CERN–TH/2001-135
May 22, 2001
hep-ph/0105215

The width difference of B_s mesons

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Next-to-leading order QCD corrections to the width difference $\Delta\Gamma$ in the B_s-meson system are presented. I further discuss how $\Delta\Gamma$ can be used to detect new physics.

Presented at the

5th International Symposium on Radiative Corrections
(RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000

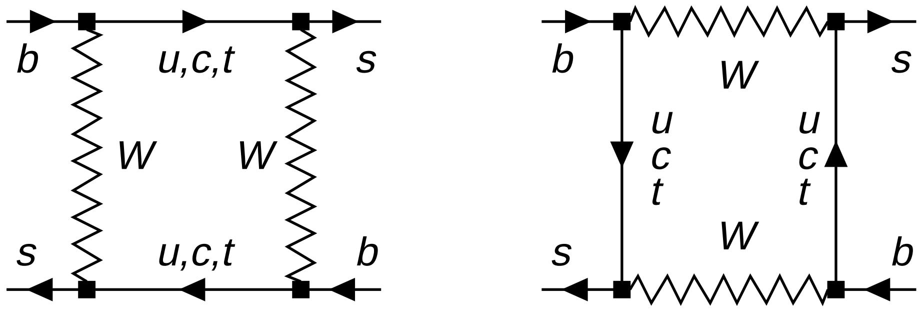

Figure 1: Lowest order contribution to $B_s - \overline{B}_s$ mixing in the Standard Model.

1 Introduction

Currently the prime focus of experimental elementary particle physics is the investigation of the flavor sector of the Standard Model. Transitions between different fermion generations originate from the Higgs-Yukawa sector, which is poorly tested so far. The experimental effort is not only devoted to a precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], which parameterizes the flavor-changing couplings. Flavor-changing neutral currents (FCNC) also provide an ideal testing ground to search for new physics, because they are highly suppressed in the Standard Model: FCNC's are loop-induced, involve the weak coupling constant and the heavy W boson, are suppressed by small CKM elements or the GIM mechanism [2] and further often suffer from a helicity-suppression, because flavor-changing couplings only involve left-handed fields. Therefore experiments in flavor physics are much more sensitive to new physics than the precision tests of the gauge sector performed in the LEP/SLD/Fermilab-Run-I era. Decays of B mesons are especially interesting: they allow us to determine three of the four CKM parameters, their rich decay spectrum helps to overconstrain the CKM matrix, they have theoretically clean CP asymmetries (as opposed to $K \to \pi\pi$ decays), information from B_d, B_s and B^+ decays can be combined using $SU(3)_F$ symmetry, the large b quark mass permits the use of heavy quark symmetries and the heavy quark expansion, and in many extensions of the Standard Model third generation fermions are most sensitive to new physics.

While B_s mesons cannot be studied at the B factories running on the $\Upsilon(4S)$ resonance [3], they are copiously produced at hadron colliders [4]. B_s mesons mix with their antiparticles. Therefore the two mass eigenstates B_H and B_L (for "heavy" and "light"), which are linear combinations of B_s and $\overline{B}_s$, differ in their mass and width. In the Standard Model $B_s - \overline{B}_s$ mixing is described in the lowest order by the box diagrams depicted in Fig. 1. The dispersive part of the $B_s - \overline{B}_s$ mixing amplitude is called M_{12}. In the Standard Model it is dominated by box diagrams with internal top quarks. The absorptive part is denoted by Γ_{12} and mainly stems from box diagrams with light charm quarks. Γ_{12} is generated by decays into final states which are common to B_s and $\overline{B}_s$. While M_{12} can receive sizable contributions from new physics, Γ_{12} is induced by the CKM-favored tree-level decay $b \to c\bar{c}s$ and is insensitive to new physics. Experimentally $B_s - \overline{B}_s$ mixing manifests itself in damped oscillations between the B_s and $\overline{B}_s$ states. We denote the mass and width differences between B_H and B_L by

$$\Delta m = M_H - M_L \,, \qquad \Delta\Gamma \;=\; \Gamma_L - \Gamma_H .$$

By solving the eigenvalue problem of $M_{12} - i\Gamma_{12}/2$ one can relate Δm and $\Delta\Gamma$ to M_{12} and Γ_{12}:

$$\Delta m = 2\,|M_{12}|, \qquad \Delta\Gamma \;=\; 2\,|\Gamma_{12}|\cos\phi, \tag{1}$$

where ϕ is defined as

$$\frac{M_{12}}{\Gamma_{12}} = -\left|\frac{M_{12}}{\Gamma_{12}}\right| e^{i\phi}. \tag{2}$$

Δm equals the $B_s - \overline{B}_s$ oscillation frequency and has not been measured yet. In deriving (1) terms of order $|\Gamma_{12}/M_{12}|^2$ have been neglected. ϕ in (2) is a CP-violating phase, which is tiny in the Standard Model, so that $\Delta\Gamma_{\rm SM} = 2|\Gamma_{12}|$. Unlike in the case of B_d mesons, the Standard Model predicts a sizable width difference $\Delta\Gamma$ in the B_s system, roughly between 5 and 30% of the average total width $\Gamma = (\Gamma_L + \Gamma_H)/2$. The decay of an untagged B_s meson into the final state f is in general governed by two exponentials:

$$\Gamma[f,t] \propto e^{-\Gamma_L t}\,|\langle f|\,B_L\,\rangle|^2 + e^{-\Gamma_H t}\,|\langle f|\,B_H\,\rangle|^2 \,. \tag{3}$$

If f is a flavor-specific final state like $D_s^- \pi^+$ or $X\ell^+\nu$, the coefficients of the two exponentials in (3) are equal. A fit of the corresponding decay distribution to a single exponential then determines the average width Γ up to corrections of order $(\Delta\Gamma)^2/\Gamma$. In the Standard Model CP violation in $B_s - \overline{B}_s$ mixing is negligible, so that we can simultaneously choose B_L and B_H to be CP eigenstates and the $b \to c\bar{c}s$ decay to conserve CP. Then B_H is CP-odd and cannot decay into a CP-even double-charm final state f_{CP+} like $(J/\psi\phi)_{L=0,2}$, where L denotes the quantum number of the orbital angular momentum. Thus a measurement of the B_s width in $B_s \to f_{CP+}$ determines Γ_L. By comparing the two measurements one finds $\Delta\Gamma/2$. CDF will perform this measurement with $B_s \to D_s^- \pi^+$ and $B_s \to J/\psi\phi$ in Run-II of the Tevatron [5].

2 QCD corrections

Weak decays of B mesons involve a large range of different mass scales: first there is the W boson mass M_W, which appears in the weak $b \to c\bar{c}s$ decay amplitude. The second scale in the problem is the mass m_b of the decaying b quark. Finally there is the QCD scale parameter Λ_{QCD}, which sets the scale for the strong binding forces in the B_s meson. QCD corrections associated with these scales must be treated in different ways. To this end one employs a series of operator product expansions, which factorize the studied amplitude into short-distance Wilson coefficient and matrix elements of local operators, which comprise the long-distance physics. Here in the first step the W-mediated $b \to c\bar{c}s$ decay amplitude is matched to matrix elements of local four-quark operators. We need the two $|\Delta B| = 1$ current-current operators

$$Q_1 = \bar{c}_i \gamma_\mu (1-\gamma_5) b_j \bar{s}_j \gamma^\mu (1-\gamma_5) c_i \qquad Q_2 = \bar{c}_i \gamma_\mu (1-\gamma_5) b_i \bar{s}_j \gamma^\mu (1-\gamma_5) c_j, \tag{4}$$

where i, j are color indices. Q_2 is pictorially obtained by contracting the W line in the $b \to c\bar{c}s$ amplitude to a point. Q_1 emerges, once gluon exchange between the two quark lines is included. In the effective hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{r=1}^{2} C_r Q_r \tag{5}$$

the Wilson coefficients C_r are determined in such a way that the Standard Model amplitude is reproduced by $\langle\, c\bar{c}s\,|\mathcal{H}_{eff}|\, b\,\rangle$ up to terms of order m_b^2/M_W^2. The Fermi constant G_F and the CKM elements have been factored out in (5). The C_r's contain the short-distance physics associated with the scale M_W. QCD corrections to the Wilson coefficients can be computed in perturbation theory. The renormalization group evolution of the C_r's down to the scale $\mu_1 = \mathcal{O}(m_b)$ sums the large logarithms $\ln(\mu_1/M_W)$ to all orders in perturbation theory. The minimal way to do this is the leading log approximation which reproduces all term of order $\alpha_s^n \ln^n(\mu_1/M_W)$, $n = 0, 1, \ldots$, of the full Standard Model transition amplitude. The next-to-leading order (NLO) corrections to the coefficients comprise the terms of order $\alpha_s^{n+1} \ln^n(\mu_1/M_W)$ and have been calculated in [6]. We remark that there are also penguin operators in the effective hamiltonian $\mathcal{H}_{eff}$. We have omitted them in (5), because their coefficients are very small. Their impact is discussed in [7,8].

$\Delta\Gamma_{\rm SM} = 2|\Gamma_{12}|$ is related to $\mathcal{H}_{eff}$ by the optical theorem:

$$\Delta\Gamma_{\rm SM} \;=\; 2|\Gamma_{12}| \;=\; \left| -\frac{1}{M_{B_s}} \,\text{Abs}\, \langle \bar{B}_s |\, i \int d^4x \; T\, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B_s \rangle \right|. \tag{6}$$

Here 'Abs' denotes the absorptive part of the amplitude, which is obtained by retaining only the imaginary part of the loop integration. The corresponding leading-order diagrams are shown in Fig. 2. In the next step of our calculation we perform an operator product expansion of the RHS in (6) in order to describe Γ_{12} in terms of matrix elements of local $|\Delta B| = 2$ operators:

$$\begin{aligned} & |\text{Abs}\, \langle \bar{B}_s |\, i \int d^4x \; T\, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B_s \rangle| \\ &= \; -\frac{G_F^2 m_b^2}{12\pi} \left| V_{cb}^* V_{cs} \right|^2 \cdot \\ & \quad \left[F\left(\frac{m_c^2}{m_b^2}\right) \langle \bar{B}_s | Q | B_s \rangle + F_S\left(\frac{m_c^2}{m_b^2}\right) \langle \bar{B}_s | Q_S | B_s \rangle \right] \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right) \right]. \end{aligned} \tag{7}$$

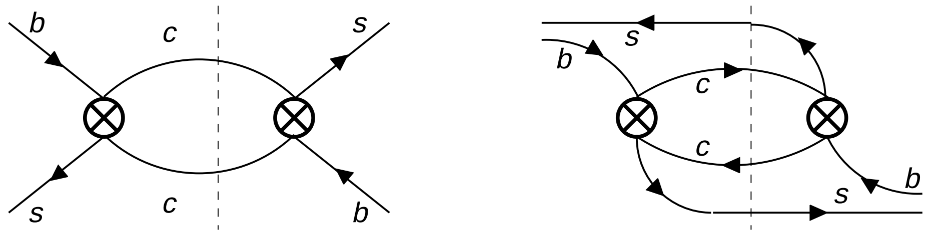

Figure 2: Leading-order diagrams for Γ_{12}

The two dimension-6 operators appearing in (7) are

$$Q = \bar{s}_i \gamma_\mu (1-\gamma_5) b_i \bar{s}_j \gamma^\mu (1-\gamma_5) b_j, \qquad Q_S = \bar{s}_i (1+\gamma_5) b_i \bar{s}_j (1+\gamma_5) b_j. \tag{8}$$

In the leading order of QCD the RHS of (7) is pictorially obtained by simply shrinking the $(c, \overline{c})$ loop in Fig. 2 to a point. Our second operator product expansion is also called *heavy quark expansion* (HQE), which has been developed long ago by Shifman and Voloshin [9]. The new Wilson coefficients F and F_S also depend on the charm quark mass m_c, which is formally treated as a hard scale of order m_b, since $m_c \gg \Lambda_{QCD}$. Strictly speaking, the HQE in (7) is an expansion in $\Lambda_{QCD}/\sqrt{m_b^2 - 4m_c^2}$. For the calculation of F and F_S it is crucial that these coefficients do not depend on the infrared structure of the process. In particular they are independent of the QCD binding forces in the external B_s and $\overline{B}_s$ states in (7), so that they can be calculated in perturbation theory at the parton level. The non-perturbative long-distance QCD effects completely reside in the hadronic matrix elements of Q and Q_S. It is customary to parametrize these matrix as

$$\begin{aligned} \langle \bar{B}_s | Q(\mu_2) | B_s \rangle &= \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B(\mu_2) \\ \langle \bar{B}_s | Q_S(\mu_2) | B_s \rangle &= -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(m_b(\mu_2) + m_s(\mu_2))^2} B_S(\mu_2). \end{aligned} \tag{9}$$

Here M_{B_s} and f_{B_s} are mass and decay constant of the B_s meson. The quark masses m_b and m_s in (9) are defined in the $\overline{\mathrm{MS}}$ scheme. In the so called vacuum insertion approximation $B(\mu_2)$ and $B_S(\mu_2)$ are equal to 1. $\mu_2 = \mathcal{O}(m_b)$ is the scale at which the $|\Delta B| = 2$ operators are renormalized. It can be chosen different from μ_1. The dependence of $\Delta\Gamma$ on the unphysical scales μ_1 and μ_2 diminishes order-by-order in perturbation theory. The residual dependence is usually used as an estimate of the theoretical uncertainty. The μ_1-dependence cancels between the $|\Delta B| = 1$ Wilson coefficients $C_{1,2}$ in (5) and the radiative corrections to F and F_S in (7). The terms in F and F_S which depend on μ_2 cancel with corresponding terms in $B(\mu_2)$ and $B_S(\mu_2)$. The scale μ_2 enters a lattice calculation of these non-perturbative parameters when the lattice quantities are matched to the continuum.

The leading-order calculation of $\Delta\Gamma$ requires the calculation of the diagrams in Fig. 2 and has been performed long ago [10]. Subsequently corrections of order Λ_{QCD}/m_b to (7) have been computed in [7]. The next-to-leading order calculation requires the calculation of the diagrams depicted in Fig. 3 [8]. The motivations for this cumbersome calculation are

1) to verify the infrared safety of F and F_S,

2) to allow for an experimental test of the HQE,

3) a meaningful use of lattice results for hadronic matrix elements,

4) a consistent use of $\Lambda_{\overline{MS}}$,

5) to reduce the sizable μ_1-dependence of the LO,

6) the large size of QCD corrections, typically of order 30%.

The disappearence of infrared effects from the Wilson coefficients F and F_S mentioned in point 1) is necessary for any meaningful operator product expansion. Yet early critics of the HQE had found power-like infrared divergences in individual cuts of diagrams of Fig. 3. In response the cancellation of these divergences has been shown [11], long ago before we have performed the full NLO calculation. However, there are also logarithmic infrared divergences. We found IR-singularities to cancel via two mechanisms:

- Bloch-Nordsiek cancellations among different cuts of the same diagram,
- factorization of IR-singularities, which end up in $\langle \bar{B}_s|Q|B_s\rangle$, $\langle \bar{B}_s|Q_S|B_s\rangle$.

Point 2) above addresses the conceptual basis of the HQE, which is sometimes termed *quark-hadron duality*. It is not clear, whether the HQE reproduces all QCD effects completely. Exponential terms like $\exp(-\kappa m_b/\Lambda_{QCD})$, for example, cannot be reproduced by a power series [12]. The relevance of such terms can at present only be addresses experimentally, by confronting HQE-based predictions with data. The only QCD information contained in the LO prediction for $\Delta\Gamma$ is the coefficients of $\alpha_s^n \ln^n M_W$, associated with hard gluon exchange along the W-mediated $b \to c\bar{c}s$ amplitude. The question of quark-hadron duality, however, addresses the non-logarithmic QCD corrections, which belong to the NLO. While it is certainly very interesting to find violations of quark-hadron duality in B physics, it will be hard to detect them in $\Delta\Gamma$: in the LO diagrams in Fig. 2 the heavy c,$\bar{c}$ quarks recoil back-to-back against each other and are fast in the b rest frame. The inclusive $b \to c\bar{c}s$ decay is more sensitive to uncontrolled long-distance effects, because in some parts of the phase space the c,$\bar{c}$ quarks move slowly in the b rest frame or with respect to each other. Still the HQE prediction for $b \to c\bar{c}s$ [13] agrees with experiment [14]. If one further takes into account that $\Delta\Gamma$ has an overall hadronic uncertainty associated with $f_{B_s}^2 B$ and $f_{B_s}^2 B_S$, it appears very unlikely that violations of quark-hadron duality can be detected in $\Delta\Gamma$. Points 3) and 4) are related to the fact that leading-order predictions are not sensitive to the renormalization scheme, which impedes the lattice-continuum matching of

Table 1:

μ_1	$m_b/2$	m_b	$2m_b$
$-F_S$	0.867	1.045	1.111
$-F_S^{(0)}$	1.729	1.513	1.341
F	0.042	0.045	0.049
$F^{(0)}$	0.030	0.057	0.103

Table 2: Numerical values of the Wilson coefficients F and F_S for $m_c^2/m_b^2 = 0.085$. Leading-order results are indicated with the superscript (0). The precise definition of our renormalization scheme can be found in [8].

the non-perturbative parameters. Likewise the μ_2 dependence of this matching procedure cannot be addressed in the leading-order. The μ_1-dependence of $\Delta\Gamma$ is huge in the leading order. It is reduced in the NLO, but still remains sizable. The results for F and F_S can be found in Tab. 2. The reduction of the μ_1 dependence can be verified from the table. The numerical values of the NLO coefficients depend on the renormalization scheme. The precise definition of this scheme involves the subtraction prescription for the ultraviolet poles (dimensional regularization with $\overline{\mathrm{MS}}$ subtraction [15]), the treatment of γ_5 (for which we have used the NDR scheme) and the chosen definitions of the evanescent operators [16], which can be found in [8]. The lattice-continuum matching must be done in the same renormalization scheme, so that all scheme dependences cancel in the prediction for $\Delta\Gamma$.

Including the corrections of order Λ_{QCD}/m_b [7] our NLO prediction reads

$$\frac{\Delta\Gamma_{\mathrm{SM}}}{\Gamma} = \left(\frac{f_{B_s}}{245\ \mathrm{MeV}}\right)^2 \left[\,(0.234 \pm 0.035)\, B_S(m_b) - 0.080 \pm 0.020\,\right]. \qquad (10)$$

Here $m_b(m_b) + m_s(m_b) = 4.3\,\mathrm{GeV}$ (in the $\overline{\mathrm{MS}}$ scheme) and $m_c^2/m_b^2 = 0.085$ has been used. Since F is small, the uncertainty in B is irrelevant, and the term involving FB has been absorbed into the constant -0.080 ± 0.020 in (10). Recently the KEK–Hiroshima group succeeded in calculating f_{B_s} in an unquenched lattice QCD calculation with two dynamical fermions [17]. The result is $f_{B_s} = (245 \pm 30)$ MeV. A recent quenched lattice calculation has found $B_S(m_b) = 0.87 \pm 0.09$ [18] for the $\overline{\mathrm{MS}}$ scheme. A similar result has been obtained in [19]. With these numbers one finds from (10):

$$\frac{\Delta\Gamma_{\mathrm{SM}}}{\Gamma} = 0.12 \pm 0.06. \qquad (11)$$

Here we have conservatively added the errors from the two lattice quantities linearly.

3 New physics

In the presence of new physics $\arg M_{12}$ and thereby ϕ in (2) can assume any value. Non-standard contributions to ϕ can be measured from CP-asymmetries, which requires the resolution of the rapid B_s–$\overline{B}_s$ oscillations and tagging, i.e. the discrimination between B_s and $\overline{B}_s$ mesons at the time $t = 0$ of their production. From (1) one verifies that a non-vanishing ϕ also affects $\Delta\Gamma$, which can be measured from untagged data samples and therefore involves better efficiencies than tagged studies. Of course in the search for new physics $\Delta\Gamma$ is only competitive with CP asymmetries, which determine $\sin\phi$, if ϕ is not too close to 0 or $\pm\pi$. Nevertheless the information on ϕ from both tagged and untagged data should be combined.

As discussed at the end of Sect. 1, $\Delta\Gamma$ is most easily found from the lifetimes measured in the decays of an untagged B_s sample into a flavor-specific final state and into a CP-specific final state f_{CP}, respectively. In the presence of a non-zero CP-violating phase ϕ the mass eigenstates B_L and B_H are no more CP eigenstates, so that now both exponentials in (3) contribute to the decay $B_s \to f_{CP}$. Then this method determines [20,21]:

$$\Delta\Gamma\cos\phi = \Delta\Gamma_{\rm SM}\cos^2\phi. \tag{12}$$

As first pointed out in [20], one can determine $|\cos\phi|$ without using the theoretical input in (10): if one is able to resolve both exponentials of (3) in the time evolution of a B_s decay into a flavor-specific final state, one will measure the true $|\Delta\Gamma|$. By comparing with (12) one can then solve for $|\cos\phi|$. This method, however, requires to distinguish $\cosh((\Delta\Gamma)t/2)$ from 1 and is very difficult to carry out. In [21] a different method has been proposed, which only requires to measure lifetimes and branching ratios: first define CP eigenstates $B_s^{\rm odd}$ and $B_s^{\rm even}$ such that $B_s^{\rm odd} \not\to D_s^+ D_s^-$. Then define

$$\Delta\Gamma_{\rm CP} \;\; = \;\; \Gamma\,(B_s^{\rm even}) - \Gamma(B_s^{\rm odd}). \tag{13}$$

$\Delta\Gamma_{\rm CP}$ is related to Γ_{12} as

$$\Delta\Gamma_{\rm CP} \;\; = \;\; 2|\Gamma_{12}|.$$

Hence $\Delta\Gamma_{\rm CP}$ equals $\Delta\Gamma_{\rm SM}$, but is not affected by the new physics phase ϕ at all! By measuring both $\Delta\Gamma_{\rm CP}$ and $\Delta\Gamma\cos\phi$ one can infer $|\cos\phi|$ from (12). Loosely speaking, $\Delta\Gamma_{\rm CP}$ is measured by counting the CP-even and CP-odd double-charm final states in B_s decays:

$$\Delta\Gamma_{\rm CP} \;\; = \;\; 2\,\Gamma \sum_{f\in X_{c\overline{c}}} Br\,(\stackrel{(-)}{B}_s \to f\,)\,(1-2\,x_f)\,\left[1\,+\,\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma}\right)\right]. \tag{14}$$

Here $Br\,(\stackrel{(-)}{B}_s \to f\,)$ is the branching ratio of an untagged B_s meson into the final state f, Γ is the average B_s width, the sum runs over all double-charm final states and x_f is the

CP-odd component of the final state f, e.g. x_f is 0 for a CP-even state and equals 1 for a CP-odd state. In the Shifman-Voloshin limit [22] one can show that $\Delta\Gamma_{\rm CP}$ is exhausted by the $D_s^{(*)+} D_s^{(*)-}$ final states [23]. Moreover these four final states are purely CP-even in this limit. ALEPH has measured the sum of these branching ratios [24] and found, relying on the SV limit,

$$\Delta\Gamma_{\rm CP} \;\approx\; 2\, Br\,(\overset{(-)}{B}_s \!\to D_s^{(*)+} D_s^{(*)-}) \;\; = \;\; 0.26^{+0.30}_{-0.15}. \tag{15}$$

In the future one can extend this method by including all detected double-charm final states into the sum in (14) and determine the CP-odd fraction x_f of each final state by measuring the B_s lifetime in the studied mode [21].

Acknowledgments

I thank Gudrun Hiller for inviting me to this conference. I gratefully appreciate the financial support from the conference.

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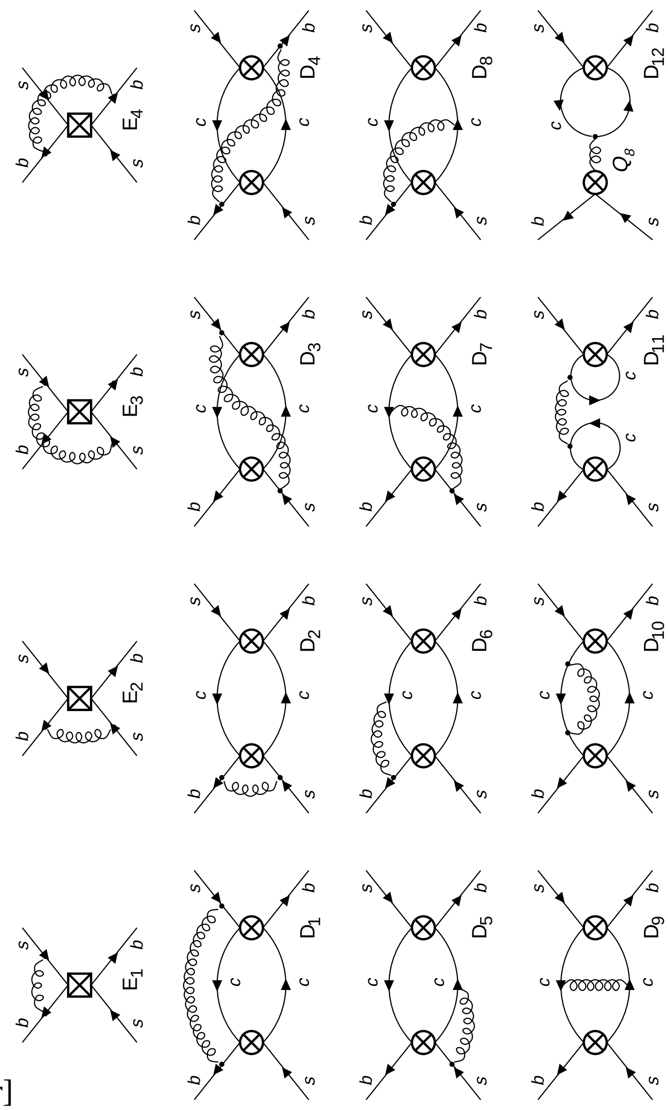

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Figure 3: Next-to-leading-order diagrams for Γ_{12}. Q_8 is the chromomagnetic penguin operator.