Gluon Pair Production from a Space-Time Dependent Classical Chromofield via Vacuum Polarization

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Abstract

We investigate the production of gluon pairs from a space-time dependent classical chromofield via vacuum polarization within the framework of the background field method of QCD. The investigation of the production of gluon pairs is important in the study of the evolution of the quark-gluon plasma in ultra-relativistic heavy-ion collisions at RHIC and LHC.

I. INTRODUCTION

Ultra-relativistic heavy-ion collisions at RHIC and LHC will provide the best opportunity to study the color deconfined state of matter, namely the quark-gluon plasma (QGP). The space-time evolution of the QGP can be split into different stages: 1. the pre-equilibrium, 2. the equilibrium, and 3. the hadronization stage. One of the central problems in these experiments is to study how partons are formed and how their distribution function evolves in space-time to form an equilibrated quark-gluon plasma (if at all). High momentum partons ($p_T \geq 1GeV$), *i.e.* minijets are calculated using pQCD. Soft Parton Production is treated differently. There exist various model approaches: 1) In the HIJING model soft parton production is treated via string formations. 2) In the color flux-tube model, an extension of the model named before, they are treated via the creation of a classical chromofield. When partons and a classical chromofield are simultanously present, a relativistic non-abelian transport equation has got to be solved.

II. FIELD AND PARTICLE DYNAMICS

The space-time evolution of the partons can be studied by solving relativistic non-abelian transport equations for quarks and gluons [1,2]. As the chromofield exchanges color with quarks and gluons, color is a dynamical quantity. The time evolution of the classical color charge follows Wong's equations [3]:

$$\frac{dQ^a}{d\tau} = gf^{abc}u_\mu Q^b A^{c\mu}. \tag{1}$$

There is also a non-abelian version of the Lorentz-force equation:

$$\frac{dp^\mu}{d\tau} = gQ^a F^{a\mu\nu} u_\nu \tag{2}$$

Taking the above equations into account, one finds the relativistic non-abelian transport equation [1,2]:

$$[p_\mu \partial^\mu + gQ^a F^a_{\mu\nu} p^\nu \partial^\mu_p + gf^{abc} Q^a A^b_\mu p^\mu \partial^c_Q] f(x,p,Q) = C + S. \tag{3}$$

Note that there are seperate transport equations for quarks, anti-quarks, and gluons. The single-particle distribution function $f(x,p,Q)$ is defined in the 14-dimensional extended phase space of co-ordinate, momentum, and SU(3)-color. The first term on the LHS of Eq.(3) corresponds to convective flow, the second to the non-abelian generalization of the effect of the Lorentz force, and the third term describes the precession of the color charge in the presence of a classical field. On the RHS there is the collision term C and the source term for particle production S. For any system containing field and particles one has the following conservation equation:

$$\partial_\mu T^{\mu\nu}_{mat} + \partial_\mu T^{\mu\nu}_f = 0 \tag{4}$$

which is coupled with the above transport equation for the description of the QGP. The evolution of the plasma depends crucially on the source term S which contains all the information about how partons are produced from the classical chromofield.

III. PARTON PRODUCTION FROM A SPACE-TIME DEPENDENT CHROMOFIELD

The background field method of QCD is a suitable method to describe the production of partons from the QCD vacuum via vacuum polarization in the presence of a classical chromofield. Let us apply the background field method of QCD in order to describe the production of $q\bar{q}$-pairs.

The situation is similar to that of e^+e^--pair production described by Schwinger [4] in QED. For a space-time dependent classical field A_{cl} the amplitude for e^+e^--pair production (see Fig.(1)) from the vacuum is given by:

$$M = < k_1, k_2 | S^{(1)} | 0 > \quad = \quad -ie\bar{u}(k_1)\gamma_\mu A^\mu_{cl}(K = k_1 + k_2) v(k_2) \tag{5}$$

What, by the general formula:

$$W^{(1)} = \int \frac{d^3k_1}{(2\pi)^3 2k^0_1} \frac{d^3k_2}{(2\pi)^3 2k^0_2} \int d^4K (2\pi)^4 \delta^{(4)}(K - k_1 - k_2) \sum_{spin} |M|^2 \tag{6}$$

leads to the pair-production probability [5]:

$$W^{(1)}_{e^+e^-} = \frac{\alpha}{3} \int d^4K \ (1 - \frac{4m^2_e}{K^2})^{\frac{1}{2}} (1 + \frac{2m^2_e}{K^2}) [|K \cdot A_{cl}(K)|^2 - K^2 |A_{cl}(K)|^2] \tag{7}$$

where the d^4K-integral is defined for $K^2 > 4m^2_e$. Similarly, carrying out the same procedure in the non-abelian theory one finds for the amplitude for $q\bar{q}$-pair production:

$$M = ig\bar{u}^i(k_1)\gamma_\mu T^a_{ij} A^{a\mu}_{cl}(k_1 + k_2) v^j(k_2) \tag{8}$$

and for the corresponding probability [6]:

$$W^{(1)}_{q\bar{q}} = \sum_f \frac{\alpha_s}{6} \int d^4K \ (1 - \frac{4m^2_f}{K^2})^{\frac{1}{2}} (1 + \frac{2m^2_f}{K^2}) \ [|K \cdot A^a_{cl}(K)|^2 - K^2 |A^a_{cl}(K)|^2]. \tag{9}$$

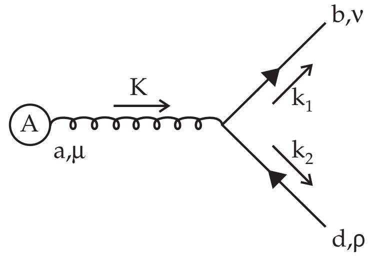

Fig. 1 Vacuum polarization diagram for the production of fermions in lowest order

A. Gluon-Pair Production from a Space-Time Dependent Chromofield

As conventional QCD cannot describe the interaction between a classical chromofield and a quantum gluon, one has to fall back on the background field method of QCD. This problem did not arise in QED, as there is no direct interaction between the classical field and the photon. That method was first introduced by DeWitt [7] and further developped by 't Hooft [8]. In the background field method of QCD, one defines:

$$A^{a\mu} = A^{a\mu}_{cl} + A^{a\mu}_{q}, \tag{10}$$

where A_{cl} will not be quantized. So the generating functional excluding quarks is:

$$Z[J, A_{cl}] = \int [dA_q] \det M_G \exp(i[S[A_q + A_{cl}] - \frac{1}{2\alpha} G \cdot G + J \cdot A_q]), \tag{11}$$

with the classical action:

$$S[A_q + A_{cl}] = -\frac{1}{4} \int d^4x \; (F^{a\mu\nu})^2, \tag{12}$$

where the field-tensor is defined as:

$$F^{a\mu\nu} = \partial^\mu (A^{a\nu}_q + A^{a\nu}_{cl}) - \partial^\nu (A^{a\mu}_q + A^{a\mu}_{cl}) + g \; f^{abc} \; (A^{b\mu}_q + A^{b\mu}_{cl})(A^{c\nu}_q + A^{c\nu}_{cl}). \tag{13}$$

The gauge fixing term G^a is chosen following 't Hooft:

$$G^a = \partial^\mu A^{a\mu}_q + g \; f^{abc} \; A^{b\mu}_{cl} A^{c\mu}_q. \tag{14}$$

The matrix element of M_G is given by:

$$(M_G(x, y))^{ab} = \frac{\delta(G^a(x))}{\delta\theta^b(y)} \tag{15}$$

which is the functional derivative of the gauge fixing term with respect to the infinitesimal change of the gauge parameter θ of the gauge transformation

$$\delta A_q^{a\mu} = -f^{abc}\,\theta^b(A_q^{c\mu} + A_{cl}^{c\mu}) \;+\; \frac{1}{g}\,\partial^\mu\theta^a. \tag{16}$$

Writing det M_G as functional integral over the ghost field, one obtains for the generating functional:

$$Z[J, A_{cl}, \xi, \xi^*] = \int [dA_q][d\chi][d\chi^*]$$
$$\times \exp(i[S[A_q + A_{cl}] + S_{ghost} - \frac{1}{2\alpha}G\cdot G + J\cdot A_q + \chi^*\xi + \xi^*\chi]), \tag{17}$$

where ξ and ξ^* are source functions for the ghosts and the ghost-part of the action is given by:

$$S_{ghost} = -\int d^4x\chi_a^\dagger[\Box^2\delta^{ab} + g\overleftarrow{\partial}_\mu f^{abc}(A_{cl}^{c\mu} + A_q^{c\mu})$$
$$-gf^{abc}A^{c\mu}\partial_\mu + g^2 f^{ace} f^{edb} A_{cl\mu}^c(A_{cl}^{d\mu} + A_q^{d\mu})]\chi_b. \tag{18}$$

Feynman rules involving a classical chromofield, gluons and ghosts can now be constructed from the above generating functional [9]. The vertices involving the coupling of two gluons to the classical field are given by:

$$(V_{1A})_{\mu\nu\rho}^{abd} = gf^{abd}[-2g_{\mu\rho}K_\nu + g_{\nu\rho}(k_1 - k_2)_\mu + 2g_{\mu\nu}K_\rho] \tag{19}$$

for coupling to the classical field once and by

$$(V_{2A})_{\mu\nu\lambda\rho}^{abcd} = -ig^2[f^{abx}f^{xcd}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda} + g_{\mu\nu}g_{\lambda\rho})$$
$$+f^{adx}f^{xbc}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$$
$$+f^{acx}f^{xbd}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\rho}g_{\nu\lambda})] \tag{20}$$

for coupling to the classical field twice.

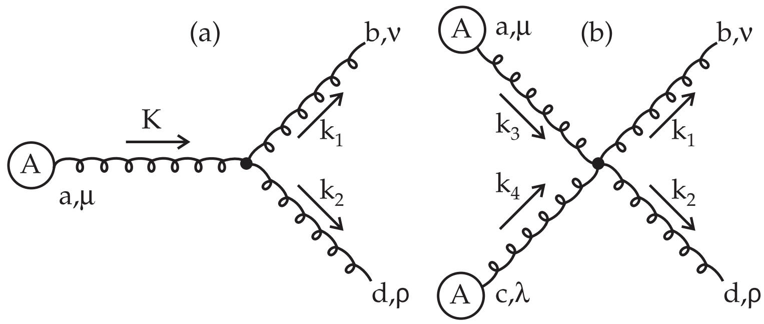

Fig. 2 Vacuum polarization diagrams for the production of gluons in lowest order.

Note that the above vertices are different from the three and four gluon vertices used in conventional QCD and that from hereon the classical field is denoted only by A not A_{cl}. The gluon production amplitude $M =< k_1k_2|S^{(1)}|0 >$ is defined in a way so that $S^{(1)}$ contains all interaction terms of the Lagrangian density involving two Q-fields, *i.e.*:

$$S^{(1)} = S_G^{(1)} + S_{GF}^{(1)}$$
$$= i\int d^4x(-\frac{1}{2}F^a_{\mu\nu}[A]gf^{abc}Q^{b\mu}Q^{c\nu}$$
$$-\frac{1}{2}(\partial_\mu Q^a_\nu - \partial_\nu Q^a_\mu)gf^{abc}(A^{b\mu}Q^{c\nu} + Q^{b\mu}A^{c\nu})$$
$$-\frac{1}{4}g^2 f^{abc}f^{ab'c'}(A^b_\mu Q^c_\nu + Q^b_\mu A^c_\nu)(A^{b'\mu}Q^{c'\nu} + Q^{b'\mu}A^{c'\nu}))$$
$$+i\int d^4x(-\partial_\lambda Q^{a\lambda}gf^{abc}A^b_\kappa Q^{c\kappa}$$
$$-\frac{1}{2}g^2 f^{abc}f^{ab'c'}A^b_\lambda Q^{c\lambda}A^{b'}_\kappa Q^{c'\kappa}). \tag{21}$$

The total amplitude $M = M_{1A} + M_{2A}$ consists of a contribution by the three-vertex (see Fig.(2)(a)):

$$M_{1A} = \frac{(2\pi)^2}{2}\int d^4K\delta^{(4)}(K - k_1 - k_2)$$
$$A^{a\mu}(K)\epsilon^{b\nu}(k_1)\epsilon^{d\rho}(k_2)(V_{1A})^{abd}_{\mu\nu\rho} \tag{22}$$

and one by the four-vertex (see Fig.(2)(b)):

$$M_{2A} = \frac{1}{4}\int d^4k_3 d^4k_4\delta^{(4)}(k_1 + k_2 - k_3 - k_4)$$
$$A^{a\mu}(k_3)A^{c\lambda}(k_4)\epsilon^{b\nu}(k_1)\epsilon^{d\rho}(k_2)(V_{2A})^{abcd}_{\mu\nu\lambda\rho}. \tag{23}$$

The above amplitudes include all the weight factors needed in order to retrieve the corresponding Lagrangian density. Now, we again calculate the pair production probability:

$$W = \sum_{spin}\int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}|M|^2. \tag{24}$$

To obtain the correct physical gluon polarizations in the final state we use:

$$\sum_{spin}\epsilon^\nu(k_1)\epsilon^{*\nu'}(k_1) = \sum_{spin}\epsilon^\nu(k_2)\epsilon^{*\nu'}(k_2) = -g^{\nu\nu'} \tag{25}$$

for the spin-sum and afterwards deduct the corresponding ghost contributions.
The probability for the gluon part becomes:

$$W^g = \frac{10}{8}\alpha_S\int d^4K$$
$$((A^a(K)\cdot A^{*a}(K))K^2 - (A^a(K)\cdot K)(A^{*a}(K)\cdot K))$$
$$+\frac{3ig\alpha_S}{4}\int d^4K d^4k_3$$
$$f^{aa'c'}[(A^a(K)\cdot A^{*a'}(-k_3))(A^{*c'}(K-k_3)\cdot K)]$$
$$+\frac{\alpha_S g^2}{16}\int d^4k_3 d^4k_3' d^4K$$
$$((A^a(k_3)\cdot A^c(K-k_3))(A^{*a'}(k_3')\cdot A^{*c'}(K-k_3'))$$
$$\times(f^{abx}f^{xcd} + f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd} + f^{a'dx'}f^{x'c'b})$$
$$+12f^{acx}f^{a'c'x}\times$$
$$(A^a(k_3)\cdot A^{*a'}(k_3'))(A^c(K-k_3)\cdot A^{*c'}(K-k_3'))). \tag{26}$$

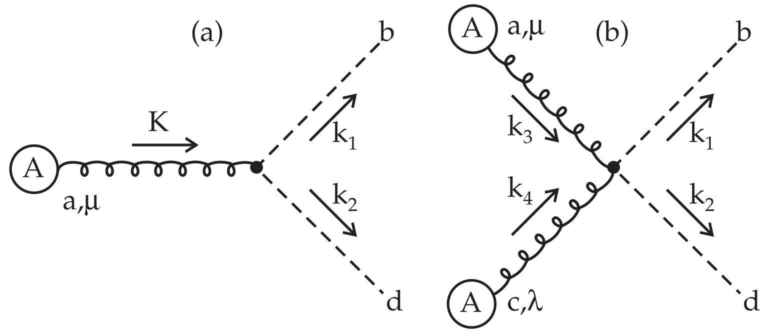

Fig. 3 Vacuum polarization diagram for the production of ghosts in lowest order

Now, we calculate the ghost part. The vertices involving two ghosts and one classical field and two ghosts and two classical fields respectively are given by:

$$(V_{1A}^{FP})_{\mu}^{abd} = +g f^{abd}(k_1 - k_2)_\mu \tag{27}$$

and:

$$(V_{2A}^{FP})_{\mu\lambda}^{abcd} = -ig^2 g_{\mu\lambda}(f^{abx}f^{xcd} + f^{adx}f^{xcb}). \tag{28}$$

The corresponding amplitude for the ghosts reads:

$$(M^{FP})^{bd} = (M_{1A}^{FP})^{bd} + (M_{2A}^{FP})^{bd} \tag{29}$$

with (see Fig.(3)(a)):

$$(M_{1A}^{FP})^{bd} = \frac{(2\pi)^2}{2} \int d^4K (2\pi)^4 \delta^{(4)}(k_1 + k_2 - K)\ A^{a\mu}(K)(V^{FP})_{\mu}^{abd} \tag{30}$$

and (see Fig.(3)(b)):

$$(M_{2A}^{FP})^{bd} = \frac{1}{4} \int d^4k_3 d^4k_4 \delta^{(4)}(k_1 + k_2 - k_3 - k_4) \\ A^{a\mu}(k_3) A^{c\lambda}(k_4)(W^{FP})_{\mu\lambda}^{abcd}. \tag{31}$$

The probability in this case is simply:

$$W^{FP} = \int \frac{d^3k_1}{(2\pi)^3 2k_1^0} \frac{d^3k_2}{(2\pi)^3 2k_2^0} (M^{FP})^{bd}(M^{FP})^{*bd}, \tag{32}$$

which becomes:

$$W^{FP} = -\frac{\alpha_S}{8} \int d^4K \delta^{(4)}(K - k_1 - k_2) \\ ((A^a(K) \cdot A^a(K))K^2 - (A^a(K) \cdot K)(A^a(K) \cdot K)) \\ -\frac{\alpha_S g^2}{32} \int d^4K d^4k_3 d^4k_3' \\ (A^a(k_3) \cdot A^c(K - k_3))(A^{a'}(k_3') \cdot A^{c'}(K - k_3')) \times \\ (f^{abx}f^{xcd} + f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd} + f^{a'dx'}f^{x'c'b}). \tag{33}$$

The real gluon-pair production probability is given by $W_{gg} = W^g - W^{FP}$.

Instead of the probabilities for pair production, one can also consider the corresponding source terms which then ultimatively enter the transport equation. The source terms are equal to the probability per unit of time and per unit volume of the phase space. Some calculations yield [10]:

$$\frac{dW_{q\bar{q}}^{(1)}}{d^4xd^3k} = \frac{g^2 m}{(2\pi)^5 \omega} \; A_\mu^a(x) \; e^{ik\cdot x} \int d^4x_2 \; A_\nu^a(x_2) \; e^{-ik\cdot x_2}$$
$$(i[k^\mu (x-x_2)^\nu + (x-x_2)^\mu k^\nu + k\cdot(x-x_2) g^{\mu\nu}]$$
$$(\frac{K_0(m\sqrt{-(x-x_2)^2}) m\sqrt{-(x-x_2)^2} + 2K_1(m\sqrt{-(x-x_2)^2})}{[\sqrt{-(x-x_2)^2}]^3})$$
$$-m^2 g^{\mu\nu} \frac{K_1(m\sqrt{-(x-x_2)^2})}{\sqrt{-(x-x_2)^2}}). \tag{34}$$

for the quarks and:

$$\frac{dW_{gg}}{d^4xd^3k} = \frac{1}{(2\pi)^5 k^0} \int d^4x' e^{ik\cdot(x-x')} \frac{1}{(x-x')^2}$$
$$\times\{\frac{3}{4} g^2 A^{a\mu}(x) A^{a\mu'}(x') [3k_\mu k_{\mu'} - 8 g_{\mu\mu'} k^\nu i \frac{(x-x')_\nu}{(x-x')^2}$$
$$+5(k_\mu i \frac{(x-x')_{\mu'}}{(x-x')^2} + k_{\mu'} i \frac{(x-x')_\mu}{(x-x')^2}) + \frac{6 g_{\mu\mu'}}{(x-x')^2} - 12 \frac{(x-x')_\mu (x-x')_{\mu'}}{(x-x')^4}]$$
$$-3ig^3 A^{a\mu}(x') A^{c\lambda}(x') A^{a'\mu'}(x) f^{a'ac} K_\lambda g_{\mu\mu'}$$
$$-\frac{1}{16} g^4 A^{a\mu}(x) A^{c\lambda}(x) A^{a'\mu'}(x') A^{c'\lambda'}(x')$$
$$[g_{\mu\lambda} g_{\mu'\lambda'} (f^{abx} f^{xcd} + f^{adx} f^{xcb})(f^{a'bx'} f^{x'c'd} + f^{a'dx'} f^{x'c'b})$$
$$+24 g_{\mu\mu'} g_{\lambda\lambda'} f^{acx} f^{a'c'x}]\}. \tag{35}$$

for the gluons. It can be checked that the above results are gauge invariant with respect to type-(I)-gauge transformations [10].

IV. DISCUSSION

The above results are still to complicated in order to directly get an idea about their content, so we look at them for a special, purely time dependent model field.

$$A^{a3}(t) = A_{in} e^{-|t|/t_0}, \; t_0 > 0, \; a = 1, ..., 8, \tag{36}$$

and all other components are equal to zero. Many other forms could have been taken. We have chosen this option just to get a feeling for how the source term in the phase-space behaves. The actual form of the decay of the classical field can only be determined from a self consistent solution of the relativistic non-abelian transport equations. The above choice yields:

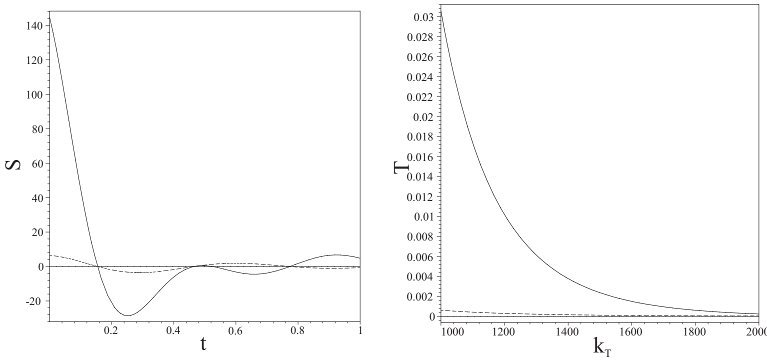

Fig. 4 Source term S [MeV] for quarks (dashed) and gluons (solid) production respectively versus time t [fm/c] and time-integrated source-tem T for quarks and gluons versus k_T [MeV] for the above choice of the model field.

$$\frac{dW_{q\bar{q}}}{d^4xd^3k} = 16\frac{\alpha_S}{(2\pi)^2}(A_{in})^2 e^{2i\omega t}e^{-|t|/t_0}\frac{t_0}{1+4\omega^2 t_0^2}\frac{m_T^2}{\omega^2}, \tag{37}$$

with $m_T^2 = m^2 + k_T^2$ where k_T is the transverse momentum and:

$$\begin{aligned}\frac{dW_{gg}}{d^4xd^3k} = & \frac{24\alpha_S}{(2\pi)^2}(A_{in})^2 e^{2ik^0 t}e^{-|t|/t_0}\frac{t_0}{1+4(k^0)^2 t_0^2}(-3 - \frac{k_T^2}{(k^0)^2}) \\ & + \frac{36\alpha_S^2}{2\pi}(A_{in})^4 e^{2ik^0 t}e^{-2|t|/t_0}\frac{t_0}{1+(k^0)^2 t_0^2}\frac{1}{(k^0)^2}.\end{aligned} \tag{38}$$

We choose the following parameters: $\alpha_S = 0.15$, $A_{in} = 1.5GeV$, $k_T = 1.5GeV$, $y = 0$, and $t_0 = 0.5fm$. Additionally, the quarks are considered to be massless. On the LHS of Fig.(4), the oscillatory behavior of the source terms S seems to indicate that there exist periods of particle creation and particle annihilation which follow each other periodically. This oscillatory behavior of the source term will play a crucial role once it is included in a self consistent transport calculation. It can also be seen in the Figure that there are considerably more gluons produced than quarks. On the RHS of Fig.(4), the time-integrated source terms T can be regarded as a measure for the net-production of particles in an infinitesimal volume around any given point in the phase-space. It does not show the oscillatory behavior which gives a totally different picture for different times. In future, we will include these source terms in the transport equation in order to study the production and equilibration of the QGP at RHIC and LHC.

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