TACHYONIC PREHEATING AND SPONTANEOUS SYMMETRY BREAKING

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We discuss the recent scenario of tachyonic preheating at the end of inflation as a consequence of a tachyonic mass term in the scalar field responsible for spontaneous symmetry breaking. We use 3D lattice simulations to expore this very non-perturbative and non-linear phenomenon, which occurs due to the spinodal instability of the scalar field. Tachyonic preheating is so efficient that symmetry breaking typically completes within a single oscillation of the field distribution as it rolls towards the minimum of its effective potential.

During the last few years we have learned that the coherent oscillations of a scalar field may induce explosive particle production within a dozen oscillations, due to a nonperturbative process called *preheating*[1]. Usually preheating is associated with broad parametric resonance in the presence of a coherently oscillating inflaton field [1], but other mechanisms are also possible. In a recent letter[2] we studied what we called *tachyonic preheating*, which occurs due to the spinodal instabilities in the scalar field responsible for symmetry breaking. Spontaneous symmetry breaking is one of the fundamental ingredients of modern theories of elementary particle physics. In the context of the evolution of the universe it has often been considered as associated with first or second order *thermal* phase transitions. We explored a new scenario[2] in which symmetry breaking occurs at zero temperature, at the end of a period of inflation, when the tachyonic mass term $-m^2\phi^2/2$ appears suddenly, i.e. on a time scale that is much shorter than the time required for symmetry breaking to occur, and which induces the spinodal growth of quantum fluctuations. Spontaneous symmetry breaking is a strongly nonlinear and nonperturbative effect. It usually leads to the production of particles with large occupation numbers inversely proportional to the coupling constants [2]. As a result, the perturbative description, including the Hartree and $1/N$ approximations, has limited applicability. For instance, it cannot describe adecuately the rescattering of the created particles and other important features such as production of topological defects. Thus, for further theoretical understanding of these issues one should go beyond perturbation theory. This is the reason why we used the new methods of lattice simulations, based on the observation that quantum states of bose fields with large occupation numbers can be interpreted as classical waves and their dynamics can be fully analyzed by solving relativistic wave equations on a lattice [3,4]. A significant advantage of these methods as compared to other lattice simulations of quantum processes is that the semi-classical nature of the effects under investigation allows us to have a clear visual picture of all the processes involved.

We will show that tachyonic preheating can be extremely efficient, both in the usual symmetry breaking model and in hybrid models [5]. In most cases it leads to the transfer of the initial potential energy density $V(0)$ into the gradient or kinetic energy of scalar particles within a single oscillation. For instance, contrary to standard expectations, the first stage of preheating in hybrid inflation [6] is typically tachyonic, which means that the stage of oscillations of a homogeneous component of the scalar fields driving inflation either does not exist at all or ends after a single oscillation. A detailed description of our results will be given in a coming publication [7].

Symmetry breaking occurs due to tachyonic instability and may be accompanied by the formation of topological defects. Here we will consider two toy models that are prototypes for many interesting applications, including symmetry breaking in hybrid inflation. The simplest model of spontaneous symmetry breaking is based on the theory with effective potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 \equiv V(0) - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \,, \tag{1}$$

where $\lambda \ll 1$. $V(\phi)$ has a minimum at $\phi = \pm v$ (or $|\phi| = v$ in the case of a complex symmetry breaking field), and a maximum at $\phi = 0$ with negative curvature $V'' = -m^2$.

The development of tachyonic instability in this model depends on the initial conditions. We will assume that initially the symmetry is completely restored so that the field ϕ does not have any homogeneous component, i.e. $\langle\phi\rangle = 0$. But then, because of the symmetry of the potential, $\langle\phi\rangle$ remains zero at all later stages, and for the investigation of spontaneous symmetry breaking one needs to find the spatial distribution of the field $\phi(x,t)$. To avoid this complication, many authors assume that there is a small but finite initial homogeneous background field $\phi(t)$, and even smaller quantum fluctuations $\delta\phi(x,t)$ that grow on top of it. This approximation may provide some interesting information, but quite often it is inadequate. In particular, it does not describe the creation of topological defects, which, as we will see, is not a small nonperturbative correction but an important part of the problem.

For definiteness, we suppose that in the symmetric phase, $\phi = 0$, there are the usual quantum fluctuations of a massive field with mode functions $\frac{1}{\sqrt{2k_0}}e^{-ik_0t+i\vec{k}\vec{x}}$, where $k_0^2 = k^2 + V''$, and then at $t = 0$ we 'switch on' the tachyonic term $-m^2\phi^2/2$. The modes with $k = |\vec{k}| < m$ will grow exponentially, $\phi_k \sim \exp(t\sqrt{m^2 - k^2})$, so the dispersion of these fluctuations can be estimated as

$$\langle\phi^2\rangle = \frac{1}{4\pi^2}\int_0^m dk\, k\, e^{2t\sqrt{m^2-k^2}} = \frac{e^{2mt}(2mt-1)+1}{16\pi^2 t^2} \,. \tag{2}$$

To get a qualitative understanding of the process of spontaneous symmetry breaking, instead of many growing waves with momenta $k < m$ let us consider a single sinusoidal wave $\delta\phi = A(t)\cos kx$ with $k \sim m$ and with initial amplitude $A(0) \sim \frac{m}{2\pi}$ in one-dimensional space. This amplitude grows exponentially until its becomes $A(t_*) \sim v = m/\sqrt{\lambda}$, which leads to the splitting of the universe into domains of size $\mathcal{O}(m^{-1})$, in which the scalar field changes from $\mathcal{O}(v)$ to $\mathcal{O}(-v)$. The gradient energy density of domain walls separating regions with positive and negative ϕ will be $\sim k^2\delta\phi^2 = \mathcal{O}(m^4/\lambda)$. This energy is of the same order as the total initial potential energy of the field $V(0) = m^4/4\lambda$. This is one of the reasons why any approximation based on perturbation theory and ignoring topological defect production cannot give a correct description of the process of spontaneous symmetry breaking. Thus a substantial part of the false vacuum energy $V(0)$ is transferred to the gradient energy of the field ϕ when it rolls down to the minimum of $V(\phi)$. Because the initial state contains many quantum fluctuations with different phases growing at a different rate, the resulting field distribution is very complicated, so it cannot give all of its gradient energy back and return to its initial state $\phi = 0$. This is could be the main reason why spontaneous symmetry breaking and the initial stage of preheating in this model may occur within a single oscillation of the field ϕ.

Consider the tachyonic growth of all fluctuations with $k < m$, i.e. those that contribute most to $\langle\phi^2\rangle$ in Eq. (2). This growth continues until $\langle\phi^2\rangle^{1/2} \sim v/2$, since at $\phi = v/\sqrt{3}$ the curvature of the effective potential vanishes and instead of tachyonic growth one has the usual oscillations of all the modes. This happens within the time $t_* \sim \frac{1}{2m} \ln \frac{\pi^2}{\lambda}$. The exponential growth of fluctuations up to that moment can be interpreted as the growth of the occupation number of particles with $k \ll m$. These occupation numbers at the time t_* grow up to $n_k \sim \exp(2mt_*) \sim \exp(\ln \pi^2/\lambda) = \frac{\pi^2}{\lambda} \gg 1$. One can easily verify that t_* depends only logarithmically on the choice of the initial distribution of quantum fluctuations. For small λ the fluctuations with $k \ll m$ have very large occupation numbers, and therefore they can be interpreted as classical waves of the field ϕ. The dominant contribution to $\langle\phi^2\rangle$ in Eq. (2) at the moment t_* is given by the modes with wavelength $l_* \sim 2\pi k_*^{-1} \sim \sqrt{2}\pi m^{-1} \ln^{1/2}(2\pi^2/\lambda) > m^{-1}$. As a result, at the moment when the fluctuations of the field ϕ reach the minimum of the effective potential, $\langle\phi^2\rangle^{1/2} \sim v$, the field distribution looks rather homogeneous on a scale $l \lesssim l_*$. On average, one still has $\langle\phi\rangle = 0$. This implies that the universe becomes divided into domains with two different types of spontaneous symmetry breaking, $\phi \sim \pm v$. The typical size of each domain is $l_*/2 \sim \frac{\pi}{\sqrt{2}} m^{-1} \ln^{1/2} \frac{2\pi^2}{\lambda}$, which differs only logarithmically from our estimate m^{-1}. At later stages the domains grow in size and percolate (eat each other up), and spontaneous symmetry breaking becomes established on a macroscopic scale.

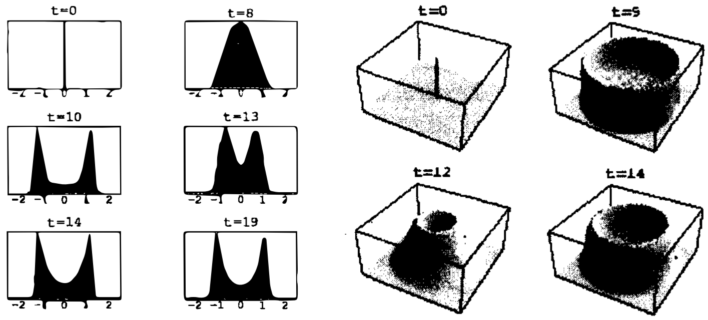

Figure 1: Left panel: The process of symmetry breaking in the model (1) for $\lambda = 10^{-4}$. In the beginning the distribution is very narrow. Then it spreads out and shows two maxima which oscillate about $\phi = \pm v$ with an amplitude much smaller than v. These maxima never come close to the initial point $\phi = 0$. The values of the field are shown in units of v. Right panel: The process of symmetry breaking in the model (1) for a complex field ϕ. The field distribution falls down to the minimum of the effective potential at $|\phi| = v$ and experiences only small oscillations with rapidly decreasing amplitude $|\Delta\phi| \ll v$.

When the field rolls down to the minimum of its effective potential, its fluctuations scatter off each other as classical waves. It is difficult to study this process analytically, but fortunately one can do it numerically using the method of lattice simulations developed in Refs. [3,4]. We performed our simulations on lattices with either 128^3 and 256^3 gridpoints. Figure 1 illustrates the dynamics of symmetry breaking in the model (1). It shows the probability distribution $P(\phi, t)$, that is the fraction of the volume containing the field ϕ at a time t if at $t = 0$ one begins with the probability distribution concentrated near $\phi = 0$, with the quantum mechanical dispersion $\langle\phi^2\rangle = m^2/4\pi^2$ as in (2).

As we see from this figure, after the first oscillation the probability distribution $P(\phi, t)$ becomes narrowly concentrated near the two minima of the effective potential corresponding to $\phi = \pm v$. In this sense one can say that symmetry breaking completes within one oscillation. To demonstrate that this is not a strong coupling effect, we show the results for the model (1) with $\lambda = 10^{-4}$. Note that only when the distribution stabilizes and the domains become large can one use the standard language of perturbation theory describing scalar particles as excitations on a (locally) homogeneous background. That is why the use of the nonperturbative approach based on lattice simulations was so important for our investigation.

The dynamics of spontaneous symmetry breaking in this model is better illustrated by a computer generated movie that can be found at http://physics.stanford.edu/gfelder/hybrid/1.gif. It consists of an animated sequence of images similar to the one shown in Fig. 1. These images show the whole process of spontaneous symmetry breaking from the growth of small Gaussian fluctuations of the field ϕ to the creation of domains with $\phi = \pm v$. Similar results can be obtained for the theory of a complex scalar field ϕ with the potential (1). For example, the behavior of the probability distribution $P(\phi_1, \phi_2, t)$ in the theory of a complex scalar field $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ is also shown in Fig. 1. As we can see, after a single oscillation this probability distribution has stabilized at $|\phi| \sim v$. A computer generated movie illustrating this process can also be found at http://physics.stanford.edu/gfelder/hybrid/2.gif. We also performed 3D lattice simulations of symmetry breaking in hybrid models of inflation and found that, contrary to original expectations, symmetry breaking also occurs within a single oscillation, thus making tachyonic preheating a generic feature of potentials with a negative curvature direction in the potential[2].

In summary, the new 3D lattice methods developed during the last few years in application to the theory of reheating after inflation have been applied to the theory of spontaneous symmetry breaking. These methods have for the first time allowed us not only to calculate correlation functions and spectra of produced particles, but to actually *see* the process of spontaneous symmetry breaking and to reveal some of its rather unexpected features, like production of topological defects, percolation of domains, and thermalization.

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