CONTROL OF COHERENT INSTABILITIES BY LINEAR COUPLING

R. Cappi, E. Métral, D. Möhl, CERN, Geneva, Switzerland

Abstract

One of the main challenges in the design of high-energy colliders is the very high luminosity necessary to provide significant event rates. This imposes strong constraints to achieve and preserve beams of high brightness, i.e. intensity to emittance ratio, all along the injector chain. Amongst the phenomena that can blow up and even destroy the beam are transverse coherent instabilities. Two methods are widely used to damp these instabilities. The first one is Landau damping by non-linearities. The second consists in using an electronic feedback system. However, non-linearities are harmful to single-particle motion due to resonance phenomena, and powerful wideband feedback systems are expensive. It is shown in this paper that linear coupling is a further method that can be used to damp transverse coherent instabilities. The theory of collective motion is outlined, including the coupling of instability rise and damping rates, chromaticity and Landau damping. Experimental results obtained at the CERN PS are reported, which are important for its role as LHC injector. Stabilisation by coupling explains (at least in part) why existing high intensity accelerators and colliders work best when adjusted relatively close to a coupling resonance. This method could be profitably used in the design of new machines.

1 INTRODUCTION

Strong coupling between the transverse planes of a particle beam leads to an "equipartition" of the oscillation energy, including the instability growth rates in the case of coherent instability. In the presence of a frequency spread, there can also be a partition of Landau damping for "optimum" coupling [1]. Linear coupling can therefore become very effective for stabilisation when a dissymmetry, e.g. in the frequency spreads or coupling impedances, is present.

In Ref. [2], a formula for transverse coherent instabilities in the presence of linear coupling (near the coupling resonance $Q_X - Q_Y = l$) was given in the form of a 4×4 determinant. In the case of mode coupling, this formula takes into account the coupling between two adjacent modes, m and m+1. The general two-dimensional dispersion relation [3], including all the head-tail modes, is given below. It is expressed in the form of a determinant of infinite matrices, which have to be truncated in practice.

The results in the absence/presence of linear coupling, frequency spread and mode coupling have already been treated in Ref. [2]. In the present paper, the general formula is discussed in Section 2, and Section 3 is devoted to the application for the case of the PS beam for the future LHC [4].

2 THEORY

2.1 General 2D Dispersion Relation

In the presence of linear coupling (near the coupling resonance $Q_X - Q_Y = l$), the stability of intense beams can be discussed using the following infinite determinant

$$\left| \left(\Delta\omega^{x} - I_x^{-1}\, I \right) \times \left(\Delta\omega^{y} - I_y^{-1}\, I \right) - \frac{\left| \underline{\hat{K}}_0 \left(l \right) \right|^2 R^4\, \Omega_0^4}{4\, \omega_{x0}\, \omega_{y0}} I \right| = 0 \,. \tag{1}$$

Here, $\Delta\omega^{x,y}$ are matrices whose elements are given by Eq. (2), $I_{x,y}^{-1}$ are matrices whose (inverse) elements are given by Eqs. (9-10), and I is the identity matrix.

$$\Delta\omega_{m,n}^{x,y} = \left(\left| m \right| + 1 \right)^{-1} \frac{j\, e\, \beta\, I_b}{2\, m_0\, \gamma\, Q_{x0,y0}\, \Omega_0\, L} \left(Z_{x,y}^{eff} \right)_{m,n} , \tag{2}$$

with

$$\left(Z_{x,y}^{eff} \right)_{m,n} = \frac{\sum\limits_{k=-\infty}^{k=+\infty} Z_{x,y} \left(\omega_k^{x,y} \right) h_{m,n} \left(\omega_k^{x,y} - \omega_{\xi_{x,y}} \right)}{\sum\limits_{k=-\infty}^{k=+\infty} h_{m,m} \left(\omega_k^{x,y} - \omega_{\xi_{x,y}} \right)} , \tag{3}$$

$$h_{m,n}(\omega) = \frac{\tau_b^2}{\pi^4} \left(\left| m \right| + 1 \right) \times \left(\left| n \right| + 1 \right) \times F_m^n$$
$$\times \left\{ \left(\omega\, \tau_b / \pi \right)^2 - \left(\left| m \right| + 1 \right)^2 \right\}^{-1} \times \left\{ \left(\omega\, \tau_b / \pi \right)^2 - \left(\left| n \right| + 1 \right)^2 \right\}^{-1} , \tag{4}$$

$$F_{m\ even}^{n\ even} = \left(-1 \right)^{\left(\left| m \right| + \left| n \right| \right)/2} \times \cos^2 \left[\omega\, \tau_b / 2 \right] , \tag{5}$$

$$F_{m\ even}^{n\ odd} = \frac{\left(-1 \right)^{\left(\left| m \right| + \left| n \right| + 3 \right)/2}}{2\, j} \times \sin \left[\omega\, \tau_b \right] , \tag{6}$$

$$F_{m\,odd}^{n\,even} = \frac{(-1)^{(|m|+|n|+1)/2}}{2\,j} \times \sin[\omega\,\tau_b], \tag{7}$$

$$F_{m\,odd}^{n\,odd} = (-1)^{(|m|+|n|+2)/2} \times \sin^2[\omega\,\tau_b\,/\,2]. \tag{8}$$

The elements of the "dispersion matrices" $I_{x,y}$ entering into Eq. (1) are

$$I_{x,m} = \int_{\hat{x}=0}^{\hat{x}=+\infty} \int_{\hat{y}=0}^{\hat{y}=+\infty} \frac{-2\,\pi^2 \,\frac{df_{x0}(\hat{x})}{d\hat{x}}\,\hat{x}^2\,f_{y0}(\hat{y})\,\hat{y}}{\omega_c - \omega_x(\hat{x},\hat{y}) - m\,\omega_s}\,d\hat{x}\,d\hat{y}, \tag{9}$$

$$I_{y,m} = \int_{\hat{x}=0}^{\hat{x}=+\infty} \int_{\hat{y}=0}^{\hat{y}=+\infty} \frac{-2\,\pi^2 \,\frac{df_{y0}(\hat{y})}{d\hat{y}}\,\hat{y}^2\,f_{x0}(\hat{x})\,\hat{x}}{\omega_c - \omega_y(\hat{x},\hat{y}) - l\,\Omega_0 - m\,\omega_s}\,d\hat{x}\,d\hat{y}. \tag{10}$$

Notice that $I_{x,m}$ and $I_{y,m}$ are the horizontal and vertical dispersion integrals, ω_c is the coherent frequency to be determined, $\omega_x(\hat{x},\hat{y})$ and $\omega_y(\hat{x},\hat{y})$ are the transverse incoherent betatron frequencies of the particles, $f_{x0}(\hat{x})$ and $f_{y0}(\hat{y})$ are the uncorrelated distribution functions of the incoherent betatron amplitudes, ω_s is the synchrotron frequency and $m = ..., -1, 0, 1, ...$ is the head-tail mode number. Furthermore, $\underline{\hat{K}}_0(l)$ is the lth Fourier coefficient of the skew gradient $\underline{K}_0 = (e\,/\,p_0)(\partial B_x\,/\,\partial x)$, with e the elementary charge, p_0 the design momentum and B_x the horizontal magnetic field, R is the average radius of the machine, Ω_0 is the average revolution frequency of the particles, $\omega_{x0,y0} = Q_{x0,y0}\,\Omega_0$ are the unperturbed betatron frequencies, $\Delta\omega_{m,m}^{x,y}$ are the complex betatron frequency shifts given by Sacherer's formula [5], $j = \sqrt{-1}$ is the imaginary unit, β and γ are the relativistic velocity and mass factors, $I_b = N_b e\,\Omega_0\,/\,(2\,\pi)$ is the current in one bunch, m_0 is the proton rest mass, $L = \beta\,c\,\tau_b$ is the total bunch length (in metres), $Z_{x,y}$ are the coupling impedances, $\omega_k^{x,y} = (k + Q_{x0,y0})\,\Omega_0 + m\,\omega_s$ with $-\infty \le k \le +\infty$, $\omega_{\xi_{x,y}} = (\xi_{x,y}\,/\,\eta)\,Q_{x0,y0}\,\Omega_0$ are the transverse chromatic frequencies, with $\xi_{x,y} = (d\,Q_{x,y}\,/\,d\,p)(\,p_0\,/\,Q_{x0,y0})$ the chromaticities, and $\eta = \gamma_{tr}^{-2} - \gamma^{-2}$ is the slippage factor.

In the case of coasting beams, Eq. (1) still applies with m=0, the impedance removed from the summation and the bunch length L equal to $2\pi R$. The dispersion integrals for a coasting beam are given by Eqs. (9-10) with m=0, adding also the contribution of the momentum spread. The incoherent betatron frequency spread around the harmonic $k_r^{x,y}$ corresponding to the frequency ω_r of the driving impedance is given by $|\omega_r + \omega_{\xi_{x,y}}| \times |\eta| \times \Delta p/\,p_0$.

Note that for head-tail instabilities below the mode-coupling threshold, the momentum spread is not effective for Landau damping since several synchrotron periods occur during the instability rise-time, and on average the spread is zero. In the case of mode coupling, the momentum spread helps to stabilise the beam, and it has thus to be taken into account in the dispersion integral.

2.2 Situation in the Absence of both Frequency Spread and Mode Coupling

In the absence of linear coupling, the determinant of Eq. (1) is the product of the two one-dimensional determinants, each of which equals zero. Below the mode-coupling threshold, each mode m can be treated separately, and the following dispersion equations are obtained, $I_{x,m}^{-1} = \Delta\omega_{m,m}^{x}$ and $I_{y,m}^{-1} = \Delta\omega_{m,m}^{y}$. In the absence of Landau damping, the stability condition for the mth mode is Im ($\Delta\omega_{m,m}^{x,y}$) $\ge$ 0, where Im ( ) stands for imaginary part.

In the presence of linear coupling, but without frequency spread and below the mode-coupling threshold, Eq. (1) leads to the following necessary condition for stability of the mth mode,

$$V_{\text{eq}x}^{m} + V_{\text{eq}y}^{m} \le 0, \tag{11}$$

where $V_{\text{eq}x,y}^{m}$ = - Im ($\Delta\omega_{m,m}^{x,y}$) are the transverse instability growth rates. If Eq. (11) is true, it is possible to stabilise this mode by increasing the skew gradient and/or by working closer to the coupling resonance $Q_X - Q_Y = l$. The stabilising values of the modulus of the lth Fourier coefficient of the skew gradient are given by

$$\left|\underline{\hat{K}}_0(l)\right| \ge \frac{2\left[-Q_{x0}\,Q_{y0}\,V_{\text{eq}x}^{m}\,V_{\text{eq}y}^{m}\right]^{1/2}}{R^2\,\Omega_0} \times \frac{\left[\left(V_{\text{eq}x}^{m} + V_{\text{eq}y}^{m}\right)^2 + \Omega_0^2\left(Q_X - Q_Y - l\right)^2\right]^{1/2}}{-\left(V_{\text{eq}x}^{m} + V_{\text{eq}y}^{m}\right)}, \tag{12}$$

where $Q_{X,Y} = (\omega_{x0,y0} + U_{\text{eq}x,y}^{m})\,/\,\Omega_0$ are the horizontal and vertical coherent tunes in the presence of wake fields ($U_{\text{eq}x,y}^{m}$ = Re ($\Delta\omega_{m,m}^{x,y}$), where Re ( ) stands for real part), but in the absence of coupling. Furthermore, in the case of coupled-bunch instabilities of M bunches, $k = n_{x,y} + k'\,M$ in Eq. (3) with $-\infty \le k' \le +\infty$, and the coupled-bunch mode numbers are related by $n_x = n_y - l$.

Notice that, when Eq. (11) is verified, it is verified for "any" intensity.

3 EXPERIMENTS

The plot of the transverse single-bunch head-tail instability growth rates as functions of the head-tail mode number is represented in Fig. 1(a) for the PS beam for LHC [6]. In fact the initial scheme has been modified [4] and slightly different parameters are now used to cope with instabilities. The theory, based on the above model (considering both resistive-wall and broad-band impedances), predicts horizontal single-bunch instabilities with the most critical head-tail mode number $|m|=6$. To test the validity of the one-dimensional theory, the skew quadrupole current was set to have the minimum of linear coupling between the transverse planes, which turned out to be $I_{skew} \approx 0.33$ A, due to the "natural" coupling present in the PS (see Fig. 1(b) where $\underline{K}_0 = \hat{\underline{K}}_0(0)$). In this situation, a head-tail instability develops, and in accordance with theory the mode $|m|=6$ is observed (see Fig. 2, showing 6 nodes).

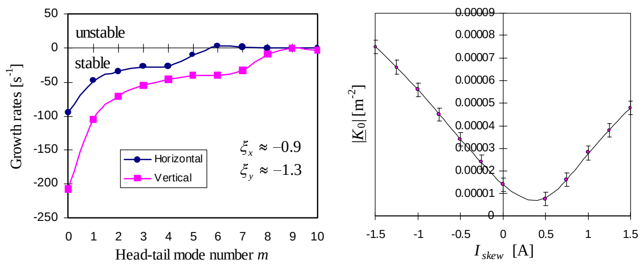

Figure 1: (a) *Transverse single-bunch head-tail instability growth rates vs. head-tail mode number for the PS nominal beam for the future LHC, in the absence of coupling*. (b) *Modulus of the normalised skew gradient, as deduced from tune separation measurements, vs. skew quadrupole current for the PS at 1 GeV kinetic energy*.

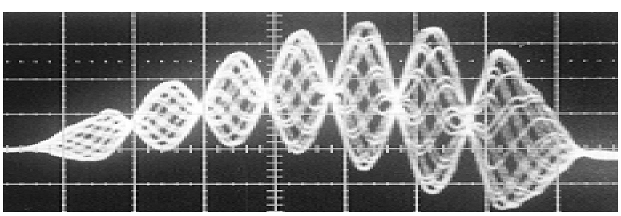

Figure 2: *ΔR signal from a radial beam-position monitor during 20 consecutive turns, in the PS with minimum coupling. Time scale: 20 ns/div*.

The beam losses due to this instability are shown in Fig. 3(a). One notes that the beam from the PSBooster (PSB) is injected into the PS in 2 batches of 3 bunches each, to overcome space charge effects in the PSB [4]. The drawback of this filling scheme is that the first batch has to wait 1.2 s at 1.4 GeV kinetic energy, and during that time about 2/3 of the beam is lost due to the above instability, if no counter measures are taken.

From Fig. 1(a), one can deduce that Eq. (11) is verified for each mode, and that it will therefore be possible to stabilise the beam by linear coupling. The fact that the mode $|m|=6$ is unstable in the horizontal plane and stable in the vertical one, is due to the different natural chromaticities ($\xi_x = -0.9$ and $\xi_y = -1.3$). In the presence of coupling, the transfer of instability growth rate is essentially here a chromaticity sharing [7].

Setting a skew quadrupole current of about –0.4 A, with the working point ($Q_X = 6.22$, $Q_Y = 6.25$), the instability is damped without any loss (see Fig. 3(b)). Furthermore, this method is reproducible and no emittance blow-up has been measured.

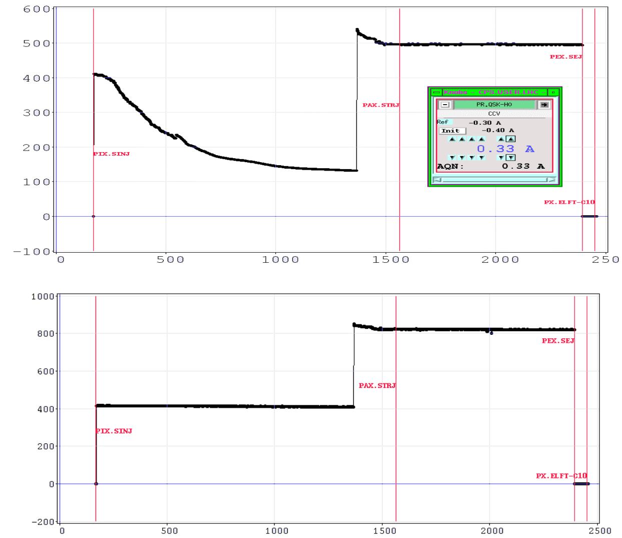

Figure 3: *Intensity of the PS ring (in units of 10^{10} protons) vs. time (in ms). (a) Without linear coupling, i.e. $I_{skew} \approx 0.33$ A. (b) With a linear coupling corresponding to $I_{skew} \approx -0.4$ A, and a working point ($Q_X = 6.22$, $Q_Y = 6.25$).*

4 CONCLUSION

A general formula for the transverse coherent instabilities in the presence of linear coupling has been given. The beneficial effect due to the transfer of the head-tail instability growth rates, which critically depend on chromaticities, has then been emphasised. This result has been verified experimentally on the PS beam for the future LHC, where a head-tail instability $|m|=6$ is damped using linear coupling, in agreement with theory. This method can be profitably used in the design of new machines, to find optimum values for the transverse tunes, the skew quadrupole and octupole currents, and the chromaticities.

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