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A REMARK ON HIGHER DIMENSION INDUCED DOMAIN WALL DEFECTS IN OUR WORLD

C.P. Korthals Altes[a,1], M. Laine[b,2]

[a] *Centre Physique Theorique, CNRS, Case 907, Luminy, F-13288 Marseille, France*

[b] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

There has been recent interest in new types of topological defects arising in models with compact extra dimensions. We discuss in this context the old statement that if only SU(N) gauge fields and adjoint matter live in the bulk, and the coupling is weak, then the theory possesses a spontaneously broken global Z(N) symmetry, with associated domain wall defects in four dimensions. We discuss the behaviour of this symmetry at high temperatures. We argue that the symmetry gets restored, so that cosmological domain wall production could be used to constrain such models.

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[1] chris.korthal-altes@cpt.univ-mrs.fr
[2] mikko.laine@cern.ch

1. Introduction

There has been recent interest in models where our four-dimensional (4d) world is a thin brane embedded in a number of other, compact dimensions. Apparently there are many alternatives as to which fields are confined to the brane and which are free to propagate in the bulk. We shall here consider the case that some non-Abelian gauge fields live in the bulk ([1] and variants thereof; for a review, see [2]). For concreteness, we mostly concentrate on the case of a single extra dimension. We then wish to ask what kind of topological defects in our 4d world such a situation may lead to.

Let us recall that the Standard Model of particle physics does not support any stable topological defects, but many extensions thereof do. For instance, Grand Unified Theories may predict the existence of monopoles. It has recently been pointed out that the existence of compact extra dimensions could also lead to various types of topological defects on our 4d brane [3].

The existence of topological defects can clearly have important implications for cosmology. Conversely, the fact that none have been observed directly or indirectly, places a number of strong constraints on theories beyond the Standard Model.

In this note we discuss one special topological defect possibly arising in the aforementioned models with compact extra dimensions. The same mechanism has previously been addressed in [4], and a somewhat analogous with two compact dimensions in [5].

2. Z(N) symmetry

It is well known that when SU(N) gauge fields are compactified with periodic boundary conditions, the system develops a global Z(N) symmetry, which is spontaneously broken for a small coupling [6, 7, 8]. This leads to the existence of domain walls. Let us briefly recall the argument.

We consider the simplest case of a flat periodic extra dimension, $y = 0...R$. As far as we can see, the statement holds also for instance for an orbifold, S_1/Z_2, with the difference that the number of effective gauge degrees of freedom is halved, since the extra symmetry removes the imaginary modes from the Fourier decomposition of a real field. Let us denote $M = R^{-1}$. SU(N) gauge theory in dimensions higher than 3+1 is not renormalizable, but for concreteness we assume that there is another cutoff scale $\Lambda \gg M$, up to which the dominant effects come just from the Yang-Mills Lagrangian. The observables we compute with this theory are in any case finite.

The partition function of the system is now

$$Z = \int_{\rm b.c.} \mathcal{D}A_\mu \exp\Big(- \int_0^R dy \int d^{d-1}x \, \frac{1}{2} {\rm Tr}\, F_{\mu\nu}^2 + ... \Big), \tag{1}$$

where the boundary conditions are periodic, $A_\mu(x,0) = A_\mu(x,R)$, and higher dimensional operators are ignored. Our convention in the following is that $D_\mu = \partial_\mu + igA_\mu$.

Consider now field transformations $U(x,y)$, $x = (x_0, ..., x_{d-2})$, which look locally just like gauge transformations, but have the property that

$$U(x,R) = zU(x,0), \tag{2}$$

where $z = \exp(i2\pi k/N) \in {\rm Z}(N)$, with $k = 0, ..., N-1$. The Lagrangian is clearly invariant under these transformation. In addition, considering transformations where U factorizes into the form on the RHS of Eq. (2) also for $y \in (R-\epsilon, R)$, the fields,

$$A_\mu \to A'_\mu = UA_\mu U^\dagger + \frac{1}{ig} U\partial_\mu U^\dagger, \tag{3}$$

remain periodic. Thus, z in Eq. (2) represents a global symmetry of the theory.

Fundamentally charged fields in the bulk, on the other hand, spoil the symmetry: after the transformation $\Phi \to \Phi' = U\Phi$, Φ' would not respect the original boundary conditions. Which of the ${\rm Z}(N)$ vacua becomes the global one, depends on the original boundary conditions [8, 9]. There might be ways of avoiding this explicit symmetry breaking, however. We might for instance imagine that most fundamentally charged fields are strictly confined to a brane as in some orbifold compactifications (see, e.g., [2]), so that they are insensitive to the extra dimension (i.e., do not couple to the corresponding covariant derivative), and the symmetry breaking effects are suppressed. Or we could imagine that the gauge group in question is not one of the Standard Model ones, and only feels adjoint matter. Later on, we shall also return briefly to the case where fundamental charges do cause an explicit symmetry breaking.

Now, all local gauge invariant operators are invariant under the transformation in Eq. (2). There is a non-local gauge invariant operator which is not invariant, however:

$$P(x) = \frac{1}{N} {\rm Tr}\, \mathcal{P} \exp(ig \int_0^R dy A_y(x,y)), \quad P(x) \to P'(x) = z^* P(x). \tag{4}$$

Because $P(x)$ is a non-local operator, its absolute value does not have a meaningful continuum limit. The phase factor of $\langle P(x) \rangle$, however, denoted by z in the following, is assumed to be physical, and acts as an order parameter for the symmetry.

This symmetry takes a more familiar form for instance in the set of gauges $\partial_y A_y = 0$. Then it just corresponds to a shift in A_y. As we see from Eq. (4), the vacuum with

$z = 1$ is obtained for $A_y = 0$, while vacua with $z = \exp(i2\pi k/N)$ for instance with $gA_y = (2\pi k/(RN)) \times \mathrm{diag}(1, 1, ..., 1 - N)$.

Despite the somewhat abstract nature of this symmetry, it does lead to domain walls carrying a finite energy density in 4d, in case the symmetry is broken[3]. The reason is that once quantum corrections are taken into account, the effective potential for A_y, or more precisely the constrained effective action for $P(x)$, develops barriers between the minima corresponding to $\arg\langle P\rangle = 2\pi k/N, k = 0, ..., N - 1$. The computation of such quantum corrections, and thus this statement, is reliable for a weak coupling.

To proceed, we may parameterise $gA_y = C = \mathrm{diag}(C_1, C_2, ..., C_N)$, $\sum_i C_i = 0$, so that $P \sim (1/N)\mathrm{Tr}\, \exp(iRC)$. In principle we want to compute the constrained effective action

$$e^{-S_{\rm eff}[C(x)]} \sim \Big\langle \Pi_x \delta\Big(\frac{1}{N}\mathrm{Tr}\, e^{iRC(x)} - P(x)\Big)\Big\rangle. \tag{5}$$

At 1-loop level, and for x-independent configurations of C, this however reduces simply to the standard effective potential $V_{\rm eff}(C)$: $S_{\rm eff} \to RL^{d-1}V_{\rm eff}(C)$, where L^{d-1} is the volume in $d - 1$ dimensions of extent L. By construction, $V_{\rm eff}$ is gauge independent.

In order to show the result for $V_{\rm eff}(C)$, we denote $C_{ij} = (C_i - C_j)/(2\pi M)$. Adding together the contributions from the gauge bosons and the ghosts in the background of C, the result can be written in the form [7, 8]

$$V_{\rm eff}(C) - V_{\rm eff}(0) = \sum_{i\neq j}\Big[U(C_{ij}) - U(0)\Big], \tag{6}$$

$$U(C_{ij}) - U(0) = (d/2 - 1)M \sum_{l=-\infty}^{\infty} \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \ln \frac{p^2 + (2\pi M)^2(l + C_{ij})^2}{p^2 + (2\pi M)^2 l^2}. \tag{7}$$

We write the logarithm in a heat kernel form $\ln(a/a_0) = \int_0^\infty (ds/s)(e^{-a_0 s} - e^{-as})$ and use the Poisson summation formula

$$\sum_{l=-\infty}^{\infty} e^{-s(2\pi M)^2(l+C_{ij})^2} = \frac{1}{2\pi M}\Big(\frac{\pi}{s}\Big)^{1/2} \sum_{l=-\infty}^{\infty} e^{-l^2/(4sM^2)+i2\pi l C_{ij}}, \tag{8}$$

obtained after the x-integration from $\sum_l f(l) = \int dx \sum_l e^{i2\pi l x} f(x)$. The s-integration can also be carried out, to arrive at

$$U(C_{ij}) - U(0) = 2(d - 2)M^d \frac{\Gamma(\frac{d}{2})}{\pi^{\frac{d}{2}}} \sum_{l=1}^{\infty} \frac{1}{l^d} \sin^2(\pi l C_{ij}). \tag{9}$$

This contribution vanishes at $C_{ij} = 0 \bmod 1$, otherwise it is positive, representing thus a barrier between the different degenerate minima.

[3]For completeness let us note that these Z(N) domain walls are not related to those found in 4d supersymmetric Yang-Mills theory (for recent reviews, see [10]).

To compute the energy density of the domain wall interpolating between two different degenerate minima, derivative terms are needed too in $S_{\rm eff}[C]$. At leading order, it is enough to keep the tree-level kinetic terms [11]. This leads to the action

$$S_{\rm eff}[C] = R\int d^{d-1}x\,\Big[\frac{1}{g^2}\sum_{\mu=0}^{d-2}{\rm Tr}\,(\partial_\mu C)^2 + V_{\rm eff}(C)\Big]. \tag{10}$$

Let us now consider two adjacent minima, $\arg\langle P\rangle = 2\pi k/N, k = 0,1$. The extremal path between $k = 0,1$ is given by $C = (2\pi Mq/N)\times {\rm diag}(1,1,...,1-N)$, where $0 \le q \le 1$ [11, 12]. We minimise $S_{\rm eff}[C]$ from Eq. (10) with these boundary conditions, assuming a planar symmetry. Converting the integral over the coordinate across the domain wall into one over q with the standard procedure, we get for the energy per unit area

$$\sigma_1 = \frac{4\pi(N-1)}{g\sqrt{N}}\int_0^1 dq\sqrt{2\Big[U(q)-U(0)\Big]}. \tag{11}$$

We are not aware of a general analytic answer for this integral, given U in Eq. (9).

We close this section with a remark. In asymptotically free theories, the effective coupling g is guaranteed to be small inside the wall (see below) but starts to grow in the wings and will be large outside. This makes for instance $U(0)$ uncomputable in confining theories. Nevertheless, the energy per unit area turns out to admit a perturbative expansion, since it is an integral over the profile and is dominated by the inside region. This infrared insensitivity has been demonstrated explicitly at next-to-leading order for $d = 4$ [11, 12], and we shall return to the case $d = 5$ below.

3. 4d at zero temperature

Because many explicit results are available, we next specialise briefly in a (2+1)d toy world, with a compact 4th dimension. In Euclidian spacetime, this situation corresponds formally to a 4d field theory at a finite temperature, and has thus been thoroughly studied, although with a different physical interpretation.

In fact, the first study was by 't Hooft [6], with yet another language. He introduced gauge invariant electric and magnetic fluxes. Then the statement is that if we consider the magnetic flux in a spatial direction (with extent L_2) orthogonal to the compact one, then for large R the extra energy related to the flux is $E_m \sim \rho L_2 \exp(-\rho L_1 R)$, where L_1 is the other of the spatial dimensions, and ρ is the electric string tension of the confining phase of pure 4d Yang-Mills. For small R, on the other hand, the effective coupling $g(2\pi M)$ is weak and we find $E_m \sim \sigma_1 L_2$, where $\sigma_1 \sim 1/R^2$ is given in Eq. (11).

Let us now recall more precisely when the perturbative computation is reliable. In the 4d case, the original gauge coupling g_4^2 is dimensionless, while that of the 3d world is to leading order $g_3^2 = g_4^2 M$. If we compute the effective potential, the only dimensionful parameters entering at 1-loop level are multiples of $2\pi M$. Thus we assume that the effective dimensionless expansion parameter related to the computation is

$$\epsilon \sim \frac{\alpha}{\pi} \sim \frac{g_4^2(2\pi M)}{4\pi^2}. \tag{12}$$

For M larger than the scale parameter in g_4^2, the expansion parameter is thus small. In the finite temperature case, this corresponds to the deconfined phase of QCD.

Then, the sum in Eq. (9) can be explicitly carried out, leading to a Bernoulli polynomial B_4 [7, 8]:

$$U(C_{ij}) - U(0) = \frac{2}{3}\pi^2 M^4 [C_{ij} \bmod 1]^2 (1 - [C_{ij} \bmod 1])^2. \tag{13}$$

Consequently [11, 12],

$$\sigma_1 = \frac{4\pi^2(N-1)}{3\sqrt{3N}} \frac{M^2}{g_4}, \quad \sigma_k = \sigma_1 \frac{k(N-k)}{N-1}. \tag{14}$$

Here $k > 1$, relevant for $N > 3$, corresponds to a profile between $z = 1$ and $z = \exp(i2\pi k/N)$. Next-to-leading order (2-loop) corrections are also known [11, 12, 13]. This domain wall energy density has effectively also been measured with lattice simulations ([14] and references therein), although again with a different interpretation.

4. 5d at zero temperature

In the physically more interesting case of a 5d theory, the original gauge coupling g_5^2 has the dimension GeV^{-1}. Let us define $g_4^2 = g_5^2 M$, and assume that the theory can be regularised such that a perturbative expansion in g_4^2 makes sense. Then, the only dimensionful mass parameters entering at 1-loop level are multiples of $2\pi M$. Thus the effective expansion parameter related to the computation is essentially the same as in Eq. (12), as long as we are inside the wall. For M in the TeV range or so, but smaller than the cutoff of the 5d theory, a weak coupling computation should thus be reliable.

Now we are not able to evaluate Eq. (11) analytically, however. We find

$$\sigma_1 = 0.622988 \times 4\pi \frac{N-1}{\sqrt{N}} \frac{M^{\frac{5}{2}}}{g_5}, \quad \sigma_k = \sigma_1 \frac{k(N-k)}{N-1}, \tag{15}$$

where the first numerical equality is in accordance with [4], and the latter part, indicating that two different domain walls tend to attract each other, follows from [12]. Expressed in terms of g_4, the dimensionful combination here is $M^{\frac{5}{2}}/g_5 = M^3/g_4$.

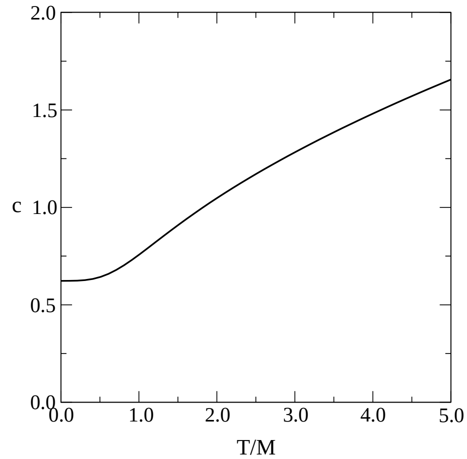

Figure 1: The coefficient c in the domain wall energy density (cf. Eq. (11)), $\sigma_1 \equiv c \times 4\pi(N-1)M^{\frac{5}{2}}/(g_5\sqrt{N})$, as a function of T/M. We have used Eqs. (11), (17). At large T/M the behaviour is $\sim (T/M)^{\frac{1}{2}}$.

5. 5d at finite temperature

Let us finally consider the 5d system at finite physical temperature T. Some finite T effects have previously been discussed for an analogous Abelian 3d system in [15].

For reference, let us recall what happens at finite temperatures for a standard Z(2) symmetry breaking model such as a 4d real scalar field with the potential $V(\phi) \sim \lambda(\phi^2 - v^2)^2$. The dominant temperature correction is now $\delta V(\phi) \sim T^2\phi^2$. Thus, if we just plot the effective potential as a function of ϕ, the barrier between the two minima gets smaller as T goes up, and vanishes at $T = T_c \sim v$. The surface energy density, obtained by the analogue of Eq. (11), decreases with increasing T. Finally, the physical correlation length, $\sim 1/\sqrt{V''(\phi)}$, increases, and diverges at $T = T_c$.

To see whether the same happens here, we compute $V_{\text{eff}}(C)$ to 1-loop order at a finite temperature T. According to the standard procedure, a path integral formulation can be obtained simply by making the Euclidian time direction τ periodic, with the period $1/T$. Then the integration measure changes as

$$\int \frac{d^{d-1}p}{(2\pi)^{d-1}} f(p_\tau, \mathbf{p}) \to T \sum_m \int \frac{d^{d-2}p}{(2\pi)^{d-2}} f(2\pi T m, \mathbf{p}). \tag{16}$$

Correspondingly, the effective potential for C is modified. Using again the Poisson transformed heat kernel expression for Eq. (7), we obtain

$$U(C_{ij}) - U(0) = 2(d-2)M^d \frac{\Gamma(\frac{d}{2})}{\pi^{\frac{d}{2}}} \sum_{l=1}^{\infty} \sin^2(\pi l C_{ij}) \Big[\frac{1}{l^d} + \sum_{m=1}^{\infty} \frac{2}{(l^2 + m^2/\rho^2)^{\frac{d}{2}}} \Big], \tag{17}$$

where $\rho = T/M$. The $T = 0$ contribution is the first term inside the square brackets.

Curiously enough, we now note that each term in the series representing the change with a finite T is *positive*. The value of the sum inside the square brackets in Eq. (17) grows like $\sim \rho$ at large ρ. Thus, introducing a temperature *increases the barrier between the minima*. Correspondingly, the domain wall energy density increases as $\sim \rho^{\frac{1}{2}}$, as shown in Fig. 1. The second derivative at the minimum also goes as $\sim \rho$, meaning that the correlation length (and the width of the wall) decrease as $\sim \rho^{-\frac{1}{2}}$.

Could this mean that the symmetry is not restored at high temperatures? In principle this is not excluded: there are well studied examples even of systems with inverse symmetry breaking [16]. It seems to us, however, that at very high temperatures $T \gg M$, the Z(N) symmetry discussed here does get restored, despite the behaviour found in the previous paragraph. This also means the perturbation theory breaks down in such a case, as we shall indicate presently.

The argument is the old one [17]. Indeed, our order parameter $\arg\langle P(x)\rangle$ lives in 3+1 dimensions, so we may think in terms of usual statistical mechanics. Then, creating a bubble of some Z(N) phase with a surface area A is Boltzmann suppressed by $\exp(-\sigma_1 A/T)$. But there are a lot of such configurations, and the corresponding entropy factor goes as $\exp(\Lambda^2 A)$, where Λ is some ultraviolet cutoff [18]. We may expect Λ^{-1} to be given by the width of the wall. According to the 1-loop discussion above, $\sigma_1 \sim (M^3/g_4)\rho^{\frac{1}{2}}$ and $\Lambda^2 \sim (g_4 M)^2\rho$, where $\rho = T/M$. The symmetry should get restored when these two opposing factors compensate for each other such that the exponent is of order unity [17]. This[4] leads to $T_c \sim M/g_4^2$.

There is another way of obtaining the same result. Imagine that we construct a dimensionally reduced effective theory by integrating out the non-zero Kaluza-Klein modes in the y-direction. The effective theory is 4d SU(N) + adjoint scalar matter, and its physics represents that of a broken Z(N) phase of the 5d theory (for a discussion in the 4d context, see [19]). Thus the symmetry is broken, as long as dimensional reduction is accurate. It can lose its accuracy only if the power suppressed higher order operators truncated from the 4d effective theory become as important as the renormalizable ones, i.e., if the lightest dynamical mass scales m within the effective theory satisfy $m \gg 2\pi M$. At finite temperatures, the confinement scale of 4d non-Abelian gauge theory is $g_4^2(2\pi T)T$ [20, 7], where the approximate (say, $\overline{\text{MS}}$) scale of g_4^2 has also been shown; the precise value has no significance here, even if it were $\propto 2\pi M$. Thus, we may expect a breakdown and a transition at $g_4^2(2\pi T_c)T_c \sim 2\pi M$, parametrically just as above. The phase diagram of the system in this language is

[4] We do not consider here the running of the effective coupling with T, since we shall argue presently that radiative corrections are parametrically subdominant for $T < T_c$.

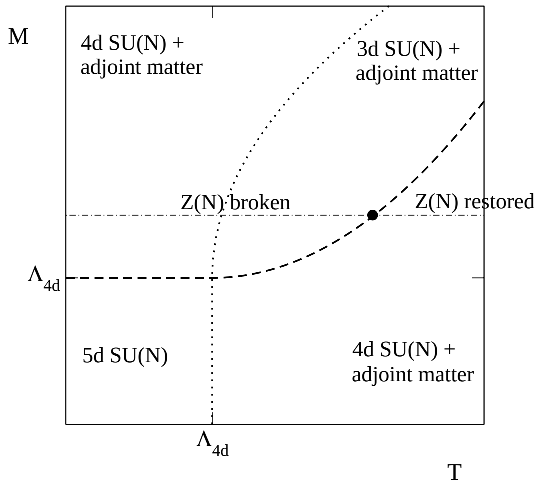

Figure 2: A schematic phase diagram, together with the low-energy effective theories in different regions. The Z(N) symmetry we have discussed is broken above the dashed line. To the right of the dotted line, another Z(N) symmetry, related to a Polyakov loop in the Euclidean (finite-T) time direction is broken; its physical interpretation is however somewhat ambiguous, so we do not dwell on it here. Our argument follows the horizontal dash-dotted line, with the blob indicating the phase transition.

illustrated in Fig. 2.

Finally, let us note that at the transition point, perturbation theory breaks down. Indeed, loops involving the temporal Matsubara zero modes obtain a dimensionless expansion parameter $\gtrsim g_5^2 T/(2\pi) \sim g_4^2 T/(2\pi M)$. This is parametrically small at $T < T_c$, but becomes of order unity at $T \sim T_c$. Thus the effective potential as well as σ_1 and the width of the wall computed above, need no longer be reliable, and σ_1 could in principle even vanish at $T = T_c$.

While we consider the arguments presented credible, they of course do not constitute a proof for the existence of a phase transition, because of the breakdown of perturbation theory. It would be interesting to study the issue non-perturbatively, at least for the 4d model discussed in Sec. 3, where a similar reasoning can be carried out, leading to the same parametric estimate for T_c.

6. Cosmology

We end with a brief cosmological consideration. It is well known that if there is a phase transition after inflation where a discrete symmetry gets broken, domain walls are generically produced [21]. They would disappear slowly because the finite speed of light imposes causality, and carry a lot of energy. In fact, they would soon be the dominant energy component, and conflict with the observed energy density fluctuations in the cosmic microwave background, unless $\sigma \lesssim (1 \text{ MeV})^3$ [21, 22].

The constraint is weaker if the walls are not absolutely stable. This can happen for instance if the Z(N) symmetry is not exact. Such is the case with some fundamental matter in the bulk; the minima corresponding to $z \neq 1$ are then lifted or lowered, depending on the boundary conditions, but they may remain metastable [8, 9]. In conventional cosmology wall domination is avoided if $\Delta\epsilon \gg \sigma^3/m_{\rm Pl}^2$, where $m_{\rm Pl} = 1.2 \times 10^{19}$ GeV, and $\Delta\epsilon$ is the energy density excess in the metastable state [22]. In the present case $\Delta\epsilon \sim M^4, \sigma \sim M^3/g_4$, so that we arrive at a rather weak constraint $M \ll g_4 m_{\rm Pl}$. The situation could change in alternative cosmologies probably more relevant in the presence of large extra dimensions.

Looking back at the first paragraph of this Section, we thus see that if the Z(N) symmetry gets restored at high T, as we have suggested in Sec. 5, then either particle physics models with only adjoint matter in the bulk, or cosmological models with temperatures above $\sim M/g_4^2$, can be excluded. Some fundamental matter in the bulk relaxes these constraints. There are many other constraints, of course, which need to be met as well; see, e.g., [2] and references therein.

7. Conclusions

It is well known that if SU(N) gauge fields and adjoint matter are compactified in an extra spacelike direction, then the theory develops a global Z(N) symmetry. If the effective coupling is weak, the symmetry is broken, resulting in domain wall configurations. These are in principle visible in the remaining flat spacetime.

In order for this observation to be physically relevant, at least two questions have to be answered. The first is, what is the effect of fundamentally charged matter? It is known that if it propagates in the extra dimension, it breaks the Z(N) symmetry explicitly. One possible way to avoid this could be to confine fundamentally charged matter strictly to a set of branes, another to consider gauge fields not belonging to the Standard Model and only interacting with adjoint matter.

The second question is, would such domain walls have any practical significance? They have a huge energy density, and thus are not produced under any normal circumstances. But if the Z(N) symmetry gets restored at high temperatures, they could be produced in the Early Universe, which would give strong constraints on particle physics models of this type, or on cosmology. We have discussed the behaviour of the

Z(N) symmetry at high temperatures, and shown that at strict 1-loop level the symmetry does *not* get restored. We have argued however that non-perturbatively it does get restored at temperatures of order M/g_4^2, where M is the mass scale related to the compact direction. Under these circumstances, either particle physics models with only adjoint matter in the bulk, or cosmologies with temperatures above $\sim M/g_4^2$, can be excluded, because cosmologically produced domain walls would induce energy density fluctuations much larger than those observed in the cosmic microwave background.

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