

The Symmetry behind Extended Flavour Democracy and Large Leptonic Mixing

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We show that there is a minimal discrete symmetry which leads to the extended flavour democracy scenario constraining the Dirac neutrino, the charged lepton and the Majorana neutrino mass term (M_R) to be all proportional to the democratic matrix, with all elements equal. In particular, this discrete symmetry forbids other large contributions to M_R , such as a term proportional to the unit matrix, which would normally be allowed by a $S_{3L} \times S_{3R}$ permutation symmetry. This feature is crucial in order to obtain large leptonic mixing, without violating 't Hooft's naturalness principle.

1 Introduction

The understanding of the observed pattern of fermion masses and mixings continues being one of the fundamental open questions in particle physics. This flavour puzzle has become even more intriguing with the recent neutrino data pointing towards neutrino oscillations, with large mixing required in order to account for the atmospheric neutrino data [1]. In the absence of a fundamental theory of flavour, one is tempted to consider specific patterns for the fermion mass matrices which could reflect the existence of a family symmetry at a higher energy scale [2]. The pattern of fermion masses and mixings may thus provide a valuable insight into the physics beyond the Standard Model (SM).

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One of the most attractive patterns for the quark mass matrices follows from the suggestion [3] that there is a $S_{3L}^q \times S_{3R}^u \times S_{3R}^d$ family permutation symmetry acting on the left-handed quark doublets, the right-handed up quarks and the right-handed down quarks, respectively. This family permutation symmetry automatically leads to quark mass matrices M_u, M_d proportional to the so-called democratic mass matrix [4], which has all elements equal to the unity. In the democratic limit, only the third generation acquires mass and the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the unit matrix. This is an interesting result since experimentally one knows that there is a strong hierarchy in the value of the quark masses, with the first two generations of quarks much lighter than the third one. Furthermore, the experimentally observed CKM matrix is close to the unit matrix, as suggested by the underlying $S_{3L}^q \times S_{3R}^u \times S_{3R}^d$ family symmetry. The first two generations acquire non-vanishing masses and a non-trivial CKM matrix is generated when the permutation symmetry is broken.

One may be tempted to extend the above scenario to the leptonic sector and assume that there is a $S_{3L}^l \times S_{3R}^{cl}$ symmetry acting on the lepton doublets and the right-handed charged leptons, respectively. If one pursues this idea, one is confronted with the problem of generating large leptonic mixing, without violating 't Hooft's naturalness principle [5]. For simplicity, let us assume for the moment the SM without right-handed neutrinos. Obviously, the $S_{3L} \times S_{3R}$ symmetry leads to a charged lepton mass matrix proportional to the democratic matrix, which we denote by Δ . However, as it has been previously pointed out [6], the most general effective Majorana mass matrix, allowed by the permutation symmetry is of the form $a\Delta + b\mathbb{I}$, where one expects a and b to be of the same order of magnitude. It follows then that independently of the ratio a/b (provided neither a nor b vanish), both the charged lepton mass matrix and the effective Majorana neutrino mass matrix are, in leading order, diagonalized by the same unitary matrix. As a result, in leading order, the leptonic mixing matrix will be given by the unit matrix. Clearly, no large angles (to solve the atmospheric neutrino problem, at least) can be generated by a small breaking of the $S_{3L} \times S_{3R}$ symmetry.

In the literature, within the framework of democratic mass matrices, examples with large lepton mixing have been given [7], by making the ad-hoc assumption that the coefficient a vanishes, which is not dictated by the permutation symmetry. More precisely, the Lagrangean does not acquire any new symmetry in the limit where a vanishes and therefore setting $a = 0$ clearly violates 't Hooft's naturalness principle. In our discussion, we have so far restricted ourselves to the case where only left-handed neutrinos are introduced. We will show in the sequel that analogous arguments also apply to the case where right-handed neutrinos are introduced and an effective left-handed Majorana mass matrix is generated through the seesaw mechanism.

In this paper we shall address the question of whether it is possible to generate large leptonic mixing using democratic-type mass matrices, without violating 't Hooft's naturalness principle. We'll show that this is indeed possible, provided we do not use a $S_{3L} \times S_{3R}$ symmetry, but rather a Z_3 symmetry, which is imposed to the quark and lepton sectors. The Z_3 symmetry constrains all fermion mass matrices to be proportional to the democratic matrix Δ , and a small perturbation of the symmetry can lead to a correct fermion spectrum and pattern of mixings. In particular, one may obtain, through the seesaw mechanism, large mixing in the leptonic sector, without violating 't Hooft's naturalness principle. The idea of extended flavour democracy (EFD), where the mass matrices of all fermions (i.e., including up and down quarks, charged leptons and neutrinos) are proportional to the democratic matrix has been previously suggested in the literature, as a phenomenological ansatz [8]. In this paper, we will show that there is an underlying symmetry which leads to the EFD scenario.

2 $S_{3L} \times S_{3R}$ symmetry, naturalness and large leptonic mixing

For simplicity, let us consider the three generation SM, with the addition of three right-handed neutrinos. The most general gauge invariant leptonic Yukawa interaction and mass terms contained in the Lagrangean, can be written as:

$$-L = Y_l^{ij} \bar{L}_i \phi l_{jR} + Y_D^{ij} \bar{L}_i \tilde{\phi} \nu_{jR} + \frac{1}{2} \nu_{iR}^T C (M_R)^{ij} \nu_{jR} + h.c., \quad (1)$$

where L_i, ϕ denote the left handed lepton and Higgs doublets, and l_{jR}, ν_{jR} the right handed charged lepton and neutrino singlets. The right-handed Majorana mass term is $SU(3) \times SU(2)_L \times U(1)$ invariant and therefore it is not protected by this symmetry. As a result, M_R is naturally large, of the order of the cutoff scale of the low-energy theory. After spontaneous symmetry breaking, we obtain the mass matrix for the charged leptons $M_l = \langle \phi \rangle Y_l$ and, besides the Majorana mass M_R , also a Dirac mass matrix for the neutrinos $M_D = \langle \phi \rangle Y_D$.

Imposing a $S_{3L} \times S_{3R}$ symmetry on the family structure of this Lagrangean and choosing, as usual, for the left as well as for the right handed leptons, real representations of this symmetry, the following textures for mass matrices are obtained:

$$M_l = \lambda' \Delta \quad ; \quad M_D = \lambda \Delta \quad ; \quad M_R = \mu (\Delta + a \mathbb{1}) \quad (2)$$

where Δ is the democratic mass matrix with all matrix elements equal to 1:

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (3)$$

It is important to notice that for the right handed heavy neutrino Majorana mass matrix, the symmetry does not forbid the existence of the extra term $a \mathbb{1}$, which, of course, will be of the same order as Δ . In the Lagrangean in Eq. (1), this term is allowed because Majorana mass terms involve only neutrino fields of the same chirality. This implies that for the M_R mass term only the S_{3R} symmetry is relevant. When $S_{3L} \times S_{3R}$ is broken, the matrices in Eq. (2) will each acquire an extra small mass term,

$$M_l = \lambda' (\Delta + \varepsilon_l P_l) \quad ; \quad M_D = \lambda (\Delta + \varepsilon_D P_D) \quad (4)$$

$$M_R = \mu (\Delta + a \mathbb{1} + \varepsilon_R P_R)$$

and the effective neutrino mass matrix will be

$$\begin{aligned} M_{eff} &= -M_D M_R^{-1} M_D^T = \\ &= -\frac{\lambda^2}{\mu} (\Delta + \varepsilon_D P_D) \cdot (\Delta + a \mathbb{1} + \varepsilon_R P_R)^{-1} \cdot (\Delta + \varepsilon_D P_D^T) \end{aligned} \quad (5)$$

In the sequel, we shall evaluate M_R^{-1} explicitly. One should emphasize the fundamental difference between the case where M_R is of the form $\Delta + \varepsilon_R P_R$ and the case where $M_R = \Delta + a \mathbb{1} + \varepsilon_R P_R$. In the first case, M_R is nilpotent and it does not have an inverse in the limit $\varepsilon_R \rightarrow 0$. As a result, when ε_R is small but non-vanishing, the contribution of the $\varepsilon_R P_R$ term to M_R^{-1} is very large. In the case of $\Delta + a \mathbb{1} + \varepsilon_R P_R$, the situation is quite different, because, due to the large extra term $a \mathbb{1}$, it has indeed an inverse when $\varepsilon_R \rightarrow 0$ and thus, only a small term of the same order in ε_R will appear in M_R^{-1} . Therefore, in the analysis of M_{eff} given in Eq. (5), it is safe to study the qualitative features of the mass spectrum and neutrino mixing by putting $\varepsilon_R = 0$. As we have argued above, this will not change qualitatively our results and will allow us to obtain an exact analytical form for M_R^{-1} . Noting that:

$$(\Delta + a \mathbb{1})^{-1} = \frac{-1}{a(3+a)} (\Delta - (3+a) \mathbb{1}) \quad (6)$$

we find, working out the product in Eq. (5), for the effective neutrino mass matrix ($\varepsilon_R = 0$):

$$M_{eff} = \lambda' (\Delta + \varepsilon_D P'_D) \quad ; \quad P'_D = \frac{1}{3} [\Delta P_D^T + P_D \Delta + o(\varepsilon_D)] \quad (7)$$

where $\lambda' = -\lambda^2/\mu(3+a)$. Of course, for $\varepsilon_R \neq 0$ a term of the order ε_R will be added to this matrix, but it will not change its form or its qualitative features.

It is clear that, in the context of the $S_{3L} \times S_{3R}$ symmetry, the lepton mixing matrix will either be close to $\mathbb{1}$, or have only a significant mixing angle in the (1, 2) sector. In the (2, 3) sector, the mixing angle will be very small (contrary to what is required by the atmospheric neutrino data), because both the effective neutrino and the charged lepton mass matrices have the same texture, namely $\Delta + \varepsilon P$. Thus, in order to have a large leptonic mixing angle in the (2, 3) sector it is crucial that the $a \mathbb{1}$ term in the heavy neutrino Majorana mass matrix is absent. This leads us to the question: is there a symmetry principle that forbids a large $a \mathbb{1}$ term in M_R , while constraining M_R , as well as all other leptonic mass matrices, to be proportional to Δ ? In the next section, we shall see that such a symmetry does indeed exist.

3 Z_3 symmetry and extended flavour democracy

Let us consider the Lagrangean of Eq. (1) and impose a Z_3 symmetry realized in the following way:

$$\begin{aligned} L_i &\rightarrow P_{ij}^\dagger L_j \\ l_{iR} &\rightarrow P_{ij} l_{jR} \quad ; \quad P = i\omega^* W \quad ; \quad W = \frac{1}{\sqrt{3}} \begin{bmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{bmatrix} \\ \nu_{iR} &\rightarrow P_{ij} \nu_{jR} \end{aligned} \quad (8)$$

where $\omega = e^{i\frac{2\pi}{3}}$. It can be readily verified that this indeed a Z_3 symmetry since $P^2 = P^\dagger$, $P^3 = \mathbb{1}$. Then, if the Lagrangean is to be invariant, each matrix M_L , M_D and M_R , must obey

$$P \cdot M \cdot P = M \quad (9)$$

Notice that we do not have $P^\dagger \cdot M \cdot P = M$. It is crucial for our results that Eq. (9) holds and it immediately follows that $\det(M) = 0$, because $\det(P)$ is not real. So M must have, at least, one zero eigenvalue.

We now prove that M has, in fact, two zero eigenvalues and is always proportional to the democratic mass matrix Δ . To do this, we write the unitary matrix W in P as $W = (1/\sqrt{3}) [\Delta + (\omega - 1)\mathbb{1}]$. It follows then that Eq. (9) is equivalent to

$$\omega^* [\Delta, M] = M \cdot (\Delta - 3\mathbb{1}) \quad (10)$$

where we have written Eq. (9) in the form $P \cdot M = M \cdot P^\dagger$ and used the property $1 + \omega + \omega^* = 0$. Because $\Delta^2 = 3\Delta$, if we multiply the right hand side of Eq. (10), on the right, by Δ , we get zero and therefore $[\Delta, M] \cdot \Delta = 0$, which implies that:

$$M\Delta = \frac{1}{3} \Delta M \Delta$$

Subsequently, if we multiply the same equation on the left by Δ we find

$$\Delta M = \frac{1}{3} \Delta M \Delta$$

Thus $[\Delta, M] = 0$, but then Eq. (10) reads

$$M = \frac{1}{3} M\Delta = \frac{1}{9} \Delta M \Delta = \lambda \Delta \quad (11)$$

where we have used the property $\Delta M \Delta = (\sum M_{ij}) \Delta$. It is also easy to check that $P \cdot \Delta \cdot P = \Delta$. Thus ³,

$$P \cdot M \cdot P = M \quad \Leftrightarrow \quad M = \lambda \Delta \quad (12)$$

Therefore, if we impose on the Lagrangean a Z_3 symmetry realized in the way indicated in Eq. (8), all leptonic matrices, M_l , M_D and M_R , are constrained to be of the democratic type, i.e., proportional to Δ . In particular, in the limit where the Z_3 symmetry holds, M_R will not contain a term $a \mathbb{1}$, since this term is not allowed by the Z_3 symmetry. It can be readily verified that Z_3 is the smallest symmetry which can lead to extended democracy in leptonic mass matrices, while forbidding the $a \mathbb{1}$ term in M_R . A Z_2 symmetry would not be sufficient.

³ Alternatively, one can derive Eq. (11), using Eq. (9), writing $M = \frac{1}{2} (P M P + P^2 M P^2)$ and substituting $P = (i\omega^*/\sqrt{3}) [\Delta + (\omega - 1)\mathbb{1}]$. This approach can be trivially generalized to a Z_n group.

4 Breaking of Z_3 and generation of large leptonic mixing

We shall now investigate what conditions have to be satisfied in order to achieve large mixing in the leptonic sector, through a small breaking of Z_3 . Generically, the breaking of Z_3 leads to leptonic matrices with the following form:

$$M_l = \lambda_l [\Delta + \varepsilon_l P_l] ; M_D = \lambda [\Delta + \varepsilon_D P_D] ; M_R = \mu [\Delta + \varepsilon_R P_R] \quad (13)$$

where the $\varepsilon_i \ll 1$ ($i = l, D, R$) and the P_i are of order 1. We assume that the perturbation P_R of the right-handed heavy Majorana neutrinos is such that the inverse of $\Delta + \varepsilon_R P_R$ exists. By noting that one has

$$\det [\Delta + \varepsilon_R P_R] = \varepsilon_R^2 (x + \varepsilon_R y) \quad (14)$$

where x and y are quadratic and cubic polynomials in the (different) elements $(P_R)_{ij}$, respectively, one readily concludes that $[\Delta + \varepsilon_R P_R]^{-1}$ is of the form:

$$Z \equiv [\Delta + \varepsilon_R P_R]^{-1} = \frac{1}{\varepsilon_R (x + \varepsilon_R y)} [L_0 + \varepsilon_R X] \quad (15)$$

where L_0 and X are matrices with respectively linear and quadratic elements in $(P_R)_{ij}$. Obviously x, y, L_0 and X are in general of order 1. It is possible to have special cases where either x or y vanish, but not both, since we require that Z exists. Furthermore, it is a general characteristic of this inverse that L_0 and X satisfy the relations:

$$\Delta L_0 = 0 \quad ; \quad (\sum X_{ij}) = x \quad ; \quad \Delta X \Delta = x \Delta \quad (16)$$

Applying these algebraic relations to the effective neutrino mass matrix formula one obtains a transparent formula for M_{eff} :

$$M_{eff} = \lambda' \left[x \Delta + \frac{\varepsilon_D^2}{\varepsilon_R} P_D L_0 P_D + \varepsilon_D (\Delta X P_D + P_D X \Delta + \varepsilon_D P_D X P_D) \right] \quad (17)$$

where $\lambda' = -\lambda^2/\mu (x + \varepsilon_R y)$.

This expression obtained for the effective neutrino mass matrix is very important because it tells us when to expect large mixing for the lepton sector in the case of an aligned hierachical spectrum for the charged leptons, Dirac and

heavy Majorana neutrinos. In general, i.e., for a generic perturbation P_R in the right-handed Majorana sector, there will be more than one element $(P_R)_{ij}$ of order 1, thus implying that the quadratic polynomial x is also of order 1. So, if the term proportional to $\varepsilon_D^2/\varepsilon_R$ is small, it is clear that the effective neutrino mass matrix will be, just like the charged lepton mass matrix, to leading order, proportional to Δ , and, thus, there will be no large mixing. Therefore, if one wants to avoid small mixing, one must have that the term proportional to $\varepsilon_D^2/\varepsilon_R$, in Eq. (17), be of order 1 or larger. Note that, since all ε_i are of $o(m_2^i/m_3^i)$, large mixing, and consequently $\varepsilon_R \leq \varepsilon_D^2$, requires that there is a strong hierarchy (in the sense that the third generation is much heavier than the first two generations) for the heavy Majorana neutrino masses. Roughly speaking, if one has that $\varepsilon_D = o(m_c/m_t)$, this result implies that at least $M_3/M_2 = o(m_t^2/m_c^2)$, which is indeed a very strong hierarchy. The only way to avoid this is to choose perturbations P_R in the right-handed Majorana sector, such that x is no longer of order 1 but much smaller. The simplest way to do this is by having only one (diagonal) element of P_R of order 1. Thus, x , which is quadratic in the different elements of P_R , will always be suppressed.

A realization of this scenario was proposed in [8] where all the perturbation matrices P_i in Eq. (13) were chosen to be diagonal matrices such that:

$$P_i = \text{diag} (0, \delta_i, 1) \quad ; \quad \delta_i = o(m_1^i/m_2^i) \quad ; \quad i = l, D, R \quad (18)$$

This choice for the breaking of the family symmetry leads to the following values for the parameters entering the general expression given by Eq. (17),

$$y = 0 \quad ; \quad x = o(M_1/M_2) \quad ; \quad XP_D = 0 \quad ; \quad P_D L_0 P_D = P \quad (19)$$

where P is, just like the P_i , diagonal with zero first entry. Thus, the effective neutrino mass matrix

$$M_{eff} = \lambda'' \left[\Delta + \frac{1}{x} \frac{\varepsilon_D^2}{\varepsilon_R} P \right] \quad (20)$$

with $\lambda'' = -\lambda^2/\mu$, is found to be of the same form as the Dirac, the right-handed Majorana neutrino and the charged lepton mass matrices. Furthermore, if the second term in Eq. (20) is to be large or of the same order as Δ , the hierarchy of the heavy Majorana neutrinos will only be $M_3/M_1 = o(m_t^2/m_c^2)$, which is a less pronounced hierarchy⁴. It was shown that with this specific

⁴ In a somewhat different context, Albright and Barr [9] used the seesaw mechanism to generate the large angle solution for the solar neutrino problem. They were also confronted with a large hierarchy for the right-handed Majorana neutrinos.

perturbation of the extended democracy, one can obtain both an experimentally acceptable light neutrino spectrum and a pattern of leptonic mixing in agreement with both the solar and atmospheric neutrino data.

Another example, where a very a strong hierarchy in the right-handed Majorana neutrino masses is also avoided is the scheme proposed in [10], where a very special perturbation P_R is assumed, leading to vanishing x . The effective neutrino mass matrix is then of the form

$$M_{eff} = \hat{\lambda} \begin{bmatrix} \frac{\varepsilon_D}{\varepsilon_R} P_D L_0 P_D + \Delta X P_D + P_D X \Delta + \varepsilon_D P_D X P_D \end{bmatrix} \quad (21)$$

Note however, that such a conspiracy in the sum of quadratic terms in the $(P_R)_{ij}$ leading to $x = 0$ seems to us likely be unstable with regard to the renormalization group evolution.

5 Conclusions

We have considered a minimal extension of the SM, where the only addition consists of the introduction of three right-handed neutrinos. We have shown that if one imposes a Z_3 symmetry on the Lagrangean, realized as in Eq. (8), all leptonic mass matrices, namely the charged lepton mass matrix M_l , the Dirac neutrino mass matrix M_D and the right-handed neutrino Majorana mass matrix M_R , are proportional to the so-called democratic mass matrix Δ . This is to be contrasted with the situation one encounters when one introduces the permutation symmetry $S_{3L} \times S_{3R}$. Although this permutation symmetry leads to M_l, M_D proportional to Δ , it allows for a M_R containing both a term proportional to Δ and a term proportional to the unit matrix. The presence of these two terms in M_R prevents the generation of large leptonic mixing.

On the contrary, in the framework of a Z_3 family symmetry, one can obtain a large leptonic mixing through a small perturbation of the Z_3 symmetry. The fact that large mixing can be obtained through a small perturbation of Z_3 , may seem surprising since, in the democratic limit, there is no leptonic mixing. The generation of large mixing is due to the fact that one is perturbing around a singular matrix Δ , where M_R has no inverse. The possibility of generating large mixing out of small mixing has already been pointed out in the literature [11]. Obviously, the Z_3 family symmetry can be trivially extended to the quark sector.

The existence of the Z_3 symmetry renders specially appealing the idea of the EFD scenario, where fermion mass matrices, both in the quark and lepton sectors, are, to leading order, all proportional to the democratic matrix. Hope-

fully, this Z_3 symmetry is the low-energy remnant of a larger family symmetry, valid at a higher energy scale.

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