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## Resummation of non-global QCD observables.<sup>1</sup>

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### Abstract

We discuss issues related to the resummation of non-global observables in QCD, those that are sensitive to radiation in only a part of phase space. Examples of such observables are certain single-hemisphere event shapes in  $e^+e^-$  and DIS. Compared to global observables (those sensitive to all emissions, e.g. the  $e^+e^-$  thrust) a new class of single-logarithmic terms arises. These have been neglected in recent calculations in the literature. For a whole set of single hemisphere  $e^+e^-$  and DIS event shapes, we analytically evaluate the first such term, at order  $\alpha_s^2$ , and give numerical results for the resummation of these terms in the large- $N_C$  limit.

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# 1 Introduction

It has long been known that in certain restricted regions of phase space the coefficients of the QCD perturbation expansion can be parametrically large. For semi-inclusive observables this is often due to the incomplete cancellation of soft and/or collinear logarithms.

A set of variables particularly sensitive to this kind of problem is event-shape distributions  $\frac{1}{\sigma} \frac{d\sigma}{dv}$  which we shall use as an illustration throughout. Typically one finds that the perturbative series is dominated by terms  $(\alpha_s^n \ln^{2n-1} v)/v$ , where  $v \ll 1$  denotes the value of the event-shape variable. In order to have a meaningful perturbative description these terms have to be resummed to all orders. This has been done for a range of observables [1–9].

Most of the observables considered so far have the property of exponentiation, implying the following structure for the resummation,

$$\Sigma(v) = \left( 1 + \sum_n C_n \bar{\alpha}_s^n \right) e^{Lg_1(\alpha_s L) + g_2(\alpha_s L) + \dots} + D(v), \quad (1)$$

where the distribution is given by  $d\Sigma/dv$ . The remainder function  $D(v)$  has the property that it goes to zero for  $v \rightarrow 0$ ;  $L = \ln 1/v$  and  $\bar{\alpha}_s = \alpha_s/2\pi$ . The terms resummed by  $Lg_1(\alpha_s L)$  are referred to as leading (or double) logarithms (LL), while those in  $g_2(\alpha_s L)$  are next-to-leading (or single) logarithms (NLL). For certain variables, rather than a single exponent one finds a sum or integral over exponents [8, 9, 7].

A property common to most of the observables considered until recently is that they are sensitive to emissions anywhere in phase space. We call these *global* event shapes. Examples are the thrust, the heavy-jet mass, the  $C$ -parameter. In this paper we are interested in *non-global* observables, those sensitive to radiation in only a part of phase space. We shall discuss mainly single-hemisphere event-shapes in  $e^+e^-$  (especially the jet-mass). Other examples include a number of DIS current-hemisphere observables and quantities such as the mass and broadening of single jets in multi-jet ensembles.

The literature on the subject of global event shapes often makes use of a combination of results in separate hemispheres to obtain the properties of the whole event [1, 2, 5]. When the separate hemisphere results are combined in this fashion the answer obtained is correct. However this can be misleading, since it turns out that the existing single-hemisphere results, applied literally to just a single hemisphere event-shape, are incomplete at NLL level. This has not been taken into account in the recently presented results for the light-jet mass and the narrow-jet broadening [7] and for the narrow-jet thrust-minor [8]. In the case of the first two variables the incompleteness of the results can be seen from a comparison of the  $\mathcal{O}(\alpha_s^2 L^2)$  term of the expansion of the resummation with fixed-order predictions from a Monte Carlo program such as Event2 [10].<sup>2</sup>

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<sup>2</sup>Here we refer to the original version of [7] where it was stated that terms down to  $\alpha_s^n L^{2n-2}$  of the expanded result were under control. In a revised version (v2) which appeared on the hep-ph arXiv shortly before the submission of our paper, the claims regarding the accuracy were revised to state that

To illustrate the point it is convenient to study a simpler variable, the hemisphere jet-mass, defined as

$$\rho = \frac{\left(\sum_{i \in \text{hemisphere}} k_i\right)^2}{\left(\sum_i E_i\right)^2}, \quad (2)$$

where the hemisphere is taken with respect to the thrust axis. Once the distribution of  $\rho$  has been determined, it can be combined with the known heavy-jet mass distribution [1] to obtain the light-jet mass distribution, using the following exact relation

$$\frac{d\sigma}{d\rho_{\text{light}}} + \frac{d\sigma}{d\rho_{\text{heavy}}} = \frac{d\sigma}{d\rho_{\text{left}}} + \frac{d\sigma}{d\rho_{\text{right}}} \equiv 2 \frac{d\sigma}{d\rho}, \quad (3)$$

where  $\rho$  is the hemisphere jet-mass. An analogous relation applies for the jet-broadenings. For the narrow-jet thrust minor the situation is more complex but considerations similar to those addressed here will apply.

One can understand the origin of the qualitative difference between the heavy-jet and single-hemisphere mass resummations by examining figure 1, which depicts two soft large-angle gluons, whose energies are ordered  $Q \gg k_1 \gg k_2$ . For the heavy-jet mass there is a cancelation between the two contributions because the emission of  $k_2$  does not affect the value of  $\rho_{\text{heavy}}$  (which is determined by the harder emission,  $k_1$ ). In the case of the right-jet mass,  $k_1$  has no effect on  $\rho_{\text{right}}$ , so the event shape receives a contribution only from the softer emission,  $k_2$ . This spoils the cancelation with the virtual diagram, figure 1a, thus leading to a term  $\alpha_s^2 L^2$ , i.e. a NLL effect. Such effects are a general feature of non-global observables.

At all orders, to NLL accuracy, the form of the resummed jet-mass distribution will be

$$\Sigma(\rho) = \left(1 + \bar{\alpha}_s C_1^{(g)}\right) \mathcal{S}(\alpha_s L) \Sigma_q(\alpha_s, L) + \bar{\alpha}_s C_1^{(g)} \Sigma_g(\alpha_s, L) + D(\rho). \quad (4)$$

$\Sigma_q$  and  $\Sigma_g$  are the resummed jet-mass cross sections which can be obtained by integrating the quantities  $J_q$  and  $J_g$  first derived in [11] and widely used in the literature. They have structures like that of the exponential factor in eq. (1). The sum of two resummed contributions arises because in a fraction  $\mathcal{O}(\alpha_s)$  of events the hard particle in the right hemisphere ( $\mathcal{H}_R$ ) is a gluon.

The new effect considered in this article is embodied by the function  $\mathcal{S}$ .<sup>3</sup> We write its expansion as

$$\mathcal{S}(\alpha_s L) = 1 + \sum_{n=2} S_n (\bar{\alpha}_s L)^n. \quad (5)$$

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only terms down to  $\alpha_s^n L^{2n-1}$  are controlled. In our notation that corresponds to controlling only the  $\alpha_s L$  term of  $g_2$ , but not terms  $\alpha_s^n L^n$  with  $n \geq 2$ , nor  $C_1$ . Here we follow the convention adopted in [1] rather than that in [7], and use the term next-to-leading to refer to logarithms in the exponent(s), not in the expansion of the exponent.

<sup>3</sup>In principle a similar function multiplies  $\Sigma_g$ , but at NLL accuracy it can be neglected.

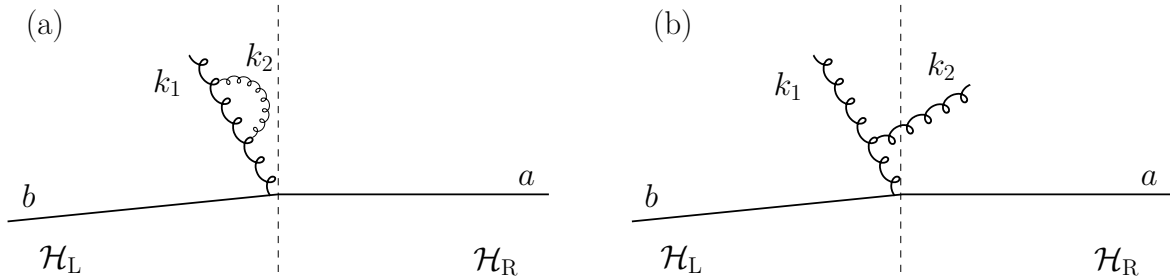


Figure 1: Kinematic configurations of interest

It is straightforward to exactly compute the first non-trivial term  $S_2$  and this is done in the following section. The full computation of  $\mathcal{S}$  involves considering an ensemble of an arbitrary number of large-angle energy-ordered soft gluons in  $\mathcal{H}_L$ , which coherently emit a single, softer gluon into  $\mathcal{H}_R$ . For reasons elucidated later it is difficult to carry out an all-orders treatment of such an effect analytically. We therefore opt to treat these effects using a Monte Carlo algorithm valid in the large- $N_C$  limit. This is outlined in section 3 and further details are given in the appendix.

Finally in section 4 we compare our results to the  $\mathcal{O}(\alpha_s^2)$  predictions of Event2. Phenomenological predictions including this effect will be shown elsewhere [12].

## 2 Fixed order calculation

First we calculate the contribution to the jet-mass distribution from the configuration in figure 1b, considering the right-hemisphere jet for concreteness. We introduce the following particle four-momenta

$$k_a = \frac{Q}{2}(1, 0, 0, 1), \quad (6a)$$

$$k_b = \frac{Q}{2}(1, 0, 0, -1), \quad (6b)$$

$$k_1 = x_1 \frac{Q}{2}(1, 0, \sin \theta_1, \cos \theta_1), \quad (6c)$$

$$k_2 = x_2 \frac{Q}{2}(1, \sin \theta_2 \sin \phi, \sin \theta_2 \cos \phi, \cos \theta_2), \quad (6d)$$

where we have labelled the quark and antiquark as  $a$  and  $b$  and defined energy fractions  $x_{1,2} \ll 1$  for the two gluons. We have ignored recoil in the kinematics, because the jet-mass is insensitive to it.

When gluon 2 is in  $\mathcal{H}_R$  the jet mass has the value  $\rho = x_2(1 - \cos \theta_2)/2$ . When only the quark is in  $\mathcal{H}_R$ ,  $\rho = 0$ .

We write the matrix element for ordered two-gluon emission as (see for example [13])

$$W = 4C_F \frac{(ab)}{(a1)(1b)} \left( \frac{C_A}{2} \frac{(a1)}{(a2)(21)} + \frac{C_A}{2} \frac{(b1)}{(b2)(21)} + \left( C_F - \frac{C_A}{2} \right) \frac{(ab)}{(a2)(2b)} \right) \quad (7a)$$

$$= C_F^2 W_1 + C_F C_A W_2, \quad (7b)$$

where  $(ij) = k_i \cdot k_j$ . The result is valid for  $1 \gg x_1 \gg x_2$  as well as for the opposite ordering of the gluons, and furthermore is completely symmetric under interchange of  $k_1$  and  $k_2$ . We have however chosen to write it in an asymmetric form to emphasise the dipole structure of the emissions, namely radiation of gluon  $k_1$  from the  $ab$  dipole, followed by the radiation of gluon  $k_2$  from the  $a1$ ,  $1b$  and  $ab$  dipoles.

The  $C_F^2$  piece of the matrix element,  $W_1$  corresponds to independent gluon emission and is included in the usual resummation of the quark jet-mass, *i.e.* in  $\Sigma_q$  of eq. (4). To study specifically the modification relative to the standard quark-jet mass result due to configurations like that shown in figure 1b, one must consider the  $C_F C_A$  part,  $W_2$ , of the emission probability above.

The integral to be considered, related to the probability for the jet mass to be less than  $\rho$ , is then

$$- C_F C_A \left( \frac{\alpha_s}{2\pi} \right)^2 \int_{-1}^0 d\cos\theta_1 \int_0^1 d\cos\theta_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_0^{2\pi} \frac{d\phi_2}{2\pi} \frac{Q^4}{16} \int_0^1 x_2 dx_2 \int_{x_2}^1 x_1 dx_1 \Theta \left( \frac{x_2}{2} (1 - \cos\theta_2) - \rho \right) W_2, \quad (8)$$

whose dominant term at small  $\rho$  is  $S_2 \bar{\alpha}_s^2 \ln^2 1/\rho$ .

The integrals over the energy fractions are straightforward. Keeping only the  $\ln^2 1/\rho$  piece and performing the azimuthal average we have

$$S_2 = -2C_F C_A \int_0^1 d\cos\theta_2 \int_{-1}^0 d\cos\theta_1 \Omega_2(\cos\theta_1, \cos\theta_2), \quad (9)$$

where the angular function  $\Omega_2$  is

$$\Omega_2 = \frac{2}{(\cos\theta_2 - \cos\theta_1)(1 - \cos\theta_1)(1 + \cos\theta_2)}. \quad (10)$$

Integrating over the polar angles we obtain

$$S_2 = -C_F C_A \frac{\pi^2}{3}. \quad (11)$$

We note that possibly related “ $\pi^2$ ” terms have been discussed in [14].

### 3 Resummation

To perform the all orders resummation of  $\mathcal{S}(\alpha_s L)$  we need to consider general configurations of soft gluons. We define the probability  $P_{\mathcal{C}}(L)$  for a given configuration  $\mathcal{C}$  at a

resolution scale  $L$  (*i.e.* not resolving gluons with energies less than  $Qe^{-L}$ ). Given  $P_{\mathcal{C}}$ , the probability  $P_{\mathcal{C}'}$  of a configuration  $\mathcal{C}'$  with one extra gluon with scale  $L' > L$ , polar angle  $\theta'$  and azimuth  $\phi'$  is (see for example [15])

$$P_{\mathcal{C}'}(L') = \bar{\alpha}_s(L') \Delta_{\mathcal{C}}(L, L') F_{\mathcal{C}}(\theta', \phi') P_{\mathcal{C}}(L), \quad (12)$$

where the form factor is given by

$$\ln \Delta_{\mathcal{C}}(L, L') = - \int_L^{L'} dL'' \int d\cos\theta d\phi \bar{\alpha}_s(L'') F_{\mathcal{C}}(\theta, \phi), \quad (13)$$

and  $F_{\mathcal{C}}(\theta, \phi)$  describes the angular and colour-structure of the radiation pattern from configuration  $\mathcal{C}$ . The angular integrations in (13) diverge whenever the angle is collinear to one of the emitting particles. In practice it is therefore convenient to introduce an angular cutoff  $\epsilon$  to regulate these divergences both in the real emissions and the virtual corrections.

We obtain  $\mathcal{S}$  by calculating the probability of there being no emissions in  $\mathcal{H}_R$  down to a scale  $L$ , divided by the corresponding probability had the only source of emissions been the original  $q\bar{q}$  pair. This gives

$$\mathcal{S}(\alpha_s L) = \frac{1}{\sqrt{\Delta_{ab}(L)}} \sum_{\mathcal{C} | \mathcal{H}_R \text{ empty}} P_{\mathcal{C}}(L), \quad (14)$$

where the sum runs over all configurations  $\mathcal{C}$  which contain no emissions in  $\mathcal{H}_R$ .

### 3.1 Large- $N_C$ limit and Monte Carlo implementation

In practice two considerations make it difficult to implement the above approach analytically. One is that the colour algebra involved in the determination of  $F_{\mathcal{C}}$  becomes progressively more complicated as the number of gluons increases. The other problem is simply that the treatment of the geometry of the many large-angle gluon ensemble quickly becomes prohibitive. The first problem can be partially solved by taking the large- $N_C$  limit. To address the issue of the geometry we shall use a Monte Carlo approach.

In the large- $N_C$  limit, one can represent gluons by pairs of colour-anticolour lines, as illustrated in figure 2. When squaring the amplitude one ignores contributions that in terms of their colour flow are topologically non-planar, because they are suppressed by powers of  $1/N_C^2$  [16]. As a result, for an ensemble of  $n$  gluons,  $F_{\mathcal{C}}$  just reduces to a sum of independent emission intensities from  $n + 1$  independent dipoles:

$$F_{\mathcal{C}}(\theta_k, \phi_k) = \sum_{\text{dipoles-}ij} \frac{2C_A}{(1 - \cos\theta_{ik})(1 - \cos\theta_{kj})}. \quad (15)$$

When the dipole  $ij$  radiates a gluon  $k$  it splits into two dipoles,  $ik$  and  $kj$ . Thus the dipole structure is determined by the history of the gluon branching.

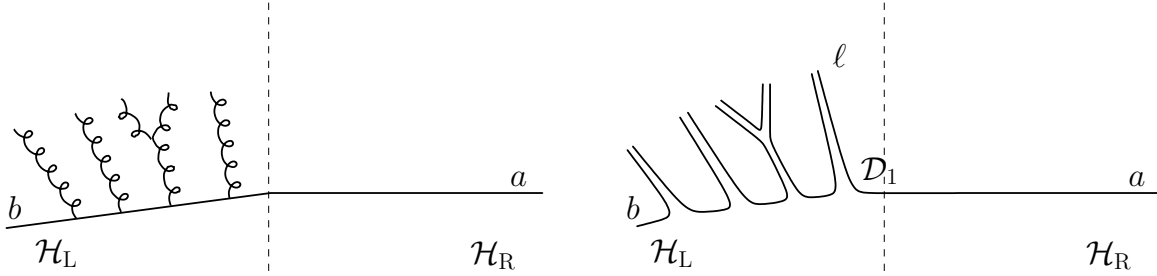


Figure 2: Left: the kind of diagram which must be considered in the calculation of  $\mathcal{S}$ . Right: the same diagram represented in the large- $N_C$  limit, with gluons shown as pairs of colour lines and quarks as single colour lines.

Such a branching pattern can be very naturally implemented using a Monte Carlo algorithm. At first sight one might envisage calculating the two factors in (14) separately, using the  $F_C$  to generate the distribution of radiation for each new configuration. However because of the collinear divergence along the direction of the quark in  $\mathcal{H}_R$ , only a tiny fraction of events would be free of emissions in  $\mathcal{H}_R$  and so contribute to the sum in (14). The sum would therefore have a large relative error, which would translate to a large absolute error on  $\mathcal{S}$  because of the division by the small quantity  $\sqrt{\Delta_{ab}(L)}$ .

Instead a more efficient procedure involves moving the division by  $\sqrt{\Delta_{ab}(L)}$  directly into the calculation of the  $P_C$ . This can be achieved using a modified radiation intensity,  $\tilde{F}_C$  (for both the emissions and the virtual corrections),

$$\tilde{F}_C(\theta, \phi) = F_C(\theta, \phi) - F_{ab}(\theta, \phi)\Theta(\theta), \quad (16)$$

where one subtracts out the radiation intensity  $F_{ab}$  which would have been produced by the original  $q\bar{q}$  pair (in the large- $N_C$  limit). One calculates quantities  $\tilde{P}_C$  using analogs of eqs. (12) and (13) with  $F_C$  replaced by  $\tilde{F}_C$  and then  $\mathcal{S}$  is simply given by

$$\mathcal{S}(\alpha_s L) = \sum_{\mathcal{C}|\mathcal{H}_R \text{ empty}} \tilde{P}_C(L). \quad (17)$$

It should be kept in mind that since  $\tilde{F}_C$  is negative in certain regions of phase space one loses a strict probabilistic interpretation for the  $\tilde{P}_C$ . Nevertheless the sum over configurations is well-defined and meaningful.

The exact details of the Monte Carlo algorithm are given in the appendix. Here we restrict ourselves to giving a parameterisation for  $\mathcal{S}$  obtained by fitting to the Monte Carlo results:

$$\mathcal{S}(\alpha_s L) \simeq \exp\left(-C_F C_A \frac{\pi^2}{3} \left(\frac{1+(at)^2}{1+(bt)^c}\right) t^2\right), \quad (18)$$

with

$$t(\alpha_s L) = \frac{1}{2\pi} \int_{e^{-L}}^1 \frac{dx}{x} \alpha_s(xQ) = \frac{1}{4\pi\beta_0} \ln \frac{1}{1-2\beta_0\alpha_s L}, \quad (19)$$

where  $\beta_0 = (11C_A - 2n_f)/(12\pi)$  and

$$a = 0.85C_A, \quad b = 0.86C_A, \quad c = 1.33. \quad (20)$$

The parameterisation should be accurate to the order of a few percent (better in most of the region) for  $t < 0.7$ , corresponding to  $1 - 2\alpha_s\beta_0L \gtrsim 0.005$ .<sup>4</sup>

Actually, for the purposes of the fit one replaces  $C_F C_A$  in (18) with  $C_A^2/2$  since the Monte Carlo works in the large- $N_C$  limit. But for use in phenomenology one wishes to have the exact colour structure at least at  $\mathcal{O}(\alpha_s^2)$ , hence the use of  $C_F C_A$  in (18).

## 4 Checks and conclusions

It is useful to check our results against fixed order results from the next-to-leading order Monte Carlo program Event2 [10]. First it is necessary to determine the constant terms  $C_1^{(g)}$  and  $C_1^{(q)}$ , which are obtained by requiring consistency between (4) and a full  $\mathcal{O}(\alpha_s)$  calculation. It is straightforward to show that they are given by

$$C_1^{(q)} = \frac{1}{2}(C_1^\tau - r_3), \quad C_1^{(g)} = \frac{r_3}{2}, \quad (21)$$

where  $C_1^\tau$  is the constant determined for the thrust in [1] and  $\bar{\alpha}_s r_3$  is the probability of there being a hard gluon along the thrust axis, as given in [7]:<sup>5</sup>

$$r_3 = C_F \left( 2 \ln^2 2 - \frac{5}{4} \ln 3 + 4 \text{Li}_2 \left( -\frac{1}{2} \right) + \frac{\pi^2}{3} - \frac{1}{6} \right) \simeq 0.917 C_F. \quad (22)$$

The plots in figure 3 show the difference between the  $\mathcal{O}(\alpha_s^2)$  exact and resummed distributions. To NLL accuracy we aim to account for terms down to  $\alpha_s^2 L^2$  in  $\Sigma$  and therefore  $\alpha_s^2 L$  in the distribution. Accordingly the difference should be independent of  $\rho$  for small  $\rho$ . One can see that this is the case for each individual colour factor. We also show that without the contribution  $\mathcal{S}$  calculated in this paper the  $C_F C_A$  part of the result is not consistent with Event2.

Using eq. (3) it is possible to combine our results for the single-hemisphere mass together with those of [1] for the heavy-jet mass in order to obtain predictions correct to single-log level for the light-jet mass (including the appropriate  $C_1$  terms).

We point out that the form computed for  $\mathcal{S}$  applies to all single-hemisphere 2-jet event-shapes that for large-angle gluons have a value of the order of the transverse momentum: in  $e^+e^-$  this means the hemisphere broadening (for the constant terms a relation analogous to (21) applies, with  $C_1^\tau$  replaced with  $C_1^{B_T}$ ). In DIS our form for  $\mathcal{S}$  applies to the current hemisphere jet-mass, the  $C$ -parameter, and the thrust and

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<sup>4</sup>The accessible range of  $t$  is limited by two issues: firstly only a small fraction of events are generated at large  $t$ , requiring considerable statistics in order to investigate that region; and secondly because an accurate determination of  $\mathcal{S}$  at large  $t$  requires a very small angular cutoff, which leads to there being many dipoles in an event, and a consequent slowing down of the evolution.

<sup>5</sup>Only in the hep-ph arXiv version 1.



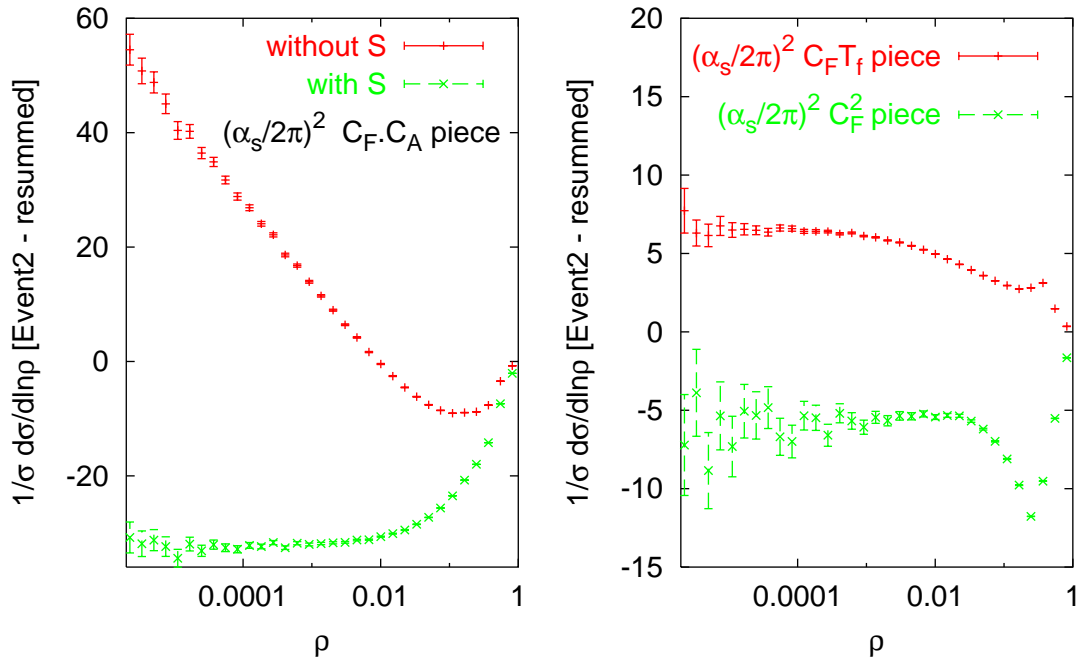


Figure 3: The difference between fixed order results from Event2 and the expansion of the resummed results. The left-hand plot shows results for the coefficient of  $\bar{\alpha}_s^2 C_F C_A$  — the two sets of points correspond to neglecting or accounting for  $\mathcal{S}$ . The right-hand plot shows the coefficients of the  $C_F T_f$  and  $C_F^2$  pieces.

broadening with respect to the thrust axis. The detailed phenomenology of the DIS variables will be considered elsewhere [12]. It indicates that the inclusion of  $\mathcal{S}$  reduces the peak height (in  $d\sigma/d\rho$ ) by about 30% before matching to the  $\mathcal{O}(\alpha_s^2)$  calculation, and by about 20% after matching (using an  $R$ -type matching [1]).

For the narrow-jet  $K_{out}$  and jet masses in multi-jet events (including hadron-hadron event shapes) different forms will apply because of the more complicated geometry of the underlying hard event, but the Monte Carlo algorithm described here can be adapted to those cases as well.

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# Appendix

Below we give details of the Monte Carlo algorithm used to calculate  $\mathcal{S}$ .<sup>6</sup>

For each dipole  $\mathcal{D}_i$ , the collinear angular cutoff  $\epsilon$  introduced in section 3 defines a rapidity interval  $\Delta\eta_i$  for emission in the dipole c.o.m. frame. We shall denote by  $\mathcal{D}_1$  the dipole which has as one of its sources the quark in  $\mathcal{H}_R$ . The evolution variable used will be  $t$ , defined in (19). This will account for the running of the coupling.

One starts an event with a quark-antiquark dipole (labelled  $ab$ ),  $t = 0$  and a weight  $w = 1$ . One then goes through the following steps to determine an event's contribution to the quantity  $-d\mathcal{S}/dt$ :

1. Establish  $\Delta\eta_{\text{tot}} = \sum_i \Delta\eta_i$ . Increase  $t$  by an amount  $\Delta t$  chosen according to a distribution proportional to

$$\exp(-2C_A\Delta\eta_{\text{tot}}\Delta t) . \quad (23)$$

2. Choose a dipole at random, such that the probability of obtaining  $\mathcal{D}_i$  is  $\Delta\eta_i/\Delta\eta_{\text{tot}}$ . Create a gluon uniformly in rapidity and azimuth in the dipole centre of mass frame, such that in the lab frame its angle relative to either of the dipole sources is larger than  $\epsilon$ .

3. There are now various possibilities:

- The gluon is in  $\mathcal{H}_L$ : insert it into dipole  $i$  so as to create two new dipoles (removing the old one from the list). Go to step 1.
- The gluon is in  $\mathcal{H}_R$  and was not emitted from  $\mathcal{D}_1$ : add  $w$  to the bin corresponding to the value of  $t$  and start a new event.
- The gluon is in  $\mathcal{H}_R$  and was emitted from  $\mathcal{D}_1$ : denote the momenta of its sources by  $p_\ell$  and  $p_a$  (the latter is just the  $\mathcal{H}_R$  quark momentum) and the new gluon momentum by  $p_g$ . One determines the value of

$$X = 1 - \frac{F_{ab}}{F_{al}} = 1 - \frac{(ab)(g\ell)}{(gb)(al)} .$$

If  $X > 0$  then with probability  $X$  we add  $w$  to current  $t$ -bin and start a new event; with probability  $1 - X$  we throw away the gluon that has just been generated and go to step 1. If  $X < 0$  then we add  $Xw$  to the current  $t$  bin, multiply  $w$  by  $1 - X$ , and go to step 1. This procedure is a simple extension to negative probabilities of the veto algorithm and corresponds to replacing  $F_C$ , used implicitly in the previous steps, with  $\tilde{F}_C$ .

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<sup>6</sup>We present a version of the algorithm which has weighted events; there exists also a slightly more efficient version of the algorithm with unweighted events, but it requires some integrations to be done analytically and is therefore less easily generalisable to observables with more complex boundaries in phase space such as jet masses in multi-jet events.

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