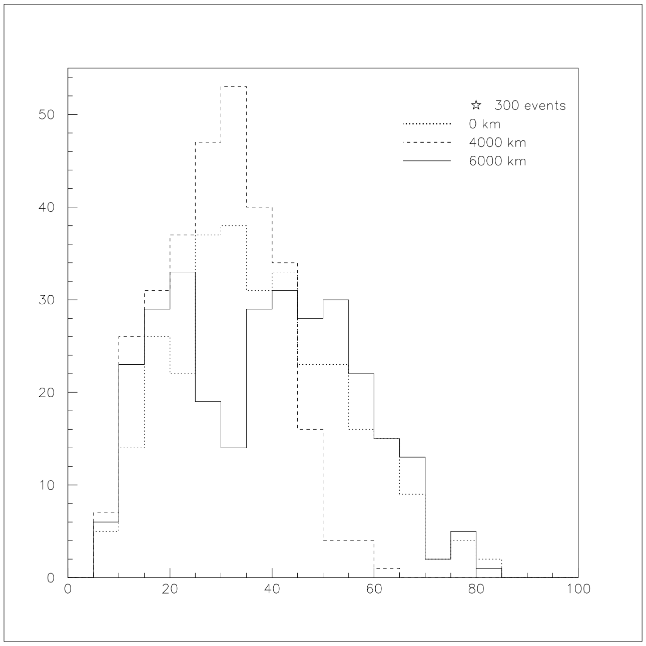

Figure 2. The number of events (y-axis) of ν_e as a function of energy in MeV (x-axis) for the LMA solution (parameters as in Fig 1). The three spectra correspond to $d = 0, 4000, 6000$ km. The spectra are normalized so that the number of events for each spectrum is 300.

supernova neutrino spectra. For a more recent detailed study of the spectra expected at various detectors, the reader is referred to [9].

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