# THEORETICAL ASPECTS OF DARK MATTER DETECTION 

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#### Abstract

Direct and indirect dark matter detection relies on the scattering of the dark matter candidate on nucleons or nuclei. Here, attention is focused on dark matter candidates (neutralinos) predicted in the minimal supersymmetric standard model and its constrained version with universal input soft supersymmetry-breaking masses. Current expectations for elastic scattering cross sections for neutralinos on protons are discussed with particular attention to satisfying all current accelerator constraints as well as insuring a sufficient cosmological relic density to account for the dark matter in the Universe.


## 1. Introduction

Minimal supersymmetric theories with R-parity conservation are particularly attractive for the study of dark matter as they predict the existence of a new stable particle which is the lightest R-odd state (the LSP). Furthermore, for parameter values of interest to resolve the gauge hierarchy problem, the LSP has an annihilation cross section which yields a relic density of cosmological interest (Ellis et al., 1984). Recent accelerator constraints have made a great impact on the available parameter space in the MSSM and in particular the constrained version or CMSSM in which all scalar masses are assumed to be unified at a grand unified scale (Ellis et al., 1998; Ellis et al., 2000a; Ellis et al., 2001b; Ellis, Nanopoulos, \& Olive, 2001). In addition, significant progress has been made concerning the relic density calculations (Ellis et al., 2000b; Ellis et al., 2001a; Gomez, Lazarides, \& Pallis, 2000). These issues were discussed in detail in John Ellis' contribution and will only be touched on briefly here.

The main impact of the null searches, particularly at LEP, is in the increase in the lower limit to the LSP mass as well as the rest of the sparticle spectrum. This has the unfortunate effect of lowering the elastic scattering cross sections for neutralinos on protons, making direct detection experiments more difficult. Nevertheless, we have now entered a period where accelerator constraints will be at a lull due to the transition from LEP to the LHC and the time required for run II at the Tevatron to

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acquire the needed luminosity*. Therefore, during the next several years we can be hopeful that direct detection experiments will improve to make meaningful inroads to the supersymmetric dark matter parameter space.

In this contribution, we will discuss the current status of the expected neutralinoproton elastic cross sections in the MSSM and CMSSM. These results will be applied to the possibility of direct dark matter detection.

As noted earlier, we will restrict our attention to regions of parameter space for which the relic density of neutralinos is of cosmological interest. For an age of the Universe $t>12 \mathrm{Gyr}$, there is a firm upper bound on the relic density $\Omega_{\chi} h^{2}<0.3$, where $\Omega_{\chi}$ is the fraction of critical density in the form of neutralinos, $\chi$, and $h$ is the Hubble parameter in units of $100 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc}$. This limit represents a strict cosmological bound on the supersymmetric parameter space. We also focus our discussion on the parameter values which lead to relic densities with $\Omega_{\chi} h^{2}>0.1$. While this does not place any bound on supersymmetry, it is a reasonable requirement for dark matter candidates. Neutralinos with a lower density could not be the dominant form of dark matter in our Galaxy, and therefore detection rates would necessarily be suppressed.

## 2. The MSSM vs. The CMSSM

As discussed here by John Ellis, the neutralino LSP is the lowest-mass eigenstate combination of the Bino $\tilde{B}$, Wino $\tilde{W}$ and Higgsinos $\tilde{H}_{1,2}$, whose mass matrix $N$ is diagonalized by a matrix $Z: \operatorname{diag}\left(m_{\chi_{1, \ldots 4}}\right)=Z^{*} N Z^{-1}$. The composition of the lightest neutralino may be written as

$$
\begin{equation*}
\chi=Z_{\chi 1} \tilde{B}+Z_{\chi 2} \tilde{W}+Z_{\chi 3} \tilde{H}_{1}+Z_{\chi 4} \tilde{H}_{2} \tag{1}
\end{equation*}
$$

We assume universality at the supersymmetric GUT scale for the gauge couplings as well as gaugino masses: $M_{1,2,3}=m_{1 / 2}$, so that $M_{1} \sim \frac{5}{3} \tan ^{2} \theta_{W} M_{2}$ at the electroweak scale (note that this relation is not exact when two-loop running of gauge sector is included, as done here).

We also assume GUT-scale universality for the soft supersymmetry-breaking scalar masses $m_{0}$ of the squarks and sleptons. In the case of the CMSSM, the universality is extended to the soft masses of the Higgs bosons as well. We further assume GUT-scale universality for the soft supersymmetry-breaking trilinear terms $A_{0}$. In the MSSM, we treat as free parameters $m_{1 / 2}$ (we actually use $M_{2}$ which is equal to $m_{1 / 2}$ at the unification scale), the soft supersymmetry-breaking scalar mass scale $m_{0}$ (which in the present context refers only to the universal sfermion masses at the unification scale), $A_{0}$ and $\tan \beta$. In addition, we treat $\mu$ and the pseudoscalar Higgs mass $m_{A}$ as independent parameters, and thus the two

[^0]Higgs soft masses $m_{1}$ and $m_{2}$, are specified by the electroweak vacuum conditions, which we calculate using $m_{t}=175 \mathrm{GeV}$. In contrast, in the CMSSM, $m_{1}$ and $m_{2}$ are set equal to $m_{0}$ at the GUT scale and hence $\mu$ (up to a sign) and $m_{A}$ are calculated quantities, their values being fixed by the electroweak symmetry breaking conditions.

## 3. Elastic Scattering Cross Sections

The MSSM Lagrangian leads to the following low-energy effective four-fermion Lagrangian suitable for describing elastic $\chi$-nucleon scattering (Falk, Ferstl, \& Olive, 1999):

$$
\begin{align*}
\mathcal{L} & =\bar{\chi} \gamma^{\mu} \gamma^{5} \chi \bar{q}_{i} \gamma_{\mu}\left(\alpha_{1 i}+\alpha_{2 i} \gamma^{5}\right) q_{i}+\alpha_{3 i} \bar{\chi} \chi \overline{q_{i}} q_{i} \\
& +\alpha_{4 i} \bar{\chi} \gamma^{5} \chi \overline{q_{i}} \gamma^{5} q_{i}+\alpha_{5 i} \bar{\chi} \chi \bar{\chi} \chi \bar{q}_{i} \gamma^{5} q_{i}+\alpha_{6 i} \bar{\chi} \gamma^{5} \chi \bar{q}_{i} q_{i} \tag{2}
\end{align*}
$$

This Lagrangian is to be summed over the quark generations, and the subscript $i$ labels up-type quarks $(i=1)$ and down-type quarks $(i=2)$. The terms with coefficients $\alpha_{1 i}, \alpha_{4 i}, \alpha_{5 i}$ and $\alpha_{6 i}$ make contributions to the elastic scattering cross section that are velocity-dependent, and may be neglected for our purposes. In fact, if the CP-violating phases are absent as assumed here, $\alpha_{5}=\alpha_{6}=0$ (Falk, Ferstl, \& Olive, 2000; Chattopadhyay, Ibrahim \& Nath, 2000). The coefficients relevant for our discussion are the spin-dependent coefficient, $\alpha_{2}$,

$$
\begin{align*}
\alpha_{2 i}= & \frac{1}{4\left(m_{1 i}^{2}-m_{\chi}^{2}\right)}\left[\left|Y_{i}\right|^{2}+\left|X_{i}\right|^{2}\right]+\frac{1}{4\left(m_{2 i}^{2}-m_{\chi}^{2}\right)}\left[\left|V_{i}\right|^{2}+\left|W_{i}\right|^{2}\right] \\
& -\frac{g^{2}}{4 m_{Z}^{2} \cos ^{2} \theta_{W}}\left[\left|Z_{\chi_{3}}\right|^{2}-\left|Z_{\chi_{4}}\right|^{2}\right] \frac{T_{3 i}}{2} \tag{3}
\end{align*}
$$

and the spin-independent or scalar coefficient, $\alpha_{3}$,

$$
\begin{align*}
\alpha_{3 i}= & -\frac{1}{2\left(m_{1 i}^{2}-m_{\chi}^{2}\right)} \operatorname{Re}\left[\left(X_{i}\right)\left(Y_{i}\right)^{*}\right]-\frac{1}{2\left(m_{2 i}^{2}-m_{\chi}^{2}\right)} \operatorname{Re}\left[\left(W_{i}\right)\left(V_{i}\right)^{*}\right] \\
& -\frac{g m_{q i}}{4 m_{W} B_{i}}\left[\operatorname{Re}\left(\delta_{1 i}\left[g Z_{\chi 2}-g^{\prime} Z_{\chi 1}\right]\right) D_{i} C_{i}\left(-\frac{1}{m_{H_{1}}^{2}}+\frac{1}{m_{H_{2}}^{2}}\right)\right. \\
& \left.+\operatorname{Re}\left(\delta_{2 i}\left[g Z_{\chi 2}-g^{\prime} Z_{\chi 1}\right]\right)\left(\frac{D_{i}^{2}}{m_{H_{2}}^{2}}+\frac{C_{i}^{2}}{m_{H_{1}}^{2}}\right)\right] \tag{4}
\end{align*}
$$

where

$$
X_{i} \equiv \eta_{11}^{*} \frac{g m_{q_{i}} Z_{\chi 5-i}^{*}}{2 m_{W} B_{i}}-\eta_{12}^{*} e_{i} g^{\prime} Z_{\chi 1}^{*}
$$

$$
\begin{align*}
Y_{i} & \equiv \eta_{11}^{*}\left(\frac{y_{i}}{2} g^{\prime} Z_{\chi 1}+g T_{3 i} Z_{\chi 2}\right)+\eta_{12}^{*} \frac{g m_{q_{i}} Z_{\chi 5-i}}{2 m_{W} B_{i}} \\
W_{i} & \equiv \eta_{21}^{*} \frac{g m_{q_{i}} Z_{\chi 5-i}^{*}}{2 m_{W} B_{i}}-\eta_{22}^{*} e_{i} g^{\prime} Z_{\chi 1}^{*} \\
V_{i} & \equiv \eta_{22}^{*} \frac{g m_{q_{i}} Z_{\chi 5-i}}{2 m_{W} B_{i}}+\eta_{21}^{*}\left(\frac{y_{i}}{2} g^{\prime} Z_{\chi 1}+g T_{3 i} Z_{\chi 2}\right) \tag{5}
\end{align*}
$$

where $y_{i}, T_{3 i}$ denote hypercharge and isospin, and

$$
\begin{align*}
\delta_{1 i}=Z_{\chi 3}\left(Z_{\chi 4}\right), & \delta_{2 i}=Z_{\chi 4}\left(-Z_{\chi 3}\right) \\
B_{i}=\sin \beta(\cos \beta), & A_{i}=\cos \beta(-\sin \beta) \\
C_{i}=\sin \alpha(\cos \alpha), & D_{i}=\cos \alpha(-\sin \alpha) \tag{6}
\end{align*}
$$

for up (down) type quarks. We denote by $m_{H_{2}}<m_{H_{1}}$ the two scalar Higgs masses, and $\alpha$ denotes the Higgs mixing angle. Finally, the sfermion mass-squared matrix is diagonalized by a matrix $\eta$ : $\operatorname{diag}\left(m_{1}^{2}, m_{2}^{2}\right) \equiv \eta M^{2} \eta^{-1}$, which can be parameterized for each flavour $f$ by an angle $\theta_{f}$ :

$$
\left(\begin{array}{cc}
\cos \theta_{f} & \sin \theta_{f}  \tag{7}\\
-\sin \theta_{f} & \cos \theta_{f}
\end{array}\right) \equiv\left(\begin{array}{ll}
\eta_{11} & \eta_{12} \\
\eta_{21} & \eta_{22}
\end{array}\right)
$$

The spin-dependent part of the elastic $\chi$-nucleus cross section can be written as

$$
\begin{equation*}
\sigma_{2}=\frac{32}{\pi} G_{F}^{2} m_{r}^{2} \Lambda^{2} J(J+1) \tag{8}
\end{equation*}
$$

where $m_{r}$ is again the reduced neutralino mass, $J$ is the spin of the nucleus, and

$$
\begin{equation*}
\Lambda \equiv \frac{1}{J}\left(a_{p}\left\langle S_{p}\right\rangle+a_{n}\left\langle S_{n}\right\rangle\right) \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{p}=\sum_{i} \frac{\alpha_{2 i}}{\sqrt{2} G_{f}} \Delta_{i}^{(p)}, a_{n}=\sum_{i} \frac{\alpha_{2 i}}{\sqrt{2} G_{f}} \Delta_{i}^{(n)} \tag{10}
\end{equation*}
$$

The factors $\Delta_{i}^{(p, n)}$ parameterize the quark spin content of the nucleon. A recent global analysis of QCD sum rules for the $g_{1}$ structure functions (Mallot, 1999), including $\mathcal{O}\left(\alpha_{s}^{3}\right)$ corrections, corresponds formally to the values

$$
\begin{align*}
\Delta_{u}^{(p)} & =0.78 \pm 0.02, \quad \Delta_{d}^{(p)}=-0.48 \pm 0.02 \\
\Delta_{s}^{(p)} & =-0.15 \pm 0.02 \tag{11}
\end{align*}
$$

The scalar part of the cross section can be written as

$$
\begin{equation*}
\sigma_{3}=\frac{4 m_{r}^{2}}{\pi}\left[Z f_{p}+(A-Z) f_{n}\right]^{2} \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{f_{p}}{m_{p}}=\sum_{q=u, d, s} f_{T q}^{(p)} \frac{\alpha_{3 q}}{m_{q}}+\frac{2}{27} f_{T G}^{(p)} \sum_{c, b, t} \frac{\alpha_{3 q}}{m_{q}} \tag{13}
\end{equation*}
$$

and $f_{n}$ has a similar expression. The parameters $f_{T q}^{(p)}$ are defined by

$$
\begin{equation*}
m_{p} f_{T q}^{(p)} \equiv\langle p| m_{q} \bar{q} q|p\rangle \equiv m_{q} B_{q} \tag{14}
\end{equation*}
$$

whilst $f_{T G}^{(p)}=1-\sum_{q=u, d, s} f_{T q}^{(p)}$ (Shifman, Vainshtein, \& Zakharov, 1978). We observe that only the products $m_{q} B_{q}$, the ratios of the quark masses $m_{q}$ and the ratios of the scalar matrix elements $B_{q}$ are invariant under renormalization and hence physical quantities.

We take the ratios of the quark masses from (Leutwyler, 1996):

$$
\begin{equation*}
\frac{m_{u}}{m_{d}}=0.553 \pm 0.043, \quad \frac{m_{s}}{m_{d}}=18.9 \pm 0.8 \tag{15}
\end{equation*}
$$

In order to determine the ratios of the $B_{q}$ and the products $m_{q} B_{q}$ we use information from chiral symmetry applied to baryons. Following (Cheng, 1989), we have:

$$
\begin{equation*}
z \equiv \frac{B_{u}-B_{s}}{B_{d}-B_{s}}=\frac{m_{\Xi^{0}}+m_{\Xi^{-}}-m_{p}-m_{n}}{m_{\Sigma^{+}}+m_{\Sigma^{-}}-m_{p}-m_{n}} \tag{16}
\end{equation*}
$$

Substituting the experimental values of these baryon masses, we find

$$
\begin{equation*}
z=1.49 \tag{17}
\end{equation*}
$$

with an experimental error that is negligible compared with others discussed below. Defining

$$
\begin{equation*}
y \equiv \frac{2 B_{s}}{B_{d}+B_{u}} \tag{18}
\end{equation*}
$$

we then have

$$
\begin{equation*}
\frac{B_{d}}{B_{u}}=\frac{2+((z-1) \times y)}{2 \times z-((z-1) \times y)} \tag{19}
\end{equation*}
$$

The experimental value of the $\pi$-nucleon $\sigma$ term is (Gasser, Leutwyler \& Sanio, 1991; Knecht, 1999):

$$
\begin{equation*}
\sigma \equiv \frac{1}{2}\left(m_{u}+m_{d}\right) \times\left(B_{d}+B_{u}\right)=45 \pm 8 \mathrm{MeV} \tag{20}
\end{equation*}
$$

and octet baryon mass differences may be used to estimate that (Gasser, Leutwyler \& Sanio, 1991; Knecht, 1999)

$$
\begin{equation*}
\sigma=\frac{\sigma_{0}}{(1-y)}: \quad \sigma_{0}=36 \pm 7 \mathrm{MeV} \tag{21}
\end{equation*}
$$

The larger value of $\sigma=65 \mathrm{MeV}$ (Olsson, 2000; Pavan et al., 1999) considered by (Arnowitt, Dutta, \& Santoso, 2000) leads to scattering cross section which are larger by a factor of about 3 . Comparing (20) and (21), we find a central value of $y=0.2$, to which we assign an error $\pm 0.1$, yielding

$$
\begin{equation*}
\frac{B_{d}}{B_{u}}=0.73 \pm 0.02 \tag{22}
\end{equation*}
$$

The formal error in $y$ derived from (20) and (21) is actually $\pm 0.2$, which would double the error in $B_{d} / B_{u}$. We have chosen the smaller uncertainty because we consider a value of $y$ in excess of $30 \%$ rather unlikely.

The numerical magnitudes of the individual renormalization-invariant products $m_{q} B_{q}$ and hence the $f_{T q}^{(p)}$ may now be determined:

$$
\begin{align*}
& f_{T u}^{(p)}=0.020 \pm 0.004, \quad f_{T d}^{(p)}=0.026 \pm 0.005 \\
& f_{T s}^{(p)}=0.118 \pm 0.062 \tag{23}
\end{align*}
$$

where essentially all the error in $f_{T s}^{(p)}$ arises from the uncertainty in $y$. The corresponding values for the neutron are

$$
\begin{align*}
& f_{T u}^{(n)}=0.014 \pm 0.003, \quad f_{T d}^{(n)}=0.036 \pm 0.008 \\
& f_{T s}^{(n)}=0.118 \pm 0.062 \tag{24}
\end{align*}
$$

It is clear already that the difference between the scalar parts of the cross sections for scattering off protons and neutrons must be rather small.

## 4. Results

We begin by discussing the results for the CMSSM (Ellis, Ferstl, \& Olive, 2000). For fixed $\tan \beta$ and sign of $\mu$, we scan over experimentally and cosmologically allowed regions in the $m_{1 / 2}-m_{0}$ plane. Results here are shown for $A_{0}=0$. The combination of the cosmological constraint $\Omega h^{2}<0.3$ and the constraint from the Higgs mass, $m_{H_{2}}>113 \mathrm{GeV}$, eliminates low values of $\tan \beta \lesssim 5$ (Ellis et al., 2001b). For the value of $\tan \beta=10$, we show in Figure 1 the elastic scattering cross section for spin-dependent ( $\mathrm{a}, \mathrm{b}$ ) and scalar ( $\mathrm{c}, \mathrm{d}$ ) processes as a function of the neutralino mass. Although it is barely discernible, the thicknesses of the central curves in the panels show the ranges in the cross section for fixed $m_{\chi}$ that are induced by varying $m_{0}$. At large $m_{\chi}$ where coannihilations are important, the range in the allowed values of $m_{0}$ is small and particularly little variation in the cross section is expected. The shaded regions show the effects of the uncertainties in the input values of the $\Delta_{i}^{(p)}(11)(\mathrm{a}, \mathrm{b})$ and in the $f_{T}^{(p)}(23)(\mathrm{c}, \mathrm{d})$. For the results of analogous analyses, see (Accomando et al., 2000; Corsetti \& Nath, 2000; Bottino et al., 2001).

In addition we show the constraint coming from $m_{H_{2}}>113 \mathrm{GeV}$ for $A_{0}=0$ which restricts one to relatively large neutralino masses. For the cases where $\mu>$ 0 , there is a potential upper limit to the neutralino mass coming from the recent BNL E821 experiment ( Brown et al., 2001), which reports a new value for the anomalous magnetic moment of the muon: $g_{\mu}-2 \equiv 2 \times a_{\mu}$ that is apparently discrepant with the Standard Model prediction at the level of $2.6 \sigma$. This limit is also displayed on Figure 1 (Ellis, Nanopoulos, \& Olive, 2001; Arnowitt, Dutta, \& Santoso, 2001). For $\tan \beta=10, \mu>0$, the theory is quite predictive in both the LSP mass and scattering cross section. No value of $\tan \beta$ is compatible with the BNL E821 result if $\mu<0$.

The scalar cross section is, in general, more sensitive to the sign of $\mu$ than is the spin-dependent cross section. Notice that, in Figure 1c for $\tan \beta=10$ and $\mu<0$, there is a cancellation. Higgs exchange is dominant in $\alpha_{3}$ and for $\mu<0$, both $Z_{\chi 3}$ and $Z_{\chi 4}$ are negative, as is the Higgs mixing angle $\alpha$. Inserting the definitions of $\delta_{1 i(2 i)}$, we see that there is a potential cancellation of the Higgs contribution to $\alpha_{3}$ for both up-type and down-type quarks. Whilst there is such a cancellation for the down-type terms, which change from positive to negative as one increases $m_{\chi}$, such a cancellation does not occur for the up-type terms, which remain negative in the region of parameters we consider. The cancellation that is apparent in the figure is due to the cancellation in $\alpha_{3}$ between the up-type contribution (which is negative) and the down-type contribution, which is initially positive but decreasing, eventually becoming negative as we increase $m_{\chi}$.

At higher values of $\tan \beta$, one can in principle expect larger elastic cross sections (Accomando et al., 2000; Lahanas, Nanopoulos, \& Spanos, 2000). In Figure 2 , we show the spin dependent ( $\mathrm{a}, \mathrm{b}$ ) and the scalar ( $\mathrm{c}, \mathrm{d}$ ) cross section for $\tan \beta=$ $35, \mu<0(\mathrm{a}, \mathrm{c})$ and for $\tan \beta=50, \mu>0(\mathrm{~b}, \mathrm{~d})$. In this figure, lower values of $m_{\chi}$ have been cut off (and are not shown) due to the constraint imposed by measurements of $b \rightarrow s \gamma$.

As was discussed in detail in (Ellis et al., 2001a), a new feature in the $m_{1 / 2}-m_{0}$ plane with acceptable relic density appears at large $\tan \beta$. At large $m_{1 / 2} \sim 1000$ GeV , it becomes possible for neutralinos to annihilate through s-channel $H_{1}$ or pseudoscalar, $A$, exchange. In fact there is a slice in the plane where $2 m_{\chi} \approx m_{H_{1}, A}$ and the relic density becomes uninterestingly small. At smaller and larger $m_{1 / 2}$ surrounding this pole region, there are regions where the relic density falls in the desired range. This leads to two separate regions in Figure 2 at lower $m_{\chi}$. The third region in Figure 2 at higher $m_{\chi}$ corresponds to the cosmological region allowed by coannihilation (Ellis et al., 2000b). For more further details on $H_{1}, A$-pole and coannihilation, see the contribution of John Ellis in these proceedings. As in the case $\tan \beta=10, \mu<0$, the scalar cross section at higher $\tan \beta$ also exhibits the cancellation feature discussed above. However, because the cosmological regions are multivalued in $m_{0}$ as a function of $m_{1 / 2}$, the cancellation occurs at a different value of $m_{\chi}$ for each of three regions just discussed. This leads (unfortunately) to a broad region in the $\sigma-m_{\chi}$ plane where the cross section is very small.


Figure 1. $(a, b)$ : The spin-dependent cross section for the elastic scattering of neutralinos on protons as a function of the LSP mass for $\tan \beta=10$. The central curves are based on the inputs (11), and their thicknesses are related to the spreads in the allowed values of $m_{0}$. The shaded regions correspond to the uncertainties in the hadronic inputs (11). (c,d): The spin-independent scalar cross section for the elastic scattering of neutralinos on protons as a function of the LSP mass for $\tan \beta=10$. The central curves are based on the inputs (23), their thicknesses are again related to the spread in the allowed values of $m_{0}$, and the shaded regions now correspond to the uncertainties in the hadronic inputs (23). The supplementary lower limits imposed on $m_{\chi}$ in this and the next figure reflect improvements in the LEP lower limit on $m_{h}$, and the upper limits for $\mu>0$ are due to $g_{\mu}-2$, which is incompatible with $\mu<0$.


Figure 2. As in Figure 1 for $\tan \beta=35$. Here the three distinct regions correspond to the two sides of the $H_{1}, A$ annihilation poles, and the coannihilation region at higher values of $m_{\chi}$.

In the MSSM, in addition to scanning over the gaugino and sfermion masses at fixed $\tan \beta$, one can treat $\mu$ and the pseudoscalar mass $m_{A}$ as free parameters as well. In (Ellis, Ferstl, \& Olive, 2001), we performed a scan over the following parameter space: $0<m_{0}<1000 ; 80<|\mu|<2000 ; 80<M_{2}<1000 ; 0<$ $m_{A}<1000 ;-1000<A<1000$. Of the $90,000(70,000)$ points scanned for $\tan \beta=10$ and $\mu>0(\mu<0)$, only 6208 (4772) survived all of the experimental and cosmological constraints. In the CMSSM, the LSP is nearly always predicted to be Bino of very high purity. However, in the MSSM, when $|\mu| \lesssim M_{2}$, the LSP may have a dominant Higgsino component. In these cases, coannihilation (Griest \& Seckel, 1991) greatly suppresses their relic density and when combined with the experimental constraints on the parameter space, Higgsino dark matter can be excluded as a viable option (Ellis et al., 1998; Ellis et al., 2000a).

The LEP chargino and Higgs cuts remove many points with low $m_{\chi}$ and/or large elastic scattering cross sections. The sfermion mass cut is less important. The
constraint that $\chi$ be the LSP removes quite a large number of points, populated more or less evenly in the cross section plots. There is a somewhat sparse set of points with very small cross sections which give some measure of how low the cross section may fall in some special cases. These reflect instances where particular cancellations take place, as discussed above. The lower boundary of the densely occupied regions offers an answer to the question how low the elastic scattering cross sections may reasonably fall, roughly $\sigma \sim 10^{-9} \mathrm{pb}$ for the spin-dependent cross section and $\sim 10^{-10} \mathrm{pb}$ for the spin-independent cross section.

Our resulting predictions for the spin-dependent elastic neutralino-proton cross section for $\tan \beta=10$ are shown in Figure 3(a,b), where a comparison with the CMSSM is also made. The raggedness of the upper and lower boundaries of the dark (blue) shaded allowed region reflect the coarseness of our parameter scan, and the relatively low density of parameter choices that yield cross sections close to these boundaries. it should be noted that the low values of $m_{\chi}$ in these plots, that yield relatively high spin-dependent cross section, have now been excluded by improvements in the Higgs mass limit. As $m_{\chi}$ increases, the maximum allowed value of $\sigma_{\text {spin }}$ decreases, though not as rapidly as in the previous CMSSM case (Ellis, Ferstl, \& Olive, 2000). The hadronic uncertainties are basically negligible for this spin-dependent cross section, as seen from the light (yellow) shading.

The analogous results for the spin-independent elastic neutralino-proton cross section are shown in Figure 3(c,d), where comparisons with the CMSSM case are also made. We see a pattern that is similar to the spin-dependent case. For small $m_{\chi}$, the spin-independent scalar cross section, shown by the dark (blue) shaded region, may be somewhat higher than in the CMSSM case, shown by the (red and turquoise) diagonal strip, whilst it could be much smaller. For large $m_{\chi}$, the cross section may be rather larger than in the CMSSM case, but it is always far below the present experimental sensitivity. Overall, we note that the hadronic uncertainties, denoted by the light (yellow) bands, are somewhat larger in the spin-independent case than in the spin-dependent case.

## 5. Conclusions

As one can see from scanning the figures, the predicted elastic scattering cross section in the CMSSM and in the more general MSSM, are relatively small. For the spin-dependent processes, the cross sections fall in the range $\sigma \sim 10^{-4}-10^{-8}$ pb , whereas for the scalar cross sections, we find $\sigma<10^{-6} \mathrm{pb}$ with an uncertain lower limit due to possible cancellations. These should be compared with current sensitivities of existing and future experiments (Gaitskell \& Mandic, 2001). The UKDMC detector is sensitive to $\sigma \gtrsim 0.5 \mathrm{pb}$ for the spin-dependent cross section. DAMA and CDMS are sensitive to $\sigma \gtrsim 2 \times 10^{-6} \mathrm{pb}$ for the scalar cross section. This is close to the upper limits we find for reasonable supersymmetric models. The future looks significantly brighter. When CDMS is moved to the


Figure 3. As in Figure 1. The main (blue) shaded regions summarize the envelopes of possible values found in our scan, for points respecting the LEP constraints, discarding points with $\Omega_{\chi} h^{2}>0.3$, and rescaling points with $\Omega_{\chi} h^{2}<0.1$. The small light (yellow) shaded extensions of this region reflect the hadronic matrix element uncertainties. The concave (red and turquoise) strips are those found previously assuming universal Higgs scalar masses (Ellis, Ferstl, \& Olive, 2000).

Soudan mine, its sensitivity will drop to between $10^{-8}$ and $10^{-7} \mathrm{pb}$ and GENIUS claims to be able to reach $10^{-9} \mathrm{pb}$. At those levels, direct detection experiments will either discover supersymmetric dark matter or impose serious constraints on supersymmetric models.

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[^0]:    * We can however, expect improvements in the uncertainties in rare B decays and the measurement of the anomalous magnetic moment of the muon, both of which will have an impact on the allowed supersymmetric parameter space.

