

CERN-TH/2001-166  
UNIL-IPT-01-09  
hep-th/0106255

# Supergravity, Non-Conformal Field Theories and Brane-Worlds

Tony Gherghetta<sup>a</sup> and Yaron Oz<sup>b</sup>

*<sup>a</sup>Institute of Theoretical Physics, University of Lausanne,  
CH-1015 Lausanne, Switzerland*

*<sup>b</sup>Theory Division, CERN  
CH-1211 Geneva 23, Switzerland*

## Abstract

We consider the supergravity dual descriptions of non-conformal super Yang-Mills theories realized on the world-volume of  $Dp$ -branes. We use the dual description to compute stress-energy tensor and current correlators. We apply the results to the study of dilatonic brane-worlds described by non-conformal field theories coupled to gravity. We find that brane-worlds based on D4 and D5 branes exhibit a localization of gauge and gravitational fields. We calculate the corrections to the Newton and Coulomb laws in these theories.

June 2001

# 1 Introduction

The AdS/CFT correspondence relates a conformal field theory in  $d$  dimensions to string theory on  $AdS_{d+1}$  space [1, 2, 3] (For a review see [4]). The radial coordinate of  $AdS_{d+1}$ , often denoted by  $U$ , is interpreted as an energy scale from the point of view of the  $d$ -dimensional conformal field theory. The canonical example is the  $\mathcal{N} = 4$  SCFT in four-dimensions realized on the worldvolume of D3-branes. The dual description being type-IIB string theory on  $AdS_5 \times S^5$ . The worldvolume theory of  $Dp$ -branes when  $p \neq 3$  is a non-conformal super Yang-Mills theory (SYM) with sixteen supercharges. These theories have dual descriptions which vary as we move in the energy scale  $U$  [5]. They have been much less studied compared to the conformal field theories.

Another line of research is the study of brane-world scenarios [6, 7]. We will consider a brane-world to be a brane (not necessarily a  $Dp$ -brane) located at  $U = U_*$ . When  $U_*$  is infinite and the brane is a  $Dp$ -brane we recover the above, where the  $Dp$ -brane theory is a Yang-Mills field theory without gravity. When  $U_*$  is finite, the theory on the brane is a Yang-Mills theory coupled to gravity with a UV cutoff  $\Lambda = U_*$ . As in the duality between gauge theory and gravity, the brane-world case with  $p = 3$  has been extensively studied, as a conformal field theory coupled to gravity system. This brane-world scenario exhibits a localization of gravity on the brane-world. However a bulk gauge field is not localized in this scenario.

In this paper we will consider brane-worlds based on  $Dp$ -branes when  $p \neq 3$ . We will start by considering the supergravity dual descriptions and use it to compute stress-energy tensor and current correlators of the non-conformal field theories. We will then apply the results to the study of dilatonic brane-worlds described by non-conformal field theories coupled to gravity. We will find that brane-worlds based on D4 and D5 branes exhibit a localization of gauge and gravitational fields, and we calculate the corrections to the Newton and Coulomb laws in these theories.

The paper is organized as follows. In section 2 we will consider the worldvolume theory of  $N$  coincident  $Dp$ -branes in type-II string theory using the dual supergravity description. We will compute correlation functions of the stress-energy tensor and global symmetry currents. In section 3 we will consider brane-world scenarios with a brane located at  $U = U_*$ . When  $p \neq 3$  the theory on the brane-world is dual to a  $(p + 1)$ -dimensional non-conformal super Yang-Mills theory coupled to gravity with a cutoff  $\Lambda = U_*$ . These brane-world scenarios are sometimes called dilatonic domain walls. We will analyse these brane-world scenarios with and without additional compactification of brane worldvolume coordinates. We will use the results of the previous section in order to compute the corrections to Newton and Coulomb laws. We will also derive these corrections in a slightly different approach and discuss the localisation of gauge and gravitational fields. We find that brane-worlds based on D4 and D5 branes exhibit a localization of both gauge and gravitational fields. Section 4 is devoted to a more detailed discussion on brane-world scenarios based on dilatonic domain walls. Some details of Green's functions computations are outlined

in the appendix.

## 2 Dp-branes

In this section we will consider the worldvolume theory of  $N$  coincident Dp-branes in type-II string theory using the dual supergravity description. We will compute correlation functions of the stress-energy tensor and global symmetry currents.

### 2.1 The dual supergravity description

Consider a system of  $N$  coinciding flat Dp-branes of type-II string theory. We denote the ten-dimensional space coordinates by  $x_0, \dots, x_9$ . The  $p + 1$  worldvolume coordinates are taken to be  $x_0, \dots, x_p$ . The field theory limit of this Dp-brane system is defined by taking the string length,  $l_s \rightarrow 0$  while keeping the Yang-Mills coupling,  $g_{YM}^2 = g_s l_s^{p-3}$  fixed, where  $g_s$  is the string coupling [5]. For  $p < 6$  this is a limit that decouples the open string degrees of freedom attached to the Dp-branes worldvolume from the bulk closed string degrees of freedom. The worldvolume theory is a  $(p + 1)$ -dimensional  $U(N)$  super Yang-Mills theory with sixteen supercharges. In the following we will consider the large  $N$  t' Hooft limit, where we denote the t' Hooft parameter by  $\lambda = g_{YM}^2 N$ .

This system has a conjectured dual string description [5]. In the supergravity approximation the (string-frame) metric describing the system is given by

$$\begin{aligned}
 ds^2 &= l_s^2 \left( \frac{U^{\frac{1}{2}(7-p)}}{\lambda^{\frac{1}{2}}} dx_{\parallel}^2 + \frac{\lambda^{\frac{1}{2}}}{U^{\frac{1}{2}(7-p)}} dU^2 + \lambda^{\frac{1}{2}} U^{\frac{1}{2}(p-3)} d\Omega_{8-p}^2 \right) , \\
 e^{\phi} &= \frac{\lambda^{\frac{1}{4}(7-p)}}{N} U^{\frac{1}{4}(7-p)(p-3)} .
 \end{aligned} \tag{1}$$

Note that we have omitted numerical factors that will be irrelevant for our discussions later. The  $p + 1$  coordinates of the Dp-brane worldvolume are denoted by  $x_{\parallel}$ , and have dimension of length. The radial coordinate  $U$  has dimensions of mass, and from the Dp-branes worldvolume field theory point of view it plays the role of an energy scale. Large values of  $U$  correspond to the UV regime and small values correspond to the IR regime of the Yang-Mills theory. The angular coordinates of the  $(8 - p)$ -dimensional sphere are denoted by  $\Omega_{8-p}$ . They are dimensionless in the above notation. The field  $\phi$  is the ten-dimensional dilaton.

We will be mostly interested in the non-conformal cases  $p \neq 3$ . In these cases the metric (1) has the isometry group  $ISO(1, p) \times SO(9 - p)$ . From the Yang-Mills theory point of view  $ISO(1, p)$  is the Poincare symmetry while  $SO(9 - p)$  is the R-symmetry group. The Yang-Mills coupling is dimensionful,  $[g_{YM}^2] = [L]^{p-3}$ , and we can define a dimensionless expansion parameter of the system

$$g_{eff}^2 = g_{YM}^2 N U^{p-3} = \lambda U^{p-3} . \tag{2}$$

The Yang-Mills theory is strongly coupled when  $g_{eff} > 1$ .

The scalar curvature of the metric (1) is given by

$$l_s^2 \mathcal{R} \sim \frac{1}{g_{eff}} . \quad (3)$$

We see from (3) that the curvature expansion in the dimensionless parameter  $l_s^2 \mathcal{R}$  on the supergravity side corresponds to a strong coupling expansion of the gauge theory in the inverse of the effective Yang-Mills bare coupling. It is a good expansion when  $l_s^2 \mathcal{R} \ll 1$ , i.e. in the regime where the Yang-Mills field theory is strongly coupled. When  $g_{eff}^2 \ll 1$  we have a large curvature. In this regime we cannot trust the supergravity description. However, the Yang-Mills perturbation theory in  $g_{eff}$  is valid in this regime. The overlap regime is when  $g_{eff} \sim 1$ .

The effective string coupling  $e^\phi$  in (1) can be expressed in terms of  $g_{eff}$  as

$$e^\phi \sim \frac{g_{eff}^{\frac{1}{2}(7-p)}}{N} . \quad (4)$$

We see from (4) that the string loop expansion corresponds to the  $1/N$  expansion of the Yang-Mills theory. This expansion is good when the effective string coupling is small  $e^\phi \ll 1$ .

Thus, the regime where the supergravity description of the system given by (1) is valid is dictated by the conditions that both the curvature (3) and effective string coupling (4) are small. This implies that

$$1 \ll g_{eff}^2 \ll N^{\frac{4}{7-p}} . \quad (5)$$

We will work in this regime. Moreover, we will consider the cases  $p < 6$ . When  $p \geq 6$  the above field theory limit is no longer a decoupling limit, and the brane worldvolume theory does not decouple from the bulk physics. This can be seen, for instance, by the fact that the eleven-dimensional Planck length  $l_{11}^3 = g_s l_s^3$  is fixed in the limit. Therefore, gravity does not decouple in eleven-dimensions, which provides the proper description of the system in UV [5]. Another way is to compute the absorption cross section for graviton scattering on the branes and see that it does not vanish in the limit [8]. From the brane-world scenario we will see that gravity and gauge fields cannot be localized.

## 2.2 Stress-energy tensor correlators

Let us now use the dual supergravity description in order to compute the two-point function of the stress-energy tensor of the Yang-Mills field theory on the  $Dp$ -branes worldvolume. Unlike the conformal case  $p = 3$ , the two-point function changes as we vary the length scale  $|x| \equiv U^{-1}$ . The regime of validity of the supergravity computation is given by (5). As noted above this is a strong coupling regime of the

gauge theory. The results that we will find will match, up to a numerical factor, with the perturbative Yang-Mills expectation in the crossover regime.

The basic idea of the correspondence between string theory and gauge theory is the identification, in the supergravity approximation, of the generating functional of the connected Green's functions in the gauge theory with the minimum of the supergravity action subject to certain boundary conditions [2, 3]. For each operator on the Yang-Mills side we identify a field on the supergravity (string) side. The bulk field that couples to the stress-energy tensor of the gauge theory is the graviton. In the computation of the generating functional for the stress-energy tensor correlators we need to solve the graviton field equation in the background (1) subject to some boundary conditions and insert it in the supergravity action. One decomposes the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \mu, \nu = 0, \dots, 9 \quad (6)$$

where  $\bar{g}_{\mu\nu}$  denotes the background metric (1) and  $h_{\mu\nu}$  denotes the perturbation. The stress-energy tensor  $T_{\mu\nu}$  of the brane worldvolume field theory couples to the bulk graviton as

$$S_{int} \sim \int d^{p+1}x T^{\mu\nu} h_{\mu\nu} . \quad (7)$$

We can decompose the graviton field as

$$h_{\mu\nu}(x, U, \Omega) = \epsilon_{\mu\nu} e^{ik \cdot x} h(U) Y(\Omega_{8-p}) , \quad (8)$$

where  $\epsilon_{\mu\nu}$  is a constant polarization tensor, and  $x$  are the  $p + 1$  worldvolume coordinates. The graviton is polarized along the brane worldvolume coordinates, i.e.  $\epsilon_{\mu\nu} = 0, \mu, \nu = p + 1, \dots, 9$ . We use the transverse traceless gauge, i.e.  $\partial_\mu h^{\mu\nu} = 0$  and  $h^\mu{}_\mu = 0$ , and consider the s-wave mode of the graviton, i.e. it does not vary along the directions of the  $(8 - p)$ -dimensional sphere. We expand to quadratic order in the supergravity action for the graviton. The type-II supergravity equation for the graviton modes  $h(U)$  is related to a minimally coupled scalar field [8]. More precisely, for certain polarization and momentum vectors the graviton equation is the minimal scalar equation

$$f''(U) + \frac{8-p}{U} f'(U) - k^2 \lambda U^{p-7} f(U) = 0 . \quad (9)$$

In the following we will suppress the index structure of the stress-energy tensor.

We consider the equation (9) in the range  $0 \leq U \leq U_0$ . This equation is solved by Bessel functions and we choose the regular solution that is normalized to one at  $U = U_0$ . Substituting the solution into the supergravity action, the generating functional is obtained as a pure boundary term. We construct the “flux factor”  $\mathcal{F}_T$

$$\mathcal{F}_T = \frac{N^2}{\lambda^2} \left[ f(U) U^{8-p} \partial_U f(U) \right]_0^{U_0} , \quad (10)$$

which yields the momentum-space two-point function and we get that

$$\langle T(k) T(-k) \rangle = \mathcal{F}_T . \quad (11)$$

By choosing the  $U_0$  independent non-analytic term in  $\mathcal{F}_T$ , and Fourier transforming we obtain the two-point function in  $x$ -space. When  $p < 5$  we get

$$\langle T(x)T(0) \rangle = \frac{N^2}{\lambda^{(2-\alpha)}} \frac{1}{|x|^{2\alpha+p+1}} = \frac{N^\alpha}{g_{YM}^{2(2-\alpha)}} \frac{1}{|x|^{2\alpha+p+1}}, \quad \alpha = \frac{7-p}{5-p}. \quad (12)$$

For  $p = 5$  the two-point function reads

$$\langle T(x)T(0) \rangle = \frac{N^2}{\lambda^{19/4}} \frac{1}{|x|^{5/2}} e^{-|x|/\sqrt{\lambda}}, \quad (13)$$

where we have only written the leading large  $|x|$  behaviour. We have summarised the results in Table 1.

Two cases which will be of particular importance for the brane-world scenarios are  $p = 4$  and  $p = 5$ . It is therefore of interest to give some more details on the computation in these two cases.

#### D4-branes

The minimal scalar equation (9) reads

$$f''(U) + \frac{4}{U} f'(U) - k^2 \lambda U^{-3} f(U) = 0. \quad (14)$$

Define the coordinate  $z = 2 \left(\frac{\lambda}{U}\right)^{\frac{1}{2}}$  which has dimensions of length. The regular normalizable solution of (14) is given by

$$f(z) = (kz)^3 K_3(kz), \quad (15)$$

where  $K_3(kz)$  is the modified Bessel function. The relevant non-analytic piece of the “flux factor” (10) reads

$$\mathcal{F}_T = N^2 \lambda k^6 \log(k). \quad (16)$$

The Fourier transform of (16) yields the result

$$\langle T(x)T(0) \rangle = \frac{N^3 g_{YM}^2}{|x|^{11}}. \quad (17)$$

At the crossover regime between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{-1} \simeq 1$  we have the matching to the expected  $U(N)$  gauge theory result with  $N^2$  degrees of freedom

$$\langle T(x)T(0) \rangle = \frac{N^2}{|x|^{10}}. \quad (18)$$

The matching agrees up to a numerical factor that is independent of the rank of the gauge group  $N$  and the Yang-Mills coupling  $g_{YM}$ .

### D5-branes

The minimal scalar equation (9) reads

$$f''(U) + \frac{3}{U}f'(U) - k^2\lambda U^{-2}f(U) = 0 . \quad (19)$$

Define  $U\lambda^{\frac{1}{2}} = \exp(z\lambda^{-\frac{1}{2}})$ , where  $z$  has dimensions of length. The solutions of (19) are given by

$$f(z) = \exp \left[ -z\lambda^{-\frac{1}{2}} \left( 1 \pm \sqrt{1 + \lambda k^2} \right) \right] . \quad (20)$$

After the normalization the relevant non-analytic piece of the “flux factor” (10) reads

$$\mathcal{F}_T = \frac{N^2}{\lambda^3} \sqrt{1 + \lambda k^2} . \quad (21)$$

Fourier transforming (21) yields (13).

At the crossover regime between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{-2} \simeq 1$  we have the matching to the expected  $U(N)$  gauge theory result with  $N^2$  degrees of freedom

$$\langle T(x)T(0) \rangle = \frac{N^2}{|x|^{12}} . \quad (22)$$

As before, the matching agrees up to a numerical factor that is independent of the rank of the gauge group  $N$  and the Yang-Mills coupling  $g_{YM}$ .

At a length scale  $|x| \ll \frac{g_{YM}}{\sqrt{N}}$  the effective string coupling is large and the weakly coupled description is given by the S-dual NS5-brane background. The minimally coupled scalar equation in this background is again (19) and the result (13) is obtained again.

### Dp-branes

For general  $p$  the correlator  $\langle T(x)T(0) \rangle$  as given in Table 1, matches at the crossover region between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{p-3} \simeq 1$  with the expected Yang-Mills result

$$\langle T(x)T(0) \rangle = \frac{N^2}{|x|^{2(p+1)}} . \quad (23)$$

We note that the result in Table 1 for the stress-energy two-point function when  $p = 1$  was obtained in [9].

## 2.3 Current-current correlators

Global symmetry currents of the brane worldvolume field theory  $J_\mu$  couple to massless bulk gauge fields  $A^\mu$  as

$$S_{int} \sim \int d^{p+1}x J_\mu A^\mu . \quad (24)$$

$p$	$\langle T(x)T(0) \rangle$
0	$\frac{N^{7/5}}{g_{YM}^{6/5}} \frac{1}{ x ^{19/5}}$
1	$\frac{N^{3/2}}{g_{YM}} \frac{1}{ x ^5}$
2	$\frac{N^{5/3}}{g_{YM}^{2/3}} \frac{1}{ x ^{19/3}}$
3	$\frac{N^2}{ x ^8}$
4	$\frac{N^3 g_{YM}^2}{ x ^{11}}$
5	$\frac{N^2}{\lambda^{19/4}} \frac{1}{ x ^{5/2}} e^{- x /\sqrt{\lambda}}$

Table 1: The supergravity results for the  $N$  D $p$ -branes  $\langle T(x)T(0) \rangle$  correlator.

We may distinguish two types of such gauge fields. Those that arise from the isometries of the space on which we reduce, in our case the  $(8 - p)$ -dimensional sphere in (1). These couple to  $SO(9 - p)$  R-symmetry currents of the D $p$ -branes gauge theory. We can have additional gauge fields that couple to other global symmetry currents as, for instance, in the five-dimensional fixed points with  $E_n$  global symmetry analysed in [10]. We will consider the former type in the following.

In order to compute the current correlators we need to solve the graviton equation for the components of the graviton with one index along the brane worldvolume and one index along the  $(8 - p)$ -dimensional sphere. Alternatively we can first reduce the supergravity metric (1) on the  $(8 - p)$ -dimensional sphere and solve the  $p + 2$ -dimensional gauge field equation. We will pursue the second procedure since it will also shed light later on brane-world scenarios based on various domain-wall solutions in the literature.

The dimensional reduction of the D $p$ -brane supergravity solutions along the sphere have been performed in [11, 12] using another coordinate system. We will repeat the same procedure here, except that we will use the coordinates of the supergravity solution (1). It is useful to retain the  $U$  coordinate because it has the interpretation of an energy scale of the brane worldvolume field theory.

The dimensional reduction is performed by using the ansatz

$$ds_E^2 = l_s^2 \left( e^{a\varphi} ds_{p+2}^2 + e^{-\frac{p}{8-p}a\varphi} d\Omega_{8-p}^2 \right) \quad (25)$$

where  $\varphi$  is a scalar field and  $ds_E^2$  is the ten-dimensional D $p$ -branes metric written in the Einstein frame, i.e. it is related to the string frame metric (1) by  $ds_E^2 = e^{-\frac{\phi}{2}} ds^2$ .



The constant  $a$  is given by

$$a = \frac{1}{2} \left( \frac{8-p}{p} \right)^{\frac{1}{2}} . \quad (26)$$

By comparing the coefficient of the  $d\Omega_{8-p}^2$  term of the two metrics  $ds_E^2$  in (25) and  $ds^2$  in (1) we can read the form of the scalar  $\varphi$

$$e^\varphi = \left[ \frac{1}{N} \lambda^{\frac{1}{4}(3-p)} U^{-\frac{1}{4}(3-p)^2} \right]^{\left( \frac{8-p}{p} \right)^{\frac{1}{2}}} . \quad (27)$$

Substituting this expression into (25) we obtain the  $(p+2)$ -dimensional metric

$$ds_{p+2}^2 = N^{\frac{4}{p}} \lambda^{-\frac{3}{p}} U^{\frac{9}{p}-1} \left( dx_{||}^2 + \lambda U^{p-7} dU^2 \right) . \quad (28)$$

The  $(p+2)$ -dimensional gauge field bulk action is then given by

$$S_{gauge} = \int d^{p+2}x \sqrt{g_{p+2}} e^{-\frac{4}{\sqrt{p(8-p)}}\varphi} F^2 \equiv \int d^{p+2}x \sqrt{g_{p+2}} \frac{1}{g_{bulk}^2} F^2 , \quad (29)$$

where

$$g_{bulk}^2 = N^{-\frac{4}{p}} \lambda^{\frac{3}{p}-1} U^{-\frac{(3-p)^2}{p}} . \quad (30)$$

This is of importance since while the gravitational and scalar fields in the reduced action are canonical in  $(p+2)$  dimensions this is not the case for the gauge field as we see from (29). Note that for our purposes we omit the group theory factors and we write the action (29) schematically. In the following we will suppress the Lorentz index on the current

In order to solve the  $(p+2)$ -dimensional Maxwell equation, we write the  $(p+2)$ -dimensional gauge field as

$$A_\mu(x, U) = \varepsilon_\mu e^{ik \cdot x} f(U) . \quad (31)$$

Imposing the gauge conditions  $\partial^\mu A_\mu = 0$ ,  $\mu = 0, \dots, p+1$  and  $A_U = 0$  we obtain

$$f''(U) + \frac{3}{U} f'(U) - k^2 \lambda U^{p-7} f(U) = 0 . \quad (32)$$

As before, we consider equation (32) in the range  $0 \leq U \leq U_0$ . We solve this equation and choose a regular solution that is normalized to one at  $U = U_0$ . The generating functional for the current correlators is obtained as a pure boundary term. Again we construct the ‘‘flux factor’’  $\mathcal{F}_A$

$$\mathcal{F}_A = \frac{N^2}{\lambda} \left[ f(U) U^3 \partial_U f(U) \right]_0^{U_0} , \quad (33)$$

and we have that

$$\langle J(k) J(-k) \rangle = \mathcal{F}_A . \quad (34)$$

By choosing the  $U_0$  independent non-analytic term in  $\mathcal{F}_A$ , and Fourier transforming we obtain for  $p < 5$

$$\langle J(x)J(0) \rangle = \frac{N^2}{\lambda^{(2-\alpha)}} \frac{1}{|x|^{2\alpha+p-1}} = \frac{N^\alpha}{g_{YM}^{2(2-\alpha)}} \frac{1}{|x|^{2\alpha+p-1}}, \quad \alpha = \frac{7-p}{5-p}, \quad (35)$$

and for  $p = 5$

$$\langle J(x)J(0) \rangle = \frac{N^2}{\lambda^{15/4}} \frac{1}{|x|^{5/2}} e^{-x/\sqrt{\lambda}}. \quad (36)$$

The results are summarized in Table 2. Consider in detail the  $p = 4$  and  $p = 5$  cases.

### D4-branes

Equation (32) reads

$$f''(U) + \frac{3}{U} f'(U) - k^2 \lambda U^{-3} f(U) = 0. \quad (37)$$

As before, define  $z = 2 \left(\frac{\lambda}{U}\right)^{\frac{1}{2}}$  which has dimensions of length. The regular normalized solution of (37) is given by

$$f(z) = (kz)^2 K_2(kz), \quad (38)$$

where  $K_2(kz)$  is the modified Bessel function. The relevant piece of the “flux factor” (10) reads

$$\mathcal{F}_A = N^2 \lambda k^4 \log(k). \quad (39)$$

Fourier transforming (39) yields

$$\langle J(x)J(0) \rangle = \frac{N^3 g_{YM}^2}{|x|^9}. \quad (40)$$

At the crossover region between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{-1} \simeq 1$  we have the matching to the expected gauge theory result

$$\langle J(x)J(0) \rangle = \frac{N^2}{|x|^8}. \quad (41)$$

The matching is up to a numerical factor that is independent of the rank of the gauge group  $N$  and the Yang-Mills coupling  $g_{YM}$ . We have omitted the group theory factors and R-charges as well.

### D5-branes

Equation (32) is identical to the minimal scalar equation (19). We consider the same solutions (20) and construct the “flux factor” which now reads

$$\mathcal{F}_A = \frac{N^2}{\lambda^2} \sqrt{1 + \lambda k^2}. \quad (42)$$

Fourier transforming (42) yields (36).

At the crossover region between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{-2} \simeq 1$  we have the matching to the expected gauge theory result

$$\langle J(x)J(0) \rangle = \frac{N^2}{|x|^{10}} . \quad (43)$$

As before, the matching is up to a numerical factor that is independent of the rank of the gauge group  $N$  and the Yang-Mills coupling  $g_{YM}$ .

Similarly as for the stress-energy tensor the result (36) is also valid for large string coupling. This is because the S-dual supergravity background of  $N$  NS-5 branes leads to the same equation of motion as (19).

### Dp-branes

Note that the correlator  $\langle J(x)J(0) \rangle$  for general  $p$ , as given in Table 2, matches at the crossover region between the supergravity description and the perturbative Yang-Mills description  $g_{YM}^2 N |x|^{p-3} \simeq 1$  with the expected Yang-Mills result

$$\langle J(x)J(0) \rangle = \frac{N^2}{|x|^{2p}} . \quad (44)$$

Note also that, up to numerical constants that are independent of  $g_{YM}$  and  $N$ , the two-point functions of the stress-tensor  $T$  and the current  $J$  are related for  $p < 5$  by

$$\langle J(x)J(0) \rangle = |x|^2 \langle T(x)T(0) \rangle , \quad (45)$$

and for  $p = 5$  by

$$\langle J(x)J(0) \rangle = \lambda \langle T(x)T(0) \rangle . \quad (46)$$

## 3 Brane-worlds

We use the notation brane-world for a brane (not necessarily a Dp-brane) located at  $U = U_*$ . When  $p \neq 3$  and the brane is a Dp-brane, the theory on the brane-world is described by a  $(p + 1)$ -dimensional non-conformal super Yang-Mills theory coupled to gravity with a cutoff  $\Lambda = U_*$ . In general, the brane-world can have extra degrees of freedom.

In this section we will analyse these brane-world scenarios with and without additional compactification of brane worldvolume coordinates. These brane-world scenarios are sometimes called dilatonic domain walls. To make the connection we note that in  $p + 2$  dimensions the RR  $(p + 2)$ -form field  $F_{p+2} = dA_{p+1}$  can be dualized and replaced by a cosmological constant. The  $p + 2$ -dimensional supergravity action reduced from ten-dimensions has domain wall solutions. Some of these supersymmetric solutions are the ten-dimensional Dp-branes solutions reduced on the  $(8 - p)$ -dimensional sphere [13, 14, 12].

$p$	$\langle J(x)J(0) \rangle$
0	$\frac{N^{7/5}}{g_{YM}^{6/5}} \frac{1}{ x ^{9/5}}$
1	$\frac{N^{3/2}}{g_{YM}} \frac{1}{ x ^3}$
2	$\frac{N^{5/3}}{g_{YM}^{2/3}} \frac{1}{ x ^{13/3}}$
3	$\frac{N^2}{ x ^6}$
4	$\frac{N^3 g_{YM}^2}{ x ^9}$
5	$\frac{N^2}{\lambda^{15/4}} \frac{1}{ x ^{5/2}} e^{- x /\sqrt{\lambda}}$

Table 2: The supergravity results for the  $N$  D $p$ -branes  $\langle J(x)J(0) \rangle$  correlator.

A further reduction to be discussed is the compactification of some of the D $p$ -branes worldvolume coordinates. In particular the compactification of one coordinate of the D4-brane worldvolume on the circle,  $S^1$  or two of the D5-brane worldvolume on a torus,  $S^1 \times S^1$  leads to a four-dimensional brane-world scenarios. We will discuss in more detail the relation to the brane-world scenarios based on the dilatonic domain wall solutions in the discussion section.

### 3.1 Newton's law

The  $(p+1)$ -dimensional gravitational coupling  $\kappa_{p+1}$  is defined in terms of the ten-dimensional gravitational coupling  $\kappa_{10}$  via

$$\frac{1}{\kappa_{10}^2} \int d^{10}x \sqrt{g_{10}} e^{-2\phi} \mathcal{R}_{10} = \frac{1}{\kappa_{p+1}^2} \int d^{p+1}x \sqrt{g_{p+1}} \mathcal{R}_{p+1} . \quad (47)$$

The LHS is the ten-dimensional gravitational action in the string frame metric (1) while the RHS is the  $(p+1)$ -dimensional gravitational action in the Einstein frame. Using (1) and (47) we obtain

$$G_{p+1} \equiv \kappa_{p+1}^2 = \frac{g_{YM}^2}{NU_*^2} , \quad (48)$$

where  $U_*$  is the UV cutoff. Note that when  $U_* \rightarrow \infty$ , the gravitational coupling (48) vanishes and gravity decouples from the brane, as expected.

Consider the correction to Newton's law in the brane-world scenarios.

### D4-branes

Consider a brane-world scenario described by a five-dimensional SYM theory coupled to gravity with a cutoff at  $U = U_*$ . The correction to the Newton's law can be computed from the graviton propagator  $G_{graviton}(k)$  [15], as in Figure 1. The first part is the free graviton propagation while the second part is the leading non-CFT correction

$$\begin{aligned} G_{graviton}(k) &\sim \frac{1}{k^2} + \frac{1}{k^2} \kappa_5 \langle T(k) T(-k) \rangle \kappa_5 \frac{1}{k^2} \\ &= \frac{1}{k^2} \left( 1 + N^3 g_{YM}^2 G_5 k^4 \log(k) \right), \end{aligned} \quad (49)$$

where we have used (16), and  $G_5$  is given by (48). Fourier transforming in five dimensions yield

$$G_{graviton}(x) \sim \frac{1}{x^3} \left( 1 + N^3 g_{YM}^2 \frac{G_5}{x^4} \right). \quad (50)$$

From (50) we can read the modification to the five-dimensional Newton's force law  $\frac{1}{r^3}$ ,

$$F_{Newton} = \frac{G_5 m_1 m_2}{r^3} \left( 1 + \frac{N^2 \lambda G_5}{r^4} \right). \quad (51)$$

### D5-branes

Consider a six-dimensional brane-world scenario. The correction to the Newton's law can be computed as before from the graviton propagator  $G_{graviton}(k)$  as in Figure 1. Again, the first part is the free graviton propagation while the second part is the leading non-CFT correction. Using (21) we get

$$G_{graviton}(k) \sim \frac{1}{k^2} \left( 1 + \frac{N^2 G_6}{\lambda^3} \frac{\sqrt{1 + \lambda k^2}}{k^2} \right), \quad (52)$$

where  $G_6$  is given by (48). Fourier transforming in six dimensions yields at large  $x$

$$G_{graviton}(x) \sim \frac{1}{x^4} \left( 1 + c \frac{N^2 G_6}{\lambda^2} \left( \frac{x}{\sqrt{\lambda}} \right)^{\frac{3}{2}} e^{-x/\sqrt{\lambda}} \right). \quad (53)$$

where by  $c$  we denote a numerical constant which we neglected. The modification to the six-dimensional Newton's force law  $\frac{1}{r^4}$  reads

$$F_{Newton} = \frac{G_6 m_1 m_2}{r^4} \left( 1 + c \frac{N^2 G_6}{\lambda^2} \left( \frac{r}{\sqrt{\lambda}} \right)^{\frac{3}{2}} e^{-r/\sqrt{\lambda}} \right). \quad (54)$$

### Dimensional reduction

In order to arrive at four-dimensional brane-world scenarios one can further compactify the worldvolume coordinates. In the D4-brane case we compactify one coordinate on a circle while in the D5-brane case we compactify two coordinates on

Figure 1: *The graviton propagator  $G_{graviton}(k)$  where the first part is the free graviton propagation while the second part is the leading non-CFT correction.*

a torus. Consider first the D4-branes scenario where we compactify on a circle of radius  $R$ . At distances  $|x| = U^{-1}$  in the regime

$$R \ll |x| \ll g_{YM}^2 N , \quad (55)$$

we can neglect the momenta along the circle direction and we are still in the supergravity regime defined by (5). Thus, Fourier transforming in four dimensions, the gravitational force is then

$$F_{Newton} = \frac{G_4 m_1 m_2}{r^2} \left( 1 + \frac{N^2 \lambda G_4 R}{r^4} \right) , \quad (56)$$

where  $G_4 R = G_5$ .

Consider next the D5-brane scenario where we compactify on two circles of radius  $R$ . At distances in the regime

$$R \ll |x| \ll (g_{YM}^2 N)^{\frac{1}{2}} , \quad (57)$$

we can neglect the momenta along the circle direction and we are still in the supergravity regime defined by (5). Fourier transforming in four dimensions, the gravitational force reads

$$F_{Newton} = \frac{G_4 m_1 m_2}{r^2} \left( 1 + c \frac{N^2 G_4 R^2}{\lambda^2} \left( \frac{r}{\sqrt{\lambda}} \right)^{-\frac{1}{2}} e^{-r/\sqrt{\lambda}} \right) , \quad (58)$$

where  $G_4 R^2 = G_6$ .

### 3.2 Coulomb's law

The effective  $(p + 1)$ -dimensional gauge coupling,  $g_{brane}$ , on the  $p$ -brane can be obtained via

$$\int d^{p+2} x \sqrt{g_{p+2}} \frac{1}{g_{bulk}^2} F_{(p+2)}^2 = \int d^{p+1} x \frac{1}{g_{brane}^2} F_{(p+1)}^2 , \quad (59)$$

where  $F_{(p+2)}$  and  $F_{(p+1)}$  denote the field strength of the gauge field in  $p + 2$  and  $p + 1$  dimensions, respectively. The LHS is the  $(p + 2)$ -dimensional gauge field action after

the dimensional reduction (25) while the RHS is the effective  $(p + 1)$ -dimensional gauge field action. Using (30) we obtain for  $p > 3$

$$g_{brane}^2 = \frac{1}{N^2 U_*^{p-3}} , \quad (60)$$

where  $U_*$  is the UV cutoff.

Consider the correction to Coulomb's law in these brane-world scenarios.

#### D4-branes

Consider a five-dimensional brane-world scenario. The correction to the Coulomb's law can be computed from the propagator for a vector field  $G_{vector}(k)$  as in Figure 2. The first part is the free vector propagation while the second part is the leading non-CFT correction

$$\begin{aligned} G_{vector}(k) &\sim \frac{1}{k^2} + \frac{1}{k^2} g_5 \langle J(k) J(-k) \rangle g_5 \frac{1}{k^2} \\ &= \frac{1}{k^2} \left( 1 + N^3 g_{YM}^2 g_5^2 k^2 \log(k) \right) , \end{aligned} \quad (61)$$

where we have used (39). The gauge coupling  $g_5$  is given by (60) for  $p = 4$ .

Fourier transforming in five dimensions yields

$$G_{vector}(x) \sim \frac{1}{x^3} \left( 1 + N^3 g_{YM}^2 \frac{g_5^2}{x^2} \right) . \quad (62)$$

The modification to the five-dimensional Coulomb's force law  $\frac{1}{r^3}$  reads

$$F_{Coulomb} = \frac{g_5^2 q_1 q_2}{r^3} \left( 1 + \frac{N^2 \lambda g_5^2}{r^2} \right) . \quad (63)$$

#### D5-branes

Consider a six-dimensional brane-world scenario. The correction to the Coulomb's law can be computed as before from the vector propagator  $G_{vector}(k)$  as in Figure 2. Again, the first part is the free graviton propagation while the second part is the leading non-CFT correction. Using (42) we obtain

$$G_{vector}(k) \sim \frac{1}{k^2} \left( 1 + \frac{N^2 g_6^2 \sqrt{1 + \lambda k^2}}{\lambda^2 k^2} \right) , \quad (64)$$

where  $g_6$  is given by (60) for  $p = 5$ . The modification to the six-dimensional Coulomb's force law  $\frac{1}{r^4}$  reads

$$F_{Coulomb} = \frac{g_6^2 q_1 q_2}{r^4} \left( 1 + c \frac{N^2 g_6^2}{\lambda} \left( \frac{r}{\sqrt{\lambda}} \right)^{\frac{3}{2}} e^{-r/\sqrt{\lambda}} \right) . \quad (65)$$

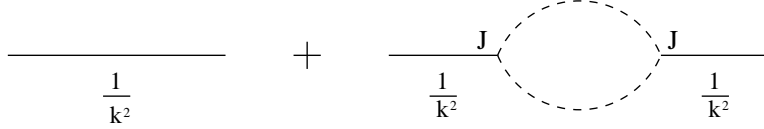


Figure 2: The vector field propagator  $G_{vector}(k)$  where the first part is the free vector propagation while the second part is the leading non-CFT correction.

### Dimensional reduction

Again, in order to arrive at four-dimensional brane-world scenarios one can further compactify the worldvolume coordinates. In the D4-brane case we compactify one coordinate on a circle while in the D5-brane case we compactify two coordinates on a torus. Consider first the D4-branes scenario where we compactify on a circle of radius  $R$ . At distances satisfying (55) the Coulomb force reads

$$F_{Coulomb} = \frac{g_4^2 q_1 q_2}{r^2} \left( 1 + \frac{N^2 \lambda g_4^2 R}{r^2} \right). \quad (66)$$

where  $g_4^2 R = g_5^2$ .

Consider next the D5-branes scenario where we compactify on two circles of radius  $R$ . At distances satisfying (57) we get

$$F_{Coulomb} = \frac{g_4^2 q_1 q_2}{r^2} \left( 1 + c \frac{N^2 g_4^2 R^2}{\lambda} \left( \frac{r}{\sqrt{\lambda}} \right)^{-\frac{1}{2}} e^{-r/\sqrt{\lambda}} \right). \quad (67)$$

where  $g_4^2 R^2 = g_6^2$ .

### 3.3 Graviton and gauge field localisation

In the previous calculation of the Newton and Coulomb laws, a massless mode with behaviour  $1/k^2$  was assumed to be localised on the brane worldvolume. However, one can explicitly check the localisation properties of the bulk graviton and gauge fields on the  $p$ -brane located at  $U = U_*$  by solving the equation of motion in the background geometry (1), and computing the Green's function  $\mathcal{G}_k$ .

By invoking the bulk-boundary correspondence, the bulk Green's function can be used to obtain an alternative derivation of the two-point functions  $\langle \mathcal{O}\mathcal{O} \rangle$  in the dual theory, considered in Section 2. Since the Green's function represents the sum over all connected diagrams we obtain the relation

$$\langle \mathcal{O}(k)\mathcal{O}(-k) \rangle = \frac{k^2}{\kappa} - \mathcal{G}_k^{-1}, \quad (68)$$

where for the stress-energy tensor  $T_{\mu\nu}$  we use the graviton propagator ( $\kappa = -2G_{p+1}$ ), while for the vector current  $J_\mu$  we use the gauge field propagator ( $\kappa = (3-p)g_{p+1}^2$ ).



For  $p < 6$  there always exists a term in (68) which does not depend on the cutoff  $U_*$ . The results for the stress-energy tensor and current correlator, obtained in this way, are all in agreement with those found in section 2.

Let us now consider the graviton and gauge field Green's function to explicitly check the localisation properties on the  $p$ -brane.

### 3.3.1 Graviton

Consider a bulk graviton in the effective  $p + 2$ -dimensional brane-world. The minimally coupled scalar equation (9) determines the mass and localisation properties of the  $p + 1$ -dimensional Kaluza-Klein modes, where  $k^2 = -m^2$ . The differential operator (9) is self-adjoint and the solutions  $f_m$  form a complete set which satisfy the orthonormal relation

$$\int_0^{U_*} dU U f_m f_{m'} = \delta_{mm'} . \quad (69)$$

When  $m = 0$  we find that the equation of motion (9) always has a solution  $f_0(U) = \text{constant}$ , and there is always a normalisable mode. This is consistent with (48).

To check whether gravity is actually localized on the  $p$ -brane we must compute the Green's function for two sources on the brane. In order to do this the first question that needs to be addressed are the boundary conditions. This is particularly important because except for the case  $p = 3$ , there is singular behaviour at  $U = 0$ . This singularity corresponds to the massless open string degrees of freedom that are missing in the description. We expect the singularity to be resolved once the required degrees are added, and we will continue to impose Hartle-Hawking boundary conditions at  $U = 0$ , as in the  $p = 3$  case. Thus following the procedure outlined in the Appendix the expression for the scalar Green's function when  $U = U' = U_*$  is given by

$$\mathcal{G}_k(U_*, U_*) = -i \frac{g_{YM}^4}{\sqrt{\lambda k}} U_*^{\frac{1}{2}(p-9)} \frac{H_\alpha^{(1)} \left( i(\alpha - 1) \sqrt{\lambda k} U_*^{\frac{1}{2}(p-5)} \right)}{H_{\alpha-1}^{(1)} \left( i(\alpha - 1) \sqrt{\lambda k} U_*^{\frac{1}{2}(p-5)} \right)} , \quad (70)$$

where  $H^{(1)}$  is the Hankel function of the first kind. When  $p < 5$  the Green's function can be written in the form  $\mathcal{G}_k(U_*, U_*) = \mathcal{G}_k^{(0)} + \mathcal{G}_k^{(KK)}$ , where the zero mode contribution  $\mathcal{G}_k^{(0)}$  and the Kaluza-Klein contribution  $\mathcal{G}_k^{(KK)}$  are given by

$$\mathcal{G}_k^{(0)} = -2 \frac{G_{p+1}}{k^2} , \quad (71)$$

$$\mathcal{G}_k^{(KK)} = i \frac{g_{YM}^4}{\sqrt{\lambda k}} U_*^{\frac{1}{2}(p-9)} \frac{H_{\alpha-2}^{(1)} \left( i(\alpha - 1) \sqrt{\lambda k} U_*^{\frac{1}{2}(p-5)} \right)}{H_{\alpha-1}^{(1)} \left( i(\alpha - 1) \sqrt{\lambda k} U_*^{\frac{1}{2}(p-5)} \right)} . \quad (72)$$

We see from the form of  $\mathcal{G}_k^{(0)}$  that one obtains Newton's law on the  $p$ -brane located at  $U_*$ . This is consistent with the fact that the graviton zero mode is also localised

there. The corresponding corrections to the  $(p + 1)$ -dimensional Newton's law are obtained from  $\mathcal{G}_k^{(KK)}$  and one obtains the result found earlier for the D4-brane.

When  $p = 5$  the Green's function has the form

$$\mathcal{G}_k(U_*, U_*) = -\frac{G_6}{k^2} \left(1 + \sqrt{1 + \lambda k^2}\right), \quad (73)$$

and again there is a localised massless mode  $(1/k^2)$  at  $U = U_*$ . The corresponding corrections to Newton's law arising from the Kaluza-Klein continuum lead to exponentially suppressed corrections, and agrees with the result found earlier.

When  $p \geq 6$  one finds that there is an equal and opposite contribution arising from  $\mathcal{G}_k^{(KK)}$  that precisely cancels the  $\mathcal{G}_k^{(0)}$  contribution. The remaining leading contribution from  $\mathcal{G}_k^{(KK)}$  does not lead to the usual Newton's law. Thus for  $p \geq 6$  we do not obtain Newton's law and there is no localised mode in the brane-world with  $1/k^2$  behaviour.

### 3.3.2 Gauge fields

The equation of motion for the  $(p + 1)$ -dimensional Kaluza-Klein gauge fields is governed by (32) where  $k^2 = -m^2$ . The differential operator in (32) is self-adjoint and the solutions  $f_m$  form a complete set which satisfy the orthonormal relation

$$\int_0^{U_*} dU U^{p-4} f_m f_{m'} = \delta_{mm'}. \quad (74)$$

Notice that when  $m = 0$  there is always a constant solution to Eq. (32). Thus only for  $p \geq 4$  will the zero mode be normalisable. This is consistent with (60).

Analogous to the graviton case we can obtain the Green's function for two charged (under the gauge field) sources localised on the  $p$ -brane. Following the procedure outlined in the Appendix one obtains the gauge field Green's function

$$\begin{aligned} \mathcal{G}_k(U_*, U_*) &= -i \frac{\sqrt{\lambda}}{N^2 k} U_*^{\frac{1}{2}(1-p)} \frac{H_{\alpha-1}^{(1)} \left( i(\alpha-1) \sqrt{\lambda} k U_*^{\frac{1}{2}(p-5)} \right)}{H_{\alpha-2}^{(1)} \left( i(\alpha-1) \sqrt{\lambda} k U_*^{\frac{1}{2}(p-5)} \right)} \\ &\equiv \mathcal{G}_k^{(0)}(U_*) + \mathcal{G}_k^{(KK)}(U_*), \end{aligned} \quad (75)$$

where for  $p \neq 5$  we have

$$\mathcal{G}_k^{(0)} = (3-p) \frac{g_{p+1}^2}{k^2}, \quad (76)$$

$$\mathcal{G}_k^{(KK)} = i \frac{\sqrt{\lambda}}{N^2 k} U_*^{\frac{1}{2}(1-p)} \frac{H_{\alpha-3}^{(1)} \left( i(\alpha-1) \sqrt{\lambda} k U_*^{\frac{1}{2}(p-5)} \right)}{H_{\alpha-2}^{(1)} \left( i(\alpha-1) \sqrt{\lambda} k U_*^{\frac{1}{2}(p-5)} \right)}, \quad (77)$$

and  $g_{p+1} = 1/(NU_*^{(p-3)/2})$  is the effective gauge coupling for  $p > 3$  on the  $p$ -brane. When  $p < 3$ , the leading contribution  $\mathcal{G}_k^{(0)}$  is positive and cancels against an equal and opposite contribution from  $\mathcal{G}_k^{(KK)}$ . Thus, there is no localised mode with  $1/k^2$  behaviour.

For  $p = 3$ , the zero mode contribution  $\mathcal{G}_k^{(0)}$  vanishes and the leading Kaluza-Klein contribution is

$$\mathcal{G}_k^{(KK)} = \frac{1}{N^2 \log(k)} \frac{1}{k^2} \equiv \frac{e_{eff}^2(k)}{k^2} . \quad (78)$$

As noticed in [16, 17, 18], even though the bulk zero mode is nonnormalizable, one can still think of a 4d gauge field with an effective gauge coupling that runs logarithmically to zero in the infrared.

When  $p = 4$  the zero mode contribution  $\mathcal{G}_k^{(0)}$  is non-vanishing and there is a localised gauge field on the brane. The corrections to Coulomb's law arise from the Kaluza-Klein continuum and agrees with the result obtained earlier for the D4-brane.

The D5-brane case is similar to that encountered for the minimally coupled scalar. The Green's function is now

$$\mathcal{G}_k(U_*, U_*) = -\frac{g_6^2}{k^2} \left( 1 + \sqrt{1 + \lambda k^2} \right) , \quad (79)$$

and one obtains the six-dimensional Coulomb's law. These results are identical to the graviton case except for the couplings and again we see the mass gap in the Kaluza-Klein spectrum of the gauge field.

When  $p > 5$  the contribution  $\mathcal{G}_k^{(0)}$  is again cancelled by an equal and opposite contribution arising from  $\mathcal{G}_k^{(KK)}$ . Thus there is no localised gauge field on the  $p$ -brane with  $1/k^2$  behaviour.

## 4 Discussion

In the following we will discuss in more detail the relation between our work and the brane-world scenarios based on dilatonic domain wall solutions. Consider domain wall solutions in  $D$ -dimensions which are solutions of the field equations derived from the action

$$S = \int d^D x \sqrt{g} \left[ \mathcal{R} - \frac{1}{2} (\partial\varphi)^2 - 2\Lambda e^{b\varphi} \right] + \int d^{D-1} x \mathcal{L} . \quad (80)$$

The field  $\varphi$  is a  $D$ -dimensional scalar field,  $\Lambda$  is a bulk cosmological constant and  $b$  is a constant parameterised in terms of a quantity  $\Delta$

$$b^2 = \Delta + \frac{2(D-1)}{(D-2)} . \quad (81)$$

The lagrangian  $\mathcal{L}$  is a delta-function source added to the bulk action. It provides a cutoff for the the boundary of the bulk solution.

When  $b = 0$  one obtains AdS solutions where the dilaton is constant and the dual description is some CFT. More precisely, if we use only a slice of AdS, the description of the brane-world scenario is in terms of a CFT coupled to gravity with a cutoff. For example, when  $D = 5$  then  $\Delta_{\text{AdS}_5} = -8/3$ , and one obtains the solution [6]. This can originate, for instance, from a ten-dimensional  $AdS_5 \times S^5$  solution of type-IIB supergravity, with an  $\mathcal{N} = 4$  SCFT dual description. In the truncated brane-world version, the dual conformal field theory is modified by introducing an ultraviolet cutoff, and gauging the Poincare symmetry [18, 19].

Similarly, when  $D = 6$  one obtains a six-dimensional AdS solution [20], where  $\Delta_{\text{AdS}_6} = -5/2$ . This can arise as a reduction of a ten-dimensional warped AdS<sub>6</sub> solution corresponding to the D4-D8 brane system [10]. The dual field theory is a five-dimensional CFT. A five-dimensional domain wall solution can be obtained by compactifying one of the brane dimensions on a circle  $S^1$ , and analysed in terms of the corresponding dual theory in [21]. Other similar solutions were obtained in [22].

Domain wall solutions can also be constructed where  $b \neq 0$ , and the dilaton is no longer constant. One example is in  $D = 5$  and  $\Delta = -12/5$  with a varying dilaton [13, 14, 11, 12], which can be obtained from the near-horizon metric of D4-branes compactified on  $S^1 \times S^4$ . Following the results in section 3.1, the Newtonian law obtained from the dimensional reduction of the D4-brane solution is consistent with the corrections obtained in [12]. Our derivation indicates that the dual theory of this domain wall solution is the five-dimensional SYM theory compactified on  $S^1$ . In comparison to the Randall-Sundrum scenario where the correction to the Newton potential is proportional to  $1/r^3$ , here the corrections are  $1/r^5$ . We also considered the gauge fields in this domain wall background. Gauge fields that arise from the isometries of  $S^4$  have a zero mode localised on the domain wall. The corrections to the Coulomb law arising from the Kaluza-Klein continuum are given by (63), and are related to the current correlators of the dual field theory.

Another  $D = 5$  solution, with  $\Delta = -2$ , arises from the near-horizon metric of D5-branes compactified on  $T^2 \times S^3$ . This case, analysed in section 3.1, gives rise to a Newtonian force with exponentially suppressed corrections (see also [12]). Our analysis indicates that the dual field theory is a six-dimensional SYM compactified on  $T^2$ . The exponentially suppressed corrections have two equivalent interpretations. In the supergravity description, they arise because the Kaluza-Klein continuum is separated from the zero mode by a mass gap. In the six-dimensional SYM theory they correspond to a mass gap of the theory developed in the strongly coupled regime. As before, we can study the gauge fields in the domain wall background. Those that arise from the isometries of  $S^3$  have a zero mode localised on the domain wall. The Coulomb law corrections are exponentially suppressed, and they are related to the current correlators of the six-dimensional SYM theory.

# Acknowledgements

The work of TG is supported by the FNRS, contract no. 21-55560.98.

## Appendix: Green's function in the extremal Dp-brane geometry

We will follow the derivation of the Green's function presented in Ref. [23], except that we will consider the extremal Dp-brane geometry. Let us first consider the case of the minimally coupled scalar  $\phi$  in ten dimensions. Introducing a source function  $\mathcal{J}$ , one obtains

$$\partial_M(\sqrt{-g}g^{MN}\partial_N\phi(X)) = \sqrt{-g}\mathcal{J}(X) . \quad (82)$$

where  $X = (x, U, \theta_i)$ , with  $x$  denoting the  $p + 1$  worldvolume coordinates. The corresponding Green's function for (82) can then be defined as

$$\phi(X) = \int d^{10}X' \sqrt{-g} G(X; X') \mathcal{J}(X') . \quad (83)$$

If we now consider the Fourier transform of the Green's function

$$G(X; X') = \sum_{\{n_i\}} e^{i\{n_i\}\theta_i} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} e^{ik \cdot (x-x')} \mathcal{G}_k(U, U') , \quad (84)$$

where the discrete Fourier transform is over the hyperspherical coordinates of  $S^{8-p}$ , then the Fourier component  $\mathcal{G}_k(U, U')$  must satisfy the equation

$$\left[ \partial_u(U^{8-p}\partial_u) - \lambda k^2 U + m_{\theta_i}^2 U^{6-p} \right] \mathcal{G}_k(U, U') = g_{YM}^4 \delta(U - U') . \quad (85)$$

The standard procedure for solving Eq. (85) is to use the solution to the homogeneous equation in the regions  $U < U'$  and  $U > U'$ , and then impose matching conditions at  $U = U'$ . If we now restrict to the case where  $m_{\theta_i}^2 = 0$  and write

$$\mathcal{G}_k(U, U') = \theta(U - U') \mathcal{G}_> + \theta(U' - U) \mathcal{G}_< , \quad (86)$$

the solution to the homogeneous equation for  $U > U'$  and  $p \neq 5$  is given by

$$\mathcal{G}_>(U, U') = iA_>(U') U^{\frac{1}{2}(p-7)} \left[ J_{\alpha-1}(q_*) H_{\alpha}^{(1)}(q) - H_{\alpha-1}^{(1)}(q_*) J_{\alpha}(q) \right] , \quad (87)$$

with  $\alpha = (7 - p)/(5 - p)$  and where we have imposed the Neumann condition  $\partial_u \mathcal{G}_k(U, U')|_{U=U_*} = 0$ . We have also defined  $q = i(\alpha - 1)\sqrt{\lambda}kU^{(p-5)/2}$ , and  $H_{\alpha}^{(1)}$  is the Hankel function of the first kind of order  $\alpha$ . As noted earlier there is a singularity at  $U = 0$ , but since the solution can be made finite there we will continue to impose the Hartle-Hawking boundary condition. Thus for  $U < U'$  we obtain

$$\mathcal{G}_<(U, U') = iA_<(U') U^{\frac{1}{2}(p-7)} H_{\alpha}^{(1)}(q) . \quad (88)$$

The unknown functions  $A_{<}(U')$  and  $A_{>}(U')$  are determined by imposing matching conditions at  $U = U'$ . Continuity of  $\mathcal{G}_k$  at  $U = U'$  leads to the condition

$$\mathcal{G}_{>}|_{U=U'} = \mathcal{G}_{<}|_{U=U'} , \quad (89)$$

while the discontinuity in  $\partial_u \mathcal{G}_k$  gives the condition

$$(\partial_u \mathcal{G}_{>} - \partial_u \mathcal{G}_{<}) \Big|_{U=U'} = g_{YM}^4 U'^{p-6} . \quad (90)$$

This leads to the solutions

$$A_{<}(U') = \frac{\pi g_{YM}^4 U'^{\frac{1}{2}(p-7)} H_{\alpha}^{(1)}(q')}{5-p H_{\alpha-1}^{(1)}(q_*)} , \quad (91)$$

$$A_{>}(U') = i \frac{\pi g_{YM}^4 U'^{\frac{1}{2}(p-7)} \left[ \frac{J_{\alpha-1}(q_*)}{H_{\alpha-1}^{(1)}(q_*)} H_{\alpha}^{(1)}(q') - J_{\alpha}(q') \right]}{5-p} . \quad (92)$$

Finally, substituting these functions into the equations for  $\mathcal{G}_{>}$  and  $\mathcal{G}_{<}$  gives the expression for the Green's function in the extremal  $Dp$ -brane geometry

$$\begin{aligned} \mathcal{G}_k(U, U') &= i \frac{\pi g_{YM}^4 (UU')^{\frac{1}{2}(p-7)} \frac{H_{\alpha}^{(1)}(q_{<})}{H_{\alpha-1}^{(1)}(q_*)}}{5-p} \\ &\quad \times \left[ J_{\alpha-1}(q_*) H_{\alpha}^{(1)}(q_{>}) - H_{\alpha-1}^{(1)}(q_*) J_{\alpha}(q_{>}) \right] , \end{aligned} \quad (93)$$

where in  $q_{>}(q_{<})$  we have defined  $U_{>}(U_{<})$  to be the greater (lesser) of  $U$  and  $U'$ . The Green's function (93) is the general expression when  $p \neq 5$  for a minimally coupled scalar in the extremal  $Dp$ -brane geometry. Notice also that from the pole condition  $H_{\alpha-1}^{(1)}(2i\sqrt{\lambda}k/(5-p)U_*^{\frac{1}{2}(p-5)}) = 0$ , there is a branch cut singularity at  $k = 0$ . This represents the Kaluza-Klein continuum beginning at  $m = 0$ , where  $k^2 = -m^2$ . When we restrict the coordinates to the location of the  $p$ -brane at  $U = U' = U_*$  we obtain the expression (70).

When  $p = 5$  the solutions to the homogeneous equation are no longer Bessel functions (20). Imposing the same boundary conditions as for  $p \neq 5$  gives the Green's function

$$\mathcal{G}_k(U, U') = -\frac{g_{YM}^2}{N} \frac{1+\gamma}{2\gamma k^2} \frac{1}{UU'} \left( \frac{U_*}{U_{>}} \right)^{-\gamma} \left[ (\gamma-1) \left( \frac{U_*}{U_{<}} \right)^{\gamma} + (\gamma+1) \left( \frac{U_*}{U_{<}} \right)^{-\gamma} \right] , \quad (94)$$

where  $\gamma = \sqrt{1 + \lambda k^2}$ . When  $U = U' = U_*$  we recover the expression (73). Notice also that from the pole condition  $k^2 = 0$  there is a massless mode. In addition from the pole condition  $\gamma = 0$ , there is also a branch cut singularity beginning at  $m = 1/\sqrt{\lambda}$ . Thus when  $p = 5$ , the Kaluza-Klein continuum is separated from the zero mode by a mass gap  $1/\sqrt{\lambda}$ .

The Green's function for the gauge field can be obtained by following a similar procedure to the minimally coupled scalar. In the case of  $p \neq 5$  the expression is

$$\mathcal{G}_k(U, U') = i \frac{\pi g_{YM}^2}{N(5-p)} \frac{1}{UU'} \frac{H_{\alpha-1}^{(1)}(q_<)}{H_{\alpha-2}^{(1)}(q_*)} \times \left[ J_{\alpha-2}(q_*) H_{\alpha-1}^{(1)}(q_>) - H_{\alpha-2}^{(1)}(q_*) J_{\alpha-1}(q_>) \right], \quad (95)$$

while for  $p = 5$  it is simply

$$\mathcal{G}_k(U, U') = -\frac{1}{N^2} \frac{1+\gamma}{2\gamma k^2} \frac{1}{UU'} \left( \frac{U_*}{U_>} \right)^{-\gamma} \left[ (\gamma-1) \left( \frac{U_*}{U_<} \right)^\gamma + (\gamma+1) \left( \frac{U_*}{U_<} \right)^{-\gamma} \right]. \quad (96)$$

Again we see that the  $p+1$ -dimensional Kaluza-Klein spectrum has the same characteristics as that found for the minimally coupled scalar. When  $U = U' = U_*$  we obtain the results (75) for  $p \neq 5$ , and (79) for  $p = 5$ .

## References

- [1] J. Maldacena, “*The large  $N$  limit of superconformal field theories and supergravity,*” Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].
- [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “*Gauge theory correlators from non-critical string theory,*” Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].
- [3] E. Witten, “*Anti-de Sitter space and holography,*” Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].
- [4] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “*Large  $N$  field theories, string theory and gravity,*” Phys. Rept. **323** (2000) 183 [hep-th/9905111].
- [5] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, “*Supergravity and the large  $N$  limit of theories with sixteen supercharges,*” Phys. Rev. **D 58** (1998) 046004 [hep-th/9802042].
- [6] L. Randall and R. Sundrum, “*An alternative to compactification,*” Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].
- [7] H. Verlinde, “*Holography and compactification,*” Nucl. Phys. B **580**, 264 (2000) [hep-th/9906182].
- [8] M. Alishahiha, H. Ita and Y. Oz, “*Graviton scattering on  $D6$  branes with  $B$  fields,*” JHEP**0006** (2000) 002 [hep-th/0004011].
- [9] A. Hashimoto and N. Itzhaki, “*A comment on the Zamolodchikov  $c$ -function and the black string entropy,*” Phys. Lett. B **454** (1999) 235 [hep-th/9903067].
- [10] A. Brandhuber and Y. Oz, “*The  $D4$ - $D8$  brane system and five dimensional fixed points,*” Phys. Lett. B **460** (1999) 307 [hep-th/9905148].
- [11] D. Youm, “*Solitons in brane worlds,*” Nucl. Phys. B **576** (2000) 106 [hep-th/9911218]; D. Youm, “*Solitons in brane worlds II,*” Nucl. Phys. B **576** (2000) 139 [hep-th/0001018]; D. Youm, “*Bulk fields in dilatonic and self-tuning flat domain walls*” Nucl. Phys. B **589** (2000) 315 [hep-th/0002147]; D. Youm, “*Localization of Gravity on Dilatonic Domain Walls: Addendum to “Solitons in Brane Worlds”,*” Nucl. Phys. B **596** (2001) 289 [hep-th/0007252].
- [12] M. Cvetič, H. Lu and C. N. Pope, “*Domain walls with localised gravity and domain-wall/QFT correspondence,*” Phys. Rev. D **63** (2001) 086004 [hep-th/0007209].
- [13] H. J. Boonstra, K. Skenderis and P. K. Townsend, “*The domain wall/QFT correspondence,*” JHEP **9901**, 003 (1999) [hep-th/9807137].



- [14] K. Behrndt, E. Bergshoeff, R. Halbersma and J. P. van der Schaar, “*On domain-wall/QFT dualities in various dimensions,*” *Class. Quant. Grav.* **16**, 3517 (1999) [hep-th/9907006].
- [15] J. Maldacena, unpublished; E. Witten, unpublished; S. S. Gubser, “*AdS/CFT and gravity,*” *Phys. Rev. D* **63** (2001) 084017 [hep-th/9912001].
- [16] A. Pomarol, “*Gauge bosons in a five-dimensional theory with localized gravity,*” *Phys. Lett. B* **486** (2000) 153 [hep-ph/9911294].
- [17] N. Kaloper, E. Silverstein and L. Susskind, “*Gauge symmetry and localized gravity in M theory,*” *JHEP* **0105** (2001) 031 [hep-th/0006192].
- [18] N. Arkani-Hamed, M. Porrati and L. Randall, “*Holography and phenomenology,*” hep-th/0012148;
- [19] R. Rattazzi and A. Zaffaroni, “*Comments on the holographic picture of the Randall-Sundrum model,*” *JHEP* **0104** (2001) 021 [hep-th/0012248].
- [20] T. Gherghetta and M. Shaposhnikov, “*Localizing gravity on a string-like defect in six dimensions,*” *Phys. Rev. Lett.* **85** (2000) 240 [hep-th/0004014].
- [21] E. Ponton and E. Poppitz, “*Gravity localization on string-like defects in codimension two and the AdS/CFT correspondence,*” *JHEP***0102** (2001) 042 [hep-th/0012033].
- [22] T. Gherghetta, E. Roessl and M. Shaposhnikov, “*Living inside a hedgehog: Higher-dimensional solutions that localize gravity,*” *Phys. Lett. B* **491** (2000) 353 [hep-th/0006251]; T. Gherghetta, “*Localizing gravity on a 3-brane in higher dimensions,*” hep-th/0010276.
- [23] S. B. Giddings, E. Katz and L. Randall, “*Linearized gravity in brane backgrounds,*” *JHEP* **0003** (2000) 023 [hep-th/0002091].