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Search for the Higgs boson: theoretical perspectives

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Abstract

We present a short review of experimental and theoretical constraints on the mass of the Standard Model Higgs boson. We briefly illustrate the unsatisfactory aspects of the standard theory, and we present some general considerations about possible non-standard scenarios.

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1 Introductory remarks

It is now an experimentally well-established fact that the $SU(2) \times U(1)$ gauge symmetry of electroweak interactions is spontaneously broken to $U(1)_{\rm em}$. We observe the three Goldstone modes corresponding to the broken part of the gauge group as the longitudinal polarization states of $W^\pm$ and Z^0. However, the details of the mechanism that induces spontaneous breaking of the gauge symmetry are still unknown. In the original formulation of the Standard Model, spontaneous symmetry breaking is achieved by means of a scalar $SU(2)$ doublet ϕ with unit hypercharge, whose classical potential

$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4 , \quad m^2 < 0 \tag{1}$$

has a minimum for $|\phi|^2 = -m^2/(2\lambda) \equiv v^2/2$, which corresponds, at the quantum level, to a non-invariant ground state (assuming the validity of perturbation theory in the scalar sector). The value of v is fixed by the measurement of the β decay rate, $v \simeq 246$ GeV. Only one physical degree of freedom in the Higgs sector remains in the spectrum, a scalar H with tree-level mass $m_H^2 = 2\lambda v^2$. Large values of m_H correspond to strong interactions in the Higgs sector. Indeed, the decay width of the standard model Higgs into a pair of gauge bosons

$$\Gamma(H \to VV) = \frac{3}{32\pi} \frac{m_H^3}{v^2} \tag{2}$$

becomes approximately equal to m_H for $m_H \sim 1.4$ TeV. In this case, the Higgs boson can no longer be considered as a particle. In the following, we will concentrate on the conventional scenario in which the Higgs sector is within the perturbative regime, and the Higgs boson mass is not too large.

The Higgs boson has not been detected so far. The lower limit on the Higgs boson mass from direct searches is about 113 GeV [1)], but there are strong indications that, if it exists, its mass should not be too much larger than this. In fact, a global fit to precision observables indicates that the Higgs boson of the minimal standard model is a light particle: the minimum value of χ^2, shown in fig. 1, is obtained for $m_H = 98$ GeV, with $m_H < 212$ GeV at 95% confidence level.

In the following, we will present a short and simple review of our present theoretical knowledge on the Higgs boson. We will then recall the problems that the introduction of scalar particles induces, and we will present some considerations about how extensions of the Standard Model may appear in future experiments.

2 Theoretical constraints on the Higgs boson mass

In this section, we will briefly review the arguments that lead to conclude that values of the Standard Model Higgs mass in the 100–200 GeV range are favoured on theoretical grunds.

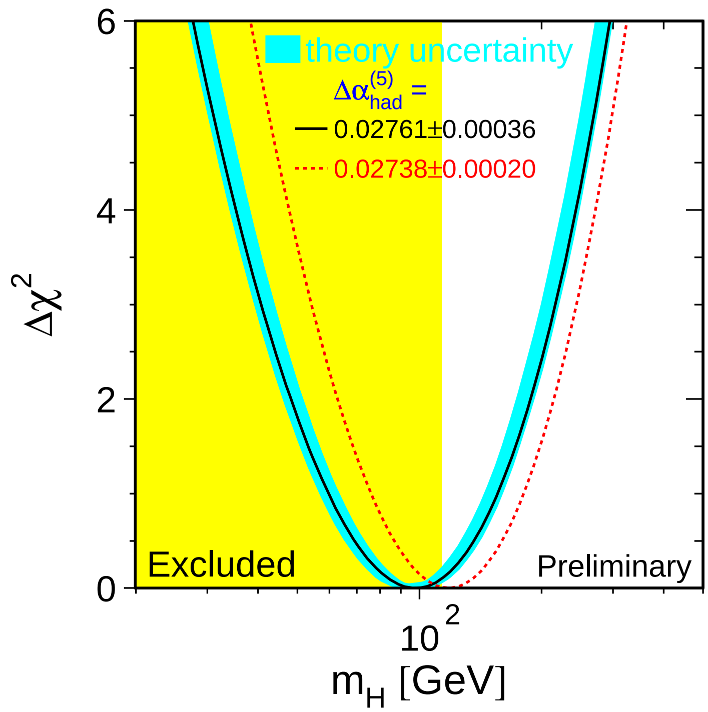

Figure 1: *Fit of the Higgs boson mass from electroweak precision data.*

A lower bound on m_H originates from the requirement that the scalar potential be bounded from below even after the inclusion of radiative corrections. In practice, it turns out that this requirement is fulfilled if the running quartic coupling $\lambda(\mu)$ stays positive, at least up to a certain scale $\mu \sim \Lambda$, the maximum energy scale at which the theory can be considered reliable. In fig. 2 the running of λ is shown for three different values of the initial condition, given at $\mu = m_Z$. Clearly, the smaller is $\lambda(m_Z)$, the smaller becomes the scale Λ at which λ becomes negative and the scalar potential unbounded. Since $m_H^2 \simeq 2\lambda(m_Z)v^2$, this implies a Λ-dependent lower bound on m_H, which is shown in fig. 3 as a function of Λ for $m_t = 174.3$ GeV. Observe that the lower bound never becomes larger than about 130 GeV.

As the dashed lines in fig. 3 show, a value of the Higgs mass just above the present exclusion limit (say $m_H = 115$ GeV) seems to imply the presence of new phenomena at a scale $\Lambda \sim 10^6$ GeV. This is in fact not strictly true, for different reasons. First, the stability lower bound of fig. 3 turns out to be extremely sensitive to the value of the top quark mass m_t, which enters the calculation because the evolution of λ also depends on the top Yukawa coupling. This can be seen from fig. 4, where the stability bound is shown for different values of m_t. For $m_t = 164$ GeV, which is only about two standard deviations smaller than the central value of the Tevatron measurements, the scalar potential of the Standard Model is bounded from below up to energies of the order of the unification scale, $\sim 10^{16}$ GeV.

Furthermore, the stability lower bound can be released by allowing metasta-

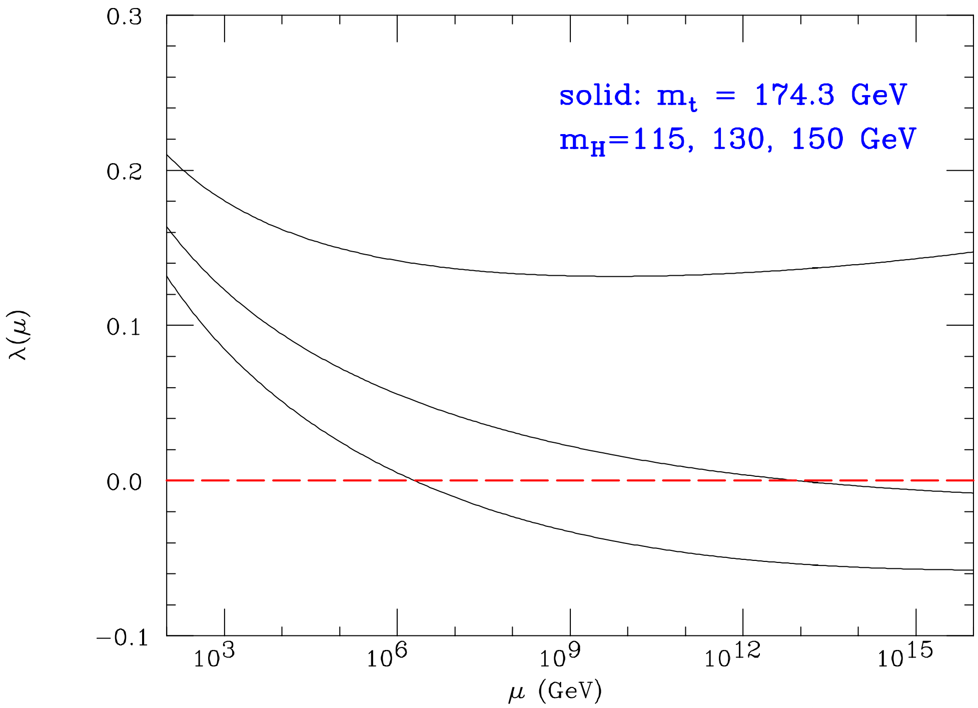

Figure 2: *The running of the scalar quartic coupling λ of the Standard Model, for three different values of the Higgs boson mass.*

bility of the ground state, instead of requiring its absolute stability, provided the lifetime of the metastable vacuum is larger than the age of the Universe, $T \sim 10^{10}$ yrs. The decay probability of the false vacuum per unit volume and per unit time is given, to one loop accuracy, by

$$\frac{\Gamma}{V} = \frac{e^{-S_1[h]}}{V} = \frac{S_0^2[h]}{4\pi^2} e^{-S_0[h]} \left| \frac{Det'(S_0''[h])}{Det(S_0''[0])} \right|^{-1/2}, \tag{3}$$

where $h(x)$ – the *bounce* – is the solution of classical field equations that interpolates between the true and the metastable vacuum state, $S_0[h]$ ($S_1[h]$) the corresponding value of the tree-level (one-loop) euclidean action, and Det' indicates that the functional determinant is to be calculated with the zero eigenvalues omitted. The metastability bound has been computed recently to one loop accuracy (see ref. [2]) and references therein). The results are summarized in fig. 5, where the dashed and dot-dashed curves represent the stability bound, $\lambda = 0$, and the metastability bound, respectively. One finds that for $m_H = 115$ GeV and m_t at its central value, the metastability bound is violated at a much higher Λ than the absolute stability bound. If $m_t = 173$ GeV,[2] less than 1 σ away from the central value, the lower bound is respected up to the grand unification scale. Therefore, even if the Higgs boson mass is close to its present exclusion limit, one cannot conclude that

[2]This value is affected by an uncertainty of ± 2 GeV, due to higher order QCD corrections.

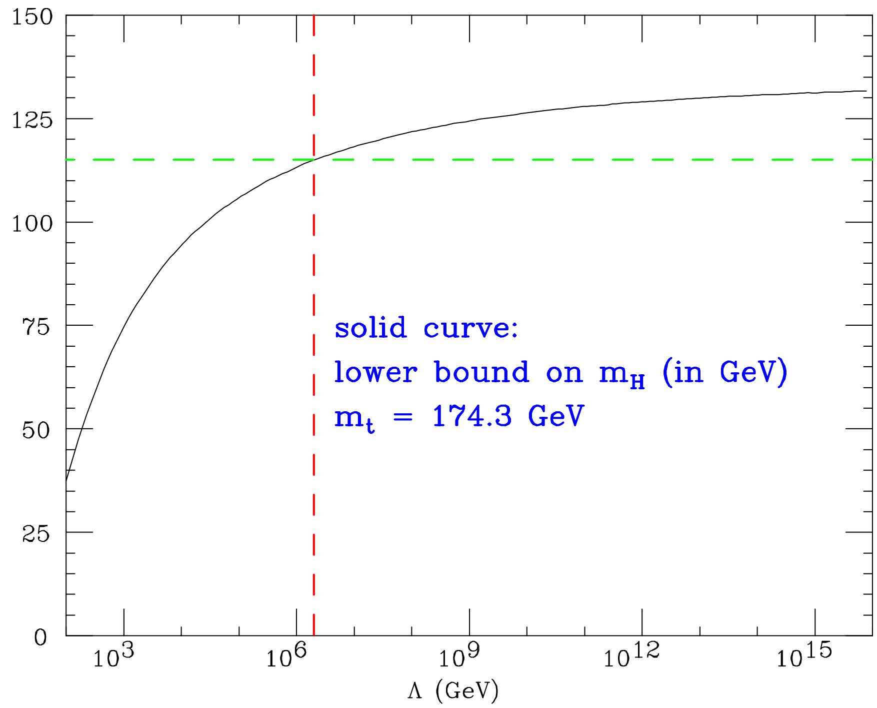

Figure 3: *The stability lower bound on the Higgs mass, for* $m_t = 174.3$ *GeV, as a function of the cut-off* Λ.

new physics is necessarily present at relatively low energies, on the basis of internal consistency arguments only.

A long-known upper bound on the Higgs mass comes from unitarity of the scattering matrix. Consider elastic scattering of longitudinally polarized Z bosons:

$$Z_L Z_L \to Z_L Z_L. \tag{4}$$

The corresponding amplitude, in the limit $s \gg m_Z^2$, can be computed using the equivalence theorem:

$$\mathcal{M} = -\frac{m_H^2}{v^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right], \tag{5}$$

and the unitarity bound on the $J = 0$ partial amplitude takes the form

$$|\mathcal{M}_0|^2 \to \left[\frac{3}{16\pi}\frac{m_H^2}{v^2}\right]^2 < \frac{s}{s-4m_Z^2}, \tag{6}$$

which, for $s \gg m_Z^2$, implies

$$m_H < \sqrt{\frac{16\pi}{3}}\, v \sim 1 \text{ TeV}. \tag{7}$$

Slightly more restrictive bounds (~ 800 GeV) are obtained considering other scattering processes, such as $Z_L W_L \to Z_L W_L$.

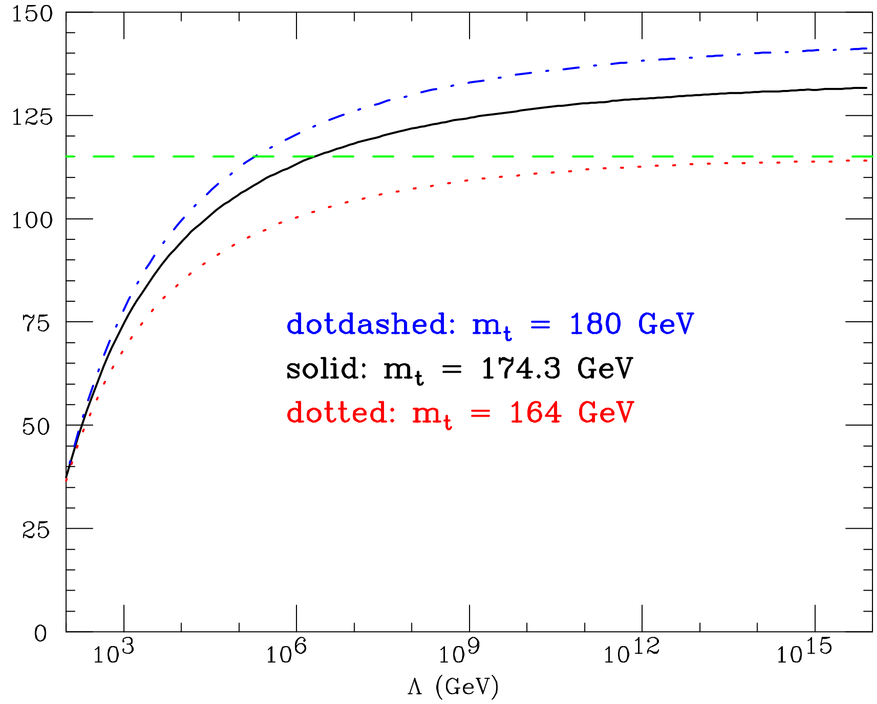

Figure 4: *Lower bound on the Higgs mass (in* GeV*) as a function of the cut-off* Λ*, for different values of* m_t*.*

A less rigorous, but more severe constraint is the so-called triviality bound. The coupling λ has a Landau pole; this can be seen explicitly by looking at the solution of the renormalization group equation for λ in the simplified case when gauge and Yukawa couplings are neglected. One finds

$$\lambda(\mu^2) = \frac{\lambda(m_Z^2)}{1 - \frac{3}{4\pi^2}\lambda(m_Z^2)\log\frac{\mu^2}{m_Z^2}} \qquad \text{(no gauge and Yukawa couplings)} \tag{8}$$

which has a singularity for

$$\mu^2 = m_Z^2 \exp\left[\frac{4\pi^2}{3\lambda(m_Z^2)}\right]. \tag{9}$$

The location of the Landau pole in the real case is at a different value of μ^2, but the qualitative behaviour is the same. The theory is no longer perturbative when μ approaches this value, and the one-loop approximation is no longer reliable. Should this behaviour persist also at higher perturbative orders, one should conclude that the theory is consistent at all energy scales only if $\lambda = 0$, that is, if the theory is a trivial one. There is no rigorous proof that this is the case for the Standard Model, but lattice computations indicate that the $\lambda\phi^4$ theory is indeed a trivial one. This is of course unacceptable in the Standard Model: we need a non-zero quartic coupling to implement the spontaneous breaking of the gauge symmetry. Therefore,

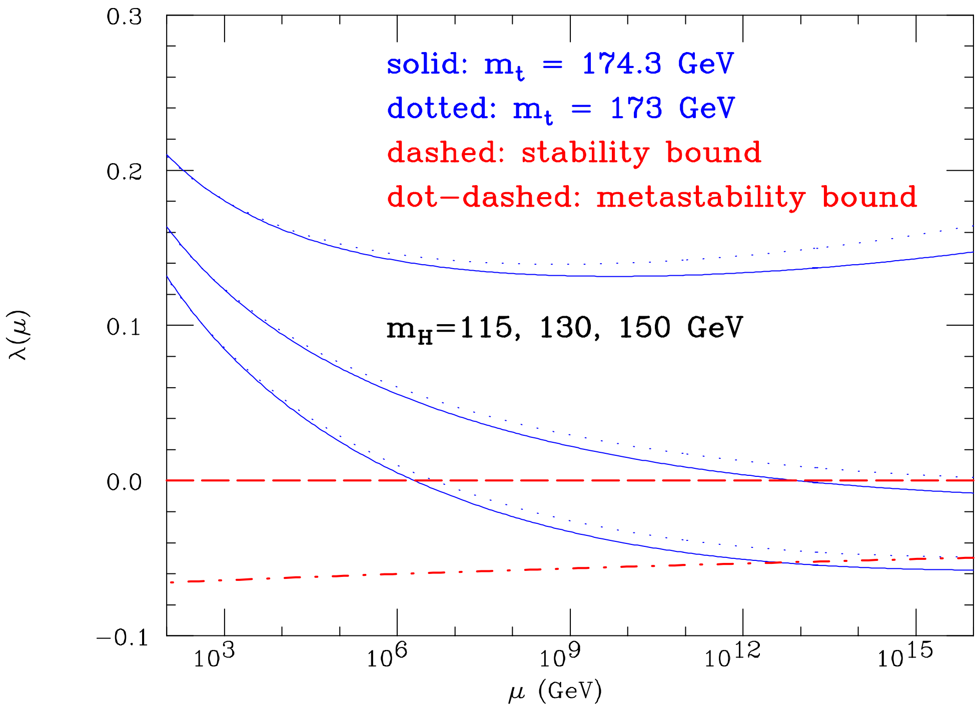

Figure 5: *Running of λ for three different values of m_H and $m_t = 174.3$ GeV (solid) and $m_t = 173$ GeV (dotted). The stability (dashed) and metastability (dot-dashed) lower bounds are also shown.*

we are forced to admit that the Standard Model is only an effective theory, valid up to some energy scale Λ, defined as the scale at which $\lambda(\mu)$ leaves the perturbative regime. Larger values of the initial condition $\lambda(m_Z)$ correspond to smaller values of Λ; conversely, the requirement that λ stay within the perturbative domain for all scales $\mu < \Lambda$, gives a Λ-dependent upper bound on m_H. Of course, this upper bound depends on how we define the perturbative domain, and therefore it is to some extent arbitrary. The triviality upper bound obtained imposing the conditions $\lambda < 1$ and $\lambda < 10$ are shown in fig. 6. We see that in both cases the triviality bound is much more stringent than the unitarity limit; an extremely severe upper bound of about 180 GeV is found if the validity of the Standard Model is pushed up to the grand unification scale.

To summarize, spontaneous gauge symmetry breaking induced by the Higgs mechanism with one scalar doublet and perturbative coupling is an extremely appealing solution: it is relatively simple, and it is consistent with present theoretical constraints. Furthermore, it should be noted that it can accommodate a consistent description (even though not an explanation) of the observed pattern of flavor violation (GIM suppression, FCNC phenomena, CP violation). At the same time, there are different indications that such a theory cannot be valid up to arbitrarily large energy scales, and that it should therefore be considered as the low-energy approximation of some more fundamental scenario.

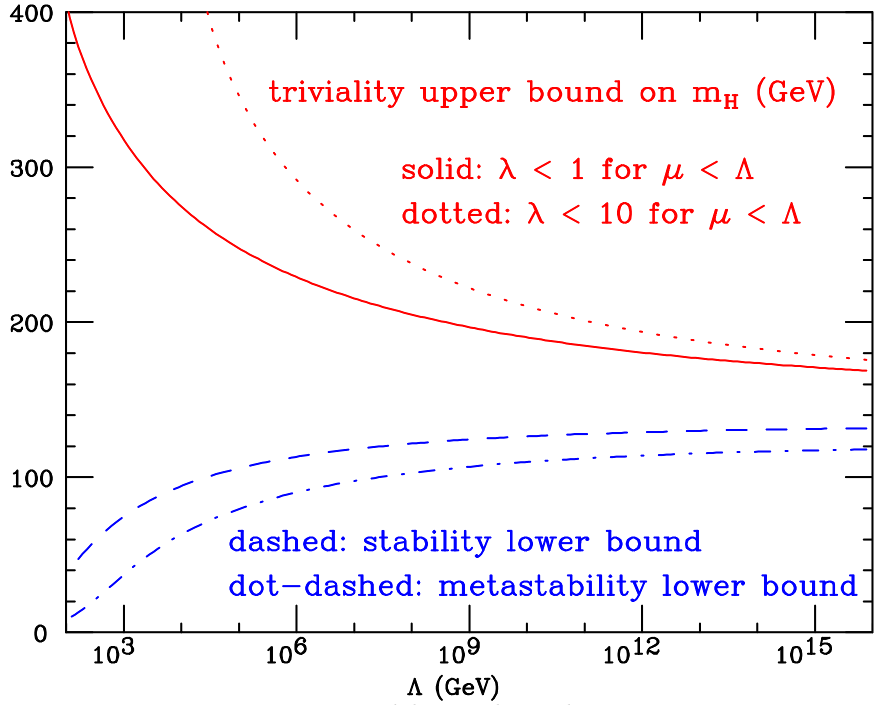

Figure 6: *Upper and lower bounds on m_H.*

3 How will new physics look like?

It is natural to ask what is the energy scale Λ at which we should expect non-standard phenomena to take place. A related question is whether it is possible to build a reasonable (i.e., consistent with data) extension of the Standard Model, where the upper bound of about 200 GeV on m_H is evaded, and the Higgs mass is close to the unitarity bound. Assuming that there are no non-standard degrees of freedom at the weak scale, one can parametrize physics at scales well below Λ by extending the Standard Model lagrangian, as suggested in ref. [3], in the following way:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda^p} \mathcal{O}_i^{(4+p)}, \tag{10}$$

where $\mathcal{O}_i^{(4+p)}$ are all the operators of dimension $4+p$, $p \geq 1$, consistent with the classical symmetries of $\mathcal{L}_{\rm SM}$, that one can build with the Standard Model fields. The upper bound on m_H obtained from the global fit to precision observables obviously holds under the assumption that Λ is large enough, so that the Standard Model is a good approximation of the true theory at presently explored energies. The non-standard term in eq. (10) must be non-renormalizable, since $\mathcal{L}_{\rm SM}$ already contains all renormalizable operators allowed by the Standard Model symmetries. The lowest

m_h	115 GeV		300 GeV		800 GeV	
c_i	-1	$+1$	-1	$+1$	-1	$+1$
$\mathcal{O}_{WB}$	9.7	10	7.5	—	—	—
$\mathcal{O}_{H}$	4.6	5.6	3.4	—	2.8	—
$\mathcal{O}_{LL}$	7.9	6.1	—	—	—	—
$\mathcal{O}'_{HL}$	8.4	8.8	7.5	—	—	—
$\mathcal{O}'_{HQ}$	6.6	6.8	—	—	—	—
$\mathcal{O}_{HL}$	7.3	9.2	—	—	—	—
$\mathcal{O}_{HQ}$	5.8	3.4	—	—	—	—
$\mathcal{O}_{HE}$	8.2	7.7	—	—	—	—
$\mathcal{O}_{HU}$	2.4	3.3	—	—	—	—
$\mathcal{O}_{HD}$	2.2	2.5	—	—	—	—

Table 1: *Fitted values of Λ (in TeV) at 95% C.L. for each of the operators in eq. (11), for different values of m_H.*

(six) dimension operators (flavor-universal, B, L, CP-conserving) are listed below:

$$\begin{array}{ll} \mathcal{O}_{WB} = (H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu} & \mathcal{O}_{H} = |H^\dagger D_\mu H|^2 \\ \mathcal{O}_{LL} = \frac{1}{2}(\bar{L}\gamma_\mu \tau^a L)^2 & \mathcal{O}'_{HL} = i(H^\dagger D_\mu \tau^a H)(\bar{L}\gamma_\mu \tau^a L) \\ \mathcal{O}'_{HQ} = i(H^\dagger D_\mu \tau^a H)(\bar{Q}\gamma_\mu \tau^a Q) & \mathcal{O}_{HL} = i(H^\dagger D_\mu H)(\bar{L}\gamma_\mu L) \\ \mathcal{O}_{HQ} = i(H^\dagger D_\mu H)(\bar{Q}\gamma_\mu Q) & \mathcal{O}_{HE} = i(H^\dagger D_\mu H)(\bar{E}\gamma_\mu E) \\ \mathcal{O}_{HU} = i(H^\dagger D_\mu H)(\bar{U}\gamma_\mu U) & \mathcal{O}_{HD} = i(H^\dagger D_\mu H)(\bar{D}\gamma_\mu D). \end{array} \tag{11}$$

The authors of ref. [3] have obtained the values of Λ for each operator in eq. (11) by fitting the data using eq. (10) with one operator at a time, and with fixed values of m_H. The results, at 95% C.L., are shown in table 1, where a blank means that no value of Λ could be found such that $\chi^2 < \chi^2_{SM} + 3.85$. Observe that the values of Λ are generally quite large. Observe also that fits to data are increasingly difficult with increasing m_H; for most operators, no value of Λ can be found that allows a Higgs boson much larger than 200 GeV. A fit is possible, for m_H in the range 300–500 GeV, for a few operators and Λ of the order of a few TeV. Building well-motivated models that give rise precisely to those effective operators is a difficult task. Some examples are known, and have been reviewed recently in ref. [5], in which non-standard physics compensates the effect of a heavy Higgs, and the fit to precision data is as good as in the Standard Model. In some cases, they lead to observable effects at the next generation of high energy experiments.

4 Hierarchy, naturalness, and fine tuning

Apart from the considerations of the previous sections, there is a very simple reason why the Standard Model is generally believed to be just an effective low-energy theory: at very high energies, new phenomena take place, that are not described by the Standard Model (gravitation is an obvious example). However, one would like to understand why the weak scale is so much smaller than other relevant energy scales, such as the Planck mass or the unification scale. This *hierarchy* problem is especially difficult to solve within the Standard Model, because of the unnaturalness of the Higgs mass. As we have seen, we have solid arguments to believe that the Higgs mass is of the same order of the weak scale; however, it is not *naturally* small, in the sense that there is no approximate symmetry that prevents it from receiving large radiative corrections. As a consequence, it naturally tends to become as heavy as the heaviest degree of freedom in the underlying theory (and therefore, maybe, of the order of the Planck mass or of the unification scale), unless the parameters are accurately chosen. It is instructive to see explicitly how this phenomenon arises in a simple example. Consider a theory of two scalars interacting through the potential

$$V_0(\phi,\Phi) = \frac{m^2}{2}\phi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\phi^2\Phi^2 \tag{12}$$

(which is the most general renormalizable potential, if symmetry under $\phi \to -\phi$, $\Phi \to -\Phi$ is required), and assume $M^2 \gg m^2 > 0$. In order to check whether this mass hierarchy is conserved at the quantum level, let us compute one-loop radiative corrections to m^2 by taking the second derivatives of the effective potential at its minimum, $\phi = \Phi = 0$. We get

$$m^2_{\text{one loop}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right), \tag{13}$$

where the running mass $m^2(\mu^2)$ obeys the renormalization group equation

$$\mu^2\frac{\partial m^2(\mu^2)}{\partial\mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right). \tag{14}$$

Corrections to m^2 proportional to M^2 appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$. The only way to preserve the hierarchy $m^2 \ll M^2$ is carefully choosing the parameters, so that

$$\lambda m^2 \sim \delta M^2, \tag{15}$$

but this requires fixing the renormalized parameters of the theory with an unnaturally high accuracy :

$$\frac{\delta}{\lambda} \sim \frac{m^2}{M^2} \tag{16}$$

This is what is usually called a *fine tuning* of the parameters. The situation is similar when $m^2 < 0$, $M^2 \gg |m^2| > 0$. In this case, the the tree-level potential has a minimum at $\Phi = 0$, $\phi^2 = -6m^2/\lambda \equiv v^2$, and the symmetry $\phi \to -\phi$ is spontaneously broken. The physical degrees of freedom in this case are Φ, with mass $m_\Phi^2 = M^2$, and $\phi' = \phi - v$ with mass $m_{\phi'}^2 = -2m^2 = \lambda v^2/3$. At one loop, v^2 is given by the minimization condition

$$m^2 + \frac{\lambda}{6}v^2 + \frac{1}{32\pi^2}\left[\lambda\left(m^2 + \frac{\lambda}{2}v^2\right)\left(\log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} - 1\right)\right.$$
$$\left. + \delta\left(M^2 + \frac{\delta}{2}v^2\right)\left(\log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2} - 1\right)\right] = 0. \qquad (17)$$

Following the same procedure as in the unbroken case, one finds

$$m_{\phi'}^2 = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2}\left[\lambda^2 \log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} + \delta^2 \log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2}\right] \qquad (18)$$

with $v \sim M$ without a suitable tuning of the parameters. These simple examples show that squared masses of scalar particles receive radiative corrections proportional to the squared masses of the other degrees of freedom in the theory. Therefore, without a suitable fine tuning of the parameters, they naturally become as large as the largest energy scale in the theory. This is related to the fact that no extra symmetry is recovered when scalar masses vanish, in contrast to what happens, for example, for fermion masses.

In the case of the Standard Model Higgs, we are already faced with this problem, as pointed out in ref. 4). The correction to m_H^2 due to a loop of top quarks is given by

$$\delta m_H^2(\mathrm{top}) = \frac{3G_F m_t^2}{\sqrt{2}\pi^2}\Lambda^2 \simeq (0.27\,\Lambda)^2, \qquad (19)$$

where we are assuming that the scale Λ that characterizes non-standard physics acts as a cut-off for the loop momentum. We have seen in the previous section that, if one assumes that no new degrees of freedom are present around the Fermi scale, Λ cannot be smaller than a few TeV. With $\Lambda \sim 5$ TeV eq. (19) gives

$$\delta m_H^2(\mathrm{top}) \sim (1.5\ \mathrm{TeV})^2, \qquad (20)$$

which is two orders of magnitude larger than the indirect value of m_H from the global fit to precision observables. There is an apparent paradox: precision tests favour a small value for the Higgs mass, but at the same time, through the analysis with effective operators, indicate that the scale Λ of non standard physics is too large to be compatible with the fitted value of m_H.

Supersymmetry offers a solution to the naturalness problem, provided the mass splittings within supermultiplets are not much larger than the Fermi scale. In fact, in supersymmetric models quadratically divergent radiative corrections to scalar masses are absent, as a consequence of the fact that supermultiplets contain both bosonic and fermionic degrees of freedom. In particular, the contribution to m_H^2 of a loop of *s-top* $\tilde{t}$ has the effect of replacing Λ^2 in eq. (19) with

$$m_{\tilde{t}}^2 \log\frac{\Lambda^2}{m_{\tilde{t}}^2} \qquad (21)$$

without affecting fits to precision observables. This is the strongest argument in favour of supersymmetry at the weak scale.

The Higgs sector of supersymmetric models has some specific features. At least two Higgs doublets must be introduced; their neutral components take non-zero vacuum expectation values v_1, v_2 (the notation $\tan\beta = v_2/v_1$ is usually adopted). After spontaneous breaking of the gauge symmetry, five physical degrees of freedom are left in the spectrum, usually denoted by h, H, A (neutral) and $H^{\pm}$ (charged). The quartic scalar coupling λ in supersymmetric models is replaced by a combination of the squared weak gauge couplings g, g'. This has two important consequences: first, the scalar potential is bounded from below by construction; second, Higgs and weak vector boson masses are related. At tree level, it can be shown that, in a wide class of supersymmetric models, the lightest Higgs h must be lighter than the Z boson. Radiative corrections shift this upper bound by an amount proportional to $G_F m_t^2$; for $\tan\beta \gtrsim 4$ one finds

$$m_h^2 \simeq m_Z^2 + \frac{3}{\sqrt{2}\pi^2} G_F m_t^4 \log\frac{m_{\tilde{t}}^2}{v^2}, \qquad (22)$$

where $m_{\tilde{t}}$ is the mass of the scalar partners of the top quark. For $m_{\tilde{t}} = 1$ TeV, eq. (22) gives $m_h \simeq 118$ GeV.

5 Conclusions and outlook

We have reviewed the basic features of the Standard Model Higgs boson, and compared them with current experimental information. The present exclusion limit for the Higgs bosons from direct searches is $m_H > 113$ GeV, but there are many indirect indications that the mass of the Standard Model Higgs may lie just above this limit, and most likely below ~ 200 GeV.

The possibility of a Higgs boson with a larger mass, whose effects are compensated by some kind of non-standard physics, has also been investigated in the literature, and reviewed here. It seems quite unlikely that this could happen in well-motivated theories, but the possibility is not ruled out.

A Higgs boson with a mass in the range 100–200 GeV is affected by the problem of the hierarchy between the weak scale and the scale of new physics, which is known to be larger than ~ 5 TeV if it is assumed that no new degree of freedom is present at accessible scales. The hierarchy problem has now become so compelling that it can be cast in the form of a paradox. Supersymmetry at the Fermi scale is still the most appealing candidate for its solution.

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