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Majorana Neutrinos and Same-Sign Dilepton Production at LHC and in Rare Meson Decays

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Abstract

We discuss same-sign dilepton production mediated by Majorana neutrinos in high-energy proton-proton collisions $pp \to \ell^+\ell'^+X$ for $\ell,\ \ell' = e,\ \mu,\ \tau$ at the LHC energy $\sqrt{s} = 14$ TeV, and in the rare decays of K, D, D_s, and B mesons of the type $M^+ \to M'^-\ell^+\ell'^+$. For the pp reaction, assuming one heavy Majorana neutrino of mass m_N, we present discovery limits in the $(m_N, |U_{\ell N}U_{\ell' N}|)$ plane where $U_{\ell N}$ are the mixing parameters. Taking into account the present limits from low energy experiments, we show that at LHC for the nominal luminosity $L = 100$ fb^{-1} there is no room for observable same-sign dilepton signals. However, increasing the integrated luminosity by a factor 30, one will have sensitivity to heavy Majorana neutrinos up to a mass $m_N \lesssim 1.5$ TeV only in the dilepton channels $\mu\mu$ and $\mu\tau$, but other dilepton states will not be detectable due to the already existing strong constraints. We work out a large number of rare meson decays, both for the light and heavy Majorana neutrino scenarios, and argue that the present experimental bounds on the branching ratios are too weak to set reasonable limits on the effective Majorana masses.

1 Introduction

Recent results from the KEK to Kamioka long baseline neutrino experiment (K2K) [1] strengthen the neutrino oscillation interpretation of the atmospheric neutrino anomaly observed earlier by the Superkamiokande detector [2]. These, as well as the solar neutrino deficit measurements reported in a number of experiments [3, 4, 5, 6] yield valuable information on the neutrino mass differences and mixing angles [7]. The simplest scheme which accounts for these results is that in which there are just three light neutrino mass eigenstates with a mass hierarchy analogous to the quarks and charged leptons, and the observed phenomena of neutrino oscillations can be accommodated by a mixing matrix in the lepton sector [8] — analogous to the well-studied quark rotation matrix [9]. If, however, the LSND result [10] is confirmed, it would imply a fourth, sterile, neutrino ν_s , separated in mass from the other neutrinos by typically $0.4 \div 1$ eV. In that case, there might be even more such (sterile) neutrinos.

While impressive, and providing so far the only evidence of new physics, the solar and atmospheric neutrino experiments do not probe the nature of the neutrino masses, i.e., they can not distinguish between the Dirac and Majorana character of the neutrinos. The nature of neutrino mass is one of the main unsolved problems in particle physics and there are practically no experimental clues on this issue. If neutrino are Dirac particles, then their masses can be generated just like the quark and charged lepton masses through weak SU(2)-breaking via the Yukawa couplings, $m_D = h_\nu v/\sqrt{2}$, where h_ν is the Yukawa coupling and $v = \sqrt{2}\langle\phi^0\rangle = 246$ GeV is the usual Higgs vacuum expectation value. In that case, to get ≤ 1 eV neutrino masses, one has $h_\nu \leq 10^{-11}$, which raises the question about the extreme smallness of the neutrino Yukawa coupling.

If neutrinos are Majorana particles then their mass term violates lepton number by two units $\Delta L = \pm 2$ [11]. Being a transition between a neutrino and an antineutrino, it can be viewed equivalently as the annihilation or creation of two neutrinos. In terms of Feynman diagrams, this involves the emission (and absorption) of two like-sign W-boson pairs (W^-W^- or W^+W^+). If present, it can lead to a large number of processes violating lepton number by two units, of which neutrinoless double beta decay ($\beta\beta_{0\nu}$) is a particular example. The seesaw models [12] provide a natural framework for generating a small Majorana neutrino mass which is induced by mixing between an active (light) neutrino and a very heavy Majorana sterile neutrino of mass M_N. The light state has a naturally small mass $m_\nu \sim m_D^2/M_N \ll m_D$, where m_D is a quark or charged lepton mass. There is a heavy Majorana state corresponding to each light (active) neutrino state. Typical scale for M_N in Grand unified theories (GUT) is of order the GUT-scale, though in general, there exists a large number of seesaw models in which both m_D and M_N vary over many orders of magnitude, with the latter ranging somewhere between the TeV scale and the GUT-scale [13].

If M_N is of order GUT-scale, then it is obvious that there are essentially no low energy effects induced by such a heavy Majorana neutrino state. However, if M_N is allowed to be much lower, or if the light (active) neutrinos are Majorana particles, then the induced effects of such Majorana neutrinos can be searched for in a number of rare processes. Among them neutrinoless double beta decay, like-sign dilepton states produced in rare meson decays and in hadron-hadron, lepton-hadron, and lepton-lepton collisions, and $e \to \mu$-conversions have been extensively studied. (See, e.g., the papers: $\beta\beta_{0\nu}$ [14, 15, 16], $K^+ \to \pi^-\mu^+\mu^+$ [17, 18, 19, 20, 21, 22], $pp \to \ell^\pm\ell^\pm X$ [23], $pp \to \ell^\pm\ell^\pm W^\mp X$ [24], $e^\pm p \to \overset{(-)}{\nu_e} \ell^\pm\ell'^\pm X$ [25, 26], the nuclear $\mu^- \to e^+$ [27, 28] and $\mu^- \to \mu^+$ [29] conversion.)

Of the current experiments which are sensitive to the Majorana nature of neutrino, the neutrinoless double beta decay, which yields an upper limit on the ee element of the Majorana mass matrix, is already quite stringent. The present best limit posted by the Heidelberg-Moscow experiment [14] is: $\langle m_{ee}\rangle = |\sum_i \eta_i U_{ei}^2 m_i| < 0.26\,(0.34)$ eV at 68% (95%) C.L., where η_i is the charge conjugation phase factor of the Majorana neutrino mass eigenstate $\nu_i = \eta_i \nu_i^c$. Despite some dependence of the actual limit on the nuclear matrix elements, this limit severely compromises the sensitivity of future e^-e^- colliders [31, 32], as well as of searches in the e^-e^- final states, such as $pp \to e^-e^-X$, induced by a Majorana neutrino, discussed here. Hence, it is exceedingly important to push the $\beta\beta_{0\nu}$-frontier; currently there are several proposals being discussed in literature, which will increase the sensitivity to $\langle m_{ee}\rangle$ by one to two orders of magnitude [15, 16, 30], with the GENIUS proposal reaching $\langle m_{ee}\rangle \sim 10^{-3}$ eV [16]. Likewise, precision electroweak physics experiments severely constrain the mixing elements, namely $\sum_N |U_{\ell N}|^2$, with $\ell = e,\ \mu,\ \tau$ [33, 34, 35].

Taking into account these constraints, we investigate in this paper the sensitivity to the Majorana neutrino induced effects involving same-sign dilepton production. We work out the following two processes in detail:

(i) Dilepton production in the high-energy proton-proton collision

$$pp \to \ell^+\ell'^+X \tag{1}$$

with $\ell,\ \ell' = e,\ \mu,\ \tau$ at LHC; and

(ii) in rare meson decays of the type

$$M^+ \to M'^-\ell^+\ell'^+ \tag{2}$$

for $M = K,\ D,\ D_s,\ B$.

We obtain discovery limits for heavy Majorana neutrinos involved in the process (1) at the LHC energy $\sqrt{s} = 14$ TeV. Using the present limits on the branching ratios of rare decays (2) we set the upper bounds on effective Majorana masses. From the existing bounds on the elements of the effective Majorana mass matrix, the indirect constraints on the branching ratios in question are deduced.

2 Dilepton production in high-energy pp collisions

We have calculated the cross section for the process (1) at high energies,

$$\sqrt{s} \gg m_W, \tag{3}$$

via an intermediate heavy Majorana neutrino N in the leading effective vector-boson approximation [36] neglecting transverse polarizations of W bosons and quark mixing. We use the simple scenario for neutrino mass spectrum

$$m_{N_1} \equiv m_N \ll m_{N_2} < m_{N_3}, ...,$$

and single out the contribution of the lightest Majorana neutrino assuming

$$\sqrt{s} \ll m_{N_2}.$$

The cross section for the process in question is then parameterized by the mass m_N and the corresponding neutrino mixing parameters $U_{\ell N}$ and $U_{\ell' N}$:

$$\sigma\left(pp \to \ell^+\ell'^+X\right) = C\left(1 - \frac{1}{2}\delta_{\ell\ell'}\right)\left|U_{\ell N}U_{\ell' N}\right|^2 F\left(E, m_N\right) \ , \tag{4}$$

with

$$C = \frac{G_F^4 m_W^6}{8\pi^5} = 0.80 \text{ fb} \ ,$$

and

$$F\left(E, m_N\right) = \left(\frac{m_N}{m_W}\right)^2 \int_{z_0}^1 \frac{dz}{z} \int_z^1 \frac{dy}{y} \int_y^1 \frac{dx}{x} p\left(x, xs\right) p\left(\frac{y}{x}, \frac{y}{x}s\right) h\left(\frac{z}{y}\right) w\left(\frac{s}{m_N^2}z\right) \ . \tag{5}$$

Here, $z_0 = 4m_W^2/s$, $E = \sqrt{s}$, and

$$w\left(t\right) = 2 + \frac{1}{t+1} - \frac{2\left(2t+3\right)}{t\left(t+2\right)}\ln\left(t+1\right)$$

is the normalized cross section for the subprocess $W^+W^+ \to \ell^+\ell'^+$ (in the limit (3) it is obtained from the well-known cross section for $e^-e^- \to W^-W^-$ [31] using crossing symmetry). The function $h(r)$ defined as

$$h\left(r\right) = -\left(1+r\right)\ln r - 2\left(1-r\right)$$

is the normalized luminosity (multiplied by r) of W^+W^+ pairs in the two-quark system [36], and

$$p\left(x, Q^2\right) = x\sum_i q_i\left(x, Q^2\right) = x\left(u + c + t + \bar{d} + \bar{s} + \bar{b}\right)$$

is the corresponding quark distribution in the proton.

In the numerical calculation of the cross section (4) the CTEQ6 Fortran codes for the parton distributions [37] have been used. The reduced cross section (5) as a function of the neutrino mass m_N is shown in Fig. 1 for the LHC energy $\sqrt{s} = 14$ TeV.

We assume the mixing constraints obtained from the precision electroweak data [34]

$$\sum\left|U_{eN}\right|^2 < 6.6\times10^{-3}, \quad \sum\left|U_{\mu N}\right|^2 < 6.0\times10^{-3}\left(1.8\times10^{-3}\right),$$
$$\sum\left|U_{\tau N}\right|^2 < 1.8\times10^{-2}\left(9.6\times10^{-3}\right). \tag{6}$$

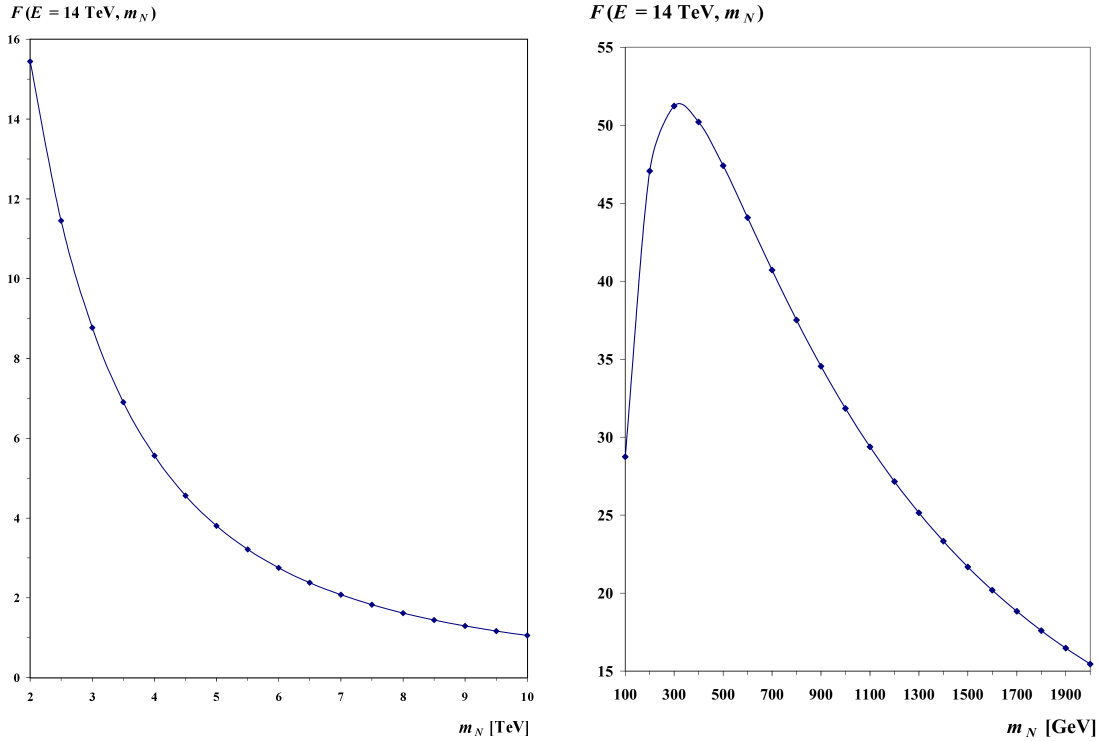


Figure 1: *Left*: The reduced cross section $F(E, m_N)$ defined in the text for dilepton production as a function of the heavy Majorana mass m_N at LHC with $E = 14$ TeV. *Right*: The same as the left figure but for lighter Majorana neutrinos.

The bound on the mixing matrix elements involving fermions depends on the underlying theoretical scenario. A mixture of known fermions with new heavy states (here, a Majorana neutrino) can in general induce both flavour changing (FC) and non-universal flavour diagonal (FD) vertices among the light states. The FC couplings are severely constrained for most of the charged fermions by the limits on rare processes [38]. In Refs. [34, 35], the FD vertices are constrained by the electroweak precision data, which we shall use here. There are two limits obtained on the FD vertices, called in [34, 35] the single limit and joint limit, obtained by allowing just one fermion mixing to be present or allowing simultaneous presence of all types of fermion mixings, respectively. The resulting constraints are more stringent in the single limit case (see numbers in parentheses on the r.h.s. in Eq. (6)) than in the joint limit case, where the constraints are generally relaxed due to possible accidental cancellations among different mixings. In our analysis, we shall use the conservative constraints for the joint limit case.

We must also include the constraint from the double beta decay $\beta\beta_{0\nu}$, mentioned above. For heavy neutrinos, $m_N \gg 1$ GeV, the bound is [31]

$$\left| \sum_{N(heavy)} U_{eN}^2 \frac{\eta_N}{m_N} \right| < 5 \times 10^{-5} \text{ TeV}^{-1}. \tag{7}$$

In calculating the cross sections for the $\ell\tau$ and $\tau\tau$ processes, we have used the effective value

$$|U_{\tau N}|^2_{eff} = \mathrm{B}_{\tau\mu}\,|U_{\tau N}|^2 < 3.1 \times 10^{-3} \tag{8}$$

with $\mathrm{B}_{\tau\mu} = \mathrm{Br}\,(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau) = 0.1737$ [38], as this τ-decay mode is most suitable for the like-sign dilepton detection at LHC (see, e.g., [26]).

Combining the constraints of Eqs. (6), (8), and (7) and taking a nominal luminosity $L = 100$ fb^{-1}, we obtain $\sigma L < 1$ for all $\ell\ell'$ channels. Therefore at the LHC there is no room for observable signals for same-sign dilepton processes $pp \to \ell\ell' X$ $(\ell, \ell' = e, \mu, \tau)$ due to the existing constraints for the mixing elements $|U_{\ell N}|^2$ from the precision electroweak data and neutrinoless double beta decay.

Let us take into account a recent proposal to increase the instantaneous LHC luminosity $\mathcal{L}$ to a value of 10^{35} cm^{-2}s^{-1} [39], i.e., a total luminosity $L = \mathcal{L} \times 1$ year $\simeq$ 3200 fb^{-1}.

Using the upgraded LHC luminosity and demanding $n = 1,\ 3$ events for discovery (i.e., $\sigma L > n$), we present the two-dimensional plot for the discovery limits in Fig. 2 for the case of identical same-sign leptons ($\ell = \ell'$).

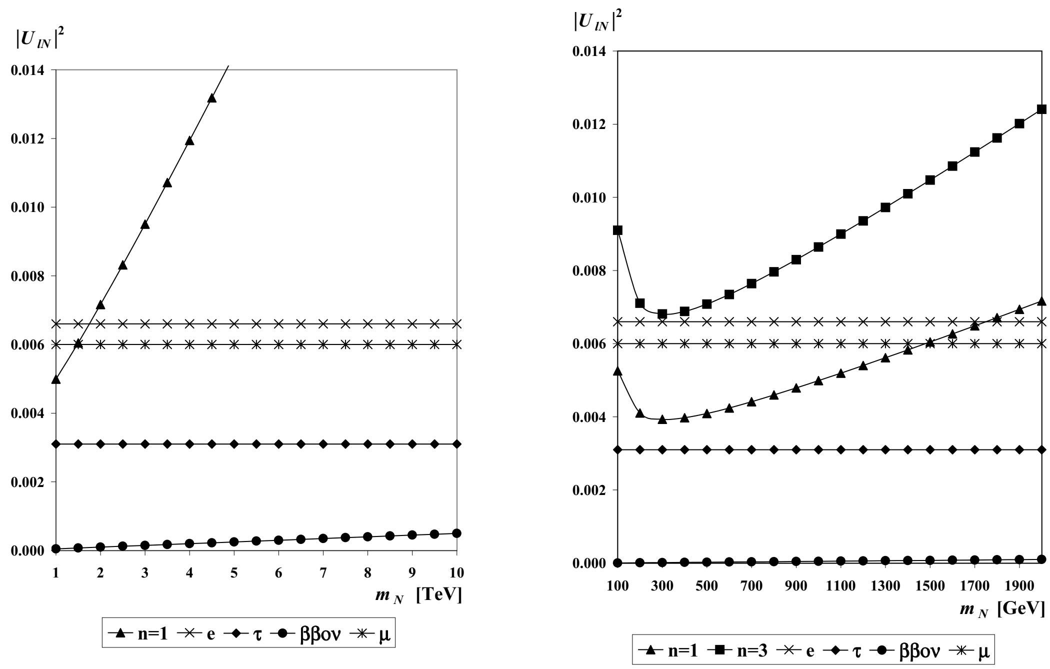


Figure 2: *Left*: Discovery limits for $pp \to \ell^+\ell^+ X$ as functions of m_N and $|U_{\ell N}|^2$ for $\sqrt{s} = 14$ TeV, $L = 3200$ fb^{-1} and various values of n, the number of events. We also superimpose the experimental limit from $\beta\beta_{0\nu}$-decay (Eq. (7)), as well as the experimental limits on $|U_{\ell N}|^2$ [horizontal lines for $\ell = e,\ \mu$ (Eq. (6)), and τ (Eq. (8))]. *Right*: The same as the left figure but for lighter Majorana neutrinos.

Discovery limits for the case of distinct same-sign leptons, $\ell\ell' = e\mu,\ e\tau,\ \mu\tau$, are shown in Fig. 3.

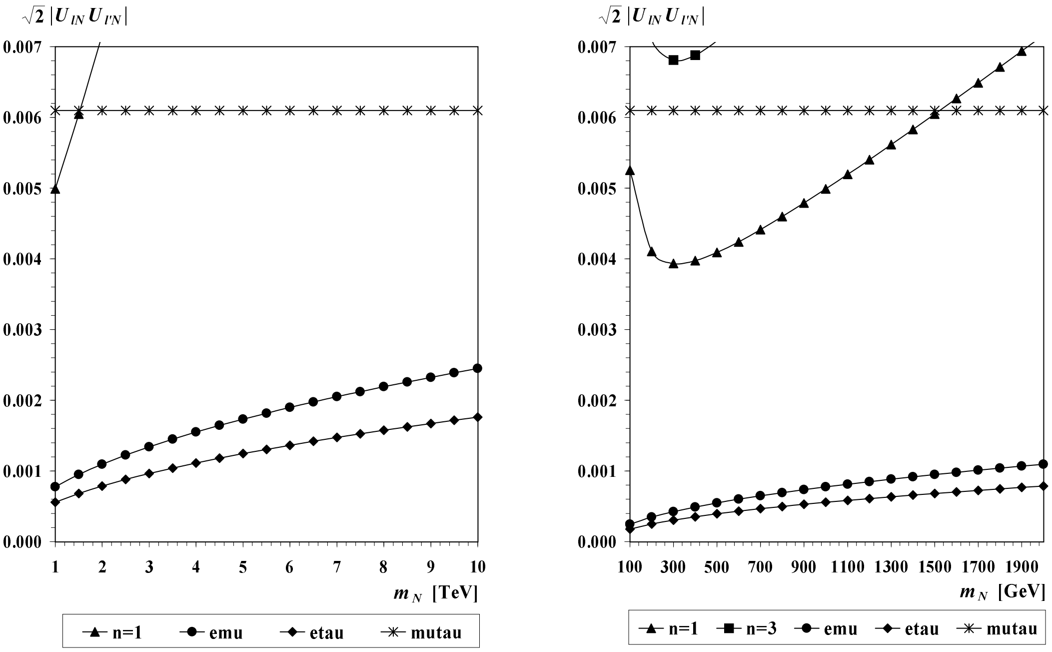


Figure 3: *Left*: Discovery limits for $pp \to \ell^+\ell'^+X$, $\ell\ell' = e\mu,\ e\tau,\ \mu\tau$ as functions of m_N and $\sqrt{2}\,|U_{\ell N}U_{\ell' N}|$ for $\sqrt{s} = 14$ TeV, $L = 3200$ fb^{-1} and various values of n, the number of events. We also superimpose the limits on $\sqrt{2}\,|U_{\ell N}U_{\ell' N}|$ obtained from the experimental limits [Eqs. (7), (6), and (8)]. *Right*: The same as the left figure but for lighter Majorana neutrinos.

¿From Figs. 2 and 3 we see that the existing strong constraints for the mixing elements $|U_{\ell N}|^2$ allow a possibility to observe only the same-sign $\mu\mu$ and $\mu\tau$ processes after increasing the nominal LHC luminosity $L = 100$ fb^{-1} by a factor of about 30. For this case, LHC experiments will have a sensitivity to heavy Majorana neutrinos of mass $m_N \lesssim 1.5$ TeV.

Before concluding this section, we would like to add a comment on Ref. [26] where *ostensibly* new and improved bounds on the effective Majorana masses

$$\langle m_{\ell\ell'}\rangle = \left|\sum_N U_{\ell N}U_{\ell' N}m_N\eta_N\right| \tag{9}$$

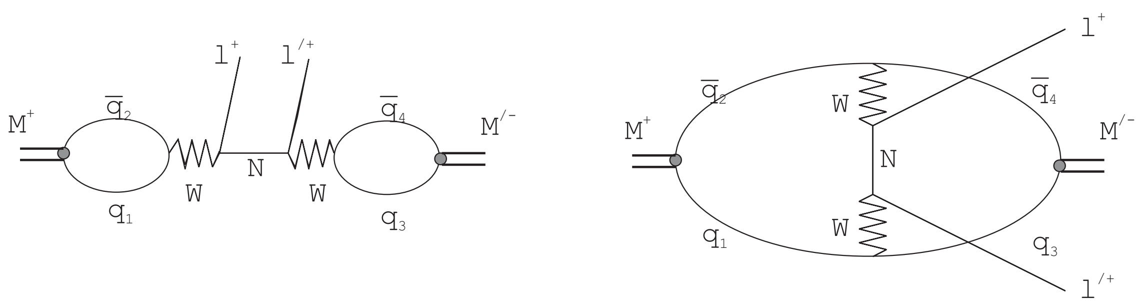
have been obtained using the HERA data on *ep* collisions. The authors in Ref. [26] have assumed that the cross sections for the processes $e^\pm p \to \overset{(-)}{\nu_e}\,\ell^\pm\ell'^\pm X$ are proportional to $\langle m_{\ell\ell'}\rangle^2$. This, however, is true only for light Majorana neutrinos. For the heavy Majorana neutrino case, the cross sections depend on the factor $\left\langle m_{\ell\ell'}^{-1}\right\rangle^2$, where

$$\left\langle m_{\ell\ell'}^{-1} \right\rangle = \left| \sum_N U_{\ell N} U_{\ell' N} \eta_N \frac{1}{m_N} \right|, \tag{10}$$

i.e. the effective *inverse* Majorana masses. As a consequence, Eq. (6) of Ref. [26] does not give new physical bounds on the light Majorana neutrino masses (as the resulting cross section is too small), and for the heavy Majorana neutrino case, their formula is not applicable (see also comments on Tables 2 and 3 below). Hence, contrary to the claims in Ref. [26], HERA data does not place any new limit on the Majorana mass matrix.

3 Rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$

We now take up rare meson decays of mesons of the type (2) mediated by Majorana neutrinos. We shall take the mesons in the initial and final state to be pseudoscalar. The lowest order amplitude of the process is given by the sum of the tree and the box diagrams shown in Fig. 4 taken from Ref. [22] (for earlier work, see Refs. [22, 17, 19]).



(tree) (box)

Figure 4: Feynman diagrams for the rare meson decay $M^+ \to M'^- \ell^+ \ell'^+$. Here N is a Majorana neutrino, bold vertices correspond to Bethe–Salpeter amplitudes for mesons as bound states of a quark and an antiquark. There are also two crossed diagrams with interchanged lepton lines.

It is well-known that the tree diagram amplitude can be expressed in a model independent way in terms of the measured decay constants of the pseudoscalar mesons in the initial and final state, f_M and $f_{M'}$. On the other hand, the box diagram depends in general on the details of hadron dynamics. In the usual folklore, the tree diagram dominates and is often used to set limits on effective Majorana masses [21]. Calculations based on various quark models for mesons have shown that this dominance really holds in the decay $K^+ \to \pi^- \mu^+ \mu^+$ for rather small neutrino masses [17]. It can be explained, at least partly, by a colour suppression factor $1/N_c = 1/3$ present in the box amplitude. Indeed, we see in Fig. 4 that in the quark loops in the tree diagram the colour summation takes place independently in the two loops. Not so in the box diagram, where the colour of the quark q_1 (antiquark $\overline{q}_2$) must be the same as the colour of the quark q_3 (antiquark $\overline{q}_4$) in the lower (upper) part of the box diagram. We also note that in some cases there is Cabibbo suppression of the tree or box amplitude due to smallness of the corresponding CKM matrix elements.

In this paper, we calculate branching ratios for rare meson decays (2) for two limiting cases of heavy and light Majorana neutrinos. In particular, we find that for the case of heavy neutrinos, $m_N \gg m_M$, the box contribution to the decay amplitude can also be expressed through f_M and $f_{M'}$ independent of a specific structure of the Bethe–Salpeter vertex for the meson in question.

The width of the rare decay $M^+(P) \to M'^-(P')\ell^+(p)\ell'^+(p')$ is given by

$$\Gamma_{\ell\ell'} = \left(1 - \frac{1}{2}\delta_{\ell\ell'}\right) \int (2\pi)^4 \, \delta^{(4)} \left(P' + p + p' - P\right) \frac{\left|A_t + A_b\right|^2}{2m_M} \frac{d^3P' d^3p \, d^3p'}{2^3 \left(2\pi\right)^9 P'^0 p^0 p'^0} \ . \quad (11)$$

Here A_t (A_b) is the tree(box)-diagram amplitude expressed in the Bethe–Salpeter formalism of Ref. [40] as

$$A_i = \frac{1}{\left(2\pi\right)^8} \int d^4q d^4q' H^{(i)}_{\mu\nu} L^{\mu\nu}_i \ , \quad (12)$$

where the lepton tensor is given by

$$\begin{aligned} L^{\mu\nu}_i &= \frac{g^4}{4} \frac{g^{\mu\alpha}}{p_i^2 - m_W^2} \frac{g^{\nu\beta}}{p_i'^2 - m_W^2} \sum_N U_{\ell N} U_{\ell' N} m_N \eta_N \\ &\times \left(\overline{v^c}\left(p\right) \left[\frac{\gamma_\alpha \gamma_\beta}{\left(p_i - p\right)^2 - m_N^2} + \frac{\gamma_\beta \gamma_\alpha}{\left(p_i - p'\right)^2 - m_N^2} \right] \frac{1 + \gamma^5}{2} v\left(p'\right) \right) , \end{aligned} \quad (13)$$

and η_N are the charge conjugation phase factors; $i = t, \ b$, implying tree and box contributions, respectively, and

$$p_t = P, \ p_t' = P'; \ p_b = \frac{1}{2}\left(P - P'\right) + q' - q, \ p_b' = \frac{1}{2}\left(P - P'\right) - q' + q.$$

The hadron tensors

$$\begin{aligned} H^{(t)}_{\mu\nu} &= \mathrm{Tr}\left\{ \chi^P\left(q\right) V_{12} \gamma_\mu \frac{1 + \gamma^5}{2} \right\} \mathrm{Tr}\left\{ \overline{\chi}^{P'}\left(q'\right) V_{43} \gamma_\nu \frac{1 + \gamma^5}{2} \right\} , \\ H^{(b)}_{\mu\nu} &= \mathrm{Tr}\left\{ \chi^P\left(q\right) V_{13} \gamma_\mu \frac{1 + \gamma^5}{2} \overline{\chi}^{P'}\left(q'\right) V_{42} \gamma_\nu \frac{1 + \gamma^5}{2} \right\} \end{aligned} \quad (14)$$

are expressed in terms of the elements of the CKM matrix, V_{jk} (the subscripts on V correspond to the quark line numbering in Fig. 4), and the model-dependent Bethe–Salpeter amplitudes χ^P for the mesons [40],

$$\chi^P\left(q\right) = \gamma^5 \left(1 - \delta_M \not{P}\right) \varphi(P, q) \, \phi_G \ ,$$

where $\delta_M = (m_1 + m_2)/M^2$, M is the mass of the meson having a quark q_1 and an antiquark $\overline{q}_2$, $m_{1,2}$ are the quark masses, $q = (p_1 - p_2)/2$ is the quark-antiquark relative 4-momentum, $P = p_1 + p_2$ is the total 4-momentum of the meson; the function $\varphi(P, q)$ is model dependent, and ϕ_G is the $SU(N_f) \times SU(N_c)$–group factor.

Table 1: The pseudoscalar decay constants f_M for the indicated mesons

Meson	f_M [MeV]
π^-	130.7
$K^\pm$	159.8
D^+	228
D_s^+	251
B^+	200

The decay constant f_M of the meson M is expressed through the amplitude χ^P as follows:

$$if_M P^\mu = \left\langle 0 \left| \overline{q}_2\left(0\right)\gamma^\mu\gamma^5 q_1\left(0\right)\right| M\left(P\right)\right\rangle = -i\sqrt{N_c}\int\frac{d^4q}{\left(2\pi\right)^4}\mathrm{Tr}\left\{\gamma^\mu\gamma^5\chi^P\left(q\right)\right\},$$

$$f_M = 4\sqrt{N_c}\,\delta_M\int\frac{d^4q}{\left(2\pi\right)^4}\,\varphi\left(P,q\right), \tag{15}$$

where the sum over colour indices is implied. The values of the pseudoscalar coupling constants f_M experimentally measured (for π and K [38]) and the central values calculated using lattice QCD (for D, D_s, and B mesons [41]) are shown in Table 1. We have neglected the errors on these quantities, as we shall see that this will not compromise our conclusions in any significant way. For all mesons in question, $m_M \ll m_W$, and we can use the leading current-current approximation in the lepton tensors (13).

Below we consider two limiting cases of heavy and light Majorana neutrinos.

Heavy neutrinos: $m_N \gg m_M$

For this case, the tree and box lepton tensors are equal to each other in the leading order of the expansion in $1/m_W^2$:

$$L_t^{\mu\nu} = L_b^{\mu\nu} = -16 g^{\mu\nu} L(p,p')\ ,$$
$$L(p,p') = G_F^2\sum_N U_{\ell N}U_{\ell' N}\eta_N\frac{1}{m_N}\left(\overline{v^c}\left(p\right)\frac{1+\gamma^5}{2}v\left(p'\right)\right). \tag{16}$$

¿From Eqs. (12), (14), (15), and (16) we obtain the total amplitude of the decay

$$A = A_t + A_b = 4K_V f_M f_{M'}(P\cdot P')L(p,p')\ ,$$
$$K_V = V_{12}V_{43} + \frac{1}{N_c}V_{13}V_{42}\ , \tag{17}$$

which is model independent in this limit.

Using Eqs. (11) and (17) we calculate the decay width:

$$\Gamma_{\ell\ell'} = \frac{G_F^4 m_M^7}{128\pi^3} f_M^2 f_{M'}^2 \left|K_V\right|^2 \left\langle m_{\ell\ell'}^{-1}\right\rangle^2 \Phi_{\ell\ell'}. \tag{18}$$

Here the effective inverse Majorana neutrino mass $\left\langle m_{\ell\ell'}^{-1}\right\rangle$ is introduced in the form of Eq. (10) and $\Phi_{\ell\ell'}$ is the reduced phase space integral. For identical leptons

$$\Phi_{\ell\ell} = \int_{4z_0}^{z_1} dz\,(z-2z_0)\left[\left(1-\frac{4z_0}{z}\right)(z_1-z)(z_2-z)\right]^{1/2}(1+z_3-z)^2. \tag{19}$$

For the case of ℓ and ℓ' being distinct leptons, assuming $m_{\ell'}/m_\ell \ll 1$, in the leading approximation, we have

$$\Phi_{\ell\ell'} = 2\int_{z_0}^{z_1} \frac{dz}{z}(z-z_0)^2\left[(z_1-z)(z_2-z)\right]^{1/2}(1+z_3-z)^2, \tag{20}$$

where the variable of integration is $z = (P-P')^2/m_M^2$ and the parameters z_k are defined as

$$z_0 = \frac{m_\ell^2}{m_M^2},\; z_1 = \left(1-\sqrt{z_3}\,\right)^2,\; z_2 = \left(1+\sqrt{z_3}\,\right)^2,\; z_3 = \frac{m_{M'}^2}{m_M^2}.$$

Using Eqs. (18), (19), (20), Table 1 and the experimental values for the total decay widths of mesons from [38], we have calculated the branching ratios

$$\mathrm{B}_{\ell\ell'}(M) = \frac{\Gamma\left(M^+ \to M'^-\ell^+\ell'^+\right)}{\Gamma\left(M^+ \to all\right)}. \tag{21}$$

Comparing the results with experimental bounds on $\mathrm{B}_{\ell\ell'}$ taken from [42] (for K decays), [43] (for D and D_s decays), and [38] (for $D^+ \to K^-\ell^+\ell'^+$, $B^+ \to \pi^-\ell^+\ell'^+$, and $B^+ \to K^-\ell^+\ell'^+$ decays), the upper bounds for the effective inverse Majorana masses (10) have been obtained. The results are shown in Table 2.

We can also obtain the indirect upper bounds on the branching ratios using the constraint (7) from $\beta\beta_{0\nu}$ on the $\langle m_{ee}^{-1}\rangle$ element of the effective inverse mass matrix. For other elements, assuming one heavy neutrino scenario, we set

$$\left\langle m_{\ell\ell'}^{-1}\right\rangle < (84.1\ \mathrm{GeV})^{-1},$$

using the current mass limits on neutral heavy leptons of the Majorana type [44]. The corresponding indirect bounds are shown in the last column of Table 2.

Our estimate for the K decay branching ratio,

$$\mathrm{B}_{\mu\mu}(K) \equiv \mathrm{B}\left(K^+ \to \pi^-\mu^+\mu^+\right) = 2.5\times 10^{-10}\ \mathrm{MeV}^2 \cdot \left\langle m_{\mu\mu}^{-1}\right\rangle^2 \tag{22}$$

should be compared with the corresponding updated result of Dib et al. [22]. Transcribed to our notations, the relevant decay width is given by the following expression

$$\Gamma^{(DGKS)}\left(K^+ \to \pi^-\mu^+\mu^+\right) = 7.0\times 10^{-32}\cdot m_K^3\left\langle m_{\mu\mu}^{-1}\right\rangle^2,$$

Table 2: Bounds on $\left\langle m_{\ell\ell'}^{-1}\right\rangle^{-1}$ and indirect bounds on the branching ratios $\mathrm{B}_{\ell\ell'}(M)$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by Majorana neutrinos (with $m_N \gg m_M$) and present experimental bounds

Rare decay	Exp. upper bounds on $\mathrm{B}_{\ell\ell'}(M)$	Theor. estimate for $\mathrm{B}_{\ell\ell'}(M)/\left\langle m_{\ell\ell'}^{-1}\right\rangle^{2}$ [MeV^2]	Bounds on $\left\langle m_{\ell\ell'}^{-1}\right\rangle^{-1}$ [keV]	Ind. bounds on $\mathrm{B}_{\ell\ell'}(M)$
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	8.6×10^{-10}	1200	2.2×10^{-30}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	2.5×10^{-10}	300	3.5×10^{-20}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	8.4×10^{-10}	1300	1.2×10^{-19}
$D^+ \to \pi^- e^+ e^+$	9.6×10^{-5}	2.2×10^{-9}	4.8	5.5×10^{-30}
$D^+ \to \pi^- \mu^+ \mu^+$	1.7×10^{-5}	2.0×10^{-9}	11	2.8×10^{-19}
$D^+ \to \pi^- e^+ \mu^+$	5.0×10^{-5}	4.2×10^{-9}	9.2	5.9×10^{-19}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	2.2×10^{-9}	4.3	5.5×10^{-30}
$D^+ \to K^- \mu^+ \mu^+$	1.2×10^{-4}	2.1×10^{-9}	4.1	3.0×10^{-19}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	4.3×10^{-9}	5.7	6.1×10^{-19}
$D_s^+ \to \pi^- e^+ e^+$	6.9×10^{-4}	1.9×10^{-8}	5.2	4.8×10^{-29}
$D_s^+ \to \pi^- \mu^+ \mu^+$	8.2×10^{-5}	1.8×10^{-8}	15	2.5×10^{-18}
$D_s^+ \to \pi^- e^+ \mu^+$	7.3×10^{-4}	3.6×10^{-8}	7.1	5.1×10^{-18}
$D_s^+ \to K^- e^+ e^+$	6.3×10^{-4}	2.2×10^{-9}	1.9	5.5×10^{-30}
$D_s^+ \to K^- \mu^+ \mu^+$	1.8×10^{-4}	2.0×10^{-9}	3.4	2.8×10^{-19}
$D_s^+ \to K^- e^+ \mu^+$	6.8×10^{-4}	4.2×10^{-9}	2.5	5.9×10^{-19}
$B^+ \to \pi^- e^+ e^+$	3.9×10^{-3}	$(0.3 \div 1.9) \times 10^{-9}$	$0.3 \div 0.7$	4.8×10^{-30}
$B^+ \to \pi^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.3 \div 1.9) \times 10^{-9}$	$0.2 \div 0.5$	2.7×10^{-19}
$B^+ \to \pi^- e^+ \mu^+$	6.4×10^{-3}	$(0.6 \div 3.8) \times 10^{-9}$	$0.3 \div 0.8$	5.4×10^{-19}
$B^+ \to \pi^- \tau^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-9}$		1.7×10^{-19}
$B^+ \to \pi^- e^+ \tau^+$		$(1.0 \div 6.2) \times 10^{-10}$		4.0×10^{-19}
$B^+ \to \pi^- \mu^+ \tau^+$		$(1.0 \div 6.2) \times 10^{-10}$		8.8×10^{-20}
$B^+ \to K^- e^+ e^+$	3.9×10^{-3}	$(0.2 \div 1.5) \times 10^{-10}$	$0.07 \div 0.20$	3.8×10^{-31}
$B^+ \to K^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.2 \div 1.5) \times 10^{-10}$	$0.05 \div 0.13$	2.1×10^{-20}
$B^+ \to K^- e^+ \mu^+$	6.4×10^{-3}	$(0.5 \div 2.9) \times 10^{-10}$	$0.09 \div 0.21$	4.1×10^{-20}
$B^+ \to K^- \tau^+ \tau^+$		$(1.3 \div 8.4) \times 10^{-12}$		1.2×10^{-21}
$B^+ \to K^- e^+ \tau^+$		$(0.7 \div 4.4) \times 10^{-11}$		6.2×10^{-21}
$B^+ \to K^- \mu^+ \tau^+$		$(0.7 \div 4.4) \times 10^{-11}$		6.2×10^{-21}

yielding a branching ratio

$$\mathrm{B}_{\mu\mu}^{(DGKS)}(K) = 1.6 \times 10^{-10}\ \mathrm{MeV}^2 \cdot \left\langle m_{\mu\mu}^{-1} \right\rangle^2 .$$

We note that including the box diagram gives the correction factor (see Eq. (17)) $(1 + 1/N_c)^2 = 16/9$, i.e. the numerical coefficient 1.6 in the above equation gets replaced by 2.8, which is close to the numerical coefficient 2.5 in our Eq. (22).

In addition, a rough estimate of the branching ratio,

$$\mathrm{B}_{\mu\mu}(K) \sim 0.2 \times 10^{-(13\pm 2)} \left(\left\langle m_{\mu\mu}^{-1} \right\rangle \cdot 100\ \mathrm{MeV} \right)^2 ,$$

obtained in [19] (and confirmed in [20]) is also in agreement with Eq. (22).

¿From Table 2 we see that the present experimental bounds on the branching ratios of the rare meson decays are too weak to give interesting bounds on the Majorana mass. From them and Eq. (18), we see that the direct bounds on the effective masses $\left\langle m_{\ell\ell'}^{-1} \right\rangle^{-1}$ of *heavy* Majorana neutrinos are much *smaller* than the difference of meson masses, $m_M - m_{M'}$. The indirect bounds estimated on $\mathrm{B}_{\ell\ell'}$, as discussed above, and given in the fifth column are so small that they can not be realistically tested in any current or planned experiments.

Light neutrinos: $m_N \ll m_\ell\, ,\ m_{\ell'}$

In this case, assuming the tree diagram dominance, the lepton tensor of Eq. (13), neglecting m_N^2 in the dominators, can be approximated as

$$L_t^{\mu\nu} = 8G_F^2 \sum_N U_{\mu N}^2 m_N \eta_N \left(\overline{v^c}\,(p) \left[\frac{\gamma^\mu \gamma^\nu}{(p_K - p)^2} + \frac{\gamma^\nu \gamma^\mu}{(p_K - p')^2} \right] \frac{1+\gamma^5}{2} v\,(p') \right) .$$

Using Eq. (11) with $A \simeq A_t$, we obtain the width of the decay (2) for the case of light Majorana neutrinos

$$\Gamma_{\ell\ell'} = \frac{G_F^4 m_M^3}{16\pi^3} f_M^2 f_{M'}^2 \left| V_{12} V_{43} \right|^2 \left\langle m_{\ell\ell'} \right\rangle^2 \phi_{\ell\ell'}, \qquad (23)$$

where $\langle m_{\ell\ell'} \rangle$ is the effective Majorana mass (9). Here the phase space integral $\phi_{\ell\ell'}$ has a rather complicated expression but in the realistic limit of massless leptons, $m_\ell / m_M \to 0$ and $m_{\ell'}/m_M \to 0$, it can be approximated as

$$\begin{aligned} \phi_{\ell\ell'} \simeq \left(1 - \frac{1}{2}\delta_{\ell\ell'} \right) \varphi\,(z_3)\,; \quad \varphi\,(z_3) &= \int_0^{z_1} dz z \left[(z_1 - z)\,(z_2 - z) \right]^{1/2} \\ &= (1 - z_3) \left[2z_3 + \frac{1}{6}\,(1 - z_3)^2 \right] + z_3\,(1 + z_3) \ln z_3, \end{aligned}$$

where z_k are the same as in Eq. (20).

Using Eq. (23), we have calculated the branching ratios (21) and obtained the direct upper bounds on the elements of the effective Majorana mass matrix. The indirect limits on the branching ratios have been also obtained with use of a rather stringent constraint on the ee element,

$$\langle m_{ee}\rangle < 1.0 \text{ eV},$$

¿from the $\beta\beta_{0\nu}$ Heidelberg-Moscow experiment [14] (see also comments in [45]) and a weaker constraint on other matrix elements used in [22]

$$\langle m_{\ell\ell'}\rangle < 9 \text{ eV} ,$$

which has been deduced in the three light neutrino scenario assuming the upper bound of 3 eV on the mass of the known neutrino [46] (more stringent but model dependent bounds on $\langle m_{\ell\ell\prime}\rangle$ have been obtained in [47]).

Our results are shown in Table 3. We note that for the $D^+ \to K^- \ell^+ \ell'^+$ decays, the tree diagram is strongly Cabibbo suppressed and the box diagram must be included even for the case of light neutrinos. We have obtained rough estimates of these decay widths assuming that the reduced (i.e., without CKM and colour factors) tree and box amplitudes are of the same order and replacing in Eq. (23) the factor $|V_{12}V_{43}|^2$ by $|K_V|^2$ (see Eq. (17)). It gives a numerical correction factor of about 57.

We conclude once again that the present experimental bounds on the branching ratios of the rare meson decays are too weak to set reasonable limits on the effective masses of light Majorana neutrinos.

As for the heavy Majorana neutrino case, one of our results,

$$\mathrm{B}\left(K^+ \to \pi^- \mu^+ \mu^+\right) = 1.4 \times 10^{-20} \text{ MeV}^{-2} \cdot \langle m_{\mu\mu}\rangle^2 , \tag{24}$$

is close to the corresponding result (in our notations) of Ref. [22],

$$\Gamma^{(DGKS)}\left(K^+ \to \pi^- \mu^+ \mu^+\right) = 4.0 \times 10^{-31} \cdot m_K^{-1} \langle m_{\mu\mu}\rangle^2 ,$$

or

$$\mathrm{B}^{(DGKS)}_{\mu\mu}(K) = 1.5 \times 10^{-20} \text{ MeV}^2 \cdot \left\langle m^{-1}_{\mu\mu}\right\rangle^2 ,$$

and the minimal value of a rough estimate of Refs. [19, 20],

$$\mathrm{B}_{\mu\mu}(K) \sim 0.2 \times 10^{-(13\pm 2)} \left(\langle m_{\mu\mu}\rangle /100 \text{ MeV}\right)^2 ,$$

gives the same order of magnitude of the branching ratio as Eq. (24).

¿From Eq. (24) we obtain the constraint (see Table 3) $\langle m_{\mu\mu}\rangle < 470$ GeV, which is almost the same as the one obtained in [21]: $\langle m_{\mu\mu}\rangle \lesssim 500$ GeV. But we have to stress, in contrast to the conclusion of Ref. [21], that there is no reasonable limit at all that emerges ¿from rare decays, as Eq. (24) is valid for $m_N \ll m_\mu \simeq 100$ MeV, and since $|U_{\mu N}| < 1$ by unitarity, the obvious inequality (see Eq. (9)) $\langle m_{\mu\mu}\rangle < |\sum_N m_N|$ holds. We note that Ref. [21] was also criticized on the same grounds in Refs. [20, 22].

Table 3: Bounds on $\langle m_{\ell\ell'}\rangle$ and indirect bounds on the branching ratios $\mathrm{B}_{\ell\ell'}(M)$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by Majorana neutrinos (with $m_N \ll m_\ell, m_{\ell'}$) and present experimental bounds

Rare decay	Exp. upper bounds on $\mathrm{B}_{\ell\ell'}(M)$	Theor. estimate for $\mathrm{B}_{\ell\ell'}(M)/\langle m_{\ell\ell'}\rangle^2$ [MeV^{-2}]	Bounds on $\langle m_{\ell\ell'}\rangle$ [TeV]	Ind. bounds on $\mathrm{B}_{\ell\ell'}(M)$
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	5.1×10^{-20}	0.11	5.1×10^{-32}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	1.4×10^{-20}	0.47	1.1×10^{-30}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	6.2×10^{-20}	0.09	5.0×10^{-30}
$D^+ \to \pi^- e^+ e^+$	9.6×10^{-5}	1.2×10^{-21}	280	1.2×10^{-33}
$D^+ \to \pi^- \mu^+ \mu^+$	1.7×10^{-5}	1.2×10^{-21}	120	9.7×10^{-32}
$D^+ \to \pi^- e^+ \mu^+$	5.0×10^{-5}	2.3×10^{-21}	150	1.9×10^{-31}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	2.3×10^{-21}	230	2.3×10^{-33}
$D^+ \to K^- \mu^+ \mu^+$	1.2×10^{-4}	2.3×10^{-21}	230	1.8×10^{-31}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	4.5×10^{-21}	170	3.7×10^{-31}
$D_s^+ \to \pi^- e^+ e^+$	6.9×10^{-4}	1.5×10^{-20}	210	1.5×10^{-32}
$D_s^+ \to \pi^- \mu^+ \mu^+$	8.2×10^{-5}	1.5×10^{-20}	74	1.2×10^{-30}
$D_s^+ \to \pi^- e^+ \mu^+$	7.3×10^{-4}	3.1×10^{-20}	150	2.5×10^{-30}
$D_s^+ \to K^- e^+ e^+$	6.3×10^{-4}	5.6×10^{-22}	1100	5.6×10^{-34}
$D_s^+ \to K^- \mu^+ \mu^+$	1.8×10^{-4}	5.6×10^{-22}	570	4.5×10^{-32}
$D_s^+ \to K^- e^+ \mu^+$	6.8×10^{-4}	1.1×10^{-21}	780	8.9×10^{-32}
$B^+ \to \pi^- e^+ e^+$	3.9×10^{-3}	$(0.3 \div 1.8) \times 10^{-23}$	$(1.5 \div 3.6) \times 10^4$	1.8×10^{-35}
$B^+ \to \pi^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.3 \div 1.8) \times 10^{-23}$	$(2.2 \div 5.5) \times 10^4$	1.5×10^{-33}
$B^+ \to \pi^- e^+ \mu^+$	6.4×10^{-3}	$(0.6 \div 3.6) \times 10^{-23}$	$(1.3 \div 3.3) \times 10^4$	2.9×10^{-33}
$B^+ \to \pi^- \tau^+ \tau^+$		$(1.5 \div 9.6) \times 10^{-25}$		7.8×10^{-35}
$B^+ \to \pi^- e^+ \tau^+$		$(0.4 \div 2.4) \times 10^{-23}$		1.9×10^{-33}
$B^+ \to \pi^- \mu^+ \tau^+$		$(0.4 \div 2.4) \times 10^{-23}$		1.9×10^{-33}
$B^+ \to K^- e^+ e^+$	3.9×10^{-3}	$(0.2 \div 1.2) \times 10^{-24}$	$(0.6 \div 1.4) \times 10^5$	1.2×10^{-36}
$B^+ \to K^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.2 \div 1.2) \times 10^{-24}$	$(0.9 \div 2.2) \times 10^5$	9.7×10^{-35}
$B^+ \to K^- e^+ \mu^+$	6.4×10^{-3}	$(0.4 \div 2.4) \times 10^{-24}$	$(0.5 \div 1.3) \times 10^5$	1.9×10^{-34}
$B^+ \to K^- \tau^+ \tau^+$		$(1.0 \div 6.1) \times 10^{-25}$		4.9×10^{-35}
$B^+ \to K^- e^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-24}$		9.7×10^{-35}
$B^+ \to K^- \mu^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-24}$		9.7×10^{-35}

4 Conclusion

We have examined two processes mediated by Majorana neutrinos: the production of like-sign dileptons $\ell^+\ell'^+$ in proton-proton collisions at the LHC energy and in the rare decays of K^+, D^+, D_s^+, and B^+ mesons of the type $M^+ \to M'^- \ell^+ \ell'^+$. We find that at LHC for the nominal luminosity $L = 100$ fb^{-1} there is no room for observable same-sign dilepton signals due to the existing constraints from the precision electroweak data and neutrinoless double beta decay. But increasing the nominal LHC luminosity by a factor of about 30 will allow a possibility to observe the same-sign $\mu\mu$ and $\mu\tau$ processes mediated by a heavy Majorana neutrino of mass $m_N \lesssim 1.5$ TeV. Data from HERA do not have an impact on the Majorana mass matrix — despite claims to the contrary [26]. However, precision electroweak data may lead to more severe constraints on the fermion mixing angles than worked out in [34, 35] and used by us. As for the rare meson decays, present direct bounds on their branching ratios are too weak to set reasonable limits on effective Majorana masses, $\langle m_{\ell\ell'} \rangle$ (for light neutrinos) and $\left\langle m_{\ell\ell'}^{-1} \right\rangle^{-1}$ (for heavy ones). Therefore, to have an impact on the Majorana mass matrix, a very substantial improvement of the experimental reach on the lepton-number violating rare meson decays is needed. Conversely, if a same sign dilepton signal is seen in any of the meson decay channels listed in Tables 2 and 3 in foreseeable future, it will be due to new physics other than the one induced by Majorana neutrinos, such as R-parity violating supersymmetry [48].

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