Electroweak Sudakov effect on processes at TeV scale *

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In Next Linear Colliders at TeV scale, electroweak double logarithmic corrections, which come from the infrared behaviors of theory can not be neglected. It is well known that in QED and QCD, double logarithmic corrections are resummed to all orders, and these corrections can be exponentiated, resulting in the Sudakov form factor. However it is never trivial that double logarithmic corrections in electroweak theory can be exponentiated, because of the spontaneous breaking of symmetry and the pattern of that. We discuss the electroweak double logarithmic corrections at two loop level and explain the differences of "Soft" structure between the electroweak theory and QCD (the unbroken non-abelian gauge theory).

1. Introduction

Next Linear Colliders at TeV scale are planned for new physics search. These colliders are expected to have high luminosities and we will be able to perform accurate experiments. On the theoretical side, to extract new physics beyond Standard Model (SM) from experimental data, higher order precision calculations are required.

Recently, it is pointed out that logarithmic corrections in electroweak (EW) theory are not negligible at TeV scale [3,4]. For example, EW 1-loop corrections for the process $e^+e^- \to \mu^+\mu^-$ are discussed [5]. The logarithmic corrections for this process dominate the cross section when total energy goes up to TeV region. These logarithmic corrections are classified into the ultraviolet (UV) logarithm, the single infrared (IR) logarithm which is the contribution from soft or collinear region in the loop integration, and the Sudakov type double logarithm (DL) [1] which is originated from soft and collinear region in the loop integration. Particularly, Sudakov type DL correction is of order 10% at $\sqrt{s}$ =1 TeV. These DL corrections may spoil the perturbative prescription when total energy grows up beyond TeV scale. Therefore we must control the DL corrections to obtain the reliable predictions. In QED and QCD, it is well known that we can resum the DL corrections to all orders, resulting in the Sudakov form factor [2]. However, the EW theory is more complicated than QCD (unbroken non-Abelian gauge theory) in two aspects. Firstly, the symmetry is spontaneously broken. Secondly, the pattern of the symmetry breaking is that the off-diagonal $U(1)_{em}$ part of $SU(2) \otimes U(1)$ is survived. These lead to the mass difference between the gauge bosons and the mixing of neutral gauge bosons. Therefore it is non-trivial that the EW DL corrections can be exponentiated. If we can't control these large DL corrections, the perturbative approach can not be trusted beyond TeV scale in EW theory.

This problem has been discussed to all orders by several authors [6–8]. Kühn and Penin [7] considered the process $e^+e^- \to f\bar{f}$ in the Coulomb gauge. They conclude that EW DL corrections are not exponentiated. But they have taken into account only W and Z contributions to the process. Ciafaloni and Comelli [6] considered the process $Z' \to f\bar{f}$ in the Feynman gauge. They use the Soft insertion formula which has been developed in QCD [10]. They assume the "strong energy ordering" to gauge bosons attached to a fermion line with Eikonal current, namely the energies of external boson lines are smaller than the ones of inner boson lines in the diagram. They conclude that the EW DL corrections cannot be

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exponentiated. However this method can take into account only ladder diagrams. Fadin et al. [8] discuss on the exponentiation using the IR evolution equation for the amplitude which is a function of the infrared cut-off, and this equation is analogous to the renormalization group equation. They conclude that EW DL corrections can be exponentiated. These papers disagree with each others.

In order to solve this controversy, several authors calculated explicit 2-loop DL corrections [9, 12,13]. Beenakker and Werthenbach [9] consider the process $e^+e^- \to f\bar{f}$ in the Coulomb gauge. Melles [12] consider the process $g \to f_R\bar{f}_L$ in the Feynman gauge. Since only photon and Z boson contribute to this process, we want to consider the general case including W boson contribution. We consider the process $g \to f_L\bar{f}_R$ in the Feynman gauge. In the next section, we show whether the exponentiation of EW Sudakov type DL corrections holds at the 2-loop level in this process, and discuss the difference of "Soft" structure between the EW theory and QCD (the unbroken non-abelian gauge theory).

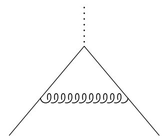
2. Explicit calculation of DL corrections

In this section, we give an explicit 2-loop calculation of DL corrections to the fermion's form factor in the Feynman gauge. The masses of W and Z bosons will be approximated to be equal $M_W \simeq M_Z \equiv M$ §. We give a fictitious small mass to photon to regularize the IR divergence and fermion is assumed to be massless. We consider the situation, $s \gg M \gg \lambda$, where s is the total energy of produced fermions. In section 2.1, we review DL corrections in QCD. In 2.2, we estimate DL corrections in EW theory. And we discuss the difference and the similarity of IR structure between EW theory and QCD.

2.1. DL corrections in QCD

There are many investigations of QCD DL corrections to the fermion's form factor [11]. The QCD 1-loop DL contribution is,

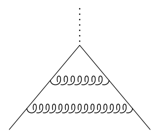
§Because we consider the leading double logarithmic contribution, we need not take into account the mass difference between W and Z boson.



$$: \quad -\frac{g_s^2}{16\pi^2}\ln^2\frac{s}{\lambda^2}C_F,$$

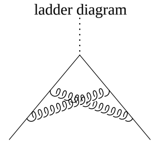
where C_F is the SU(3) Casimir operator for fundamental representation and g_s is the strong coupling constant.

Next, we consider the 2-loop DL contribution. The diagrams which contribute at 2-loop level are,



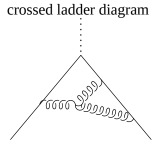
ladder diagram

$$: \quad \frac{g_s^4}{(8\pi^2)^2}\frac{1}{24}\ln^4\frac{s}{\lambda^2}C_F^2,$$



crossed ladder diagram

$$: \quad \frac{g_s^4}{(8\pi^2)^2}\frac{1}{12}\ln^4\frac{s}{\lambda^2}(C_F^2 - \frac{1}{2}C_AC_F),$$



3-point coupling diagram

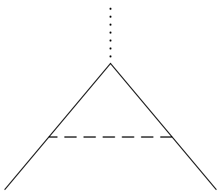
$$\times 2 \quad : \quad \frac{g_s^4}{(8\pi^2)^2}\frac{1}{12}\ln^4\frac{s}{\lambda^2}\frac{1}{2}C_AC_F,$$

where C_A is the Casimir Operator for adjoint representation. The factor 2 in the contribution of diagrams which have the triple gauge boson coupling comes from the symmetric diagram. Note that the second term in the crossed ladder diagram appears as a result of the non-abelian nature of SU(3). But, this term is cancelled out by the contribution of the 3-point coupling diagrams. Therefore, the 2-loop contribution becomes $\frac{1}{2}$(1-loop contribution)^2, and we find that the exponentiation of QCD DL corrections holds at 2-loop level.

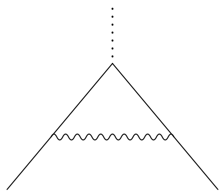
2.2. DL corrections in EW theory

We devote this section to discussion of the EW DL corrections. We consider the process of the production of the left handed fermion and the right handed antifermion from a $SU(2) \otimes U(1)$ singlet source.

First, we consider 1-loop DL contribution. The diagrams which contribute at 1-loop level are,



(a)



(b)

, where the dashed line is photon, the wavy line is W or Z boson. We present the group factor of SU(2)$\otimes$U(1) and the kinematical factor of loop integration separately. The group factors become,

$$\begin{aligned} \gamma \text{ exchange} &: \ e^2Q^2 \ , \\ W \text{ exchange} &: \ g^2 \sum_{a=1,2} T^a T^a \ , \\ Z \text{ exchange} &: \ g^2 T^3 T^3 + g'^2 Y^2 - e^2 Q^2 \ , \end{aligned}$$

where $Q = T^3 + Y$, T^a are the SU(2) generators, Y is the hypercharge, g and g' are SU(2) and U(1) coupling constants and e is the electric charge. The loop integrals in which gauge bosons are exchanged produce the following double logarithms.

$$\begin{aligned} \text{(a) diagram} &: \ -\frac{1}{16\pi^2} \ln^2 \frac{s}{\lambda^2} \\ \text{(b) diagram} &: \ -\frac{1}{16\pi^2} \ln^2 \frac{s}{M^2}. \end{aligned}$$

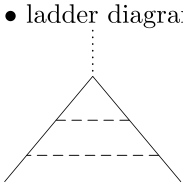
By combining the group factor and the loop kinematical factor, the result of form factor up to 1-loop is,

$$\begin{aligned} \Gamma^{(1)} = 1 \quad &- \quad \frac{1}{16\pi^2} \left(g^2 C_F + g'^2 Y^2 - e^2 Q^2 \right) \ln^2 \frac{s}{M^2} \\ &- \quad \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2} \ , \end{aligned}$$

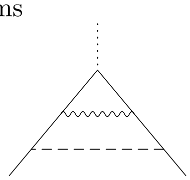
where C_F is the SU(2) Casimir operator for the fundamental representation.

Next, we consider 2-loop DL contributions. The diagrams which contribute at 2-loop level are as follows.

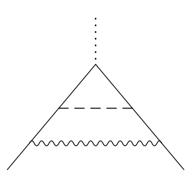
- ladder diagrams



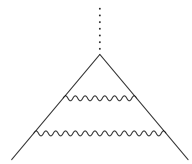
(1a.)



(1c.)

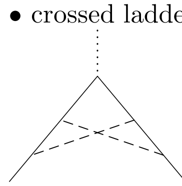


(1d.)

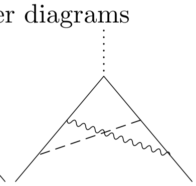


(1f.)

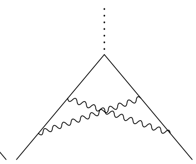
- crossed ladder diagrams



(1b.)

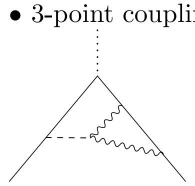


(1e.)

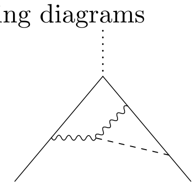


(1g.)

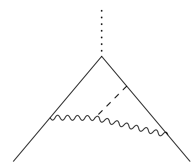
- 3-point coupling diagrams



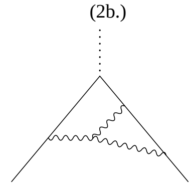
(2a.)



(2b.)



(2c.)



(2d.)

For notational simplicity, group factors and loop integral factors are written as,

$$\begin{aligned} \text{group factor} \quad &: \quad \gamma \equiv e^2 Q^2, \\ & \quad\ \ W + Z \equiv g^2 C_F + g'^2 Y^2 - e^2 Q^2. \\ \text{loop integral factor} \quad &: \quad l \equiv \ln \frac{s}{\lambda^2}, \qquad L \equiv \ln \frac{s}{M^2}. \end{aligned}$$

The contribution of each diagram reads.

$$\begin{aligned} &\text{(ladder contribution)} \\ &= \frac{1}{(8\pi^2)^2} \left[\gamma^2 \frac{1}{24} l^4 \right. \\ &+ 2\gamma \left(W + Z \right) \frac{1}{8} L^2 l^2 + \left(W + Z \right)^2 \frac{1}{24} L^4 \\ &\left. + \gamma \left(W + Z \right) \left\{ \frac{1}{6} L^4 - \frac{1}{3} L^3 l \right\} \right], \\ &\text{(crossed ladder contribution)} \\ &= \frac{1}{(8\pi^2)^2} \left[\gamma^2 \frac{1}{12} l^4 \right. \end{aligned}$$

$$+\left\{(W+Z)^2 - g^4\frac{1}{2}C_A C_F\right\}\frac{1}{12}L^4$$
$$+\gamma\,(W+Z)\left(-\frac{1}{6}L^4+\frac{1}{3}L^3 l\right)$$
$$+2g^2e^2QT^3\left(\frac{1}{6}L^4-\frac{1}{6}L^3 l\right)\Bigg],$$

(3-point coupling contribution)

$$=\frac{1}{(8\pi^2)^2}\left[g^4\frac{1}{2}C_A C_F\frac{1}{12}L^4\right.$$
$$\left.+2g^2e^2QT^3\left\{-\frac{1}{6}L^4+\frac{1}{6}L^3 l\right\}\right],$$

where C_A is the SU(2) Casimir operator for the adjoint representation. We find that even the ladder diagram contribution has an non-exponentiating term(4-th term) which does not emerge in QCD. But this term is cancelled out by the third term in the crossed ladder contribution. And the term proportional to the Casimir operator for adjoint representation appears in the crossed ladder contribution due to the non-abelian nature of SU(2). This term is cancelled out by the first term in the 3-point coupling contribution as in QCD. Other non-exponentiating terms(the 4-th term in the crossed ladder contribution and the second term in the 3-point coupling contribution) cancel each other. We have shown that non-exponentiating terms are cancelled out completely, and obtain the exponentiation of Sudakov form factor at 2-loop level as follows.

$$\Gamma^{(2)} = 1-\frac{1}{16\pi^2}\left\{(W+Z)\,L^2+\gamma l^2\right\} +\frac{1}{2!}\left[\frac{1}{16\pi^2}\left\{(W+Z)\,L^2+\gamma l^2\right\}\right]^2 .$$

3. Summary and Conclusion

We have considered the electroweak form factor at 2-loop level in the DL approximation. We have used the standard Feynman gauge. Our results have shown the exponentiation of the EW Sudakov form factor at 2-loop level like QED and QCD.

This results are very important for theoretical predictions because these support the validity of the perturvative approach in EW theory beyond TeV scale. And this EW Sudakov effect have to be taken into account on processes at TeV scale in future colliders to obtain reliable predictions.

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REFERENCES

1. V. V. Sudakov, *Sov. Phys. JETP* **3** (1956) 65.
2. See *e.g.* J. C. Collins, in *Perturbative Quantum Chromodynamics* ed. A. H. Mueller (Word Scientific, Singapore,1989) P. 573; and references therein.
3. P. Ciafaloni and D. Comelli, *Phys. Lett.* **B446** (1999) 278.
4. M. Ciafaloni, P. Ciafaloni and D. Comelli, *Phys. Rev. Lett.***84** (2000) 4810; hep-ph/0004071.
5. M. Beccaria, P.Ciafaloni, D. Comelli, F. Renard and C. Verzegnassi, *Phys. Rev.* **D61** (2000) 073005.
6. P. Ciafaloni and D. Comelli, *Phys. Lett.* **B476** (2000) 49.
7. J. H. Kühn and A. A. Penin, hep-ph/9906545.
8. V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles, *Phys. Rev.* **D61** (2000) 094002.
9. W. Beenakker and A. Werthenbach, hep-ph/0005316.
10. See *e.g.* M. Ciafaloni, in *Perturbative Quantum Chromodynamics* ed. A. H. Mueller (Word Scientific, Singapore,1989) P. 491.
11. See *e.g.* J. Carrazone, E. Poggio and H. Quinn, *Phys. Rev.* D11 2286.
J. M. Cornwall and G. Tiktopoulos, *Phys. Rev.* **D13** (1976) 3370;
J. Frenkel, M. -L. Frenkel and J. C. Taylor, *Nucl. Phys.* **B124** (1977) 268; and references therein.

12. M. Melles, hep-ph/0006077.
13. M. Hori, H. Kawamura and J. Kodaira, *Phys. Lett.*B491 (2000) 275.
14. H. Kawamura, hep-ph/0102162.