Inclusive Rare B Decays

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Abstract

We review the present status of rare B decays, focusing on inclusive decay modes and their role in our search for new physics. We also briefly discuss direct CP violation in rare B decays and the rare kaon decays $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, which offer complementary opportunities for precision flavour physics.

1 Introduction

Flavour physics deals with that part of the standard model (SM) which distinguishes between the three generations of fundamental fermions. It is still a mystery why there are exactly three generations. Also the origin of the fermion masses and their mixing is unknown; in particular, the SM does not explain the hierarchical pattern of these parameters. Flavour physics can be regarded as the least tested part of the SM. This is reflected in the rather large error bars of several flavour parameters such as the mixing parameters at the 20% level [1], which has to be compared with errors smaller than 1% in high energy electroweak precision experiments.

However, the experimental situation concerning flavour physics is drastically changing. There are several B physics experiments successfully running at the moment and, in the upcoming years, new facilities will start to explore B physics with increasing sensitivity and within various different experimental settings: Apart from the CLEO experiment (Cornell, USA), located at the Cornell Electron-Positron Storage Ring (CESR) [2], two B factories, operating at the $\Upsilon(4S)$ resonance in an asymmetric mode (fig.1), have started successfully: the BaBar experiment at SLAC (Stanford, USA) [3] and the BELLE experiment at KEK (Tsukuba, Japan) [4]. Besides the successfully running hadronic B physics program at FERMILAB (Batavia, USA) [5] there are independent B physics experiments planned at the hadronic colliders: the LHC-B experiment at CERN in Geneva [6] and the BTeV experiment at FERMILAB [7]. The main motivation for a B physics program at hadron colliders is the huge b quark production cross section with respect to the one at e^+e^- machines.

While the time of the electroweak precision physics focusing on the *gauge* sector of the SM draws to a close with the completion of the LEP experiments at CERN and the SLC experiment in Stanford, the era of precision flavour physics focusing on the *scalar* sector of the SM has just begun with the start of the B factories.

The B system represents an ideal framework for the study of flavour physics. Since the b quark mass is much larger than the typical scale of the strong interaction, long-distance strong interactions are generally less important and are under better control than in kaon physics thanks to the heavy mass expansion. Thus, for example the CP violation in the B system will yield an important independent test of the SM description of CP violation (see [8]). B meson decays also allow for a rich CKM phenomenology and a stringent test of the unitarity constraints.

The so-called rare decays are of particular interest. These processes represent flavour changing neutral currents (FCNC) and occur in the SM only at the quantum level. The inclusive rare decay modes are theoretically clean observables because no specific model is needed to describe the hadronic final states. Their role is twofold: on the one hand they are relevant to the determination of CKM matrix elements. On the other hand they are particularly sensitive to new physics beyond the SM, since additional contributions to the decay rate, in which SM
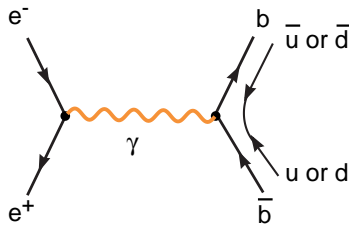
Figure 1: $e^+ e^- \to Y(4S) \to B^+ B^-, B^0 \bar{B}^0$.

particles are replaced by new particles such as the supersymmetric charginos or gluinos, are not suppressed by additional factors $\alpha/(4\pi)$ relative to the SM contribution. This makes it possible to observe new physics indirectly - a strategy complementary to the direct production of new (supersymmetric) particles. The latter production is reserved for the planned hadronic machines such as the LHC at CERN, while the indirect search of the B factories already implies significant restrictions for the parameter space of supersymmetric models and will thus lead to important clues for the direct search of supersymmetric particles. It is even possible that these rare processes lead to the first evidence of new physics by a significant deviation from the SM prediction, for example in the observables concerning direct CP violation, although it will then be difficult to identify in this way the new structures in detail. But also in the long run, after new physics has already been discovered, these decays will play an important role in analyzing in greater detail the underlying new dynamics.

Although the general focus within flavour physics is at present on B systems, kaon physics offers interesting complementary opportunities in the new physics search such as the rare decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$. They are specifically interesting in view of the current experiments at the Brookhaven laboratory (USA) and suggested experiments at FERMILAB (USA) and at KEK (Japan).

The paper is organized as follows: in Section 2 we briefly discuss the role of the strong interaction within flavour physics. In Section 3 the status of rare B decays within the SM is reviewed. In Section 4 we explore the implications of these decays for our search of physics beyond the SM. In Section 5 we discuss direct CP violation and in Section 6 the complementary role of rare kaon decays within precision flavour physics. In Section 7 we present our summary.

2   Strong interaction in B decays

Flavour physics is governed by the interplay of strong and weak interactions. One of the main difficulties in examining the observables in flavour physics is the influence of the strong interaction. As is well known, for matrix elements dominated by long-distance strong interactions there is no adequate quantitative solution available in quantum field theory. The resulting hadronic uncertainties restrict the opportunities in flavour physics significantly. The present discussion
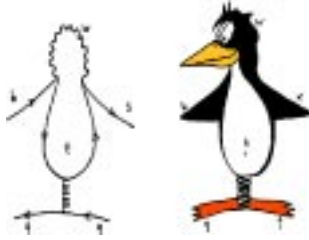
Figure 2: Penguin decays of B mesons.

on the new $g - 2$ muon data [9] also reflects this issue. While the hadronic self-energy contribution to the $g - 2$ observable can be determined by experimental data, the well-known light-by-light contribution can only be modelled at present (see for example [10]).

However, there are several fundamental tools available, which are directly based on QCD. High hopes for precise QCD predictions are placed on lattice gauge theoretical calculations. While there are competitive predictions from lattice gauge theory for form factors of semileptonic decays, pure hadronic decays are less accessible to these methods ([11]). With the help of the so-called QCD sum rules, a consistency test between hadron physics and perturbative QCD, it becomes possible to connect hadronic and fundamental QCD parameters directly. Theoretical predictions via QCD sum rules, however, always have relatively large uncertainties [12]. Another approach is the method of factorization [13]. This method has recently been systematized for nonleptonic decays in the heavy quark limit [14]. However, within this approach a quantitative method to estimate the $1/m_b$ corrections to this limit is missing. The latter contributions can be specifically large if they are chirally enhanced [15]. Further fundamental methods whose applications and precision are also somewhat restricted are chiral perturbation theory [16] and heavy quark effective theory [17].

In view of this, the goal must be to minimize theoretical uncertainties with the help of an optimized combination of different fundamental methods solely based on QCD. This can only be done for a selected number of observables in flavour physics. However, there are also observables, dominated by purely perturbative contributions, which will make precision flavour physics possible in the near future. Among them inclusive rare B decays (see fig. 2 [18]) play the most important role.

Inclusive decay modes are theoretically clean and represent a theoretical laboratory of perturbative QCD. In particular, the decay width $\Gamma(B \to X_s \gamma)$ is well approximated by the partonic decay rate $\Gamma(b \to s\gamma)$, which can be analysed in renormalization group improved perturbation theory:

$$\Gamma(B \to X_s\gamma) = \Gamma(b \to s\gamma) + \Delta^{nonpert.} \tag{2.1}$$

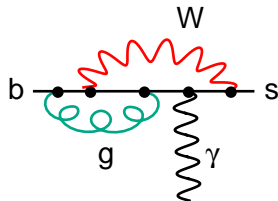Nonperturbative effects, $\Delta^{nonpert.}$, play a subdominant role and are under control

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Figure 3: QCD corrections to the decay $B \to X_s \gamma$.

thanks to the heavy mass expansion.

Thus, in general, inclusive decay modes should be preferred to exclusive ones from the theoretical point of view. The inclusive modes $B \to X_{s(d)} \gamma$ and $B \to X_{s(d)} l^+ l^-$ can be measured by the electron-positron experiments (B factories, CLEO) with their kinematic constraints and their low background, while thy are more difficult to measure at hadronic machines. Exclusive decay modes, however, are more accessible to experiments, in particular at hadronic machines. But in contrast to the inclusive modes, they have in general large nonperturbative QCD contributions. Exclusive decays such as $B_{d,s} \to \mu^+ \mu^-$, $B_d \to K^* \gamma$ and $B_d \to K^* \mu^+ \mu^-$ are distinguished observables at the LHC-B experiment.

Within inclusive B decay modes, short-distance QCD effects lead to a tremendous rate enhancement. These effects are induced by hard gluon exchange between the quark lines of the one-loop electroweak diagrams (fig. 3).

The QCD radiative corrections bring in large logarithms of the form $\alpha_s^n(m_b) \log^m(m_b/M)$, where $M = m_t$ or $M = m_W$ and $m \le n$ (with $n = 0, 1, 2, ...$). This is a natural feature in any process where two different mass scales are present. In order to get a reasonable result at all, one has to resum at least the leading-log (LL) series

$$\alpha_s^n(m_b) \log^n(m_b/M), \ (LL) \tag{2.2}$$

with the help of renormalization group techniques. Working to next-to-leading-log (NLL) precision means that one is also resumming all the terms of the form

$$\alpha_s(m_b) \left(\alpha_s^n(m_b) \log^n(m_b/M) \right), \ (NLL). \tag{2.3}$$

A suitable framework to achieve the necessary resummations of the large logs is an effective low-energy theory with five quarks, obtained by integrating out the heavy particles, which, in the SM, are the top quark and the W boson. The standard method of the operator product expansion allows for a separation of an amplitude of a weak meson decay process into two distinct parts, the long-distance contributions contained in the operator matrix elements and the short-distance physics described by the so-called Wilson coefficients (see fig. 4). In the case of B decays, the W boson and the top quark with mass bigger than the factorization scale are integrated out, that is removed from the theory as dynamical variables. The effective hamiltonian can be written

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \ \mathcal{O}_i(\mu) \tag{2.4}$$
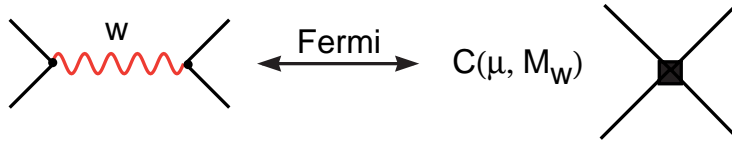
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Figure 4: Operator product expansion.

where $\mathcal{O}_i(\mu)$ are the relevant operators and $C_i(\mu, M_{heavy})$ are the corresponding Wilson coefficients. As the heavy fields are integrated out, the complete top and W mass dependence is contained in the Wilson coefficients. Working out a convenient set of quantities, both in the effective (low-energy) theory and in the full (standard model) theory, and requiring equality (matching) up to terms suppressed by higher powers of m_W or m_t, these coefficients can be determined.

Within this framework QCD corrections for the decay rates are twofold: the ingredients are the order α_s corrections to the matrix elements of the various operators and the order α_s corrections to the Wilson coefficients, of course both at the low-energy scale $\mu \approx m_b$. Only the sum of the two contributions is renormalization scheme and scale independent; in fact, from the μ-independence of the effective Hamiltonian, one can derive a renormalization group equation (RGE) for the Wilson coefficients $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} \, C_j(\mu) \quad , \tag{2.5}$$

where the matrix γ is the anomalous dimension matrix of the operators $\mathcal{O}_i$. Then there are the following three principal steps leading to the leading-log (next-to-leading-log) result within the effective field theory approach:

- Step 1: One has to match the full SM theory with the effective theory at the scale $\mu = \mu_W$, where μ_W denotes a scale of order m_W or m_t. At this scale, the matrix elements of the operators in the effective theory lead to the same logarithms as the full theory calculation. Consequently, the Wilson coefficients $C_i(\mu_W)$ only pick up small QCD corrections, which can be calculated in fixed-order perturbation theory. In the LL (NLL) program, the matching has to be worked out at the $O(\alpha_s^0)$ $(O(\alpha_s^1))$ level.

- Step 2: Then one performs the evolution of these Wilson coefficients from $\mu = \mu_W$ down to $\mu = \mu_W$, where μ_b is of the order of m_b. As the matrix elements of the operators evaluated at the low scale μ_b are free of large logarithms, the latter are contained in resummed form in the Wilson coefficients. For a LL (NLL) calculation, this RGE step has to be done using the anomalous dimension matrix up to order α_s^1 (α_s^2).

- Step 3: To LL (NLL) precision, the corrections to the matrix elements of the operators $\langle s\gamma | \mathcal{O}i(\mu) | b \rangle$ at the scale $\mu = \mu_b$ have to be calculated to order α_s^0 (α_s^1) precision.

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Finally, we stress that the step from the leading (LL) to the next-to-leading (NLL) order within the framework of the renormalization group improved perturbation theory is not only a quantitative one increasing the precision of the theoretical prediction, but also a qualitative one, which tests the validity of the perturbative approach in the given problem.

3 Inclusive decay modes

3.1 Experimental data on $B \to X_s \gamma$

Among inclusive rare B decays, the $B \to X_s \gamma$ mode is the most prominent because it is the only one that is already measured: in 1993, the first evidence for a penguin-induced B meson decay was found by the CLEO collaboration. At CESR, they measured the exclusive electromagnetic penguin process $B \to K^* \gamma$. The inclusive analogue $B \to X_s \gamma$ was also found by the CLEO collaboration through the measurement of its characteristic photon energy spectrum in 1994 (see [23]). As this process is dominated by the two-body decay $b \to s\gamma$, its photon energy spectrum is expected to be a smeared delta function centred at $E_\gamma \approx m_b/2$, where the smearing is due to perturbative gluon bremsstrahlung and to the nonperturbative Fermi motion of the b quark within the B meson. Only the high part of the photon energy spectrum is sensitive to the rare decay $B \to X_s \gamma$. Some lower cutoff in the photon energy has to be imposed in order to exclude the background, mainly from the nonleptonic charged current processes $b \to c q \bar{q}' + \gamma$ or $b \to u q \bar{q}' + \gamma$, which have a typical bremsstrahlung spectrum that is maximal at $E_\gamma = 0$ and falls off for larger values of E_γ. Therefore only the "kinematic" branching ratio for $B \to X_s \gamma$ in the range between $E_\gamma = 2.2$ GeV and the kinematic endpoint at $E_\gamma = 2.7$ GeV could be measured directly. To obtain from this measurement the total branching ratio, one has to know the fraction R of the $B \to X_s \gamma$ events with $E_\gamma \geq 2.2$ GeV. This was done in [19] where the Fermi motion of the b quark in the B meson was taken into account by using the phenomenological model of Altarelli et al. (ACCMM model) [20]. Using this *theoretical* input regarding the photon energy spectrum, the value $R = 0.87 \pm 0.06$ was used by the CLEO collaboration, leading to the CLEO branching ratio [21]

$$\mathcal{B}(B \to X_s \gamma) = (2.32 \pm 0.57_{stat} \pm 0.35_{sys}) \times 10^{-4}. \tag{3.1}$$

The first error is statistical and the second is systematic (including model dependence). This measurement was based on a sample of $2.2 \times 10^6 B\bar{B}$ events.

In 1999, CLEO has presented an improved measurement [22], which is based on 53% more data (3.3×10^6 events). They also used the slightly wider E_γ window starting at 2.1 GeV. The relative error drops almost by a factor of $\sqrt{3}$:

$$\mathcal{B}(B \to X_s \gamma) = (3.15 \pm 0.35_{stat} \pm 0.32_{sys} \pm 0.26_{mod}) \times 10^{-4}. \tag{3.2}$$

The errors represent statistics, systematics, and the model dependence, respectively.

There are also data at the Z^0 peak from the LEP experiments. The ALEPH collaboration [24] has measured the inclusive branching ratio

$$\mathcal{B}(H_b \to X_s\gamma) = (3.11 \pm 0.80_{stat} \pm 0.72_{sys}) \times 10^{-4}. \tag{3.3}$$

It should be noted that the branching ratio in (3.3) involves a weighted average of the B mesons and Λ_b baryons produced in Z^0 decays (hence the symbol H_b) different from the corresponding one given by CLEO, which has been measured at the $\Upsilon(4S)$ resonance. High luminosity is more difficult to obtain at higher e^+e^- collision energies. Thus, $B\bar{B}$ samples obtained by the LEP experiments are smaller than the one at CESR. The rate measured by ALEPH is consistent with the CLEO measurement.

Recently, CLEO presented a refined preliminary analysis (with an lower photon energy cut $E_\gamma \geq 2.0$ GeV) [25]:

$$\mathcal{B}(B \to X_s\gamma) = (2.85 \pm 0.35_{stat} \pm 0.22_{sys}) \times 10^{-4} \tag{3.4}$$

and also BELLE has presented preliminary data [25] of competitive experimental accuracy:

$$\mathcal{B}(B \to X_s\gamma) = (3.37 \pm 0.53_{stat} \pm 0.42_{sys} \pm 0.54_{mod}) \times 10^{-4}. \tag{3.5}$$

More accurate data can be expected in the near future. With the expected high luminosity of the B-factories, an experimental accuracy below 10% in the inclusive $B \to X_s\gamma$ mode appears to be possible.

The uncertainty regarding the fraction R of the $B \to X_s\gamma$ events with $E_\gamma \geq 2.2$ GeV quoted in the experimental measurement, also cited as model dependence, should be regarded as a purely theoretical uncertainty. As mentioned above, the fraction R was calculated in [19] using the phenomenological model by Altarelli et al., where the Fermi motion of the b quark in the B meson is characterized by two parameters, the average Fermi momentum p_F of the b quark and the mass m_q of the spectator quark. The error on the fraction R is essentially obtained by varying the model parameters p_F and m_q in the range for which the ACCMM model correctly describes the energy spectrum of the charged lepton in the semileptonic decays $B \to X_c\ell\nu$ and $B \to X_u\ell\nu$, measured by CLEO and AR-GUS. In [19] a first comparison between the calculated photon energy spectrum and the one measured by the CLEO collaboration was presented. The (normalized) measured photon energy spectrum and the theoretical one are in agreement for those values of p_F and m_q, that correctly describe the inclusive semileptonic CLEO data $B \to X_c\ell\nu$ and $B \to X_u\ell\nu$; at present, the data from the radiative decays is, however, not precise enough to further constrain the values of p_F and m_q. The best fit between the theoretical and measured photon energy spectrum is obtained for $p_F = 450$ MeV and $m_q = 0$. One should mention that the analysis [19] of the photon energy spectrum, in particular the calculation of the fraction R in the ACCMM model used by CLEO, does not include the full NLL information, which becomes available in the meantime.
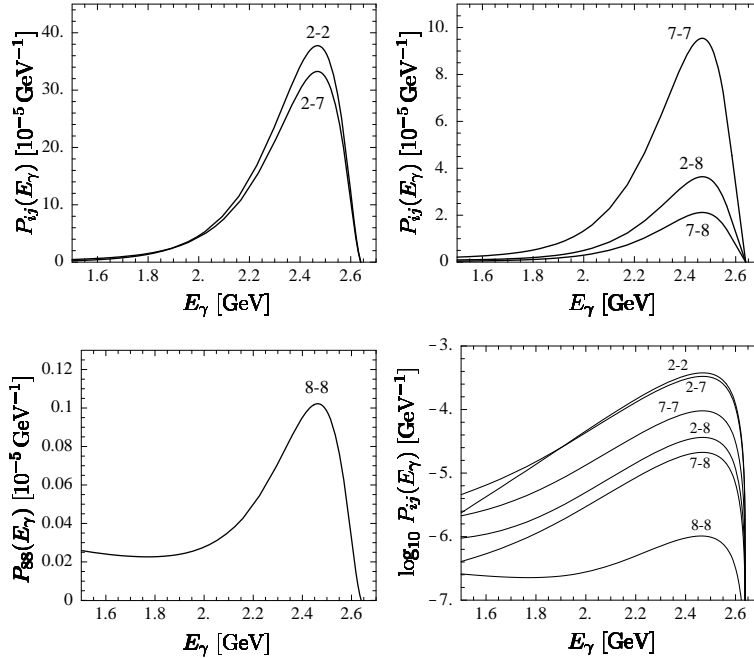
Figure 5: Different components of the photon spectrum in the $B \to X_s\gamma$ decay, from [26].

Besides this phenomenological model by Altarelli et al., more fundamental theoretical methods are available today to implement the bound state effects, namely by making use of operator product expansion techniques in the framework of heavy quark effective theory (HQET). A new analysis along these lines was presented in [26]. Unfortunately, the operator product expansion breaks down near the endpoint of the photon energy spectrum; therefore, an infinite number of leading-twist corrections has to be resummed into a nonperturbative universal shape function, which determines the light-cone momentum distribution of the b-quark in the B meson [27]. The physical decay distributions are then obtained from a convolution of parton model spectra with this shape function. At present this function cannot be calculated, but there is at least some information on the moments of the shape function, which are related to the forward matrix elements of local operators. Ansätze for the shape function, constrained by the latter information, are used. In contrast to the older analysis based on the ACCMM model, the new analysis of Kagan and Neubert [26] includes the full NLL information. Their fraction $R = 0.78^{+0.09}_{-0.11}$ (for the energy cut $E_\gamma > 2.2$ GeV) is smaller than the factor used by CLEO.

An important observation is that the shape of the photon spectrum is not sensitive to physics beyond the SM. As can be seen in fig. 5, all different contributions to the spectrum (corresponding to the interference terms of the various operators involved) have a very similar shape besides the small 8-8 contribution. This implies that we do not have to assume the correctness of the SM in the

experimental analysis and, thus, a precise measurement of the photon spectrum can be used to determine the parameters of the shape function.

Clearly, a lower experimental cut decreases the sensitivity to the parameters of the shape function (or, more generally, the model dependence). With respect to this, the ideal energy cut would be 1.6 GeV. However, in this case a better understanding of the ψ background would be mandatory. The intermediate ψ background, namely $B \to \psi X_s$ followed by $\psi \to X'\gamma$ is more than 4×10^{-4} in the 'total' branching ratio. With the present energy cut of 2.1 GeV this contribution is suppressed and estimated to be less than 5% [28].

Another future aim should be to determine the shape function (and analogously the parameter of the ACCMM model) by using the high-precision measurements of the photon energy spectrum.

3.2   NLL QCD calculations

As mentioned above, the inclusive decay $B \to X_s\gamma$ is a laboratory for perturbative QCD. Nonperturbative effects (see section 3.3) play a subdominant role and are well under control thanks to the heavy quark expansion. The short-distance QCD corrections enhance the partonic decay rate $\Gamma(b \to s\gamma)$ by more than a factor of 2. The corresponding large logarithms of the form $\alpha_s^n(m_b) \log^m(m_b/M)$, where $M = m_t$ or $M = m_W$ and $m \leq n$ (with $n = 0, 1, 2, ...$), have to be summed with the help of the renormalization group improved perturbation theory as presented in section 2.

The effective Hamiltonian relevant to $B \to X_s\gamma$ in the SM reads

$$H_{eff}(B \to X_s\gamma) = -\frac{4G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{8} C_i(\mu) \, \mathcal{O}_i(\mu) \quad , \tag{3.6}$$

where $\mathcal{O}_i(\mu)$ are the relevant operators, $C_i(\mu)$ are the corresponding Wilson coefficients, which contain the complete top- and W- mass dependence (see fig. 6), and $\lambda_t = V_{tb}V_{ts}^*$ with V_{ij}, the CKM matrix elements. The CKM dependence globally factorizes, because we work in the approximation $\lambda_u = 0$ (in the case of $B \to X_s\gamma$). One neglects the operators with dimension > 6. which are suppressed by higher powers of $1/m_W$.

Using the equations of motion for the operators, one arrives at the following basis of dimension-6 operators:

$$
\begin{aligned}
\mathcal{O}_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}) \, (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \ , \\
\mathcal{O}_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha}) \, (\bar{s}_{L\beta}\gamma_\mu c_{L\beta}) \ , \\
\mathcal{O}_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) \left[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + ... + \left(\bar{b}_{L\beta}\gamma_\mu b_{L\beta}\right) \right] \ , \\
\mathcal{O}_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta}) \left[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + ... + \left(\bar{b}_{L\beta}\gamma_\mu b_{L\alpha}\right) \right] \ , \\
\mathcal{O}_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha}) \left[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + ... + \left(\bar{b}_{R\beta}\gamma_\mu b_{R\beta}\right) \right] \ , \\
\mathcal{O}_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta}) \left[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + ... + \left(\bar{b}_{R\beta}\gamma_\mu b_{R\alpha}\right) \right] \ ,
\end{aligned}
$$

Figure 6: SM Hamiltonian in the case of $B \to X_s \gamma$.

$$\begin{aligned}
\mathcal{O}_7 &= (e/16\pi^2)\, \bar{s}_\alpha\, \sigma^{\mu\nu}\, (\overline{m}_b(\mu)P_R + \overline{m}_s(\mu)P_L)\, b_\alpha\, F_{\mu\nu}\,, \\
\mathcal{O}_8 &= (g_s/16\pi^2)\, \bar{s}_\alpha\, \sigma^{\mu\nu}\, (\overline{m}_b(\mu)P_R + \overline{m}_s(\mu)P_L)\, T^A_{\alpha\beta}\, b_\beta\, G^A_{\mu\nu}. \qquad (3.7)
\end{aligned}$$

In the dipole type operators $\mathcal{O}_7$ and $\mathcal{O}_8$, e and $F_{\mu\nu}$ (g_s and $G^A_{\mu\nu}$) denote the electromagnetic (strong) coupling constant and field strength tensor, respectively; T^a ($a = 1, 8$) denote $SU(3)$ colour generators.

The error of the leading logarithmic (LL) result [29] was dominated by a large renormalization scale dependence at the $\pm 25\%$ level, which alreday indicated the importance of the NLL series. By convention, the dependence on the renormalization scale μ_b is obtained by the variation $m_b/2 < \mu_b < 2m_b$. The former measurement of the CLEO collaboration (see (3.1)) overlaps with the estimates based on LL calculations, and the experimental and theoretical errors are comparable. In view of the expected increase in the experimental precision in the near future, it became clear that a systematic inclusion of the NLL corrections was becoming necessary. Moreover, such a NLL program is also important in order to ensure the validity of renormalization group improved perturbation theory in this specific phenomenlogical application.

This ambitious NLL enterprise was completed some years ago. This was a joint effort of many different groups ([19],[30], [31], [32], [33]),[34]). The theoretical error of the previous LL result was substantially reduced to $\pm 10\%$, and the central value of the partonic decay rate increased by about 20%.

All three steps to NLL precision listed below (2.5) involve rather difficult calculations.

- The most difficult part in Step 1 is the two-loop (or order α_s) matching of the dipole operators $\mathcal{O}_7$ and $\mathcal{O}_8$. It involves two-loop diagrams both in the full and in the effective theory. It was first worked out by Adel and Yao [32]. As this is a crucial step in the NLL program, Greub and Hurth confirmed their findings in a detailed re-calculation using a different method [34]. Two further recalculations of this result [35, 36] were presented in the meanwhile, confirming the original results in [32]. In order to match the dimension-6 operators $\mathcal{O}_7$ and $\mathcal{O}_8$, it is sufficient to extract the terms of order $m_b \frac{m_b^2}{M^2}$ ($M = m_W, m_t$) from the SM matrix elements for $b \to s\gamma$ and $b \to sg$. Terms suppressed by additional powers of m_b/M correspond to higher-dimensional operators in the effective theory. In [34] the finite parts

of the two-loop diagrams in the SM were calculated by means of the well-known method of aymptotic mass expansions, which naturally leads to a systematic expansion of Feynman diagrams in inverse powers of M.

- The order α_s^2 anomalous dimension matrix (Step 2) has been worked out by Chetyrkin, Misiak and Münz [33]. In particular, the calculation of the elements γ_{i7} and γ_{i8} ($i = 1, ..., 6$) in the $O(\alpha_s^2)$ anomalous dimension matrix involves a huge number of three-loop diagrams from which the pole parts (in the $d-4$ expansion) have to be extracted. This extraction was simplified by a clever decomposition of the scalar propagator. Moreover, the number of necessary evanescent operators was reduced by a new choice of a basis of dimension-6 operators. Using the matching result (Step 1), these authors obtained the NLL correction to the Wilson coefficient $C_7(\mu_b)$. Numerically, the LL and the NLL value for $C_7(\mu_b)$ turn out to be rather similar; the NLL corrections to the Wilson coefficient $C_7(\mu_b)$ lead to a change of the $B \to X_s \gamma$ decay rate that does not exceed 6% [33].

  It should be stressed that the result of Step 2, in particular the entries γ_{i7} and γ_{i8} ($i = 1, ..., 6$) of the anomalous dimension matrix to NLL precision, is the only part of the complete NLL enterprise which has not been confirmed by an independent group.

- Step 3 basically consists of bremsstrahlung corrections and virtual corrections. While the bremsstrahlung corrections were worked out some time ago by Ali and Greub [19] and were confirmed and extended by Pott [30], a complete analysis of the virtual two-loop corrections (up to the contributions of the four-quark operators with very small coefficients) was presented by Greub, Hurth and Wyler [31]. The latter calculation involves two-loop diagrams, where the full charm dependence has to be taken into account. By using Mellin-Barnes techniques in the Feynman parameter integrals, the result of these two-loop diagrams was obtained in the form

$$
c_0 + \sum_{n=0,1,2,...;m=0,1,2,3} c_{nm} \left(\frac{m_c^2}{m_b^2} \right)^n \log^m \frac{m_c^2}{m_b^2}, \tag{3.8}
$$

  where the quantities c_0 and c_{nm} are independent of m_c. The convergence of the Mellin-Barnes series was proved; the practical convergence of the series (3.8) was also checked explicitly. Moreover, a finite result is obtained in the limit $m_c \to 0$, as there is no naked logarithm of m_c^2/m_b^2. This observation is of some importance in the $b \to d\gamma$ process, where the u-quark propagation in the loop is not CKM-suppressed (see below). The main result of Step 3 consists in a drastic reduction of the renormalization scale uncertainty from about $\pm 25\%$ to about $\pm 6\%$. The central value was shifted by about 20%.

  In [31] these results are presented also in the 't Hooft-Veltman scheme, which may be regarded as a first step towards a cross-check of the complete NLL calculation prediction in a different renormalization scheme.
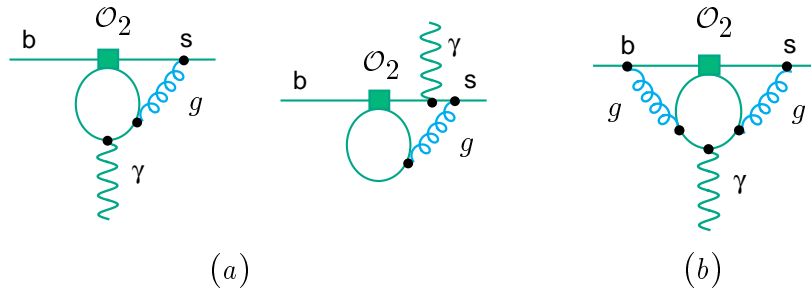
Figure 7: (a) Typical diagrams (finite parts) contributing to the matrix element of the operator $\mathcal{O}_2$ at the NLL level (Step3); (b) typical diagram (infinite part) contributing to the NLL anomalous dimension matrix (Step2); typical diagram (finite part) contributing in the NLL matching calculation shown in fig.3 (Step1).

Quite recently, the results of the matrix elements based on the operator $\mathcal{O}_2$ were confirmed by an independent group [37] with the help of the method of asymptotic expansions.

It is clear that many parts of the NLL calculations at the partonic level in the case of $b \to s\gamma$ can be straightforwardly taken over to the cases $b \to d\gamma$, $b \to s\,gluon$ and $b \to sl^+l^-$. In the latter case, however, many modifications are necessary; in particular the operator basis gets enlarged as will be discussed below.

Combining the NLL calculations of the three steps, the first complete theoretical prediction to NLL precision for the branching ratio of $B \to X_s\gamma$ was presented in [33] (see also [38]):

$$\mathcal{B}(B \to X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4}. \tag{3.9}$$

The theoretical error has two dominant sources. The μ dependence, which is now reduced to about 6%. The other main uncertainty of 5% stems from the m_c/m_b dependence. This first theoretical NLL prediction already included the nonperturbative correction scaling with $1/m_b^2$, which are rather small (at the 1% level) (see section 3.3). Surprisingly, these first NLL predictions ([33],[38]) are almost identical to the current prediction quoted in (3.17), in spite of so many important additional refinements such as the electroweak two-loop corrections and the nonperturbative corrections which will be discussed below.

3.3   Nonperturbative contributions

Within the framework of the heavy mass expansion, nonperturbative corrections to the branching ratio of decay $B \to X_s\gamma$ can be singled out. These contributions also apply to the case of the decay $B \to X_d\gamma$ and, with some modifications, to the case of the decay $B \to X_sl^+l^-$.
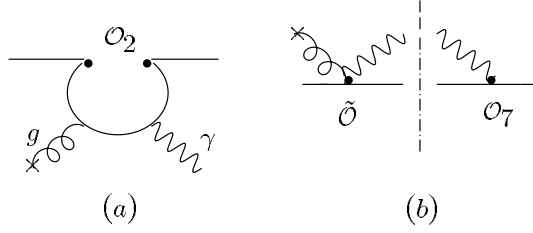
Figure 8: a)Feynman diagram from which the operator $\tilde{\mathcal{O}}$ arises. b) Relevant cut-diagram for the $(\mathcal{O}_2, \mathcal{O}_7)$-interference.

If one neglects perturbative QCD corrections and assumes that the decay $B \to X_s \gamma$ is due to the operator $\mathcal{O}_7$ only, the calculation of the differential decay rate basically amounts to working out the imaginary part of the forward scattering amplitude $T(q)$:

$$T(q) = i \int d^4x \, \langle B|T\mathcal{O}_7^+(x)\,\mathcal{O}_7(0)|B\rangle \, \exp(iqx) \quad . \tag{3.10}$$

Using the operator product expansion for $T\mathcal{O}_7^+(x)\,\mathcal{O}_7(0)$ and heavy quark effective theory methods, the decay width $\Gamma(B \to X_s\gamma)$ reads [39, 40] (modulo higher terms in the $1/m_b$ expansion):

$$\Gamma_{B \to X_s\gamma}^{(\mathcal{O}_7,\mathcal{O}_7)} = \frac{\alpha G_F^2 m_b^5}{32\pi^4} |V_{tb}V_{ts}|^2 \, C_7^2(m_b) \left(1 + \frac{\delta_{rad}^{NP}}{m_b^2}\right) \quad ,$$

$$\delta_{rad}^{NP} = \frac{1}{2}\lambda_1 - \frac{9}{2}\lambda_2 \quad , \tag{3.11}$$

where λ_1 and λ_2 are the parameters for kinetic energy and the chromomagnetic energy. Using $\lambda_1 = -0.5\,\text{GeV}^2$ and $\lambda_2 = 0.12\,\text{GeV}^2$, one gets $\delta_{rad}^{NP} \simeq -4\%$. The $B \to X_s\gamma$ decay width is usually normalized by the semileptonic one. The semileptonic decay width gets $1/m_b^2$ corrections, which are negative; thus, the nonperturbative corrections scaling with $1/m_b^2$ tend to cancel in the branching ratio $\mathcal{B}(B \to X_s\gamma)$, and only about 1% remains.

Voloshin [41] considered the nonperturbative effects when including also the operator $\mathcal{O}_2$. This effect is generated by the diagram in Fig. 8a (and by the one, not shown, where the gluon and the photon are interchanged); g is a soft gluon interacting with the charm quarks in the loop. Up to a characteristic Lorentz structure, this loop is given by the integral

$$\int_0^1 dx \int_0^{1-x} dy \, \frac{xy}{m_c^2 - k_g^2 x(1-x) - 2xy k_g k_\gamma} \quad . \tag{3.12}$$

As the gluon is soft, i.e. $k_g^2, k_g k_\gamma \approx \Lambda^{QCD} m_b/2 \ll m_c^2$, the integral can be expanded in k_g. The (formally) leading operator, denoted by $\tilde{\mathcal{O}}$, is

$$\tilde{\mathcal{O}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* C_2 \frac{eQ_c}{48\pi^2 m_c^2} \bar{s}\gamma_\mu(1-\gamma_5) g_s G_{\nu\lambda} b \, \epsilon^{\mu\nu\rho\sigma} \partial^\lambda F_{\rho\sigma} \quad . \tag{3.13}$$

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Then working out the cut diagram shown in Fig. 8b, one obtains the nonperturbative contribution $\Gamma_{B \to X_s \gamma}^{(\tilde{\mathcal{O}}, \mathcal{O}_7)}$ to the decay width, which is due to the $(\mathcal{O}_2, \mathcal{O}_7)$ interference. Normalizing this contribution by the LL partonic width, one obtains

$$\frac{\Gamma_{B \to X_s \gamma}^{(\tilde{\mathcal{O}}, \mathcal{O}_7)}}{\Gamma_{b \to s \gamma}^{LL}} = -\frac{1}{9} \frac{C_2}{C_7} \frac{\lambda_2}{m_c^2} \simeq +0.03 \quad . \tag{3.14}$$

As the expansion parameter is $m_b \Lambda_{QCD}/m_c^2 \approx 0.6$ (rather than Λ_{QCD}^2/m_c^2), it is not a priori clear whether formally higher order terms in the m_c expansion are numerically suppressed. More detailed investigations [42, 43, 44] have shown that higher order terms are indeed suppressed, because the corresponding expansion coefficients are small.

The analogous $1/m_c^2$ effect has been found independently in the exclusive mode $B \to K^* \gamma$ in ref. [45]. Numerically, the effect there is also at the few per cent level. Moreover, the analysis of the $1/m_c^2$ effects was extended to the decay $B \to X_s l^+ l^-$ in [44, 46].

As was recently emphasized by Misiak [47], an analogous systematic analysis of terms like $\Gamma_{B \to X_s \gamma}^{(\mathcal{O}_2, \mathcal{O}_2)}$ at first order in α_s is still missing. Rigorous techniques such as operator product expansions do not seem to be applicable in this case.

3.4 Theoretical status

3.4.1 $B \to X_s \gamma$

The theoretical prediction for the partonic $b \to s \gamma$ decay rate is usually normalized by the semileptonic decay rate in order to get rid of uncertainties related with the CKM matrix elements and the fifth power of the b quark mass. Moreover, an explicit lower cut on the photon energy in the bremsstrahlung correction is often made:

$$R_{quark}(\delta) = \frac{\Gamma[b \to s\gamma] + \Gamma[b \to s\gamma gluon]_\delta}{\Gamma[b \to X_c e \bar{\nu}_e]} \tag{3.15}$$

where the subscript δ means that only photons with energy $E_\gamma > (1-\delta)E_\gamma^{max} = (1-\delta)\frac{m_b}{2}$ are counted. The ratio R_{quark} is divergent in the limit $\delta \to 1$ due to the unphysical soft photon divergence in the subprocess $b \to s\gamma gluon$. In this limit only the sum of $\Gamma[b \to s\gamma]$, $\Gamma[b \to sgluon]$ and $\Gamma[b \to s\gamma gluon]$ is a reasonable physical quantity, in which all divergences cancel out. In [26] it was shown that the theoretical result is rather sensitive to the unphysical soft-photon divergence; the choice $\delta = 0.90$ was suggested as the optimized definition of the total decay rate. In the analysis presented in [31] the limit $\delta \to 1$ is taken and the singularities are removed by adding the virtual photon corrections to $b \to sgluon$.

It is suggestive to give up the concept of a total decay rate of $b \to s\gamma$ and compare theory and experiment using the same energy cut as CLEO ($E_\gamma > 2.1$ GeV). Then also the theoretical uncertainty regarding the photon energy spectrum mentioned above would occur naturally in the theoretical prediction.

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In the meanwhile detailed studies of the electroweak corrections were performed. In [48] part of the electroweak two-loop contributions, namely contributions from fermion loops in gauge boson propagators (γ and W) and from short-distance photonic loop corrections, were calculated. Moreover, it was found that the on-shell value of the fine structure constant $1/\alpha_{em} = 137$ is more appropriate for real photon emission than the value $1/\alpha_{em} = (130.3 \pm 2.3)$ used in previous analyses. The QED loop calculations in [48] confirmed this expectation. This change in α_{em} leads to a reduction of 5% in R_{quark}. In [26] the QED analysis made in [48] was improved by resumming the contributions of order $\alpha \log(\mu_b/M)(\alpha_s \log(\mu_b/M)^n$ to all orders (while in [48] only the $n = 0$ contribution was included). This resummation decreases the QED corrections. In [49] a complete analysis of the heavy top and the heavy Higgs corrections in the limit $m_W \to 0$ was made. This analysis was recently refined in [50]. A 2% reduction of the branching ratio of $B \to X_s\gamma$ due to purely electroweak corrections is found.

Using the measured semileptonic branching ratio $\mathcal{B}^{sl}_{exp.}$, the branching ratio $\mathcal{B}(B \to X_s\gamma)$ is given by

$$\mathcal{B}(B \to X_s\gamma) = R_{quark} \times \mathcal{B}^{sl}_{exp.}(1 + \Delta_{nonpert}), \qquad (3.16)$$

where the nonperturbative corrections scaling with $1/m_b^2$ and $1/m_c^2$, summed in $\Delta_{nonpert}$, have a numerical effect of $+1\%$ [39, 40] and $+3\%$ [41], respectively, on the branching ratio only.

For a comparison with the ALEPH measurement (3.3) the measured semileptonic branching ratio $\mathcal{B}(H_b \to X_{c,u}\ell\nu)$ should be used consistently. This leads to a larger theoretical prediction for the LEP experiments.

Including only the resummed QED corrections and the nonperturabtive corrections discussed in section 3.3, using the on-shell value of α_{em} and working with the convention $\delta \to 1$ in R_{quark}, one ends up with the following theoretical prediction for the $B \to X_s\gamma$ branching ratio [51]:

$$\mathcal{B}(B \to X_s\gamma) = (3.32 \pm 0.14 \pm 0.26) \times 10^{-4}, \qquad (3.17)$$

where the first error represents the uncertainty regarding the scale dependences, while the second error is the uncertainty due to the input parameters. In the second error the uncertainty due to the parameter m_c/m_b is dominant.

Quite recently, quark mass effects within the decay $B \to X_s\gamma$ were further analysed [55], in particular the definitions of the quark masses m_c and m_b in the matrix element $\langle \mathcal{O}_2 \rangle \equiv \bar{X}_s\gamma|(\bar{s}c)_{V-A}(\bar{c}b)_{V-A}|b\rangle$. Since the charm quark in the matrix element $\langle \mathcal{O}_2 \rangle$ are dominantly off-shell (see fig. 7a) the authors argue that the running charm mass should be chosen instead of the pole mass. . The latter choice was used in all previous analyses [31, 33, 35, 26, 51].

$$m_c^{\text{pole}}/m_b^{\text{pole}} \qquad \Rightarrow \qquad m_c^{\overline{\text{MS}}}(\mu)/m_b^{\text{pole}}, \ \mu \in [m_c, m_b]. \qquad (3.18)$$

Since the matrix element starts at NLL order and, thus, the renormalization scheme for m_c and m_b is an NNLL issue, one should regard this choice as an

educated guess of the NNLL corrections. However, this new choice is guided by the experience gained from many higher-order calculations in perturbation theory. Numerically, the shift from $m_c^{\text{pole}}/m_b^{\text{pole}} = 0.29 \pm 0.02$ to $m_c^{\overline{\text{MS}}}(\mu)/m_b^{\text{pole}} = 0.22 \pm 0.04$ is rather important and leads to a $+11\%$ shift of the central value of the $B \rightarrow X_s\gamma$ branching ratio. The authors of [55] quote a weighted experimental world average using the preliminary data from CLEO and BELLE, (3.4) and (3.5), and the published ALEPH data (3.3):

$$\mathcal{B}(B \rightarrow X_s\gamma) = (2.96 \pm 0.35) \times 10^{-4}. \tag{3.19}$$

With their new choice of the charm mass and with $\delta = 0.9$, their theoretical prediction for the 'total' branching ratio is

$$\mathcal{B}(B \rightarrow X_s\gamma) = (3.73 \pm 0.30) \times 10^{-4}, \tag{3.20}$$

which means that the difference between the theoretical and the experimental value is consistent with zero at the level of 1.6σ (if one assumes that a statistical interpretation of this difference is really possible). Because the choice of the renormalization scheme for m_c and m_b is a NNLL effect, one could argue for a larger theoretical uncertainty in $m_c^{\overline{\text{MS}}}(\mu)/m_b^{\text{pole}}$ which includes also the value of m_c^{pole}. A more conservative choice would then be $m_c^{\overline{\text{MS}}}(\mu)/m_b^{\text{pole}} = 0.22 \pm 0.07$ which would reduce the significance of the perceived discrepancy.

Instead of making a theoretical prediction for the branching ratio $\mathcal{B}(B \rightarrow X_s\gamma)$, one can use the experimental data and theory in order to directly determine the combination $|V_{tb}V_{ts}^*|/|V_{cb}|$ of the CKM matrix elements; in turn, one can determine $|V_{ts}|$ by making use of the relatively well known CKM matrix elements V_{cb} and V_{tb}. An update of the analysis in [56] was presented in [51]. Using the CLEO data (3.2), the ALEPH data (3.3), and the theoretical prediction (3.17), one finds [51]

$$\frac{|V_{ts}^*V_{tb}|}{|V_{cb}|} = 0.95 \pm 0.08_{exp.} \pm 0.05_{th.} \quad \text{CLEO}$$

$$\frac{|V_{ts}^*V_{tb}|}{|V_{cb}|} = 0.91 \pm 0.15_{exp.} \pm 0.04_{th.} \quad \text{ALEPH}.$$

The average of the two measurements yields

$$\frac{|V_{ts}^*V_{tb}|}{|V_{cb}|} = 0.93 \pm 0.09 \pm 0.03 = 0.93 \pm 0.10 \tag{3.21}$$

where in the very last step the theoretical and experimental errors were added in quadrature. Using $|V_{tb}| = 0.99 \pm 0.15$ from the CDF measurement and $|V_{cb}| = 0.0393 \pm 0.0028$ extracted from semileptonic B decays, one obtains [51]

$$|V_{ts}| = 0.037 \pm 0.007, \tag{3.22}$$

where all the errors were added in quadrature. This is probably the most direct determination of this CKM matrix element. With an improved measurement of

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$\mathcal{B}(B \to X_s\gamma)$ and V_{tb}, one expects to reduce the present error on $|V_{ts}|$ by a factor of 2 or even more.

Finally, some remarks on the decay mode $b \to s\,gluon$ are in order. The effective Hamiltonian is the same as in the $b \to s\gamma$ case. By replacing the photon by the gluon, the NLL QCD calculation of $b \to s\gamma$ can also be used. However, in the calculation of the matrix element of the operator $\mathcal{O}_2$, further diagrams with the nonabelian three-gluon coupling have to be calculated [52]. Numerically, one obtains $\mathcal{B}(b \to s\,gluon) = (5.0 \pm 1.0) \times 10^{-3}$, which is more than a factor of 2 larger than the former LL result $\mathcal{B}(b \to s\,gluon) = (2.2 \pm 0.8) \times 10^{-3}$ [29]. The mode $b \to s\,gluon$ represents one component to the inclusive charmless hadronic decays, $B \to X_{nocharm}$, where $X_{nocharm}$ denotes any hadronic charmless final state. A measurement of the corresponding branching ratio would allow the extraction of the ratio $|V_{ub}/V_{cb}|$, which is poorly known at present [53]. At the quark level, there are decay modes with three-body final states, $b \to q'\overline{q}'q$ ($q' = u, d, s; q = d, s$) and the modes $b \to qg$, with two-body final-state topology. The component $b \to sg$ of the charmless hadronic decays is expected to manifest itself in kaons with high momenta (of order $m_b/2$), owing to its two-body nature [54].

3.4.2 $B \to X_d\gamma$

With respect to new physics, also the $B \to X_d\gamma$ decay is of specific interest, because its CKM suppression by the factor $|V_{td}|^2/|V_{ts}|^2$ in the SM may not be true in extended models. Moreover, a future measurement of the $B \to X_d\gamma$ decay rate will help to drastically reduce the currently allowed region of the CKM-Wolfenstein parameters ρ and η.

Most of the theoretical improvements carried out in the context of the decay $B \to X_s\gamma$ (see sections 3.2 and 3.3) can straightforwardly be adapted for the decay $B \to X_d\gamma$. As for the former decay, the NLL-improved and power-corrected decay rate for $B \to X_d\gamma$ has much reduced theoretical uncertainty, which would allow a more precise extraction of the CKM parameters from the measured branching ratio.

The perturbative QCD corrections in the decay $B \to X_d\gamma$ can be treated in complete analogy to the ones in the decay $B \to X_s\gamma$. The effective Hamiltonian is the same in the processes $b \to s\gamma$ and $b \to d\gamma$ up to the obvious replacement of the s-quark field by the d-quark field. However, as λ_u for $b \to d\gamma$ is not small with respect to λ_t and λ_c, one also has to encounter the operators proportional to λ_u. The matching conditions $C_i(m_W)$ and the solutions of the RG equations, yielding $C_i(\mu_b)$, coincide with those needed for the process $B \to X_s\gamma$. The power corrections in $1/m_b^2$ and $1/m_c^2$ (besides the CKM factors) are also the same for the two modes.

The long-distance contributions from the intermediate u-quark in the penguin loops, however, are different. These are suppressed in the $B \to X_s\gamma$ mode by the unfavourable CKM matrix elements. In $B \to X_d\gamma$, there is no CKM suppression and one has to include the long-distance intermediate u-quark contributions,

which can only be modelled at present. However, these contributions are estimated to be rather small [57]. Moreover, it must be stressed that there is no spurious enhancement of the form $\log(m_u/\mu_b)$ in the perturbative contribution to the matrix elements $\langle X_d\gamma|O_{iu}|B\rangle$ $(i = 1, 2)$ as shown by the explicit calculation in [31] and also discussed in [58]. In other words, the limit $m_u \to 0$ can be taken.

The predictions for the $B \to X_d\gamma$ decay given in [59] show that for $\mu_b = 2.5$ GeV (and the central values of the input parameters) the difference between the LL and NLL results is $\sim 10\%$, increasing the branching ratio in the NLL case. For a fixed value of the CKM-Wolfenstein parameters ρ and η, the theoretical uncertainty of the branching ratio is:

$$\Delta\mathcal{B}(B \to X_d\gamma)/\mathcal{B}(B \to X_d\gamma) = \pm(6 - 10)\%. \qquad (3.23)$$

Of particular theoretical interest is the ratio of the branching ratios, defined as

$$R(d\gamma/s\gamma) \equiv \frac{\mathcal{B}(B \to X_d\gamma)}{\mathcal{B}(B \to X_s\gamma)}, \qquad (3.24)$$

in which a good part of the theoretical uncertainties cancels. This suggests that a future measurement of $R(d\gamma/s\gamma)$ will have a large impact on the CKM phenomenology.

Varying the CKM-Wolfenstein parameters ρ and η in the range $-0.1 \leq \rho \leq 0.4$ and $0.2 \leq \eta \leq 0.46$ and taking into account other parametric dependences stated above, the results (without electroweak corrections) are

$$6.0 \times 10^{-6} \quad \leq \quad \mathcal{B}(B \to X_d\gamma) \leq 2.6 \times 10^{-5} ,$$
$$0.017 \quad \leq \quad R(d\gamma/s\gamma) \leq 0.074 .$$

These quantities are expected to be measurable at the high-luminosity B facilities.

3.4.3 $B \to X_s l^+ l^-$

The inclusive $B \to X_s l^+ l^-$ decay will also be accessible at the B factories. In comparison with the $B \to X_s\gamma$ decay, it presents a complementary and also more complex test of the SM since different contributions add to the decay rate (fig. 9). Because of kinematic observables such as the invariant dilepton mass spectrum and the forward-backward asymmetry, it is particularly attractive. It is also dominated by perturbative contributions, if one eliminates $c\bar{c}$ resonances with the help of kinematic cuts.

Using heavy quark expansion, nonperturbative corrections scaling with $1/m_b^2$ and $1/m_c^2$ can be calculated quite analogously to those in the decay $B \to X_s\gamma$ [44]. However, there are also on-shell $c\bar{c}$ resonances, which one has to take into account. While in the decay $B \to X_s\gamma$ (on-shell photon) the intermediate ψ background for example, namely $B \to \psi X_s$ followed by $\psi \to X'\gamma$, is suppressed and can be subtracted from the $B \to X_s\gamma$ decay rate (see section 3.1), the $c\bar{c}$ resonances show up as large peaks in the dilepton invariant mass spectrum in the decay
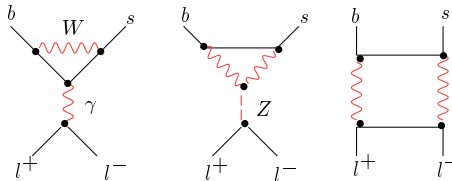
Figure 9: One-loop contributions to the decay $B \to X_s l^+ l^-$.

$B \to X_s l^+ l^-$ (off-shell photon). However, these resonances can be removed by appropriate kinematic cuts in the invariant mass spectrum: In the 'perturbative window', namely $0.05 < \hat{s} = (m_{l^+l^-}/m_b)^2 < 0.25$, theoretical predictions for the invariant mass spectrum are dominated by the purely perturbative contributions, and theoretical precision comparable with the one reached in the decay $B \to X_s \gamma$ is in principle possible.

The present status of the calculation of the perturbative contributions is the following: the effective Hamiltonian relevant to $B \to X_s l^+ l^-$ in the SM reads

$$H_{eff}(B \to X_s l^+ l^-) = -\frac{4 G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{10} C_i(\mu)\, \mathcal{O}_i(\mu) \quad , \tag{3.25}$$

Compared with the decay $B \to X_s \gamma$ (see (3.6)), the effective Hamiltonian (3.25) contains in this case two additional operators:

$$\begin{aligned}
\mathcal{O}_9 &= \frac{e^2}{16\pi^2}\, (\bar{s}\gamma_\mu P_L b)\, (\bar{l}\gamma^\mu l)\,, \\
\mathcal{O}_{10} &= \frac{e^2}{16\pi^2}\, (\bar{s}\gamma_\mu P_L b)\, (\bar{l}\gamma^\mu \gamma_5 l)\,.
\end{aligned} \tag{3.26}$$

It turns out that the first large logarithm of the form $\log(m_b/M)$ $(M = m_W)$ arises already without gluons, because the operator $\mathcal{O}_2$ mixes into $\mathcal{O}_9$ at one loop (the pair $c\bar{c}$ in $\mathcal{O}_2$ can be closed to form a loop, and an off-shell photon producing a $l\bar{l}$ pair can be radiated from a quark line). This possibility has no correspondence in the $B \to X_s \gamma$ case within the SM. Consequently, the decay amplitude is ordered according to

$$G_F\ \log(m_b/M)\ (\alpha_s(m_b)\ \log(m_b/M))^n\quad (LL), \tag{3.27}$$

$$G_F\ \log(m_b/M)\, \alpha_s(m_b)\ (\alpha_s(m_b)\ \log(m_b/M))^n\quad (NLL), \tag{3.28}$$

which should be compared with (2.2) and (2.3). To technically achieve the resummation of these terms, it is convenient to redefine magnetic, chromomagnetic and lepton-pair operators $\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9$, and $\mathcal{O}_{10}$ and the corresponding coefficients as follows [62, 63]:

$$\mathcal{O}_i^{new} = \frac{16\pi^2}{g_s^2}\mathcal{O}_i, \quad C_i^{new} = \frac{g_s^2}{16\pi^2}C_i \qquad (i = 7, ..., 10). \tag{3.29}$$

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This redefinition enables one to proceed according to the three calculational steps presented in section 2, when calculating the decay amplitude [62, 63]. In particular, the one-loop mixing of the operator $\mathcal{O}_2$ with the operator $\mathcal{O}_9^{new}$ appears formally at order g_s^2, after the reshufflings in (3.29).

The QCD calculation up to NLL precision can be found in [62, 63]. However, the *LL* term in the series accidentally turns out to be small. In order to reach the same accuracy as in the case of the NLL prediction for $B \to X_s\gamma$ one has to include the NNLL order contribution in the $B \to X_s l^+ l^-$ calculation.

Large parts of the NLL calculation in the decay $B \to X_s\gamma$, reviewed in section 3.2, can be taken over and used in the NNLL calculation within the decay $B \to X_s l^+ l^-$. However, the *complete* NNLL enterprise - following the standard three steps in the formalism of effective theories (see section 2) - is a formidable task:

- Step 1: In [60] the complete Step 1 up to NNLL precision was presented. The authors did the two-loop matching for all the operators relevant to $B \to X_s l^+ l^-$ (including a confirmation of the $B \to X_s\gamma$ NLL matching results of [32, 34, 35, 36]). The inclusion of this NNLL contribution already removes the large matching scale (μ_W) uncertainty of around 16% present in the NLL prediction of $B \to X_s l^+ l^-$. As usual the partonic decay width is normalized by the semileptonic decay width in order to get rid of uncertainties due to the fifth power in m_b:

$$R_{quark}^{l^+l^-}(\hat{s}) = \frac{1}{\Gamma(b \to X_c e\bar{\nu})} \frac{d\Gamma(b \to X_s l^+ l^-)}{d\hat{s}} \ . \tag{3.30}$$

  One finds the following partial NNLL prediction [60]:

$$\begin{aligned}
\mathcal{B}(B \to X_s l^+ l^-)_{Cut: \ \hat{s} \in [0.05, 0.25]} &= \\
&= \mathcal{B}(B \to X_c e\bar{\nu}) \int_{0.05}^{0.25} d\hat{s}\, [R_{quark}^{l^+l^-}(\hat{s}) + \delta_{1/m_b^2} R(\hat{s}) + \delta_{1/m_c^2} R(\hat{s})] \\
&= 0.104\, [(1.36 \pm 0.18_{scale}) + 0.06 - 0.02]\, 10^{-5} \\
&= (1.46 \pm 0.19_{scale})\, 10^{-6} \tag{3.31}
\end{aligned}$$

  $\delta_{1/m_b^2} R(\hat{s})$ and $\delta_{1/m_c^2} R(\hat{s})$ are the nonperturbative contributions discussed in section 3.3. The quoted error in (3.31) reflects only the μ_b scale uncertainty. This purely perturbative uncertainty should get significantly reduced by contributions within Step 3 of the NNLL program, namely the two-loop QCD corrections to the matrix element of the four-quark operators $\mathcal{O}_2$. The error due to the uncertainties in the input parameters and to other contributions was not estimated in [60], at this intermediate stage of the NNLL calculation.

- Step 2: The most important NNLL contribution from the three-loop renormalization group evolution of the Wilson coefficients from the matching scale μ_W to the low scale μ_b, namely the three-loop anomalous dimensions

corresponding to the mixing of the four-quark operators $\mathcal{O}_i$ $(i = 1...6)$ into the dipole operators $\mathcal{O}_7$ and $\mathcal{O}_8$, can be taken over from the NLL calculation in the decay $B \to X_s\gamma$ [33]. However, the analogous three-loop anomalous dimensions corresponding to the mixing of the four-quark operators into the operator $\mathcal{O}_9$ is missing. In [60] an estimate was made which suggests that the numerical influence of these missing NNLL contributions to the branching ratio of $B \to X_s l^+l^-$ is small.

- Step 3: Within the NLL $B \to X_s\gamma$ calculation the two-loop matrix elements of the four-quark operator $\mathcal{O}_2$ for an on-shell photon were calculated in [31] and quite recently confirmed in [37]. This calculation was extended to the case of an off-shell photon [61], which corresponds to a NNLL contribution relevant to the decay $B \to X_s l^+l^-$. The calculation includes also that part of the corresponding gluon bremsstrahlung which is needed to cancel infrared and collinear singularities of the virtual corrections. If one includes also this NNLL piece in the partonic NNLL prediction for the decay $B \to l^+l^-$, one gets [61]

$$\int_{0.05}^{0.25} d\hat{s}\, R_{quark}^{l^+l^-}(\hat{s}) = (1.25 \pm 0.08_{scale}) \times 10^{-5} \qquad (3.32)$$

Again the only error given corresponds to the uncertainty of the low scale μ_b. As expected the inclusion of the two-loop virtual corections to the four-quark operator $\mathcal{O}_2$ has reduced this scale ambiguity from $\pm 13\%$ down to $\pm 6.5\%$. The authors of [61] also analyse the error due to the uncertainty in the input parameter m_c/m_b and find an uncertainty of $\pm 7.6\%$ within the partonic quantity.

Within the Step 3 of the NNLL calculation, the renormalization group invariant two-loop matrix element of the operator $\mathcal{O}_9$ is not calculated yet. Because this contribution includes no logarithms, the scale dependence of the NLL prediction is not sensitive to this NNLL contribution.

One could think that within this perturbative window at low $\hat{s} \in [0.05, 0.25]$, one is only sensitive to C_7 which would be redundant information, since we already know it from the decay $B \to X_s\gamma$. However, as was explicitly shown in [62, 63], one is also sensitive to the new Wilson coefficients C_9 and C_{10} and interference terms in the low $\hat{s}$ regime with $\hat{s} = m_{l+l-}/m_b^2 \in [0.05, 0.25]$ (see fig. 10 where the various perturbative contributions to R_{quark} (with NLL precision) are plotted).

Together with the decay $B \to X_s\gamma$, the inclusive $B \to X_s l^+l^-$ decay will make precision flavour physics possible, if one can also measure the kinematic variables in the $B \to X_s l^+l^-$ decay precisely. As was first advocated in [64],

- the invariant dilepton mass spectrum

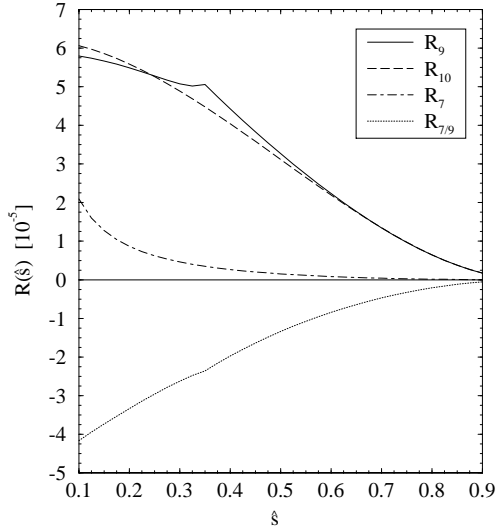$$d\Gamma(B \to X_s l^+l^-)\,/\,d\hat{s}, \qquad (3.33)$$

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Figure 10: Comparison of the different short-distance contributions to $R_{quark}(\hat{s})$ (NLL precision), from [63].

- the forward-backward charge asymmetry

$$A(s) = \int_{-1}^{1} d\cos\theta \; d^2\Gamma(B \to X_s l^+ l^-) \,/\, d\hat{s}\, d\cos\theta \;\; \mathrm{sgn}(\cos\theta) \qquad (3.34)$$

- and the decay rate of $B \to X_s\gamma$,

$$\Gamma(B \to X_s\gamma) \qquad (3.35)$$

determine the magnitude and also the sign of the three Wilson coefficients C_7, C_8, and C_{10}, and allow for a model-independent analysis of rare B decays. For the measurements of these kinematic distributions, however, high statistics will be necessary.

4 Indirect search for supersymmetry

Today supersymmetric models are given priority in our search for new physics beyond the SM. This is primarily suggested by theoretical arguments related to the well-known hierarchy problem. The decay $B \to X_s\gamma$ is sensitive to the mechanism of supersymmetry breaking because in the limit of exact supersymmetry, the decay rate would be just zero:

$$\mathcal{B}(B \to X_s\gamma) = 0. \qquad (4.1)$$

This follows from an argument first given by Ferrara and Remiddi in 1974 [65]. In that work the absence of the anomalous magnetic moment in a supersymmetric

abelian gauge theory was shown. The necessary mechanism of supersymmetry breaking, however, is unknown and leads to a proliferation of free parameters in the (unconstrained) minimal supersymmetric standard model (MSSM).

There are two types of new contributions to flavour changing neutral currents in the MSSM: CKM-induced contributions, which are induced by a charged Higgs or a chargino, and generic new contributions, which are induced by flavour mixing in the squark-mass matrix. The structure of the MSSM does not explain the suppression of flavour changing neutral currents which is observed in experiments. This is the essence of the well-known supersymmetric flavour problem.

In the framework of the MSSM there are at present three favoured concrete supersymmetric models. They solve the supersymmetric flavour problem by a specific mechanism through which the sector of supersymmetry breaking communicates with the sector accessible to experiments: in the minimal supergravity model (mSUGRA) [66], supergravity is the corresponding mediator; in the other two models this role is fulfilled by gauge interactions (GMSB) [67] and by anomalies (AMSB) [68]. Furthermore, there are other classes of models in which the flavour problem is solved by particular flavour symmetries [69].

Flavour violation thus originates from the interplay between the dynamics of flavour and the mechanism of supersymmetry breaking. The model-independent analysis of rare B and K decays therefore can contribute to the question of which mechanism ultimately breaks the supersymmetry and will thus yield important (indirect) information on the construction of supersymmetric extensions of the SM. In this context it is important to analyse the correlations between the different information from rare B and K decays.

As was already emphasized in the introduction, inclusive rare decays, as loop-induced processes, are particularly sensitive to new physics and theoretically clean. Neutral flavour transitions involving third-generation quarks, typically in the B system, do not pose yet serious threats to specific models. However, despite the relatively large experimental uncertainties, the rare decay $B \to X_s \gamma$ has already carved out some regions in the space of free parameters of most of the models in the classes mentioned above (see [70],[71] and references therein). Once more precise data from the B factories are available, this decay will undoubtedly gain efficiency in selecting the viable regions of the parameter space in the various classes of models; this may help in discriminating between the models by then proposed. In view of this, it is important to calculate the rate of this decay with theoretical uncertainties reduced as much as possible, and general enough for generic supersymmetric models.

While in the SM, the rate for $B \to X_s \gamma$ is known up to NLL in QCD, the calculation of this decay rate within supersymmetric models is still far from this level of sophistication. There are several contributions to the decay amplitude: besides the W t-quark and the H t-quark contributions, there are also the chargino, gluino and neutralino contributions. In most of the phenomenological analyses of the decay $B \to X_s \gamma$ these nonstandard contributions were not investigated with NLL precision as the SM contribution. However, as has already been pointed
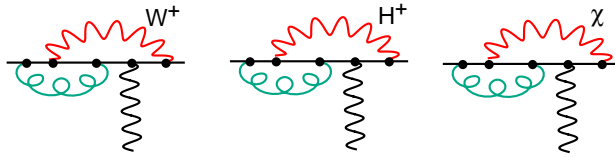
Figure 11: SM, charged Higgs and chargino contribution at the matching scale.

out, the step from the LL to the NLL precision is also necessary in order to check the validity of the perturbative approach in the model under consideration. It is possible that the restriction of the parameter space of nonstandard models, based on an LL analysis only, proves to be invalid after the NLL analysis is completed. Moreover, it was already shown in specific new physics scenarios that bounds on the parameter space of nonstandard models are rather sensitive to NLL contributions (see below).

Nevertheless, within supersymmetric models partial NLL results are available. The gluonic NLL two-loop matching contributions were recently presented [72]. A complete NLL calculation of the $B \to X_s \gamma$ branching ratio in the simplest extension of the SM, namely the Two-Higgs-Doublet Model (2HDM), is already available [35, 73]. In the 2HDM of Type II (which already represents a good approximation for gauge-mediated supersymmetric models with large $\tan \beta$ where the charged Higgs contribution dominates the chargino contribution), the $B \to X_s \gamma$ is only sensitive to two parameters of this model, the mass of the charged Higgs boson and $tan\beta$. Thus, the experimental data of the decay $B \to X_s \gamma$ allows for stringent bounds on these two parameters which are much more restrictive than the lower bound on the charged Higgs mass found in the direct search at LEP. One also finds that these indirect bounds are very sensitive to NLL QCD corrections and even to the two-loop electroweak contributions (see [35, 73]).

In [74] a specific supersymmetric scenario is presented, where in particular the possibility of destructive interference of the chargino and the charged Higgs contribution is analysed. The analysis has been done under two assumptions. First it is assumed that the only source of flavour violation at the electroweak scale is that of the SM, encoded in the CKM matrix (minimal flavour violation). Therefore, the analysis applies to mSUGRA, GMSB and AMSB models (in which the same features are assumed at the messenger scale) only when the sources of flavour violation, generated radiatively between the supersymmetry breaking scale and the electroweak scale, can be neglected with respect to those induced by the CKM matrix. The second assumption is that there exists a specific mass hierarchy, in particular the heavy gluino limit. Indeed, the NLL calculation has been done in the limit

$$\mu_{\tilde{g}} \sim O(m_{\tilde{g}}, m_{\bar{q}}, m_{\tilde{t}_1}) \gg \mu_W \sim O(m_W, m_{H^+}, m_t, m_\chi, m_{\tilde{t}_2}). \qquad (4.2)$$

The mass scale of the charginos (χ), and of the lighter stop ($\tilde{t}_2$) is the ordinary electroweak scale μ_W, while the scale μ_g is characteristic of all other strongly

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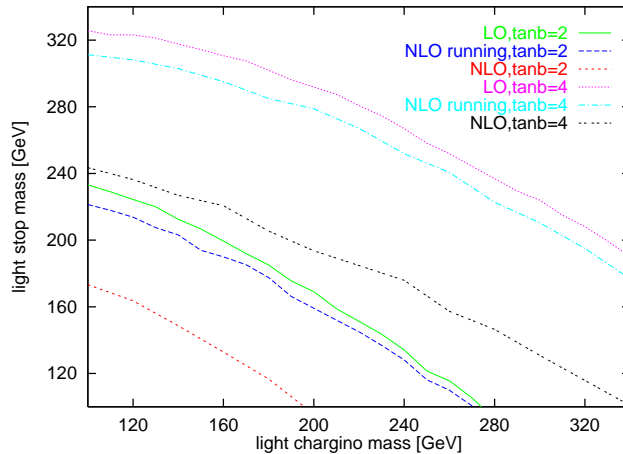
Figure 12: Upper bounds on the lighter chargino and stop masses from $B \to X_s\gamma$ data in the scenario (4.2) if a light charged Higgs mass is assumed; for $\tan\beta = 2$ (three lower curves) and 4 (three upper plots) the LL, NLL-running and NLL results (from the top to the bottom) are shown (see text), from [74].

interacting supersymmetric particles (squarks and gluinos) and is assumed to be of the order of 1 TeV. NLL QCD corrections have been calculated up to first order in $\mu_W/\mu_{\tilde{g}}$ including the important nondecoupling effects [74].

At the electroweak scale μ_W, the new contributions do not induce any new operators in this scenario. Thus, the only step in the new NLL calculation beyond the one within the SM is Step 1, the matching calculation at the scale μ_W where we encounter the two new CKM-induced contributions of the charged Higgs and the chargino (see fig. 11):

$$C_{NLL}(\mu_W) = C_{NLL}^{SM}(\mu_W) + C_{NLL}^{H^+}(\mu_W) + C_{NLL}^{\chi}(\mu_W). \tag{4.3}$$

One finds [74] that, in this specific supersymmetric scenario, bounds on the parameter space are rather sensitive to NLL contributions and they lead to a significant reduction of the stop-chargino mass region where the supersymmetric contribution has a large destructive interference with the charged-Higgs boson contribution. In fig. 12 the upper bounds on the lighter chargino and stop masses from $B \to X_s\gamma$ data in the scenario of (4.2) are illustrated if a light charged Higgs mass of $m_{H^\pm} = 100$ GeV is assumed. The stop mixing is set to $|\theta_{\tilde{t}}| < \pi/10$ which corresponds to the assumption of a mainly right-handed light stop. Moreover, $|\mu| < 500$ GeV and all heavy masses are around 1 TeV. For $\tan\beta = 2$ and 4 the results of the LL, 'NLL running' and NLL calculations are given. The result of neglecting the new NLL supersymmetric contributions to the Wilson coefficients is labelled as "NLL running" and illustrates the importance of the NLL chargino contribution [74].

Quite recently, this minimal flavour violation scenario was refined and extended to the large $tan\beta$ regime by the resummations of terms of the form

$\alpha_s^n tan^{n+1}\beta$ [75, 76]. The stability of the renormalization group improved perturbation theory was reassured for this specific scenario: the resummed NLL results in the large $tan\beta$ regime show constraints similar to the LL results (see also [77]).

For example, it is a well-known feature in the mSUGRA model, that depending on the sign of $A_t \cdot \mu$ (where A_t denotes the stop mixing parameter) the chargino contribution can interfere constructively ($A_t \cdot \mu > 0$) or destructively ($A_t \cdot \mu < 0$) with the SM and the charged Higgs contribution. Therefore, the scenario $A_t \cdot \mu > 0$ within this model requires very heavy superpartners in order to accommodate the $B \to X_s\gamma$ data. But also the case $A_t \cdot \mu < 0$ is constrained in the large $tan\beta$ regime where the chargino contribution is strongly enhanced (for details see [75, 76, 77]).

However, all these NLL analyses are valid only in the heavy gluino regime. Thus, these calculations cannot be used in particular directions of the parameter space of the above listed models in which quantum effects induce a gluino contribution as large as the chargino or the SM contributions. Nor can it be used as a model-discriminator tool, able to constrain the potentially large sources of flavour violation typical of generic supersymmetric models. A complete NLL calculation should also include contributions where the gluon is replaced by the gluino.

The flavour nondiagonal vertex gluino-quark-squark induced by the flavour violating scalar mass term and trilinear terms is particularly interesting. This is generically assumed to induce the dominant contribution to quark flavour transitions, as this vertex is weighted by the strong coupling constant g_s. Therefore, it is often taken as the only contribution to these transitions and in particular to the $B \to X_s\gamma$ decay, when attempting to obtain order-of-magnitude upper bounds on flavour violating terms in the scalar potential [78, 79]. Once the constraints coming from the experimental measurements are imposed, however, the gluino contribution is reduced to values such that the SM and the other supersymmetric contributions can no longer be neglected. Any LL and NLL calculation of the $B \to X_s\gamma$ rate in generic supersymmetric models, therefore, should then include all possible contributions.

The gluino contribution, however, presents some peculiar features related to the implementation of the QCD corrections. In ref. [80] this contribution to the decay $B \to X_s\gamma$ has been investigated in great detail for supersymmetric models with generic soft terms. The gluino-induced contributions to the decay amplitude for $B \to X_s\gamma$ are of the following form:

$$\alpha_s(m_b) \left(\alpha_s(m_b) \log(m_b/M)\right)^n \quad (LL), \tag{4.4}$$

$$\alpha_s^2(m_b) \left(\alpha_s(m_b) \log(m_b/M)\right)^n \quad (NLL). \tag{4.5}$$

In the matching calculation, all factors α_s, regardless of their source, should be expressed in terms of the α_s running with five flavours. In [80] it is shown that the relevant operator basis of the SM effective Hamiltonian gets enlarged to contain magnetic and chromomagnetic operators with an extra factor of α_s and weighted by a quark mass m_b or m_c, and also magnetic and chromomagnetic operators of lower dimensionality where the (small) factor m_b is replaced by the gluino mass.
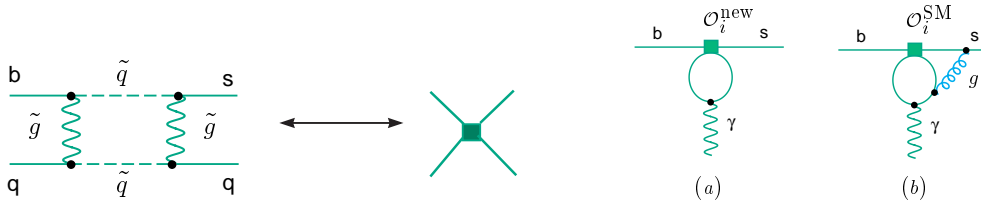
Figure 13: Matching of gluino-squark box on new scalar operators (left frame); mixing of new (scalar) operators at one-loop (a) in constrast to the vectorial operators of the SM (b) who mix at two-loop only (right frame).

Furthermore, one finds that gluino-squark boxes induce new scalar and tensorial four-quark operators, which are shown to mix into the magnetic operators without gluons. On the other hand, the vectorial four-quark operators mix only with an additional gluon into magnetic ones (fig. 13). Thus, they will contribute at the NLL order only. However, from the numerical point of view the contributions of the vectorial operators (although NLL) are not necessarily suppressed w.r.t. the new four-quark contributions; this is due to the expectation that the flavour-violation parameters present in the Wilson coefficients of the new operators are expected to be much smaller (or much more stringently constrained) than the corresponding ones in the coefficients of the vectorial operators. This feature shows that a complete order calculation is important.

In ref. [80] the effects of the LL QCD corrections on constraints on super-symmetric sources of flavour violation are analysed. To understand the sources of flavour violation that may be present in supersymmetric models in addition to those enclosed in the CKM matrix, one has to consider the contributions to the squark mass matrices

$$\mathcal{M}_f^2 = \begin{pmatrix} m_{f,LL}^2 & m_{f,LR}^2 \\ m_{f,RL}^2 & m_{f,RR}^2 \end{pmatrix} + \tag{4.6}$$

$$\begin{pmatrix} F_{f,LL} + D_{f,LL} & F_{f,LR} \\ F_{f,RL} & F_{f,RR} + D_{f,RR} \end{pmatrix} \quad , \tag{4.7}$$

where f stands for up- or down-type squarks. In the super CKM basis where the quark mass matrices are diagonal and the squarks are rotated in parallel to their superpartners, the F terms from the superpotential and the D terms turn out to be diagonal 3×3 submatrices of the 6×6 mass matrices $\mathcal{M}_f^2$. This is in general not true for the additional terms (4.6), originating from the soft supersymmetric breaking potential. Because all neutral gaugino couplings are flavour diagonal in the super CKM basis, the gluino contributions to the decay $b \to s\gamma$ are induced by the off-diagonal elements of the soft terms $m_{f,LL}^2$, $m_{f,RR}^2$, $m_{f,RL}^2$.

It is convenient to select one possible source of flavour violation in the squark sector at a time and assume that all the remaining ones are vanishing. It should be stressed that one already excludes any kind of interference effects between different sources of flavour violation in this way. Following ref. [78], all diagonal
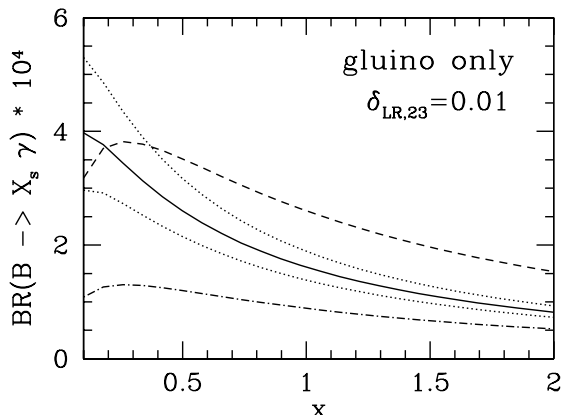
Figure 14: Gluino-induced branching ratio $\mathcal{B}(B \to X_s \gamma)$ as a function of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, obtained when the only source of flavour violation is $\delta_{LR,23}$ (see text).

entries in $m_{d,LL}^2$, $m_{d,RR}^2$, and $m_{u,RR}^2$ are set to be equal and their common value is denoted by $m_{\tilde{q}}^2$. The branching ratio can then be studied as a function of

$$\delta_{LL,ij} = \frac{(m_{d,LL}^2)_{ij}}{m_{\tilde{q}}^2}, \qquad \delta_{RR,ij} = \frac{(m_{d,RR}^2)_{ij}}{m_{\tilde{q}}^2}, \tag{4.8}$$

$$\delta_{LR,ij} = \frac{(m_{d,LR}^2)_{ij}}{m_{\tilde{q}}^2} \ (i \neq j). \tag{4.9}$$

The remaining crucial parameter needed to determine the branching ratio is $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, where $m_{\tilde{g}}$ is the gluino mass. Figures 14 and 15 show the LL QCD corrections to the gluino contribution.

In these figures the solid lines show the QCD corrected branching ratio, when only $\delta_{LR,23}$ or $\delta_{LL,23}$ are nonvanishing. The branching ratio is plotted as a function of x, using $m_{\tilde{q}} = 500\,\text{GeV}$. The dotted lines show the range of variation of the branching ratio, when the renormalization scale μ varies in the interval 2.4–9.6 GeV. Numerically, the scale uncertainty in $\mathcal{B}(B \to X_s \gamma)$ is about $\pm 25\%$. An extraction of bounds on the δ quantities more precise than just an order of magnitude or less, would, therefore, require the inclusion of NLL QCD corrections. It should be noticed, however, that the inclusion of the LL QCD corrections has already removed the large ambiguity on the value to be assigned to the factor $\alpha_s(\mu)$ in the gluino-induced operators. Before adding QCD corrections, the scale in this factor can assume all values from $O(m_b)$ to $O(m_W)$: the difference between the value of $\mathcal{B}(B \to X_s \gamma)$ obtained when taking $\alpha_s(m_b)$ and that obtained when taking $\alpha_s(m_W)$ is of the same order as the LL QCD corrections. The corresponding values of $\mathcal{B}(B \to X_s \gamma)$ for the two extreme choices of μ are indicated in Figs. 14
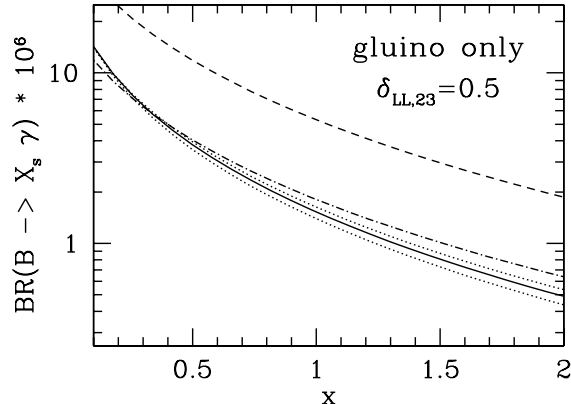
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Figure 15: Gluino-induced branching ratio $\mathcal{B}(B \to X_s\gamma)$ as a function of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, obtained when the only source of flavour violation is $\delta_{LL,23}$ (see text).

and 15 by the dot-dashed lines ($\mu = m_W$) and the dashed lines ($\mu = m_b$). The choice $\mu = m_W$ gives values for the non-QCD corrected $\mathcal{B}(B \to X_s\gamma)$ relatively close to the band obtained when the LL QCD corrections are included, if only $\delta_{LL,23}$ is nonvanishing. Finding a corresponding value of μ that minimizes the QCD corrections in the case studied in Fig. 14, when only $\delta_{LR,23}$ is different from zero, depends strongly on the value of x. In the context of the full LL result, it is important to stress that the explicit α_s factor has to be evaluated - like the Wilson coefficients - at a scale $\mu = O(m_b)$.

In spite of the large uncertainties still affecting the branching ratio $\mathcal{B}(B \to X_s\gamma)$ at LL in QCD, it is possible to extract indications of the size that the δ-quantities may maximal acquire without inducing conflicts with the experimental measurements (see [80]).

Finally, it should be emphasized that a consistent analysis of the bounds on the sfermion mass matrix should also include interference effects between the various contributions. For this issue we refer to a quite recent paper [81], where the interplay between the various sources of flavour violation and the interference effects of SM, gluino, chargino, neutralino and charged Higgs boson contributions is systematically analysed. New bounds on simple combinations of elements of the soft part of the squark mass matrices are found to be, in general, one order of magnitude weaker than the bound on the single off-diagonal element $\delta_{LR,23}$, which was derived in previous work [78, 82] by neglecting any kind of interference effects.

5 Direct CP violation in b/s transitions

Detailed measurements of CP asymmetries in rare B decays will be possible in the near future. Theoretical predictions for the *normalized* CP asymmetries of the inclusive channels (see [59, 83, 84]) within the SM lead to

$$\alpha_{CP}(B \to X_{s/d}\,\gamma) = \frac{\Gamma(\bar{B} \to X_{s/d}\gamma) - \Gamma(B \to X_{\bar{s}/\bar{d}}\gamma)}{\Gamma(\bar{B} \to X_{s/d}\gamma) + \Gamma(B \to X_{\bar{s}/\bar{d}}\gamma)} \qquad (5.1)$$

$$\alpha_{CP}(B \to X_s\gamma) \approx 0.6\%, \qquad \alpha_{CP}(B \to B_d\gamma) \approx -16\% \qquad (5.2)$$

when the best-fit values for the CKM parameters [85] are used.

The leading partonic contribution to the CP asymmetries is given by

$$\alpha_{CP}(B \to X_{s/d}\,\gamma) \simeq \frac{10^{-2}}{|C_7|^2}(1.17 \times \text{Im}\,[C_2 C_7^*] - 9.51 \times \text{Im}\,[C_8 C_7^*] \qquad (5.3)$$

$$+0.12 \times \text{Im}\,[C_2 C_8^*] - 9.40 \times \text{Im}\,\left[\epsilon_{s(d)}C_2\left(C_7^* - 0.013\,C_8^*\right)\right]);$$

$$\epsilon_s = \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \simeq -\lambda^2(\rho - i\eta), \qquad \epsilon_d = \frac{V_{ud}^* V_{ub}}{V_{td}^* V_{tb}} \simeq \frac{\rho - i\eta}{1 - \rho + i\eta}.$$

The large coefficient of the second term in (5.3) has triggered an attractive scenario in which an enhanced chromomagnetic dipole contribution, C_8, induces a large direct CP violation in the decay $B \to X_s\gamma$. Such a possible enhancement of the chromomagnetic contribution would lead to a natural explanation of the phenomenology of semileptonic B decays and also of charm production in B decays [83].

An analysis for the leptonic counterparts is presented in [86]. The normalized CP asymmetries may also be calculated for exclusive decays; however, these results are model-dependent. An example of such a calculation may be found in [87].

CLEO has already presented a measurement of the CP asymmetry in inclusive $b \to s\gamma$ decays, yielding [88]

$$\alpha_{CP}(B \to X_s\gamma) = (-0.079 \pm 0.108 \pm 0.022) \times (1.0 \pm 0.030)\,, \qquad (5.4)$$

which indicates that very large effects are already excluded.

Supersymmetric predictions for the CP asymmetries in $B \to X_{s/d}\gamma$ depend strongly on what is assumed for the supersymmetry-breaking sector and are, thus, a rather model-dependent issue. The minimal supergravity model cannot account for large CP asymmetries beyond 2% because of the constraints coming from the electron and neutron electric dipole moments [89]. However, more general models allow for larger asymmetries, of the order of 10% or even larger [90, 83]. Recent studies of the $B \to X_d\gamma$ rate asymmetry in specific models led to asymmetries between -40% and $+40\%$ [92] or -45% and $+21\%$ [91]. In general, CP asymmetries may lead to clean evidence for new physics by a significant deviation from the SM prediction. From (5.2), it is obvious that a large CP

asymmetry in the $B \to X_s\gamma$ channel or a positive CP asymmetry in the inclusive $B \to X_d\gamma$ channel would be a clear signal for new physics.

In [93] it was pointed out that the exclusive and inclusive decays of the form $b \to s\gamma$ and $b \to d\gamma$, as well as their leptonic counterparts, provide a stringent test, if the CKM matrix is indeed the only source of CP violation. Using U-spin, which is the $SU(2)$ subgroup of flavour $SU(3)$ relating the s and the d quark and which is already a well-known tool in the context of nonleptonic decays [94, 95], one derives relations between the CP asymmetries of the exclusive channels $B^- \to K^{*-}\gamma$ and $B^- \to \rho^-\gamma$ and of the inclusive channels $B \to X_s\gamma$ and $B \to X_d\gamma$. One should make use of the U-spin symmetry only with respect to the strong interactions. Moreover, within exclusive final states, the vector mesons like the U-spin doublet (K^{*-}, ρ^-) are favoured as final states because these have masses much larger than the (current-quark) masses of any of the light quarks. Thus one expects, for the ground-state vector mesons, the U-spin symmetry to be quite accurate in spite of the nondegeneracy of m_d and m_s. Defining the rate asymmetries (not the *normalized* CP asymmetries) by

$$\Delta\Gamma(B^- \to V^-\gamma) = \Gamma(B^- \to V^-\gamma) - \Gamma(B^+ \to V^+\gamma) \qquad (5.5)$$

one arrives at the following relation [93]:

$$\Delta\Gamma(B^- \to K^{*-}\gamma) + \Delta\Gamma(B^- \to \rho^-\gamma) = b_{exc}\Delta_{exc} \qquad (5.6)$$

where the right-hand side is written as a product of a relative U-spin breaking b_{exc} and a typical size Δ_{exc} of the CP violating rate difference. In order to give an estimate of the right-hand side, one can use the model result from [87] for Δ_{exc},

$$\Delta_{exc} = 2.5 \times 10^{-7} \, \Gamma_B. \qquad (5.7)$$

The relative breaking b_{exc} of U-spin can be estimated, e.g. from spectroscopy. This leads us to

$$|b_{exc}| = \frac{M_{K^*} - m_\rho}{\frac{1}{2}(M_{K^*} + m_\rho)} = 14\%. \qquad (5.8)$$

Certainly, other estimates are also possible, such as a comparison of f_ρ and f_{K^*}. In this case one finds a very small U-spin breaking. Using the more conservative value for b_{exc}, which is also compatible with sum rule calculations of form factors (see [96]), one arrives at the standard-model prediction for the difference of branching ratios

$$|\Delta\mathcal{B}(B^- \to K^{*-}\gamma) + \Delta\mathcal{B}(B^- \to \rho^-\gamma)| \sim 4 \times 10^{-8} \qquad (5.9)$$

Note that the right-hand side is model-dependent. Still (5.9) is of some use, since a value significantly above this estimate would be a strong hint that non-CKM sources of CP violation are active.

The issue is more attractive in the inclusive modes. Due to the $1/m_b$ expansion for the inclusive process, the leading contribution is the free b-quark decay. In

particular, there is no sensitivity to the spectator quark and thus one arrives at the following relation for the CP rate asymmetries [93]:

$$\Delta\Gamma(B \to X_s\gamma) + \Delta\Gamma(B \to X_d\gamma) = b_{inc}\Delta_{inc}. \tag{5.10}$$

In this framework one relies on parton-hadron duality (besides in the long-distance contribution from up-quark loops, which is found to be rather small [57]). So one can actually compute the breaking of U-spin by keeping a nonvanishing strange quark mass. However, it is a formidable task to do this for the CP asymmetries and it has not yet been done. The typical size of b_{inc} can be roughly estimated to be of the order of $|b_{inc}| \sim m_s^2/m_b^2 \sim 5 \times 10^{-4}$; $|\Delta_{inc}|$ is again the average of the moduli of the two CP rate asymmetries. These have been calculated (for vanishing strange quark mass), e.g. in [59] and thus one arrives at

$$|\Delta\mathcal{B}(B \to X_s\gamma) + \Delta\mathcal{B}(B \to X_d\gamma)| \sim 1 \cdot 10^{-9}. \tag{5.11}$$

Again, any measured value in significant deviation of (5.11) would be an indication of new sources of CP violation. Although only an estimate is given here, it should again be stressed that in the inclusive mode the right-hand side in (5.11) can be computed in a model-independent way with the help of the heavy mass expansion.

6 $\quad K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$

The rare decays $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ represent complementary opportunities for precision flavour physics. They are flavour changing current processes induced at the one-loop level (see fig. 16) and are exceptionally clean processes. In particular, the $K_L \to \pi^0\nu\bar{\nu}$ amplitude can be calculated with a theoretical uncertainty below 3% [97].

This implies the important role of these decay modes for CKM phenomenology: they play a unique role among K decays, like the $B_d \to \psi K_S$ mode among the B decays. They allow a measurement of one angle of the unitarity triangle without any hadronic uncertainties to a precision comparable to that obtained by the $B_d \to \psi K_S$ mode before the LHC era [98]:

$$\sin 2\beta = \frac{2r_s}{1 + r_s^2} \qquad r_s = \sqrt{\sigma} \frac{\sqrt{\sigma(B_1 - B_2)} - P_0(K^+)}{\sqrt{B_2}} \tag{6.1}$$

where σ is just related to the Wolfenstein parameter $\lambda = 0.22$ via $(1 - \lambda^2/2)^{-2}$; $P_0(K^+) = 0.40 \pm 0.06$ is the internal charm contribution to $K^+ \to \pi^+\nu\bar{\nu}$; this quantity is known up to next-to-leading QCD precision, and the dependence on V_{tb} is only of second order in λ; B_1 and B_2 represent here the reduced branching ratios $B_1 = B(K^+ \to \pi^+\nu\bar{\nu})/(4.11 \times 10^{-11})$ and $B_2 = B(K_L \to \pi^0\nu\bar{\nu})/(1.80 \times 10^{-10})$.

The time-integrated CP violating asymmetry in $B_d^0 \to \psi K_S$ is given by $A_{CP}(\psi K_S) = -\sin 2\beta x_d/(1 + x_d^2)$ where $x_d = \Delta m/\Gamma$ gives the size of $B_d^0 - \bar{B}_d^0$
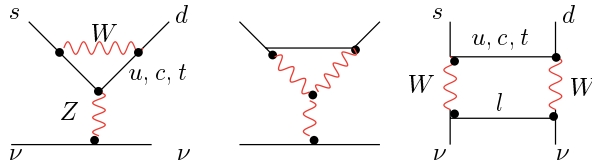
Figure 16: One-loop diagrams contributing to $K \to \pi \nu \bar{\nu}$.

mixing. With $(sin2\beta)_{\pi\nu\bar{\nu}} = (sin2\beta)_{\psi K_s}$, one obtains an interesting connection between rare K decays and B physics:

$$\frac{2r_s(B_1, B_2)}{1 + r_s^2(B_1, B_2)} = -A_{CP}(\psi K_S)\frac{1 + x_d^2}{x_d}, \qquad (6.2)$$

which must be satisfied in the SM. As was stressed in [98], all quantities in this 'golden relation' (6.2) - except for $P_0(K^+)$ - can be directly measured experimentally and the relation is almost independent of V_{cb}.

Besides their rich CKM phenomenology, the decays $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ are very sensitive to new physics beyond the SM. In addition, the theoretical information is very clean and, thus, the measurement of these decays leads to very accurate constraints on any new physics model. Moreover, there is the possibility that these clean rare decay modes themselves lead to first evidence of new physics when the measured decay rates are not compatible with the SM.

New physics contributions in $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ can be parametrized in a model-independent way by two parameters [99] which quantify the violation of the golden relation (6.2). New effects in supersymmetric models can get induced through new box diagram and penguin diagram contributions involving new particles such as charged Higgs or charginos and stops (fig. 17), replacing the W boson and the up-type quark of the SM (fig. 16).

In the constrained minimal supersymmetric standard model (MSSM), where all flavour changing effects are induced by contributions proportional to the CKM mixing angles the golden relation (6.2) is valid. Thus, the measurements of $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ still directly determine the angle β, and a sigificant violation of (6.2) would rule out this model.

At the present experimental status of supersymmetry, however, a model-independent analysis including also flavour change through the squark mass matrices is more suitable. If the new sources of flavour change get parametrized by the mass-insertion approximation, an expansion of the squark mass matrices around their diagonal, it turns out that SUSY contributions in this more general setting of the unconstrained MSSM allow for a significant violation of the golden rule. An enhancement of the branching ratios by an order of magnitude (in the case of $K^+ \to \pi^+\nu\bar{\nu}$ by a factor 3) compared with the SM values is possible, mostly due to the chargino-induced Z-penguin contribution [100]. Recent analyses [100, 101, 102] within the uMSSM focused on the correlation of rare
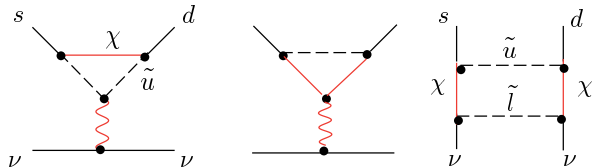
33

Figure 17: Supersymmetric contributions to $K \to \pi \nu \bar{\nu}$.

decays and ϵ'/ϵ, and led to reasonable upper bounds for the branching ratios: $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) \leq 1.2 \times 10^{-10}$, and $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) \leq 1.7 \times 10^{-10}$. which should be compared with the latest numerical SM predictions [103]: $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = (7.9 \pm 3.1) \times 10^{-11}$, $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) = (2.8 \pm 1.1) \times 10^{-11}$.

The rare decays $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ are specifically interesting in view of the suggested experiments at the Brookhaven laboratory (USA) [106] and at FERMILAB (USA) [107] [108] and at KEK (Japan) [109]. The current Brookhaven experiment E787 has already observed a single, but clean candidate event for $K^+ \to \pi^+ \nu \bar{\nu}$ in 1997 which corrsponds to the following branching ratio [104]:

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = (1.5^{+3.4}_{-1.2}) \times 10^{-10}. \tag{6.3}$$

For the $K_L \to \pi^0 \nu \bar{\nu}$ mode, there is only an upper bound available [105]:

$$\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7}. \tag{6.4}$$

7 Summary

In this paper we have reviewed the status of inclusive rare B decays, highlighting recent developments. These decays give special insight into the CKM matrix; moreover, as flavour changing neutral current processes, they are loop-induced and therefore particularly sensitive to new physics.

Decays modes such as $B \to X_s \gamma$ and $B \to X_s l^+ l^-$ (with specific kinematic cuts) represent laboratories for perturbative QCD. Nonperturbative contributions play a subdominant role and they are under control thanks to the heavy mass expansion. The inclusive rare B decays are or will be accessible at the present $e^+ e^-$ machines with their low background and their kinematic constraints (CLEO, BaBar, BELLE) and will make precision flavour physics possible in the near future.

Significant progress has been made during the last couple of years. The calculation of NLL (or even NNLL) QCD corrections to these decay modes has been performed. The theoretical uncertainty has been reduced below the 10% level. As was emphasized, the step from LL to NLL precision within the framework of the renormalization group improved perturbation theory is not only a quantitative, but also a qualitative one, which tests the validity of the perturbative approach in the given problem.

Inclusive rare B decays allow for an indirect search for new physics, a strategy complementary to the direct production of new (supersymmetric) particles, which is reserved for the planned hadronic machines such as the LHC at CERN. However, the indirect search at the B factories will imply significant restrictions for the parameter space of supersymmetric models and will thus lead to important theoretically clean information for the direct search of supersymmetric particles. Within supersymmetric models the QCD calculation of the inclusive rare B decays has not reached the sophistication of the corresponding SM calculation. However, NLL analyses in specific scenarios already show that bounds on the parameter space of nonstandard models are rather sensitive to NLL QCD contributions.

Detailed measurements of CP asymmetries in rare B decays will also be possible in the near future. They will allow for a stringent and clean test if the CKM matrix is indeed the only source of CP violation.

The rare kaon decays, $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, offer complementary opportunities for precision flavour physics. Besides the current Brookhaven experiment, several more are planned or suggested to explore these theoretically clean decay modes.

Acknowledgements

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