Determination of the high-twist contribution to the structure function $xF_3^{\nu N}$

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Abstract

We extract the high-twist contribution to the neutrino-nucleon structure function $xF_3^{(\nu+\overline{\nu})N}$ from the analysis of the data collected by the IHEP-JINR Neutrino Detector in the runs with the focused neutrino beams at the IHEP 70 GeV proton synchrotron. The analysis is performed within the infrared renormalon (IRR) model of high twists in order to extract the normalization parameter of the model. From the NLO QCD fit to our data we obtained the value of the IRR model normalization parameter $\Lambda_3^2 = 0.69 \pm 0.37$ (exp) ± 0.16 (theor) GeV^2. We also obtained $\Lambda_3^2 = 0.36 \pm 0.22$ (exp) ± 0.12 (theor) GeV^2 from a similar fit to the CCFR data. The average of both results is $\Lambda_3^2 = 0.44 \pm 0.19$ (exp) GeV^2.

1. Attempts to extract the high twist (HT) contributions to the neutrino-nucleon deep-inelastic scattering (DIS) structure functions started many years ago [1], but have not lead to the ultimate answer up to now. The main difficulty in this study is that due to the linear rise of the total interaction cross-section with the incident neutrino energy E_ν, the largest data samples have been collected in experiments at relatively high neutrino energies $E_\nu > 50$ GeV. The region of the small momentum transfered Q, which is most relevant for the study of the HT effects, is rather poorly populated by the data points coming from these experiments because of kinematical and/or methodical cuts. At lower neutrino energies experiments with very high luminosity are necessary to achieve the statistical precision in the structure function measurements sufficient for the quantitative estimation of the HT contribution.

The first results on the twist-4 contribution to the neutrino-nucleon structure function $xF_3^{\nu N}$ extracted from the analysis of data collected in a single experiment were reported in Ref. [2]. The indication on a negative sign of this contribution given in this paper was later confirmed with a better precision in Ref. [3]. Nevertheless, the experimental errors were large in both cases which did not allow for a conclusive comparison with the available theoretical models of HT. Later the CCFR experiment at Fermilab collected large statistical data sample [4], which allowed for more precise determination of the twist-4 contribution to $xF_3^{\nu N}$ [5, 6], but the precision is still poor.

The data of Ref. [7] from the IHEP-JINR Neutrino Detector can be used to improve our knowledge of the HT contribution to the DIS structure functions. This experiment used a neutrino beam of relatively low energy ($E_\nu < 30$ GeV), but collected rather large statistics (5987 neutrino and 741 antineutrino charged-current (CC) interactions). The lowest Q^2 is 0.55 GeV^2 and the HT contribution would clearly manifest itself as a power-like correction to the logarithmic-like leading twist (LT) dependence of the structure functions on Q. Meanwhile the Q range spanned by the data is limited (maximal Q^2 is 20 GeV^2) and for this reason the simultaneous determination of the power-like and logarithmic-like terms is difficult. In the analysis of Ref. [7] we fixed the HT contribution as it was defined from other experiments and performed the next-to-leading-order (NLO) QCD analysis of our data in order to constraint the LT contribution. The value of the strong coupling constant α_s, which mainly governs the Q dependence of the LT term, was determined from this analysis as $\alpha(M_Z) = 0.123^{+0.010}_{-0.013}$.

At the same time the Q dependence of the LT contribution can be well constrained using the data from other experiments. The world average of $\alpha_s(M_Z)$ is known with the precision of about 0.003 [8] and one can perform the analysis complimentary to the one of Ref. [7], i.e., fix the value of $\alpha_s(M_Z)$ at the world average and try to extract the HT contribution from the data. An additional error estimated as variation of the results of the fit under variation of the world average of $\alpha_s(M_Z)$ within its uncertainty should be ascribed thereafter. Meanwhile in most cases the total error in the HT contribution extracted using this approach would be less as compared with the results of the simultaneous fit of the HT and the LT terms to the data. The reduction of the error depends on the ratio of the error in $\alpha_s(M_Z)$ obtained in such simultaneous fit to the error in the world average. For our data this ratio is larger than 3 and for this reason we extracted the HT contribution value from these data using the fit with a fixed value of α_s.

2. The analysis is based on the data collected with three independent exposures of the IHEP-JINR Neutrino Detector [9] to the wide-band neutrino and antineutrino beams [10] of Serpukhov U-70 accelerator. The exposure to the antineutrino beam ($\overline{\nu}_\mu$-exposure) was

performed at the proton beam energy $E_p = 70$ GeV, whereas the two ν_μ-exposures were carried out at $E_p = 70$ GeV and at $E_p = 67$ GeV. The energy of the selected ν_μ $(\overline{\nu}_\mu)$ CC events was in the range of $6 < E_{\nu(\overline{\nu})} < 28$ GeV. The experimental set-up and the selection criteria of CC neutrino and antineutrino interactions are discussed in Ref. [11]. The $F_2^{(\nu+\overline{\nu})N}$ and $xF_3^{(\nu+\overline{\nu})N}$ structure functions of nucleon have been measured as a function of x averaged over all Q^2 permissible (the details of experimental procedures are described in Ref. [7]).

These data were analyzed in the NLO QCD approximation in the modified minimal-subtraction ($\overline{\text{MS}}$) renormalization-factorization scheme. The partons evolution code applied in this analysis was used earlier for the global fit of the parton distribution functions [12]. The boundary parton distributions were chosen in the form

$$xp_{\text{NS}}(x, Q_0) = A_{\text{NS}}x^{a_{\text{NS}}}(1-x)^{b_{\text{NS}}}, \quad xp_{\text{S}}(x, Q_0) = A_{\text{S}}(1-x)^{b_{\text{S}}},$$

$$xp_{\text{G}}(x, Q_0) = A_{\text{G}}(1-x)^{b_{\text{G}}} \tag{1}$$

at $Q_0^2 = 0.5$ GeV^2, where indices NS, S, and G correspond to non-singlet, singlet, and gluon distributions, respectively. These distributions were substituted in the expressions for the LT contributions to $F_{2,3}$

$$xF_3^{(\nu+\overline{\nu})N,LT}(x, Q) = \int_x^1 \frac{dz}{z} C_3^q(z, Q)p_{NS}(x/z, Q),$$

$$F_2^{(\nu+\overline{\nu})N,LT}(x, Q) = \int_x^1 \frac{dz}{z} \{C_2^q(z, Q)\left[p_{NS}(x/z, Q) + p_S(x/z, Q)\right] +$$

$$+ C_G(z, Q)p_G(x/z, Q)\} \tag{2}$$

where $C(z, Q)$ are the perturbative QCD coefficient functions in the $\overline{\text{MS}}$ scheme. The parameter A_{NS} was calculated using the constraint $\int_0^1 dx q_{\text{NS}} = 3$, and the parameter A_{G} – from the momentum-conservation constraint, while other parameters of Eq. (1) were fitted to the data. The form of Eq. (1) was checked to be flexible enough, i.e., its complication did not lead to the improvement of the fit.

The target mass (TM) corrections of $O(M^2/Q^2)$, as they are given in Ref. [13], were applied to the LT contribution. The HT contribution was parameterized in the additive form and within the infrared renormalon model (IRR) [14]. In this model the HT contribution is connected with the LT one by the known coefficient function and the only free parameter of the model is related to the total normalization. In particular, the HT contribution to $xF_3^{(\nu+\overline{\nu})N}$ reads [15, 16]

$$H_3(x, Q) = A_2'(F_3^{\nu N}) \int_x^1 \frac{dz}{z} C_3^{IRR}(z)p_{\text{NS}}(x/z, Q), \tag{3}$$

where C_3^{IRR} is the IRR model coefficient function and $A_2'(F_3^{\nu N})$ defines the total normalization. The HT contribution to $F_2^{(\nu+\overline{\nu})N}$ contains the non-singlet term similar to Eq.(3) with the normalization parameter $A_2'(F_2^{\nu N})$ and the respective coefficient function C_2^{IRR}. In addition, the singlet and gluon terms calculated in Ref. [17] also come to the expression for the HT contribution to F_2 as it is given by the IRR model, but these terms are relevant for small x only and for this reason we used the non-singlet approximation for the calculation of the IRR contribution to $F_2^{(\nu+\overline{\nu})N}$ as well as for $xF_3^{(\nu+\overline{\nu})N}$. Following Ref. [17], we describe

the general normalization of the HT contributions to $F_{2,3}$ by the parameters $\Lambda_{2,3}$, which are connected with the parameters $A_2'(F_{2,3}^{\nu N})$ by the relations

$$A_2'(F_{2,3}^{\nu N}) = -\frac{2C_F}{\beta_0}\Lambda_{2,3}^2, \qquad (4)$$

where $C_F = 4/3$ and β_0 is the first coefficient of the QCD β-function. Both ways are completely equivalent if the number of active fermions in the expression for β_0 does not depend on Q. Meanwhile in order to provide self-consistency of the analysis, we changed n_f in Eq. (4) from 3 to 4 at Q equal to the c-quark mass $m_c = 1.5$ GeV. For this reason the value of A_2' depends on Q in our case, although the numerical effect is inessential. The value of $\alpha_s(M_Z)$ was fixed at 0.118, which is close to the world average of Ref. [8]. As one can see in Fig.1 the analyzed data are insensitive to the parameter Λ_2 and we fixed it at the value of 1 GeV^2 inspired by the results of Ref. [16] on the analysis of charged leptons DIS data. The systematic errors on the data were accounted for in the covariance matrix approach, described in Ref. [18].

Table 1: The results of the fit of the IRR model to the data from different neutrino experiments. The value of χ^2 over the number of data points (NDP) is given in the last column.

Experiment	Λ_3^2[GeV^2]	Λ_2^2[GeV^2]	χ^2/NDP
IHEP-JINR	0.69 ± 0.37	1.	3/12
CCFR	0.36 ± 0.22	0.91 ± 0.77	253/222

The results of the fit are given in Table 1 and in Fig. 1. The obtained contribution to $xF_3^{(\nu+\overline{\nu})N}$ is negative which supports the earlier observation of Refs. [2, 3] and is in agreement with the results of Refs. [5, 6]. The HT contribution to $F_2^{(\nu+\overline{\nu})N}$ is negligible in the whole region of x spanned by the data. The value of Λ_3^2 is determined from our data with the 50% accuracy. For the comparison, in the NLO QCD fit to the CCFR data on the structure function $xF_3^{(\nu+\overline{\nu})N}$ the value $A_2'(F_3^{\nu N}) = -0.12 \pm 0.05$ GeV^2 was obtained in Ref. [5]. This estimate did not account for the systematic errors in the data, while the estimate accounting for systematics is $A_2'(F_3^{\nu N}) = -0.10 \pm 0.09$ GeV^2 [6], i.e., our result is the most precise estimate of the IRR normalization parameter at the moment.

The change of Λ_3^2 under variation of $\alpha_s(M_Z)$ by ± 0.003 is 0.055 GeV^2 and we consider this shift as a theoretical error in Λ_3^2. Another source of the theoretical error comes from the uncertainty due to the effect of the higher-order (HO) QCD corrections to the LT term. These effects may be especially important for our study since the data at rather low Q are involved in the analysis. The HO corrections generally make the Q dependence of the LT contribution steeper, and correspondingly lead to the decrease of the HT contribution. In order to estimate the uncertainty due to neglected HO corrections, we repeated the fit with the QCD renormalization scale changed from the nominal value of Q to $2Q$ (see Ref. [19] for a detailed argumentation of this approach). The obtained shift in the value of Λ_3^2 is 0.15 GeV^2. Note that the significant part of the error in the world average of $\alpha_s(M_Z)$ also come from the uncertainty due to neglected HO QCD corrections. For this reason the two considered sources of theoretical errors are correlated. Having no possibility to account for this correlation, we just combine both errors in quadrature and estimate the total theoretical error in Λ_3^2 as 0.16 GeV^2. One can see that the uncertainty in Λ_3 is dominated by the experimental error,
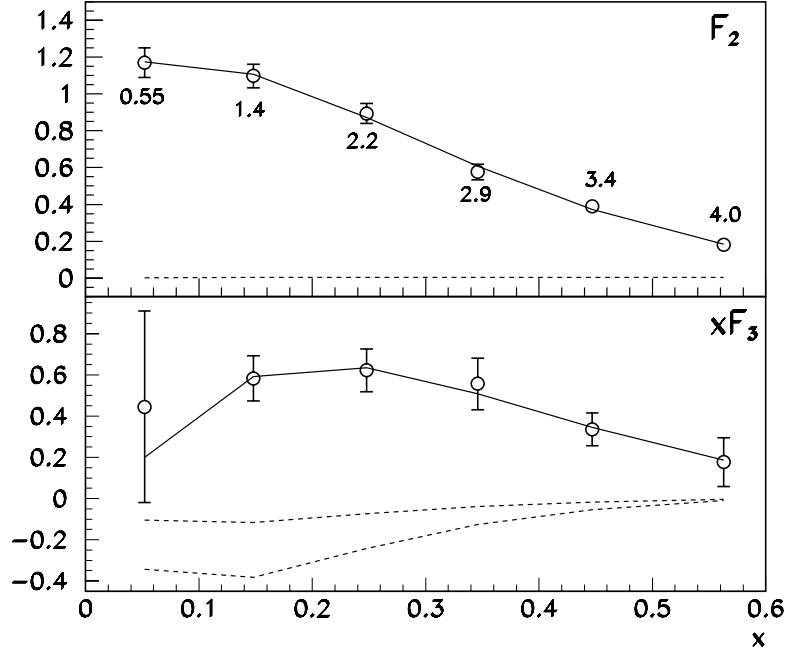
Figure 1: The x dependence of the measured structure functions $F_2^{(\nu+\overline{\nu})N}$ (upper) and $xF_3^{(\nu+\overline{\nu})N}$ (lower). The average values of Q^2 (GeV^2) for the x bins are given in the upper plot. The full curves give the result of the LT+HT fit to the data, the dashed curves correspond to the 1σ bands of the HT contributions obtained from the fit.

moreover accounting for the correlations of the separate sources of the theoretical error would decrease the latter.

3. We also extracted the HT contribution to $F_{2,3}^{\nu N}$ from the CCFR data of Ref. [4] using the approach with α_s fixed. The value of the parameter Λ_3^2 obtained from the NLO QCD analysis of those data with $x < 0.7$ is given in Table 1. The CCFR data are sensitive to the parameter Λ_2 too, although the precision is poor and the fitted value is comparable with zero within the errors[1]. The theoretical error in Λ_3^2 estimated in the same way as for the analysis of our data is 0.12 GeV^2. Results for both experiments are comparable within the errors and combining them we obtain the average

$$\Lambda_3^2 = 0.44 \pm 0.19 \text{ (exp) GeV}^2. \tag{5}$$

In order to check universality of the IRR model scales with respect to the specific choice of structure function in the DIS process, we compared this value with the results of Ref. [16] on the analysis of the charged-leptons DIS data. Since in Ref. [16] the results are given in terms of parameter A_2', we transformed our average (5) using Eq. (4). As a result, we obtain that for $n_f = 3$

$$A_2'(F_3^{\nu N}) = -0.130 \pm 0.056 \text{ (exp) GeV}^2. \tag{6}$$

This value is smaller, than $A_2'(F_2^{eN}) = -0.2$ GeV^2, given in Ref. [16], although within the errors both values are comparable. More precise conclusion about universality of the IRR model scales may be derived from the analysis of experimental data with improved statistics, which have been collected using the IHEP-JINR Neutrino Detector with a different configuration of the neutrino beam channel. The analysis of these data in order to extract the structure functions is currently in progress. The data from the NuTeV collaboration [20], after their processing have been completed, may also be used to improve the precision of the IRR scales determination. In far sight a potential neutrino factory would allow for a detailed cross-check of the IRR model predictions and, in particular, the determination of the IRR scales with an accuracy of several percent [21].

In conclusion, we extract the high-twist contribution to the neutrino-nucleon structure function $xF_3^{(\nu+\bar{\nu})N}$ from the analysis of the data collected in the first runs of the IHEP-JINR Neutrino Detector at the IHEP U-70 accelerator. We observe the negative HT contribution to the structure function $xF_3^{(\nu+\bar{\nu})N}$ which supports the earlier observations. The normalization scale of the IRR model extracted from the combined analysis of the IHEP-JINR and CCFR experiments is about 1σ lower than the one extracted from the data on the structure function F_2^{lN} for the DIS of charged leptons.

Acknowledgments

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