String Field Theory and the Fuzzy Sphere

Harald Ita and Yaron Oz

Theory Division, CERN
CH-1211, Geneva 23, Switzerland

Abstract

We use boundary string field theory to study open string tachyon conden-sation on a three-sphere closed string background. We consider the closed string background described by $SU(2)_k$ WZW model in the limit of large k. We compute the exact tachyon potential and analyse the decay modes.

June 2001

1   Introduction

Non-BPS D-brane systems and the process of open string tachyon condensation have been extensively studied in recent years [1]. In one line of research, it has been shown that by using boundary string field theory (BSFT) [2, 3, 4, 5] the study of open string tachyon condensation simplifies considerably [6, 7, 8]. In this case the system has only one field, the tachyon, and one can compute exact properties such as profiles and tensions of lower dimensional branes. So far, boundary string field theory has been applied in the cases of a flat target space. The inclusion of a nonzero closed string B-field in the BSFT framework has been analysed in [9, 10, 11], and the inclusion of the open string gauge fields has been studied in [12, 13, 14].

It is clearly of interest to extend the available methods to consider curved closed string backgrounds. This is of particular interest since it can touch on the nature of the background independence of the boundary string field theory formalism.

In this letter we will make a step into this direction. We will consider as the closed string background the three-sphere S^3. Closed strings on S^3 are described by $SU(2)_k$ WZW model. Tachyon condensation in this background has been discussed in [15]. We will be interested in the boundary string field theory description of D-brane systems wrapping 2-cycles in S^3. Exact analysis of these systems when the level k is finite is technically difficult. However, in the limit of large k the system simplifies considerably, and allows an exact analysis. We will compute the exact tachyon effective action up to two derivatives in the tachyon field, and in particular the exact tachyon potential. We will then use the results to study the possible decay products.

The letter is organised as follows. In section 2 we briefly review D-branes in $SU(2)_k$ WZW model and the structure of their function algebra in the large k limit. In section 3 we will consider the bosonic boundary string field theory approach to D-branes wrapping $S^2 \subset S^3$. We will perform exact computations in the large k limit and will obtain the exact tachyon potential. We will then discuss the decay products. In section 4 we will consider the supersymmetric string case. The relevant systems are D-branes and anti D-branes wrapping the same 2-cycle or different ones, and systems of non-BPS D-branes obtained from the branes-antibranes systems by projection. We will compute the exact tachyon potential in all these cases and analyse the decay products.

2 $SU(2)$ WZW and the fuzzy sphere

Consider closed strings propagating on a three-sphere S^3 with radius $R_{S^3} = \sqrt{\alpha' k}$, and a nonzero NSNS 3-form H field proportional to the volume form ω_{S^3}. The metric on S^3 is given by

$$ds^2_{S_3} \;=\; \alpha' k \left(d\theta^2 + \sin^2\theta (d\psi^2 + \sin^2\psi d\phi^2) \right). \tag{1}$$

This system is described by an $SU(2)_k$ WZW model.

Symmetry preserving D-branes[1] on S^3 are characterised by boundary conditions on the currents

$$J(z) = \bar{J}(\bar{z}) \quad at \quad z = \bar{z} , \tag{2}$$

where

$$J(z) = -k\partial_z g g^{-1}, \quad \bar{J}(\bar{z}) = kg^{-1}\partial_{\bar{z}}g . \tag{3}$$

Solutions to these boundary conditions can be labelled by an index $\alpha = 0, 1, ..., k$, with each D-brane having a world volume being an $SU(2)$ conjugacy class. Geometrically these conjugacy classes are two-spheres $S^2 \subset S^3$ specified by an angle θ given by

$$\theta_\alpha = \frac{\pi\alpha}{k}, \quad 0 \le \alpha \le k . \tag{4}$$

For a D-brane with label α the open string vertex operators are labeled by $V[Y_m^J]$ $J = 0, 1..., min(\alpha, k - \alpha), |m| \le J$ where m is an integer. The OPE of the vertex operators $V[Y_m^J]$ reads [16, 17]

$$V[Y_i^I](x_1)V[Y_j^J](x_2) \sim \sum_{K,k}(x_1 - x_2)^{h_K - h_I - h_J} \begin{bmatrix} I & J & K \\ i & j & k \end{bmatrix} c_{IJK}^{k,\alpha} V[Y_k^K](x_2) , \tag{5}$$

where $h_I = \frac{I(I+1)}{k+2}$ is the conformal dimension of $V[Y_i^I](x_1)$ and $[:::]$ are the Clebsch-Gordan coefficients of $SU(2)$. The structure constants $c_{IJK}^{k,\alpha}$ are given by the q-deformed $6J$ symbols of $SU(2)$ [18],

$$c_{IJK}^{k,\alpha} = \left\{ \begin{matrix} I & J & K \\ \alpha/2 & \alpha/2 & \alpha/2 \end{matrix} \right\}_q, \quad q = e^{\frac{2\pi i}{k+2}} . \tag{6}$$

We will consider the system in the limit of large k. In this case the q-deformation parameter goes to one and the structure constants $c_{IJK}^{k,\alpha}$ become the ordinary $6J$ symbols. We expand around small conformal weights, such that the OPE (5) depends on the insertion points only through the order of the operators

$$(x_1 - x_2)^{h_K - h_I - h_J} \to 1, \quad for \quad \frac{I(I+1)}{k+2}, \frac{J(J+1)}{k+2}, ... \ll 1 . \tag{7}$$

To leading order the OPE of the vertex operators then reads

$$V[Y_i^I] \cdot V[Y_j^J] = \sum_{K,k} \begin{bmatrix} I & J & K \\ i & j & k \end{bmatrix} c_{IJK}^{k,\alpha} V[Y_k^K] . \tag{8}$$

The OPE (8) for a brane α is isomorphic to the matrix algebra $M_{\alpha+1}(\mathbb{C})$ [16]

$$V[Y_i^I] \sim Y_{Ii}, \quad V[Y_i^I] \cdot V[Y_j^J] \sim Y_{Ii} * Y_{Jj} , \tag{9}$$

---

[1] We will concentrate on the branes related to conjugacy classes with trivial auto-morphism which are centred around a fixed point 'the' pole of the S^3.

where $*$ is the ordinary matrix product (see appendix A for a detailed discussion). This simplification can be used in order to calculate the expectation values of products of vertex operators via

$$\langle V[A](x_1)V[B](x_2)...V[C](x_n)\rangle \sim \ tr(A*B*...*C) \ , \tag{10}$$

where $V[A] = \sum_{Jj} A_{Jj} V[Y_j^J]$.

Note that since for a stack of N branes on the same world volume we have to add the Chan Paton matrices λ, the effective matrix algebra is enlarged from $M_{\alpha+1}(\mathbb{C})$ to $M_{N(\alpha+1)}(\mathbb{C})$.

3   The bosonic BSFT

In this section we will consider the boundary string field theory approach to D-branes wrapping $S^2 \subset S^3$ in the large k limit. We will consider the bosonic case, compute the exact tachyon potential and analyse the decay modes. In general the D-branes may have extra world volume coordinates in M^6. This can be taken into account simply by considering matrix valued functions on these coordinates. In the following we will ignore dependence on M^6.

3.1   The tachyon action

Consider the two-dimensional action

$$S = S_0 + \int_{\partial D} d\tau \mathcal{V} \ , \tag{11}$$

where S_0 denotes an open plus closed conformally invariant background (the bulk action), and $\mathcal{V}$ is a general boundary perturbation. In our case S_0 is a WZW action, while $\mathcal{V}$ is

$$\mathcal{V} = T(g(z,\bar{z}))|_{z=\bar{z}} = \sum_{I,i} T_i^I V[Y_i^I](z,\bar{z})|_{z=\bar{z}} \ . \tag{12}$$

∂D denotes the boundary of the disk D. Following [2, 3, 4, 5] one constructs the partition function on the disk

$$Z(\lambda) = \langle \exp(\int_{\partial D} d\tau \sum_{Ii} T_i^I V[Y_i^I])\rangle_{S_0} \ . \tag{13}$$

The proposed space-time action $\mathcal{S}(T)$ is defined by

$$\mathcal{S}(T) = (\beta^T \partial_T + 1)Z(T) \ , \tag{14}$$

where β^T are the β-functions of the couplings T_i^I.

We will work in the large k limit described in the previous section. The β-function for (the SU(2) part of) the tachyon reads

$$\beta_T = -T + \alpha' G^{ab} L_a L_b T + O(T^2) \ , \tag{15}$$

where $L_a, a = 1, 2, 3$ are the rescaled angular momentum operators

$$L_a = \mathbf{L}_a / \sqrt{2\alpha'}, \quad \mathbf{L}_a = [Y_a^1, .] \ , \tag{16}$$

and

$$G^{ab} = \left(\frac{2}{k} + O(1/k)^2 \right) \delta^{ab} \ . \tag{17}$$

The string partition function (13) to linear order in α' reads

$$Z(T) = C_\alpha \ tr \left[e^{-T} (1 - a_1 \alpha' L_a L^a) T \right] \ , \tag{18}$$

where α labels the S^2 conjugacy classes on the S^3 and tr is taken on the $(\alpha + 1) \times (\alpha + 1)$ matrices. The normalisation factor

$$C_\alpha = \frac{4\pi \alpha' k \ sin(\frac{\pi\alpha}{k})}{\alpha} \ , \tag{19}$$

is chosen such that for $T = 0$ one gets the mass of the brane. The mass is given by the noncommutative brane tension. The usual string coupling g_s is replaced by the non commutative string coupling $G_s = \frac{g_s}{\sin \frac{\pi\alpha}{k}}$ along the lines of [19] times the volume of the brane. a_1 is a numerical constant that will be fixed shortly.

The boundary string field theory relates the string partition function to the space-time action (14)

$$\mathcal{S}(T) \ = \ (1 + \beta_T \frac{\delta}{\delta T}) Z(T) \tag{20}$$

$$= \ C_\alpha tr \left[e^{-T} (1 + T - \alpha' (1 - a_1)(L_a L^a) T \right] \ . \tag{21}$$

Let us show that by using appropriate field redefinitions the tachyon actions, to all orders in T and to second order in "derivatives" L_a, can thus be recast in the form

$$\mathcal{S}(T) = C_\alpha tr \left[e^{-T} \left(L_a T L^a T + 1 + T \right) \right] . \tag{22}$$

We start with an action for the tachyon of the form

$$\mathcal{S}(T) \sim tr \left[e^{-T} \left(a_1(T) L_a T a_2(T) L^a T + 1 + T \right) \right] \ , \tag{23}$$

where $a_i(T)$ are some polynomials in T. We can use a field redefinition

$$T \to T - b_1(T) L_a T b_2(T) L^a T \ , \tag{24}$$

4

where $b_i(T)$ are some polynomials in T, which generates, to leading order in $L_a L^a$, an orbit of actions

$$\mathcal{S}_{orbit}(T) \sim tr \left[e^{-T} \left(a_1(T) L_a T a_2(T) L^a T + T b_1(T) L_a T b_2(T) L^a T + 1 + T \right) \right] . \quad (25)$$

These orbits of actions thus differ by a kinetic term of the form

$$\mathcal{S}_{kin} = C \, e^{-T} L_a T L^a T , \quad (26)$$

where C is a constant, which cannot be generated by field redefinitions.

Thus, the most general tachyon action is

$$\mathcal{S}(T) \sim tr \left[e^{-T} \left(\sum_n a_{1,n}(T) L_a T a_{2,n}(T) L^a T + 1 + T \right) \right] . \quad (27)$$

and as we have seen it can be reached by field redefinitions from the action with a constant in front the kinetic term. To fix the constant we can use the consistency of the field equations derived from $\mathcal{S}(T)$ and the β-function [6]

$$\beta_T \sim \frac{\delta \mathcal{S}(T)}{\delta T} . \quad (28)$$

We get

$$C = \sum_n a_{1,n}(0) a_{2,n}(0) = 1 , \quad (29)$$

and arrive at (22).

3.2 Decay modes

We have seen above that for a D-brane wrapping the α conjugacy class, the tachyon potential is given by

$$V(T) = C_\alpha tr[e^{-T} (1 + T)] . \quad (30)$$

For a single D-brane, T is a hermitian $(\alpha + 1) \times (\alpha + 1)$ matrix. For N D-branes we have to increase the matrix size to $N(\alpha + 1) \times N(\alpha + 1)$. The potential (30) is the exact (string tree level) tachyon potential in the large k limit. It has the same form as for a flat bosonic D-brane and the flat bosonic D-brane with a constant B-field in the non-commutative limit. What differs is the function algebra, which in our case is simply a matrix algebra. In the following we discuss the possible decay modes of the system. For this analysis we can neglect the kinetic term and analyse the potential.

Minima of the tachyon potential satisfy

$$V'(T) = -T e^{-T} = 0 . \quad (31)$$

5

In order to analyse (31) we can diagonalise the tachyon. Solutions of (31) are matrices with zero and infinite eigenvalues. The top of the tachyon potential is when T is the zero matrix. The absolute minimum of the tachyon potential is reached when the tachyon's eigenvalues are all infinite

$$T = \lambda \, diag\{t_1, t_2, ...t_{\alpha+1}\}, \quad \lambda \to \infty, \quad t_i > 0 \ . \tag{32}$$

It corresponds to reaching the closed string vacuum.

Intermediate decay products correspond to tachyon configurations where not all eigenvalues are infinite

$$T = \lambda \, diag\{t_1, .., t_k, 0, .., 0\}, \quad \lambda \to \infty, \quad t_i > 0 \ . \tag{33}$$

Such a configuration corresponds to the decay of the α D-brane to the $\alpha - k$ D-brane. The number of zeros is also the number of D0-branes from which the spherical D2-brane is built. In particular the trivial tachyon $T = 0$ describes the α D2-brane which is made of $\alpha + 1$ D0-branes.

In the language of perturbations (12) the analysis of decay modes is simple. Since we work in the large k limit we associate for a given tachyon perturbation an $(\alpha+1) \times (\alpha+1)$ matrix, using the results in section 2 and the details in the appendix. We then diagonalise this matrix and analyse its eigenvalues as above. This provides us with the information on the decay mode associated with the perturbation and a simple picture of the endpoint of the two-dimensional RG flow. Let us illustrate this with a simple example.

We consider a D2-brane wrapping the $\alpha = 1$ 2-sphere. The tachyon field is a hermitian 2×2-matrix T, which we can expand in terms of the matrix representation of the $\alpha = 1$ matrices $\{Y_j^J\}_{ab}$ given by (66). When T has two positive eigenvalues the system condenses to the vacuum. This happens, for instance, if $T = iY_0^0$. To get an $\alpha = 0$ brane at the endpoint of the condensation we can take a tachyon configuration of the form $T = Y_1^1 + Y_{-1}^1 - i\sqrt{2}Y_0^0$ which has eigenvalues zero and one.

We note that this simple picture is really a feature of the large k limit. For finite k, such perturbations which are typically not free, are much harder to analyse. In particular, we cannot simply look for the vanishing locus of the tachyon profile expressed via the spherical harmonics in order to analyse the decay product, as done for the free perturbations used in the flat target space case. This can be seen from the above example. The spherical harmonic $iY_0^1(\theta, \phi)$ has zeros at the poles $\psi = 0, \pi$. Thus one might suspect the $\alpha = 1$ brane to decay into two separated $\alpha = 0$ branes on the poles. However, the matrix iY_0^1 has nonzero eigenvalues $\pm 1/\sqrt{2}$.

It seems plausible to assume that for a given tachyon perturbation, the endpoint of the two-dimensional RG flow for finite k will not differ from the large k one. In such a case, while the finite k RG-flow is complicated to analyse, at least the end-points of the flow have a simple picture as described by the large large k limit above.

6

4 Supersymmetric BSFT

In this section we will consider the supersymmetric boundary string field theory approach to $D\bar{D}$-branes and non-BPS branes wrapping $S^2 \subset S^3$ in the large k limit.

4.1 The tachyon action

The boundary vertex operators for N_α branes and N_β antibranes can be constructed from [20, 21],

$$M(A_\alpha, A_\beta, T) = \begin{pmatrix} -iA_\alpha & T \\ T^\dagger & -iA_\beta \end{pmatrix}, \tag{34}$$

with,

$$A_i = \psi_a V[A_i^a] + \theta\sqrt{2/\alpha'}\left[: j_a V[A_i^a] : -\alpha' : \psi_a \psi_b : V[L^b A_i^a]\right], \quad i = \alpha, \beta \tag{35}$$

$$T = (V[T] - \theta\sqrt{2\alpha'}\left[\psi_a V[L^a T]\right] . \tag{36}$$

where ψ_a and j_a form the supercurrent $J_a(x) = i\sqrt{2}(\psi_a(x) - \sqrt{2/\alpha'}\theta j_a(x))$. A_i^a are products of Y_j^Js and Chan Paton matrices of dimensions $N_i \times N_i$, such that $V[A_i^a]^\dagger = V[A_i^a]$. The tachyon fields T are complex $(T = \frac{1}{2}(T_1 + iT_2))$ products of Y_j^Js and Chan Paton matrices of dimensions $N_\alpha \times N_\beta$. The superconnection structure appears in(34), as in [22, 23, 13, 14, 24].

The operators $L_a T$ act as the generators of the rotation group on the respective branes. For the tachyon field they are defined as $\sqrt{2\alpha'}L_a T = ((Y_\alpha)_a^1 \otimes 1_{\alpha\times\alpha})T - T((Y_\beta)_a^1 \otimes 1_{\beta\times\beta})$. This is natural as the tachyons transform in the bifundamental representation of the rotation groups on the respective branes α, β. Note that the Chan Paton factors are not changed under the rotation. The normalisation is such that the OPEs read

$$j^a(z_1)j^b(z_2) = \frac{\alpha'}{2}\frac{G^{ab}}{(z_1 - z_2)^2} + \alpha'\frac{if^{ab}{}_c j^c(z_2)}{z_1 - z_2} , \tag{37}$$

$$\psi^a(z_1)\psi^b(z_2) = \frac{G^{ab}}{z_1 - z_2}, \quad f^{ab}{}_c = \frac{1}{\sqrt{\alpha'}}\frac{2}{k}\varepsilon^{ab}{}_c . \tag{38}$$

For later use we introduce M_0 and M_1 for the θ independent and the θ dependent part of $M = M_0 + \theta M_1$. M_0 (M_1) is related to the picture (-1) (picture (0)) vertex operators of the sum of the tachyon and the gauge field.

The world sheet action reads

$$S = S_0 + S_{pert} , \tag{39}$$

$$S_{pert} = \int d\hat{\tau}\left[\hat{\bar{\eta}}^a D\hat{\eta}^a + \hat{\bar{\eta}}^a M_{ab}\hat{\eta}^b\right] , \tag{40}$$

where the boundary fields $\hat{\bar{\eta}}, \hat{\eta}$ have to be integrated over in the path integral. We use the notation $d\hat{\tau} = d\tau d\theta$, $D = \partial_\theta + \theta\partial_\tau$ and the fermionic superfields $\hat{\eta}^a = \eta^a + \theta\chi^a$.

The integration over the auxiliary fields χ rearranges the boundary action, such that one finds the path ordered product of $M_0^2 + M_1$,

$$S = S_0 + S'_{pert} , \tag{41}$$

$$S'_{pert} = \left(\int d\tau \left[\bar{\eta} \partial_\tau \eta + \bar{\eta} (M_0^2 + M_1) \eta \right) \right] . \tag{42}$$

For zero gauge fields A_1, A_2 one finds

$$M_0^2 + M_1 = \begin{pmatrix} V[T]V[T]^\dagger & -\sqrt{2\alpha'} \, \psi_a V[L^a T] \\ -\sqrt{2\alpha'} \, \psi_a V[L^a T]^\dagger & V[T]^\dagger V[T] \end{pmatrix} . \tag{43}$$

From this (43) the tachyon potential, which is the leading order term in the $1/k$ expansion, can be read. First order corrections have two origins: corrections from the OPE (5) and contributions from the off diagonal entries in (43).

$$Z(T, T^\dagger) = C_\alpha tr_\alpha \left[e^{-c_1 TT^\dagger - c_2 LT(LT)^\dagger} \right] \tag{44}$$

$$+ C_\beta tr_\beta \left[e^{-c_1 T^\dagger T - c_2 (LT)^\dagger LT} \right] + O(\frac{1}{k^2}) . \tag{45}$$

The kinetic terms are understood to be ordered in a symmetric way. The constants c_1, c_2 will be fixed below. The BSFT action $\mathcal{S}$ of the super string is conjectured to be [8] (see also [25, 26])

$$\mathcal{S}(T) = Z(T) . \tag{46}$$

Consistency with the β function can be used to fix the coefficients $c_1 = 1/4, c_2 = -\alpha'/2$. This is because to quadratic order in the tachyon the action has to reproduce the mass formula $-\alpha' m^2 = J(J+1)/k - 1/2$. The kinetic term gives

$$2 \frac{\alpha'}{2} G^{ab} L_a L_b Y_j^J = \frac{J(J+1)}{k} Y_j^J . \tag{47}$$

The constant c_2 is fixed such that the quadratic term in the tachyon potential produces the $-\frac{1}{2}$ in the open string mass formula.

Finally, the tachyon action up to second order in $1/k$ reads

$$\mathcal{S}(T)_{D\bar{D}} = C_\alpha tr_\alpha \left[e^{-\frac{1}{4} TT^\dagger} \left(-\alpha'/2 \, L_a T (L^a T)^\dagger + 1 \right) \right] \tag{48}$$

$$+ C_\beta tr_\beta \left[e^{-\frac{1}{4} T^\dagger T} \left(-\alpha'/2 \, (L_a T)^\dagger L^a T + 1 \right) \right] . \tag{49}$$

Next we will derive the tachyon action for a N_α non-BPS branes on a α sphere $S_2 \subset S_3$. To this end one gauges the $(-1)^{F_L}$ symmetry of the system of N_α brane anti brane pairs [1]. This equates the gauge fields $A_\alpha = A_{\alpha'}$ and selects the tachyon field to be real $T = T^\dagger$.

8

The boundary perturbation simplifies to

$$M(A,T) = \begin{pmatrix} -iA & T \\ T & -iA \end{pmatrix} . \tag{50}$$

Analogous steps as above lead to the tachyon action

$$\mathcal{S}(T)_{non-BPS} = \sqrt{2} C_\alpha tr_\alpha \left[e^{-\frac{1}{4}TT} \left(-\alpha'/2 \ L_a T(L^a T) + 1 \right) \right] . \tag{51}$$

4.2 Decay modes

We can distinguish three different systems:
(1) Coinciding brane-antibrane system wrapping the same conjugacy class α, which we denote by $D(\alpha)\bar{D}(\alpha)$.
(2) Brane wrapping a conjugacy class α and antibrane wrapping a different conjugacy class β, which we denote by $D(\alpha)\bar{D}(\beta)$.
(3) Non-BPS brane wrapping a conjugacy class α.
Consider the decay modes of these systems.

$D(\alpha)\bar{D}(\alpha)$ **system**

The tachyon energy of the $D2\bar{D}2$ system wrapping the α 2-cycle is given by

$$E(\alpha, T, T^\dagger) = 2C_\alpha tr \left(e^{-\frac{1}{4}TT^\dagger} \right) . \tag{52}$$

As in the bosonic case we are interested in the extrema of this potential, they fulfil the equations

$$T^\dagger e^{-\frac{1}{4}TT^\dagger} = 0, \quad e^{-\frac{1}{4}TT^\dagger}T = 0 . \tag{53}$$

The tachyon can be diagonalised by two unitary matrices U, W with $UTW^\dagger$. The global minimum corresponds to tachyon perturbations with nonzero eigenvalues that become infinite at the endpoint of the perturbation

$$T = \lambda \ diag\{t_1, t_2, ...t_{\alpha+1}\}, \quad \lambda \to \infty, \quad t_i \neq 0 . \tag{54}$$

Intermediate decay products of $D2\bar{D}2$ system wrapping a 2-cycle $\beta < \alpha$ correspond to the tachyon matrices with some zero eigenvalues. They are bi-unitary transformations of

$$T = \lambda \ diag\{t_1, .., t_k, 0, .., 0\}_{\alpha+1}, \quad \lambda \to \infty, \quad t_i \neq 0 . \tag{55}$$

Tachyons of the form (55) correspond to a perturbation from a $D2\bar{D}2$ system wrapping a 2-cycle α to a $D2\bar{D}2$ system wrapping a 2-cycle $\beta = \alpha - k$. One can also interpret the zeros of the tachyon matrix as counting the number of $D0\bar{D}0$ constituent

9

states of the condensed $D2\bar{D}2$ system. As usual the zero tachyon corresponds to the top of the potential with no condensation.

$D(\alpha)\bar{D}(\beta)$ system

We consider a brane wrapping a 2-cycle α and an antibrane wrapping a 2-cycle β. Without loss of generality we assume that $\alpha > \beta$ and that the branes are concentric. The energy of the system in this case reads

$$E(\alpha, \beta, T, T^\dagger) \;=\; C_\alpha tr_\alpha \left(e^{-\frac{1}{4}TT^\dagger} \right) + C_\beta tr_\beta \left(e^{-\frac{1}{4}T^\dagger T} \right) \; . \tag{56}$$

For extrema the conditions

$$T^\dagger e^{-\frac{1}{4}TT^\dagger} = 0 \; , \quad e^{-\frac{1}{4}TT^\dagger} T = 0 \tag{57}$$

have to hold.

We can think of the tachyon T and its conjugate $T^\dagger$ as maps $T : E_\beta \to E_\alpha$ and $T^\dagger : E_\alpha \to E_\beta$, where E_α and E_β are the vector bundles on the α and β 2-cycles corresponding to the brane and antibrane respectively. The relevant operators for the discussion are $TT^\dagger$ and $T^\dagger T$. In our case, they are matrices of size $(\alpha+1)\times(\alpha+1)$ and $(\beta+1)\times(\beta+1)$. Their zeros determine the number of $D0$-branes and $\bar{D}0$-branes constituents, respectively. Thus, the index

$$Index(E, T) = dim\ Ker\ TT^\dagger - dim\ Ker\ T^\dagger T \; , \tag{58}$$

counts the net $D0$-brane charge.

As to the decay modes, the analysis is as before. A matrix $TT^\dagger$ of the form

$$TT^\dagger = \lambda\ diag\{t_1, .., t_k, 0, ..., 0\}_{\alpha+1}, \quad \lambda \to \infty, \quad t_i \neq 0 \; , \tag{59}$$

corresponds to a perturbation that will reduce the 2-cycle α to a 2-cycle $\alpha - k$. A $T^\dagger T$ matrix of the form

$$T^\dagger T = \lambda\ diag\{t_1, .., t_k, 0, ..., 0\}_{\beta+1}, \quad \lambda \to \infty, \quad t_i \neq 0 \; , \tag{60}$$

corresponds to a perturbation that will reduce the 2-cycle β to a 2-cycle $\beta - k$.

Non-BPS branes

For a non-BPS α brane wrapping the α 2-cycle the energy reads

$$E(\alpha, T) \;=\; \sqrt{2}C_\alpha tr_\alpha \left(e^{-\frac{1}{4}TT} \right) \; , \tag{61}$$

where $T = T^\dagger$. The analysis of the decay modes is as before. A tachyon of the form

$$T = \lambda\ diag\{t_1, ...t_k, 0, ..., 0\}_{\alpha+1} \; , \tag{62}$$

initiates a flow to a non-BPS branes wrapping the $\alpha - k$ 2-cycle.

Appendix

A   Representations Of The Fuzzy Algebra

In this appendix we review some details concerning the vertex operator algebra as the matrix algebra of the fuzzy sphere [27]. Recall the OPE of the vertex operators (8)

$$V[Y_i^I] \cdot V[Y_j^J] = \sum_{K,k} \left[\begin{array}{ccc} I & J & K \\ i & j & k \end{array} \right] c_{IJK}^{k,\alpha} \, V[Y_k^K] \, . \tag{63}$$

The symbols [:::] are related to the $3j$-symbols (:::) by

$$\left[\begin{array}{ccc} I & J & K \\ i & j & k \end{array} \right] = (-1)^{k+1}\sqrt{2I+1}\sqrt{2J+1}\sqrt{2K+1} \left(\begin{array}{ccc} I & J & K \\ i & j & -k \end{array} \right). \tag{64}$$

The spherical harmonics Y_j^J are represented on the α fuzzy sphere by the $(\alpha+1) \times (\alpha+1)$ matrix

$$\{Y_j^J\}_{ab} = (-1)^{\alpha/2-j}\sqrt{2J+1} \left(\begin{array}{ccc} \alpha/2 & J & \alpha/2 \\ -a & j & b \end{array} \right) \quad a,b = -\alpha/2, -\alpha/2+1, ..., \alpha/2$$

$$tr[Y_j^J Y_i^I] = (-1)^{j+\alpha}\delta_{JI}\delta_{j,-i} \, . \tag{65}$$

For $\alpha = 1$ the matrices $\{Y_j^J\}_{ab}$ are

$$Y_0^0 = \left(\begin{array}{cc} -i/\sqrt{2} & 0 \\ 0 & -i/\sqrt{2} \end{array} \right), \tag{66}$$

$$Y_1^1 = \left(\begin{array}{cc} 0 & 0 \\ -i & 0 \end{array} \right), \ Y_0^1 = \left(\begin{array}{cc} i/\sqrt{2} & 0 \\ 0 & -i/\sqrt{2} \end{array} \right), \ Y_{-1}^1 = \left(\begin{array}{cc} 0 & i \\ 0 & 0 \end{array} \right). \tag{67}$$

The most general tachyon configuration thus reads $T = \sum_{Jj} T_j^J Y_j^J$. Its eigenvalues determine the endpoint of the respective RG flow, as discussed in the paper.

References

[1] A. Sen, "*Non-BPS states and branes in string theory*," hep-th/9904207.

[2] E. Witten, "*On background independent open string field theory*," Phys. Rev. D **46** (1992) 5467 [hep-th/9208027].

[3] E. Witten, "*Some computations in background independent off-shell string theory*," Phys. Rev. D **47** (1993) 3405 [hep-th/9210065].

[4] S. L. Shatashvili, "*On the problems with background independence in string theory*," hep-th/9311177.

[5] S. L. Shatashvili, "*Comment on the background independent open string theory*," Phys. Lett. B **311** (1993) 83 [hep-th/9303143].

[6] A. A. Gerasimov and S. L. Shatashvili, "*On exact tachyon potential in open string field theory*," JHEP **0010**, 034 (2000) [hep-th/0009103].

[7] D. Kutasov, M. Marino and G. Moore, "*Some exact results on tachyon condensation in string field theory*," JHEP **0010**, 045 (2000) [hep-th/0009148].

[8] D. Kutasov, M. Marino and G. Moore, "*Remarks on tachyon condensation in superstring field theory*," hep-th/0010108.

[9] L. Cornalba, "*Tachyon condensation in large magnetic fields with background independent string field theory*," Phys. Lett. B **504**, 55 (2001) [hep-th/0010021].

[10] K. Okuyama, "*Noncommutative tachyon from background independent open string field theory*," Phys. Lett. B **499**, 167 (2001) [hep-th/0010028].

[11] D. Nemeschansky and V. Yasnov, "*Background independent open string field theory and constant B-field*," hep-th/0011108.

[12] O. Andreev, "*Some computations of partition functions and tachyon potentials in background independent off-shell string theory*," Nucl. Phys. B **598**, 151 (2001) [hep-th/0010218].

[13] P. Kraus and F. Larsen, "*Boundary string field theory of the D D-bar system*," Phys. Rev. D **63**, 106004 (2001) [hep-th/0012198].

[14] T. Takayanagi, S. Terashima and T. Uesugi, "*Brane-antibrane action from boundary string field theory*," JHEP **0103**, 019 (2001) [hep-th/0012210].

[15] Y. Hikida, M. Nozaki and T. Takayanagi, "*Tachyon condensation on fuzzy sphere and noncommutative solitons*," Nucl. Phys. B **595**, 319 (2001) [hep-th/0008023].

[16] A. Y. Alekseev, A. Recknagel and V. Schomerus, "*Non-commutative world-volume geometries: Branes on SU(2) and fuzzy spheres*," JHEP**9909** (1999) 023 [hep-th/9908040].

[17] I. Runkel, "*Boundary structure constants for the A-series Virasoro minimal models*," Nucl. Phys. B **549** (1999) 563 [hep-th/9811178].

[18] L. Alvarez-Gaume, C. Gomez and G. Sierra, "*Quantum Group Interpretation Of Some Conformal Field Theories*," Phys. Lett. B **220** (1989) 142.

[19] K. Hashimoto and K. Krasnov, "*D-brane solutions in non-commutative gauge theory on fuzzy sphere*," hep-th/0101145.

[20] J. Fuchs, "*Superconformal Ward Identities And The WZW Model*," Nucl. Phys. B **286** (1987) 455.

[21] J. Fuchs, "*More On The Super WZW Theory*," Nucl. Phys. B **318** (1989) 631.

[22] E. Witten, "*D-branes and K-theory*," JHEP **9812**, 019 (1998) [hep-th/9810188].

[23] C. Kennedy and A. Wilkins, "*Ramond-Ramond couplings on brane-antibrane systems*," Phys. Lett. B **464**, 206 (1999) [hep-th/9905195].

[24] M. Alishahiha, H. Ita and Y. Oz, "*On superconnections and the tachyon effective action*," Phys. Lett. B **503**, 181 (2001) [hep-th/0012222].

[25] M. Marino, "*On the BV formulation of boundary superstring field theory*," JHEP **0106**, 059 (2001) [hep-th/0103089].

[26] V. Niarchos and N. Prezas, "*Boundary superstring field theory*," hep-th/0103102.

[27] J. Hoppe, "*Diffeomorphism Groups, Quantization And SU(Infinity)*," Int. J. Mod. Phys. A **4** (1989) 5235.