

Duality and Confinement in Massive Antisymmetric Tensor Gauge Theories

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Abstract

We extend the duality between massive and topologically massive antisymmetric tensor gauge theories in arbitrary space-time dimensions to include topological defects. We show explicitly that the condensation of these defects leads, in 4 dimensions, to confinement of electric strings in the two dual models. The dual phase, in which magnetic strings are confined is absent. The presence of the confinement phase explicitly found in the 4-dimensional case, is generalized, using duality arguments, to arbitrary space-time dimensions.

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1. Introduction

Antisymmetric tensor gauge theories have attracted much interest in recent years [1], [2], [3], [4] [5], since they arise in constructing gauge theories of elementary extended objects (strings, membranes,...): an antisymmetric tensor of rank $(p + 1)$ couples to elementary p -branes, a natural generalization of the coupling of the vector potential one-form in Maxwell theory to elementary point-particles (0-branes). Antisymmetric tensors also appear naturally in effective field theories for the low-energy dynamics of strings and in supersymmetric theories [6], where they play an important role in the realization of various dualities among string theories [7]. General dualities between different phases of antisymmetric tensor field theories were first established in [5]. However, topological terms were not considered there.

The study of dualities is becoming more and more important due to recent developments in string theory, where it was shown that inequivalent vacua are related by dualities based on the existence of extended objects, the D-branes [7]. Establishing a duality means that one has two equivalent descriptions of the same theory in terms of different fields. This is normally very useful because, typically, duality exchanges the coupling constant $e \rightarrow \frac{1}{e}$: strong and weak coupling are interchanged, opening up the possibility of doing perturbative calculations in all regimes of the coupling constant.

In this paper we will concentrate on Abelian gauge invariant formulations for Massive Gauge Theories (MGTs) and Topologically Massive Gauge Theories (TMGTs) for antisymmetric tensor fields.

As an alternative to the Higgs mechanism for gauge invariant masses we will consider a Stückelberg-like formulation [8], and a topological coupling, the BF term, that is a generalization to arbitrary space-time dimensions of the Chern-Simons term. These two models have been shown to be dual in [9], in case of non-compact gauge symmetry group.

The MGTs we will consider are described by the action [5] [10]

$$\begin{aligned}
 S = \int & \frac{(-1)^{p+1}}{g^2} dB_{p+1} \wedge *dB_{p+1} + \frac{(-1)^p e^2}{4} \left(\tilde{m}B_{p+1} + \frac{1}{e}dA_p \right) \wedge * \left(\tilde{m}B_{p+1} + \frac{1}{e}dA_p \right) \\
 & + j \left(\tilde{m}B_{p+1} + \frac{1}{e}dA_p \right) \wedge *J_{p+1} ,
 \end{aligned}
 \tag{1.1}$$

where A_p is an antisymmetric tensor of rank p , e is a dimensionless coupling constant, \tilde{m} is a mass parameter and J_{p+1} is a current of p -branes

$$J^{\mu_1 \dots \mu_{p+1}}(x) = \int \delta^{d+1}(x - y(\sigma)) dy^{\mu_1} \wedge \dots \wedge dy^{\mu_{p+1}} . \quad (1.2)$$

Here $y(\sigma)$ are the coordinates of the world-volume of the p -branes. S is invariant under the combined gauge transformation:

$$\begin{aligned} B_{p+1} &\rightarrow B_{p+1} + d\Lambda_p , \\ A_p &\rightarrow A_p - e\tilde{m}\Lambda_p . \end{aligned} \quad (1.3)$$

Since the term $(B_{p+1} + \frac{1}{\tilde{m}e}dA_p)$ is itself gauge-invariant, the current J_{p+1} does not need to be conserved, and we can couple the theory to open p -branes. Up to a gauge transformation, we can rewrite (1.1) as a generalized version of the Proca Lagrangian for an antisymmetric tensor field $\bar{B}_{p+1} = \tilde{m}B_{p+1} + \frac{1}{e}dA_p$:

$$\begin{aligned} S = \int &\frac{(-1)^{p+1}}{g^2} d\bar{B}_{p+1} \wedge *d\bar{B}_{p+1} + (-1)^p \tilde{m}^2 \bar{B}_{p+1} \wedge *\bar{B}_{p+1} \\ &+ j\bar{B}_{p+1} \wedge *J_{p+1} . \end{aligned} \quad (1.4)$$

In this formulation the higher-rank tensor B_{p+1} has “eaten” the tensor A_p , which is a generalization of the familiar Higgs-Stückelberg mechanism for vector fields.

The model we will consider for TMGTs is a modified form, generalized to arbitrary dimensions, of the mechanism proposed in [11] in the contest of 3-dimensional QED, where the photon acquires a mass due to the presence of a topological term, the Chern-Simons action. In our case the topological term will be a BF term of the form $B_{p+1} \wedge d\tilde{A}_p$, with an action given by:

$$\begin{aligned} \tilde{S} = \int &\frac{(-1)^{p+1}}{g^2} dB_{p+1} \wedge *dB_{p+1} + (-1)^p \tilde{m} \left(B_{p+1} \wedge d\tilde{A}_{d-p-2} \right) \\ &\frac{(-1)^p}{e^2} \left(d\tilde{A}_{d-p-2} \wedge *d\tilde{A}_{d-p-2} \right) + jB_{p+1} \wedge *J_{p+1} + \phi\tilde{A}_p \wedge *\phi_p , \end{aligned} \quad (1.5)$$

where J_{p+1} is a current of closed p -branes and ϕ_p is a current of closed $(p-1)$ -branes. In this way it is possible to obtain a gauge invariant massive gauge theory for the B_{p+1} and the \tilde{A}_{d-p-2} form without a Higgs field, as it has been shown in [12]. The action (1.5) is gauge invariant under the two independent gauge transformations:

$$\begin{aligned} B_{p+1} &\rightarrow B_{p+1} + d\Lambda_p , \\ \tilde{A}_{d-p-2} &\rightarrow \tilde{A}_{d-p-2} + d\tilde{\Lambda}_{d-p-3} . \end{aligned} \quad (1.6)$$

In [9] it has been shown that these two models can be obtained starting from the master action:

$$S_M = \int \frac{(-1)^{p-1}}{g^2} dB_{p+1} \wedge^* dB_{p+1} + H_{d-p-1} \wedge \left(\frac{1}{e} dA_p + \tilde{m} B_{p+1} \right) + \frac{(-1)^{d-p-2}}{e^2} H_{d-p-1} \wedge^* H_{d-p-1} . \quad (1.7)$$

Integrating over H_{d-p-1} leads to (1.1), while integrating over the Lagrange multiplier A_p gives (1.5).

If the antisymmetric tensors are compact variables, extended objects can also appear as topological defects in the underlying gauge theory: they are $(d - p - 2)$ -dimensional extended objects representing the world-hypervolumes of $(d - p - 3)$ -branes (instantons are (-1) -branes). The presence of topological defects can lead to modifications of the infrared perturbative behaviour: their condensation (or lack of it) can drastically change the phase structure of the theories [13].

It has been shown in [5] that the condensation of generic p -branes interpolates between massless and massive antisymmetric tensor field theories of different rank. The appearance of antisymmetric tensors of higher rank in the phase in which the p -branes condense is related (in the case of rank two) to Polyakov's confining string mechanism[14].

In this paper we will not address the problem of establishing if topological defects indeed condense and for which regimes of the coupling constants, but we will concentrate, instead, in studying the nature of the phases in which a finite condensate of topological defects exists. In what follows we will extend the duality established in [9] to include topological defects and we will analyze the effects of the condensation of these topological defects in the two dual theories.

2. Duality with Topological Defects

To study the duality between massive and topologically massive antisymmetric tensor gauge theories in case of a compact gauge symmetry, we will follow the approach of [5], and treat the topological defects explicitly. They will be represented by singular forms t_p such that

$$*t_p = V_{d-p} , \quad V_{d-p}^{\mu_1 \dots \mu_{d-p}} = \int \delta^d(x - \tilde{y}(\tilde{\sigma})) d\tilde{y}^{\mu_1} \wedge \dots \wedge d\tilde{y}^{\mu_{d-p}} , \quad (2.1)$$

with $\tilde{y}^\mu(\tilde{\sigma}^{\nu_1}, \dots, \sigma^{\nu_{d-p}})$ an open hypervolume describing the generalization to higher-dimensional topological defects of the Dirac string. The boundary of this hypervolume describes the world-hypersurface of the topological defects.

In order to make both the combined gauge symmetry (1.3) of the MGTs and the two independent gauge symmetries (1.6) of the TMGTs compact we need to introduce three different types of topological defects: t , \bar{t} and \tilde{t} .

The master action we start from is:

$$\begin{aligned}
S_M = & \int \frac{(-1)^{p-1}}{g^2} (dB_{p+1} + \bar{t}t_{p+2}) \wedge^* (dB_{p+1} + \bar{t}t_{p+2}) \\
& + H_{d-p-1} \wedge \left(\frac{1}{e} dA_p + \tilde{m}B_{p+1} + tt_{p+1} \right) \\
& + \frac{(-1)^{d-p-2}}{e^2} (H_{d-p-1} + \tilde{t}\tilde{t}_{d-p-1}) \wedge^* (H_{d-p-1} + \tilde{t}\tilde{t}_{d-p-1}) .
\end{aligned} \tag{2.2}$$

As before we will have two dual forms, A_p and \tilde{A}_{d-p-2} , with their respective dual topological defects, t_{p+1} and \tilde{t}_{d-p-1} . The B_{p+1} form and its topological defects \bar{t}_p are not dualized.

Integrating over the form H_{d-p-1} we obtain the compact version of the massive gauge theory:

$$\begin{aligned}
S = & \int \frac{(-1)^{p-1}}{g^2} (dB_{p+1} + \bar{t}t_{p+2}) \wedge^* (dB_{p+1} + \bar{t}t_{p+2}) \\
& + \frac{(-1)^p e^2}{4} \left(\frac{1}{e} dA_p + \tilde{m}B_{p+1} + tt_{p+1} \right) \wedge^* \left(\frac{1}{e} dA_p + \tilde{m}B_{p+1} + tt_{p+1} \right) \\
& + \tilde{t}\tilde{t}_{d-p-1} \wedge \left(\frac{1}{e} dA_p + \tilde{m}B_{p+1} \right) + \tilde{t}\tilde{t}_{d-p-1} \wedge t_{p+1} .
\end{aligned} \tag{2.3}$$

Here t_{p+1} enters as a singular form, due to the compactness of the gauge group, while the \tilde{t}_{d-p-1} form appears as a (non-conserved) current minimally coupled to $(\frac{1}{e}dA_p + \tilde{m}B_{p+1})$. The last term is a generalized Aharonov-Bohm interaction between the two topological defects that leads to a generalized Dirac quantization condition and does not contribute to the partition function [5].

Integration over the Lagrange multiplier A_p implies that $H_{d-p-1} = d\tilde{A}_{d-p-2}$, with an action given by:

$$\begin{aligned}
\tilde{S} = & \int \frac{(-1)^{p-1}}{g^2} (dB_{p+1} + \bar{t}t_{p+2}) \wedge^* (dB_{p+1} + \bar{t}t_{p+2}) \\
& + \frac{(-1)^{d-p-2}}{e^2} \left(d\tilde{A}_{d-p-2} + \tilde{t}\tilde{t}_{d-p-1} \right) \wedge^* \left(d\tilde{A}_{d-p-2} + \tilde{t}\tilde{t}_{d-p-1} \right) \\
& + \tilde{m}B_{p+1} \wedge d\tilde{A}_{d-p-2} + tt_{p+1} \wedge d\tilde{A}_{d-p-2} .
\end{aligned} \tag{2.4}$$

Here, the dual form \tilde{t}_{d-p-1} is the one connected to the compactness of the gauge group, while t_{p+1} appears as a conserved current ($J_{d-p-2} = (-1)^{d-1+(p+1)^2} (*dt_{p+1})$) minimally coupled to \tilde{A}_{d-p-2} . As we said, the B_{p+1} form does not participate in the duality, and so the singular form \bar{t}_{p+2} appears in the same way in both models: as a singular form due to the compactness of the gauge symmetries (1.3) and (1.6). We have so established a duality between MGTs and TMGTs in case of a compact gauge group. The two dual actions are given by the equations (2.3) and (2.4).

By integrating over the tensor field A_p in (2.3) and over \tilde{A}_{d-p-2} in (2.4), we obtain an effective action for the higher rank-tensor B_{p+1} . In the case of the MGTs described by the action (2.3), it was found in [10] that, when the topological defects t_{p+1} are dilute, this effective action still possesses a massive pole. In the case in which, instead, these topological defects are in a dense phase, the effective action we get is:

$$S_{\text{eff}} = \int \frac{(-1)^{p+1}}{g^2} (dB_{p+1} + \bar{t}_{p+2}) \wedge^* (dB_{p+1} + \bar{t}_{p+2}) . \tag{2.5}$$

The mass term for B_{p+1} is no longer present: the condensation of the topological defects t_{p+1} prevents the tensor B_{p+1} to become massive through the Higgs-Stückelberg mechanism. The same happens in the dual model (2.4) when we consider the effective action for the higher rank-tensor B_{p+1} . Again when the topological defects t_{p+1} are dilute, this effective action still possesses a massive pole, while in the case in which these topological defects are in a dense phase, the effective action is given by (2.5), preventing the tensor B_{p+1} to become massive through the topological mechanism.

A crucial role in the determination of the phase diagram is played by the space-time dimension and by the dimension of the topological defects. In what follows we will give an example of the effect of the condensation of the topological defects in the 4-dimensional case. We will explicitly show, with a separate analysis of the phase structure of the two models, that they, indeed, admit the same phases. The generalization to arbitrary dimensions will be briefly discussed at the end. A more detailed discussion of the possible phases in arbitrary space-time dimensions, and the study of the conditions for the condensation of topological defects is left for a forthcoming publication [15].

3. Condensation of Topological Defects in the 4-Dimensional Case

We will now concentrate on the phase structure of the two dual theories in 4-dimensional Euclidean space-time. We will start with the case $p = 1$, and show at the end that the result for $p = 1$ can be generalized to arbitrary rank and arbitrary space-time dimensions. For $p = 1$ the higher order antisymmetric tensor is a two form representing the Kalb-Ramond tensor $B_{\mu\nu}$, while the two lower order tensors are the Maxwell field A_μ and its dual \tilde{A}_μ . In order to study the phase structure of the two models we need to introduce an external probe: since the topologically massive gauge theory (1.5) can couple only to closed p -branes we will couple the theory to a conserved two form $J_{\mu\nu}$, that can be interpreted as the worldsheet of a closed string. The coupling with it plays the role of the (non-local) order parameter, the Wilson “surface” W_S , for the phase transitions in the theory. This is the generalization of the Wilson loop to objects of one dimension higher. Notice that, while the standard confinement of point particles is described by an area law for the surface enclosed by the worldline of the particles, the corresponding phenomenon for strings is given by a volume law for the volume enclosed by the worldsheet of the strings.

The two dual actions describing the massive and the topologically massive gauge theories are:

$$S = \int d^4x \frac{1}{2g^2} (F_\mu + \bar{t}t_\mu)^2 + \frac{e^2}{4} \left(\frac{1}{e} f_{\mu\nu} + \tilde{m} B_{\mu\nu} + tt_{\mu\nu} \right)^2 + i\tilde{t} \left(\tilde{m} B_{\mu\nu} + \frac{1}{e} f_{\mu\nu} \right) \tilde{t}_{\mu\nu} - ij B_{\mu\nu} J_{\mu\nu} . \quad (3.1)$$

and

$$\tilde{S} = \int d^4x \frac{1}{2g^2} (F_\mu + \bar{t}t_\mu)^2 + \frac{1}{e^2} (h_{\mu\nu} + \tilde{t}\tilde{t}_{\mu\nu})^2 + i\tilde{m} B_{\mu\nu} \tilde{h}_{\mu\nu} + i\tilde{t}\tilde{h}_{\mu\nu} t_{\mu\nu} - ij B_{\mu\nu} J_{\mu\nu} . \quad (3.2)$$

Here F_μ is the dual of the Kalb-Ramond field strength, $f_{\mu\nu}$ is the Maxwell field strength for A_μ and $h_{\mu\nu}$ the Maxwell field strength for \tilde{A}_μ . The dual of the Maxwell field strength will be denoted by $\tilde{f}_{\mu\nu}$ for A_μ and by $\tilde{h}_{\mu\nu}$ for \tilde{A}_μ . g, e are dimensionless coupling constants. As we said before, in order to have all gauge symmetries (1.3) and (1.6) compact, we need to introduce three different types of topological defects: \bar{t}_μ , $\tilde{t}_{\mu\nu}$ and $t_{\mu\nu}$. \bar{t}_μ is associated with the tensor $B_{\mu\nu}$, $t_{\mu\nu}$ with A_μ and $\tilde{t}_{\mu\nu}$ with \tilde{A}_μ . The last terms in (3.1) and (3.2), represent the coupling to the Wilson “surface” W_S .

To properly define the models we use a lattice regularization. The lattice we consider is a hypercubic lattice with lattice spacing l in four Euclidean dimensions, with sites denoted by x (lattice notation is defined in the Appendix). The two compact massive and topologically massive gauge theories to be considered will be described by partition functions of the Villain type [16]. With lattice regularization the role of the minimally coupled topological defects $\tilde{t}_{\mu\nu}$ in (3.1) and $t_{\mu\nu}$ in (3.2) becomes explicit: the sum over these two integer fields in the Villain formulation, in the phase in which they condense, has the effect of breaking the global gauge symmetries of the two models to a discrete group, $Z_{\tilde{t}}$ for (3.1) and Z_t for (3.2) [17].

For the theory described in (3.1) the expectation value $\langle W_S \rangle$ is given by:

$$\begin{aligned}
\langle W_S \rangle &= \frac{1}{Z_{\text{top}}} \sum_{\substack{\{\tilde{t}_{\mu\nu}, t_{\mu\nu}\} \\ \{\tilde{t}_\mu\}}} \exp(-S_{\text{top}} - W_{\text{top}} - W_0) \\
S_{\text{top}} &= \sum_{\mathbf{x}, \mu} \frac{\pi^2}{2g^2} \left(\bar{t}_\mu - \frac{g}{lm} t_\mu \right) \frac{m^2 \delta_{\mu\nu} - d_\mu \hat{d}_\nu}{m^2 - \nabla^2} \left(\bar{t}_\nu - \frac{g}{lm} t_\nu \right) + \\
&\quad + \frac{\tilde{t}^2}{2e^2} \tilde{t}_{\mu\nu} \frac{O_{\mu\nu\alpha\beta}^M}{m^2 - \nabla^2} \tilde{t}_{\alpha\beta} + \frac{i\pi \tilde{t} m}{ge} \bar{t}_\mu \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} \tilde{t}_{\nu\alpha} + \frac{i\pi \tilde{t}}{el} t_\mu \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} t_{\nu\alpha}, \\
W_0 &= \sum_{\mathbf{x}, \mu} \frac{j^2 g^2}{l^2} J_{\mu\nu} \frac{1}{m^2 - \nabla^2} J_{\mu\nu} = \sum_{\mathbf{x}, \mu} 2g^2 j^2 V_\mu \frac{M_{\mu\nu}}{m^2 - \nabla^2} V_\nu, \\
W_{\text{top}} &= \sum_{\mathbf{x}, \mu} +2i\pi j \bar{t}_\mu \frac{M_{\mu\nu}}{m^2 - \nabla^2} V_\nu + \frac{2i\pi j g}{l} t_\mu \frac{m}{m^2 - \nabla^2} V_\nu + \frac{2t j m g}{e} \tilde{t}_{\mu\nu} \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} V_\alpha.
\end{aligned} \tag{3.3}$$

Here $lK_{\mu\nu\alpha}V_\alpha = J_{\mu\nu}$, with V_μ is the volume enclosed by the surface $J_{\mu\nu}$, $m = \tilde{m}ge$, and we have reabsorbed \bar{t} and \tilde{t} into the integer fields \bar{t}_ν and $\tilde{t}_{\mu\nu}$. $t_\mu = lK_{\mu\nu\alpha}t_{\nu\alpha}$ is the physical integer degree of freedom that describes the topological defects associated with the Maxwell gauge field. It describes closed (or infinitely long) strings of magnetic charge: $\hat{d}_\mu t_\mu = 0$.

For the topologically massive theory (3.2) the expectation value $\langle W_S \rangle$ is, again, given by:

$$\begin{aligned}
\langle W_S \rangle &= \frac{1}{Z_{\text{top}}} \sum_{\substack{\{t_{\mu\nu}, \bar{t}_\mu\} \\ \{\tilde{t}_{\mu\nu}\}}} \exp(-S_{\text{top}} - W_{\text{top}} - W_0) \\
S_{\text{top}} &= \sum_{\mathbf{x}, \mu} \frac{2\pi^2}{e^2} \tilde{t}_{\mu\nu} \frac{O_{\mu\nu\lambda\omega}^M}{m^2 - \nabla^2} \tilde{t}_{\lambda\omega} + \frac{\pi^2}{2g^2} \bar{t}_\mu \frac{m^2 \delta_{\mu\nu} - d_\mu \hat{d}_\nu}{m^2 - \nabla^2} \bar{t}_\nu + \frac{e^2 \tilde{t}^2}{8l^2} t_\mu \frac{1}{m^2 - \nabla^2} t_\mu \\
&\quad + \frac{2i\pi^2 m}{eg} \bar{t}_\mu \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} \tilde{t}_{\nu\alpha} + \frac{met}{2gl} t_\mu \frac{1}{m^2 - \nabla^2} \bar{t}_\mu + \frac{i\pi t}{l} \tilde{t}_{\mu\nu} \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} t_\alpha, \\
W_0 &= \sum_{\mathbf{x}, \mu} 2g^2 j^2 V_\mu \frac{M_{\mu\nu}}{m^2 - \nabla^2} V_\nu, \\
W_{\text{top}} &= \sum_{\mathbf{x}, \mu} +2i\pi j \bar{t}_\mu \frac{M_{\mu\nu}}{m^2 - \nabla^2} V_\nu + \frac{4gjm}{e} \tilde{t}_{\mu\nu} \frac{K_{\mu\nu\alpha}}{m^2 - \nabla^2} V_\alpha + \frac{ijt\pi eg}{2l} t_\mu \frac{m}{m^2 - \nabla^2} V_\mu,
\end{aligned} \tag{3.4}$$

where $J_{\mu\nu} = lK_{\mu\nu\alpha}V_\alpha$, $m = \tilde{m}ge$, and $t_\mu = lK_{\mu\nu\alpha}t_{\nu\alpha}$ with $\hat{d}t_\mu = 0$. Now, since t_μ is minimally coupled to \tilde{A}_μ , the role of these topological defects as a conserved current of magnetic charges is evident: indeed they enter the theory as an external current coupled to the dual of the electromagnetic field A_μ .

The phase structure of (3.3), in absence of the term $i\tilde{t}(\tilde{m}B_{\mu\nu} + \frac{1}{e}f_{\mu\nu})\tilde{t}_{\mu\nu}$, was discussed in [10] (this correspond to the case in which the topological defects $\tilde{t}_{\mu\nu}$ are dilute). It was found that the condensation of t_μ leads to an expectation value of the Wilson surface of the form

$$\begin{aligned}
\langle W_S \rangle &= \frac{1}{Z_{\text{top}}} \sum_{\{\bar{t}_\mu\}} \exp(-S_{\text{top}} - W_{\text{top}} - W_0) \\
S_{\text{top}} &= \sum_{\mathbf{x}, \mu} -\frac{\pi^2}{2g^2} Q \frac{1}{\nabla^2} Q - \frac{t^2}{e^2} \tilde{t}_{\mu\nu} \tilde{t}_{\mu\nu}, \\
W_0 &= \sum_{\mathbf{x}, \mu} -\frac{g^2 j^2}{l^2} J_{\mu\nu} \frac{1}{\nabla^2} J_{\mu\nu}, \\
W_{\text{top}} &= \sum_{\mathbf{x}, \mu} -\frac{i\pi j}{l} \bar{t}_\mu \frac{\hat{K}_{\mu\nu\alpha}}{\nabla^2} J_{\nu\alpha},
\end{aligned} \tag{3.5}$$

where $Q = l\hat{d}_\mu \bar{t}_\mu$. \bar{t}_μ and t_μ are both connected with the compactness of the gauge symmetry (1.3), and, thus, they enter the theory on the same footing, as magnetic strings. Q are the monopoles that live at the end-points of the strings \bar{t}_μ . In this magnetic condensation phase, a lengthy calculation shows that the self energy of a circular charge loop of radius R is proportional to $R \ln R$, and this gives rise to logarithmic confinement

of electric strings (logarithmic confinement phase)[18]. The monopoles Q are always in a plasma phase : their condensation (after the condensation of t_μ), as it was shown in [10], gives a volume law and thus the usual confinement strings.

Let us see what is the effective action induced by the condensation of t_μ in the dual model (3.2). Also in this case the expectation value of the Wilson “surface” is:

$$\begin{aligned}
\langle W_S \rangle &= \frac{1}{Z_{\text{top}}} \sum_{\{\bar{t}_\mu\}} \exp(-S_{\text{top}} - W_{\text{top}} - W_0) \\
S_{\text{top}} &= \sum_{\mathbf{x}, \mu} -\frac{\pi^2}{2g^2} Q \frac{1}{\nabla^2} Q - \frac{4\pi^2}{e^2} \tilde{t}_{\mu\nu} \tilde{t}_{\mu\nu} , \\
W_0 &= \sum_{\mathbf{x}, \mu} -\frac{g^2 j^2}{l^2} J_{\mu\nu} \frac{1}{\nabla^2} J_{\mu\nu} , \\
W_{\text{top}} &= \sum_{\mathbf{x}, \mu} -\frac{i\pi j}{l} \bar{t}_\mu \frac{\hat{K}_{\mu\nu\alpha}}{\nabla^2} J_{\nu\alpha} ,
\end{aligned} \tag{3.6}$$

with $Q = l \hat{d}_\mu \bar{t}_\mu$ (the difference in the coefficients of $\tilde{t}_{\mu\nu}$ is due to the way the topological defects enter the Villain formulation in the two dual models). Also in this phase the self energy of a circular loop of radius R is proportional to $R \ln R$, and this gives rise to logarithmic confinement of electric strings: in this dual theory the objects that condense enter as a minimally coupled current and, therefore, one can look at this electric confinement phase as a “magnetic Higgs phase”. It is interesting to notice that in (3.2) what lead to confinement is the condensation of the topological defects that enter the theory as a minimally coupled current and, as we explained before, are associated on the lattice with the breaking the global gauge symmetry down to Z_t . A similar phase was found in [18], in the context of effective field theories for 3-dimensional Josephson junction arrays, studying a model in which the original gauge group was non-compact and only the BF coupling was periodic. Also in (3.6), as expected, the further condensation of the \bar{t}_μ gives a volume law for the expectation value of the Wilson “surface”.

Let us now analyze the effect of the condensation of $\tilde{t}_{\mu\nu}$. In both models this leads to an effective action for the Wilson surface of the type

$$W_0 = \sum_{\mathbf{x}, \mu} -\alpha J_{\mu\nu} J_{\mu\nu} , \tag{3.7}$$

with $\alpha = j^2 g^2$. This term measures the area of the Wilson “surface”. This Wilson surface can be seen as the world-sheet of a closed string and (3.7) is just the standard Nambu-Goto

action for a string with string tension $\frac{1}{\alpha}$. (3.7) correspond to an area law for surfaces and it does thus not describe any type of confinement for strings.

From this we learn that both in MGT and TMGT only the condensation of one type of topological defects, t_μ , leads to a confinement phase, while the condensation of the dual topological defects $\tilde{t}_{\mu\nu}$ has only the effect of promoting the effective action for the probe surface to a Nambu-Goto type of action. The dual phase, in which magnetic charges are confined, is absent.

In [10] it was proved that the confinement phase we found in the 4-dimensional case and for rank $p = 1$ for the MGTs described by the action (2.3), is present also in arbitrary space-time dimensions and for arbitrary p ($p \leq d - 1$). Simply invoquing the duality we know that the phase is present also in the TMGTs, described by the action (2.4), in arbitrary space-time dimensions and for arbitrary rank antisymmetric tensor fields. The role of the condensation of \tilde{t}_p will be discussed in [15].

4. Conclusions

In this paper we have shown that the duality between MGTs and TMGTs [9] can be extended also to include topological defects. We have also explicitly shown that the condensation of these defects leads, in 4-dimensions, to confinement for electric 1-branes in the two dual models. This phase can be seen as a confinement phase in one model and as a Higgs phase in the dual model in which the topological defects that condense enter as a minimally coupled current. The dual phase in which magnetic branes are confined is absent. The presence of the confinement phase explicitly found in the 4-dimensional case, can be easily generalized, using the duality between (2.3) and (2.4), to arbitrary space-time dimensions.

5. Appendix

On the lattice, we define the following forward and backward derivatives and shift operators:

$$\begin{aligned}
 d_\mu f(\mathbf{x}) &\equiv \frac{f(\mathbf{x} + \hat{\mu}l) - f(\mathbf{x})}{l}, & S_\mu f(\mathbf{x}) &\equiv f(\mathbf{x} + \hat{\mu}l), \\
 \hat{d}_\mu f(\mathbf{x}) &\equiv \frac{f(\mathbf{x}) - f(\mathbf{x} - \hat{\mu}l)}{l}, & \hat{S}_\mu f(\mathbf{x}) &\equiv f(\mathbf{x} - \hat{\mu}l).
 \end{aligned}
 \tag{5.1}$$

Summation by parts interchanges the two derivatives, with a minus sign, and the two shift operators. We also introduce the three-index lattice operators [18]:

$$K_{\mu\nu\alpha} = S_\mu S_\nu \epsilon_{\mu\phi\nu\alpha} d_\phi, \quad \hat{K}_{\mu\nu\alpha} = \epsilon_{\mu\nu\phi\alpha} \hat{d}_\phi \hat{S}_\alpha \hat{S}_\nu. \quad (5.2)$$

These operators are gauge-invariant in the sense that:

$$\begin{aligned} K_{\mu\nu\alpha} d_\alpha &= K_{\mu\nu\alpha} d_\nu = \hat{d}_\mu K_{\mu\nu\alpha} = 0, \\ \hat{K}_{\mu\nu\alpha} d_\alpha &= \hat{d}_\mu \hat{K}_{\mu\nu\alpha} = \hat{d}_\nu \hat{K}_{\mu\nu\alpha} = 0. \end{aligned} \quad (5.3)$$

Moreover they satisfy the equations:

$$\begin{aligned} \hat{K}_{\mu\nu\alpha} K_{\alpha\lambda\omega} &= K_{\mu\nu\alpha} \hat{K}_{\alpha\lambda\omega} = O_{\mu\nu\lambda\omega} = \\ &= -(\delta_{\mu\lambda} \delta_{\nu\omega} - \delta_{\mu\omega} \delta_{\nu\lambda}) \nabla^2 + (\delta_{\mu\lambda} d_\nu \hat{d}_\omega - \delta_{\nu\lambda} d_\mu \hat{d}_\omega) - (\delta_{\nu\omega} d_\mu \hat{d}_\lambda - \delta_{\mu\omega} d_\nu \hat{d}_\lambda), \\ \hat{K}_{\mu\omega\alpha} K_{\omega\alpha\nu} &= K_{\mu\omega\alpha} \hat{K}_{\omega\alpha\nu} = 2M_{\mu\nu} = -2(\delta_{\mu\nu} \nabla^2 - d_\mu \hat{d}_\nu). \end{aligned} \quad (5.4)$$

The expressions $O_{\mu\nu\lambda\omega}$ and $M_{\mu\nu}$ are lattice versions of the Kalb-Ramond and Maxwell kernels, respectively, and $\nabla^2 = d_\mu \hat{d}_\mu = \hat{d}_\mu d_\mu$ is the lattice Laplacian. We also define the operator

$$\begin{aligned} O_{\mu\nu\lambda\omega}^M &= +(\delta_{\mu\lambda} \delta_{\nu\omega} - \delta_{\mu\omega} \delta_{\nu\lambda}) m^2 - (\delta_{\mu\lambda} d_\nu \hat{d}_\omega - \delta_{\nu\lambda} d_\mu \hat{d}_\omega) \\ &+ (\delta_{\nu\omega} d_\mu \hat{d}_\lambda - \delta_{\mu\omega} d_\nu \hat{d}_\lambda). \end{aligned} \quad (5.5)$$

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