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# On Non-Abelian Structure From Matrix Coordinates

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**Abstract**

We consider the matrix quantum mechanics of $N$ D0-branes in the background of the 1-form RR field. It is observed that the transformations of matrix coordinates of D0-branes induce on the Abelian RR field symmetry transformations that are like those of non-Abelian gauge fields. The Lorentz-like equations of motion for matrix coordinates are derived. The field strengths appearing in the Lorentz-like equations transform in the adjoint representation of $U(N)$ under symmetry transformations. A possible relation between D0-brane dynamics in RR background, and the semi-classical dynamics of charged particles in Yang-Mills background is mentioned.

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One of the most interesting aspects of D$p$-brane [1] dynamics is the appearance of "matrix coordinates" as the dynamical variables describing the position of coincident D$p$-branes. From String Theory point of view, this enhancement of degrees of freedom from numbers to matrices is due to the addition of dynamics of strings stretched between D$p$-branes to the usual degrees of freedom capturing the dynamics of each D$p$-brane individually. Consequently, it is understood that the correct degrees of freedom for a system of bound state of D$p$-branes (and strings) are matrices [2].

Though the appearance of matrix coordinates is interesting, a more interesting case arises when D$p$-branes are put in non-trivial backgrounds, namely non-trivial form fields. Accordingly, the situation with matrix coordinates will be serious while it is understood that the position dependences of background fields on transverse directions of D$p$-branes also should be given by matrix coordinates. Now a natural question can be about the consequences of the enhancement of degrees of freedom on the symmetry issues, and one of the important ones symmetry transformations. As we will see for the case of D0-branes, the symmetry transformations of matrix coordinates induce on the 1-form RR field symmetry transformations which are like those of non-Abelian gauge fields. In other words, the new transformations of the RR fields are the result of "active transformations on the matrix coordinates of space."

We also investigate the covariance of the equations of motion under the symmetry transformation of the 1-form RR fields. It is observed that the field strengths appearing in the equations of motion transform under symmetry transformations as those of Yang-Mills theory, *i.e.*, in adjoint representation. Finally, we comment on a possible relation between the dynamics of D0-branes and the semi-classical dynamics of charged particles in Yang-Mills backgrounds.

To be consistent with T-duality of String Theory, Myers [3] proposed an action containing the Born-Infeld and Chern-Simons parts for the dynamics of D$p$-brane bound states (see also [4]). The proposed bosonic action for the bound state of $N$ D$p$-branes (in units $2\pi l_s^2=1$) is the sum of:

$$S_{BI} = -T_p \int d^{p+1}\sigma^a \;\mathrm{Tr}\left(\mathrm{e}^{-\phi}\sqrt{-\det(P\{E_{ab}+E_{ai}(Q^{-1}-\delta)^{ij}E_{jb}\}+F_{ab})\det(Q^i_j)}\right), \quad (1)$$

$$S_{CS} = \mu_p \int \mathrm{Tr}\left(P\{\mathrm{e}^{i\,\mathbf{i}_\Phi \mathbf{i}_\Phi}(\sum C^{(n)}\mathrm{e}^B)\}\mathrm{e}^F\right), \quad (2)$$

with following definitions:

$$E_{\mu\nu} \equiv G_{\mu\nu}+B_{\mu\nu}, \quad Q^i_j \equiv \delta^i_j + i[\Phi^i,\Phi^j]E_{kj}, \quad (3)$$
$$\mu,\nu = 0,\cdots,9, \quad a,b=0,\cdots,p, \quad i,j=p+1,\cdots,9.$$

In the above $G_{\mu\nu}$ and $B_{\mu\nu}$ are the metric and the NS-NS 2-form, and $\Phi^i$ are world-volume scalars and $N\times N$ hermitian matrices, that describe the position of D$p$-branes in the $9-p$ transverse directions. The $C^{(n)}$ is $n$-form RR field, while $F$ is the $U(N)$ field strength. In this action, $P\{\cdots\}$ denotes the pull-back of the bulk fields to the world-volume of the D$p$-branes, and Tr is trace on the gauge group. $\mathbf{i}_{\mathbf{v}}$ denotes the interior product with a vector $\mathbf{v}$; for example, $\mathbf{i}_\Phi$ acts on a 2-form $C^{(2)}=\frac{1}{2}C^{(2)}_{ij}dx^idx^j$ as

$$\mathbf{i}_\Phi C^{(2)}=\Phi^i C^{(2)}_{ij}dx^j,\qquad \mathbf{i}_\Phi\mathbf{i}_\Phi C^{(2)}=\Phi^i\Phi^j C^{(2)}_{ij}=\frac{1}{2}[\Phi^i,\Phi^j]C^{(2)}_{ij}. \tag{4}$$

Therefore $(\mathbf{i}_\Phi)^2C^{(n)}{=}0$ for the commutative case, *i.e.*, for one D$p$-brane.

Some comments on the above action are in order:

*i)* All the derivatives in the longitudinal directions should be actually covariant derivatives, *i.e.*, $\partial_a\to D_a=\partial_a+i[A_a,\ ]$ [5]. This point is true also for the pull-back quantities.

*ii)* The pull-back quantities depend on the transverse directions of the D$p$-branes only via their functional dependence on the world-volume scalars $\Phi^i$ [6], ordered by "symmetrization prescription" [3, 7, 8, 9, 10]. For example for the case of metric $G_{\mu\nu}(x^\rho)$, we can present the $\Phi$ dependences by a non-Abelian Taylor expansion as [3]

$$\begin{aligned} G_{\mu\nu}(\sigma^a,x^i)|_{x\to\Phi} &\equiv G_{\mu\nu}(\sigma^a,\Phi^i)=\exp[\Phi^i\partial_{x^i}]G_{\mu\nu}(\sigma^a,x^i)\\ &= \sum_{n=0}^{\infty}\frac{1}{n!}\Phi^{i_1}\cdots\Phi^{i_n}(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}})G_{\mu\nu}(\sigma^a,x^i)|_{x^i=0}. \end{aligned} \tag{5}$$

*iii)* This action involves a single Tr , and this Tr should be calculated by symmetrization prescription for the non-commutative quantities $F_{ab}$, $D_a\Phi^i$ and $i[\Phi^i,\Phi^j]$ [11] [3].

Let us consider the special case of D0-branes, with $G_{\mu\nu}=\eta_{\mu\nu}$ and $B_{\mu\nu}=0$. The low energy action, with only non-vanishing the RR 1-form, is given by ($\sigma^0=t$) [3]

$$S=\int dt\ \mathrm{Tr}\left(\frac{1}{2}m_0D_t\Phi^iD_t\Phi_i-\mu_0C^{(1)}_t(\Phi,t)-\mu_0D_t\Phi^iC^{(1)}_i(\Phi,t)-V(\Phi)\right), \tag{6}$$

$$V(\Phi)=-\frac{1}{4}m_0[\Phi^i,\Phi^j]^2,\qquad D_t=\partial_t+i[A_0(t),\ ], \tag{7}$$

in which $C^{(1)}_t(\Phi,t)$ and $C^{(1)}_i(\Phi,t)$ are the pull-backs of the RR 1-form $C^{(1)}_\mu(x^\nu)$ to the world-line of the D0-brane bound state. Let us first check the effect of a $U(1)$ gauge transformation of the 1-form bulk field $C^{(1)}_\mu(x^\nu)$, defined by

$$C^{(1)}_\mu(x^\nu)\to C'^{(1)}_\mu(x^\nu)=C^{(1)}_\mu(x^\nu)-\partial_\mu\Lambda(x^\nu), \tag{8}$$

[3] There is a stronger prescription, with symmetrization between all non-commutative objects $F_{ab}$, $D_a\Phi^i$, $i[\Phi^i,\Phi^j]$, and the individual $\Phi$'s appearing in the functional dependences of the pull-back fields [3, 12]. We do not use this one here, with no essential change in the conclusions.

with $\Lambda(x^\nu)$ an arbitrary function in the bulk. Under this transformation, the variation of the action (6) is

$$\delta S \sim \mu_0 \int dt\ \mathrm{Tr}\left(\partial_t \Lambda(\Phi,t) + D_t\Phi^i \partial_i \Lambda(x,t)|_{x\to\Phi}\right), \tag{9}$$

in which the symmetrization prescription is understood after the replacements $x \to \Phi$. One then obtains

$$\delta S \sim \mu_0 \int dt\ \mathrm{Tr}\left(\partial_t \Lambda(\Phi,t) + \partial_t\Phi^i \partial_i \Lambda(x,t)|_{x\to\Phi} + i[A_0,\Phi^i]\partial_i \Lambda(x,t)|_{x\to\Phi}\right). \tag{10}$$

The first two terms yield as the surface term $d\Lambda(\Phi,t)/dt$, and therefore

$$\begin{aligned}\delta S &\sim \mu_0 \int dt\ \mathrm{Tr}\left(i[A_0,\Phi^i]\partial_i \Lambda(x,t)|_{x\to\Phi}\right) &(11)\\ &\sim \mu_0 \int dt\ \mathrm{Tr}\left(iA_0[\Phi^i,\partial_i \Lambda(x,t)|_{x\to\Phi}]\right). &(12)\end{aligned}$$

At first look this term seems non-vanishing, but in fact due to symmetrization prescription, it also vanishes [8]. Hence, thanks to the symmetrization, the action (6) is invariant under $U(1)$ gauge transformations in the bulk.

Actually the action (6) is also invariant under active transformations of coordinates, as

$$\begin{aligned}\Phi^i &\to \tilde{\Phi}^i = U^\dagger(\Phi,t)\Phi^i U(\Phi,t)\\ A_0(t) &\to \tilde{A}_0(\Phi,t) = U^\dagger(\Phi,t)A_0(t)U(\Phi,t) - iU^\dagger(\Phi,t)\partial_t U(\Phi,t),\end{aligned} \tag{13}$$

with $U(\Phi,t)$ as an arbitrary $N\times N$ unitary matrix; in fact under these transformations one obtains

$$\begin{aligned}D_t\Phi^i &\to \tilde{D}_t\tilde{\Phi}^i = U^\dagger(\Phi,t)D_t\Phi^i U(\Phi,t),\\ D_tD_t\Phi^i &\to \tilde{D}_t\tilde{D}_t\tilde{\Phi}^i = U^\dagger(\Phi,t)D_tD_t\Phi^i U(\Phi,t).\end{aligned} \tag{14}$$

Now, in the same spirit as for the previously introduced $U(1)$ symmetry of eq.(8), one finds the symmetry transformations:

$$\begin{aligned}\Phi^i &\to \tilde{\Phi}^i = U^\dagger(\Phi,t)\Phi^i U(\Phi,t)\\ A_0(t) &\to \tilde{A}_0(\Phi,t) = U^\dagger(\Phi,t)A_0(t)U(\Phi,t) - iU^\dagger(\Phi,t)\partial_t U(\Phi,t),\\ C_i^{(1)}(\Phi,t) &\to \tilde{C}_i^{(1)}(\Phi,t) = U^\dagger(\Phi,t)C_i^{(1)}(\Phi,t)U(\Phi,t) - iU^\dagger(\Phi,t)\partial_i U(x,t)|_{x\to\Phi},\\ C_t^{(1)}(\Phi,t) &\to \tilde{C}_t^{(1)}(\Phi,t) = U^\dagger(\Phi,t)C_t^{(1)}(\Phi,t)U(\Phi,t) - iU^\dagger(\Phi,t)\partial_t U(\Phi,t),\end{aligned} \tag{15}$$

in which we assume that $U(\Phi,t) = \exp(-i\Lambda)$ is arbitrary up to this condition that $\Lambda(\Phi,t)$ is totally symmetrized in the $\Phi$'s. The above transformation on the 1-form RR field is similar to those of non-Abelian gauge theories, and we see that it is just the consequence of the existing matrix coordinates. In other words, a $U(1)$ theory on a matrix coordinate space has symmetry transformations like those of a non-Abelian theory.

The above observation on gauge theory associated to D0-brane matrix coordinates on its own is not a new one, and we already know another example of this kind in non-commutative gauge theories. In spaces whose coordinates satisfy the algebra

$$[x^\alpha, x^\beta] = i\theta^{\alpha\beta}, \tag{16}$$

with constant $\theta^{\alpha\beta}$, the symmetry transformations of the $U(1)$ gauge theory are like those of non-Abelian gauge theory, and are known as non-commutative $U(1)$ transformations [13, 14, 15]. Note that the above algebra is satisfied also for the transformed coordinates $\tilde{x}^\alpha \equiv U^\dagger(x,t) x^\alpha U(x,t)$.

In addition, the case we see here for D0-branes may be considered as another example of the relation between gauge symmetry transformations and transformations of matrix coordinates [16].

The last notable points are about the behaviour of $A_0(t)$ and $C_t^{(1)}(\Phi,t)$ under symmetry transformations (15). From the world-line theory point of view, $A_0(t)$ is a dynamical variable, but $C_t^{(1)}(\Phi,t)$ should be treated as a part of background, however they behave similarly under transformations. Also we see by (15) that the time, and only time dependence of $A_0(t)$, which is the consequence of dimensional reduction, should be understood up to a gauge transformation.

As expected, the action (6) looks like that of an electric charged particle in an electromagnetic background $(C_t^{(1)}(x,t), C_i^{(1)}(x,t))$. Thus in principle, one expects to obtain Lorentz-like equations of motion from this action. For the moment ignoring the potential term $V(\Phi)$, one can derive the equations of motion for $\Phi^i$ and $A_0$ as

$$\begin{aligned} m_0 D_t D_t \Phi_i &= \mu_0 \Big( E_i(\Phi,t) + \underbrace{D_t \Phi^j B_{ji}(\Phi,t)} \Big) && (17)\\ m_0 [\Phi_i, D_t \Phi^i] &= \mu_0 [C_i^{(1)}(\Phi,t), \Phi^i], && (18) \end{aligned}$$

with the definitions:

$$\begin{aligned} E_i(\Phi,t) &\equiv -\partial_i C_t^{(1)}(x,t)|_{x\to\Phi} + \partial_t C_i^{(1)}(\Phi,t), && (19)\\ B_{ji}(\Phi,t) &\equiv \partial_j C_i^{(1)}(x,t)|_{x\to\Phi} - \partial_i C_j^{(1)}(x,t)|_{x\to\Phi} && (20) \end{aligned}$$

where the symbol $\underbrace{D_t \Phi^j B_{ji}(\Phi,t)}$ denotes the average over all of positions of $D_t\Phi^j$ between the $\Phi$'s of $B_{ji}(\Phi,t)$. As mentioned, the above equations for the $\Phi$'s are like the Lorentz equation of motion, with the exceptions that two sides are $N \times N$ matrices, while the time derivatives $\partial_t$ are replaced by their covariant counterpart $D_t$ [4].

It is a good exercise to study the behaviour of eqs. (17) and (18) under gauge transformation (15). Since the action is invariant under (15), it is expected that the equations of motion change covariantly. The left-hand side of (17) changes to $U^\dagger D_t D_t \Phi U$ by (14), and therefore we should find the same change for the right-hand side. This is in fact the case, since

$$\begin{aligned}
f(\Phi,t) &\rightarrow \tilde{f}(\tilde{\Phi},t) = U^\dagger(\Phi,t) f(\Phi,t) U(\Phi,t),\\
\frac{\delta f(\Phi,t)}{\delta \Phi^i} &\rightarrow \frac{\tilde{\delta}\tilde{f}(\tilde{\Phi},t)}{\delta\tilde{\Phi}^i} = U^\dagger(\Phi,t)\frac{\delta f(\Phi,t)}{\delta\Phi^i}U(\Phi,t),\\
\frac{\partial f(\Phi,t)}{\partial t} &\rightarrow \frac{\partial\tilde{f}(\tilde{\Phi},t)}{\partial t} = U^\dagger(\Phi,t)\frac{\partial f(\Phi,t)}{\partial t}U(\Phi,t),
\end{aligned} \tag{21}$$

in which $\partial_i$ has been realized via its functional form, $\delta/\delta\Phi^i$. In conclusion, the definitions (19) and (20), lead to

$$\begin{aligned}
E_i(\Phi,t) &\rightarrow \tilde{E}_i(\tilde{\Phi},t) = U^\dagger(\Phi,t) E_i(\Phi,t) U(\Phi,t),\\
B_{ji}(\Phi,t) &\rightarrow \tilde{B}_{ji}(\tilde{\Phi},t) = U^\dagger(\Phi,t) B_{ji}(\Phi,t) U(\Phi,t),
\end{aligned} \tag{22}$$

a result consistent with the fact that $E_i$ and $B_{ji}$ are functionals of $\Phi$. We thus see that, in spite of the absence of the usual commutator term $i[A_\alpha, A_\beta]$ of non-Abelian gauge theories, in our case the field strengths transform like non-Abelian ones. We recall that these are all consequences of the matrix coordinates of D0-branes. Finally by the similar reason of vanishing of (12), both sides of (18) transform identically.

An equation of motion similar to (17) is considered in [17, 18] as a part of similarities between the dynamics of D0-branes and bound states of quarks–QCD strings [17, 18, 19]. The point is that the center-of-mass dynamics of D0-branes is not affected by the non-Abelian sector of the background, *i.e.*, the center-of-mass is "white" with respect to $SU(N)$ sector of $U(N)$. The center-of-mass coordinates and momenta are defined by:

$$\Phi^i_{c.m.} \equiv \frac{1}{N}\mathrm{Tr}\ \Phi^i, \qquad P^i_{c.m.} \equiv \mathrm{Tr}\ P^i_\Phi, \tag{23}$$

where we are using the convention Tr $\mathbf{1}_N = N$. To specify the net charge of a bound state, its dynamics should be studied in zero magnetic and uniform electric fields, *i.e.*, $B_{ji} = 0$ and $E_i(\Phi,t) = E_{0i}$ [5]; thus these fields are not involved by $\Phi$ matrices, and contain just

[4] $D_t$ is absent in the definition of $E_i$, because, the combination $i[A_0, C_i]$ has been absorbed to produce $D_t\Phi^j$ for both parts of $B_{ji}$.

[5] In a non-Abelian gauge theory an uniform electric field can be defined up to a gauge transformation, which is quite well for identification of white (singlet) states.

the $U(1)$ part. In other words, under gauge transformations $E_{0i}$ and $B_{ji}=0$ transform to $\tilde{E}_i(\Phi,t)=V^\dagger(\Phi,t)E_{0i}V(\Phi,t)=E_{0i}$ and $\tilde{B}_{ji}=0$. Thus the action (6) yields the following equation of motion:

$$(Nm_0)\ddot{\Phi}^i_{c.m.}=\mu_0 N E^i_{0(1)}, \tag{24}$$

in which the subscript (1) denotes the $U(1)$ electric field. So the center-of-mass only interacts with the $U(1)$ part of $U(N)$. From the String Theory point of view, this observation is based on the simple fact that the $SU(N)$ structure of D0-branes arises just from the internal degrees of freedom inside the bound state.

It will be interesting to mention the relation between the dynamics of D0-branes in RR background, and the semi-classical equations of motion for charged particles in Yang-Mills background. By semi-classical, as will be more clear later, we mean treating the space-time motion of charged particles classically, while describing the charge degrees of freedom, calling them "isotopic spin," quantum mechanically. The classical mechanics of the charged particles is known by the original work of Wong [20], based on an appropriate limit of the equations of motion of operators; see [21] as a good review. The starting point is the standard action of $U(N)$ gauge theory, accompanied with fermionic matter in the fundamental representation (in units $c=1,\ \hbar=1$)

$$S=\int d^dx\Big(-\bar{\psi}\gamma^\mu(\partial_\mu+iA_\mu)\psi-m\bar{\psi}\psi-\frac{1}{4g^2}\mathrm{Tr}\ F_{\mu\nu}F^{\mu\nu}\Big), \tag{25}$$

$$\mu,\nu=0,\cdots,d-1,\quad A^\mu=A^\mu_aT^a,\quad F^{\mu\nu}=F^{\mu\nu}_aT^a. \tag{26}$$

In above $T^a$, $a=1,\cdots,N^2$, are $N\times N$ matrices as generators of the $U(N)$ group, with the commutation relation $[T^a,T^b]=if^{ab}_cT^c$; we assume sum over the lower and upper indices. Consequently, the classical equation of motion for the charged particle are proposed as

$$m\frac{d^2\xi^\mu(\tau)}{d\tau^2}=\Big(F^{\mu\nu}_a(\xi)T^a_{\rm cl}(\tau)\Big)\frac{d\xi_\nu(\tau)}{d\tau}, \tag{27}$$

where $\xi^\mu(\tau)$ denote the world-line of the particle, parametrised with $\tau$. In above, $T^a_{\rm cl}(\tau)$ are numbers as the classical analogues of the matrices $T^a$, with the canonical relation $\{T^a_{\rm cl}(\tau),T^b_{\rm cl}(\tau)\}=f^{ab}_cT^c_{\rm cl}(\tau)$, and the equation of motion as

$$\frac{dT^a_{\rm cl}(\tau)}{d\tau}-\frac{d\xi_\mu}{d\tau}f^{ab}_cA^\mu_b(\xi)T^c_{\rm cl}(\tau)=0. \tag{28}$$

Thus, the particle is described by an internal vector $T^a_{\rm cl}(\tau)$ as well as its space-time coordinates $\xi^\mu(\tau)$. Also, by the equations for $T^a_{\rm cl}(\tau)$ one deduces that $d/d\tau(T^a_{\rm cl}T_{{\rm cl}a})=0$. Hence the isotopic spin of the particle performs a precessional motion.

Now we try to sketch the relation. Let us first take the simple case of one charged particle. One may define the covariant derivative along the world-line by notion of world-line gauge field $A_\tau \equiv \dot{\xi}_\mu A^\mu_{\rm cl}(\xi,\tau) = \dot{\xi}_\mu A^\mu_a(\xi) T^a_{\rm cl}(\tau)$, as follows

$$D_\tau \equiv \partial_\tau - \{A_\tau,\ \}. \tag{29}$$

We notice that, from the world-line theory point of view, $A_\tau$ is a dynamical variable. Hence the equation of motion (28) reduces to $D_\tau T^a_{\rm cl}(\tau) = 0$. Then we define the variables as $X^{\mu a}(\tau) \equiv \xi^\mu(\tau) T^a_{\rm cl}(\tau)$; for these variables we have

$$D_\tau X^{\mu a} = \dot{\xi}^\mu(\tau) T^a_{\rm cl}(\tau). \tag{30}$$

Also by using (27) we find that

$$\begin{aligned} m D_\tau D_\tau X^{\mu a} &= m\ddot{\xi}^\mu(\tau) T^a_{\rm cl}(\tau) = F^{\mu\nu}_{\rm cl}(\xi,\tau)\dot{\xi}_\nu T^a_{\rm cl}(\tau) \Rightarrow \\ m D_\tau D_\tau X^{\mu a} &= F^{\mu\nu}_{\rm cl}(\xi,\tau) D_\tau X^a_\nu, \end{aligned} \tag{31}$$

in which we are using the notation $F^{\mu\nu}_{\rm cl}(\xi,\tau) = F^{\mu\nu}_a(\xi) T^a_{\rm cl}(\tau)$. The last equation for $X^a$ are reminiscent of the D0-brane equation of motion in the RR 1-form background, with this exception that here the field strength $F^{\mu\nu}_{\rm cl}(\xi,\tau)$ depends on variables $\xi^\mu$. Here one can define a map from fields $F^{\mu\nu}_{\rm cl}(\xi,\tau)$ to new fields $\hat{F}^{\mu\nu}(X^a,\tau)$. The map defines the fields $\hat{F}^{\mu\nu}(X^a,\tau)$ by the relation

$$F^{\mu\nu}_{\rm cl}(\xi,\tau) = F^{\mu\nu}_a(\xi) T^a_{\rm cl}(\tau) \equiv \hat{F}^{\mu\nu}(X^a,\tau). \tag{32}$$

Thus we have the equations: $m D_\tau D_\tau X^{\mu a} = \hat{F}^{\mu\nu}(X^a,\tau) D_\tau X^a_\nu$. The map also may be defined at the level of gauge potentials $A^\mu_{\rm cl}(\xi,\tau)$ and $\hat{A}^\mu(X^a,\tau)$, in a Lagrangian or Hamiltonian formulation of the problem.

The last equation we obtained have already the extra index $a$ on the variables $X^{\mu a}$. One may consider $N^2$ copies of variables $\xi^\mu_a(\tau)$, with common charge variables $T^a_{\rm cl}(\tau)$. Since in this case there are $N^2$ copies of variables, and so world-lines, the definition of a unique world-line covariant derivative $D_\tau$ is not possible. The best one can think about, as the case for coincident D0-branes, is that the $N^2$ copies are nearly "identified" in space-time. Thus we assume $\xi_a(\tau) = \xi(\tau) + \delta\xi_a(\tau)$, with $\delta\xi_a \ll \xi$. Actually the thing one needs is a unique combination of $A_\tau = \dot{\xi}_{a\mu} A^\mu_{\rm cl}(\xi_a,\tau)$ as a unique world-line gauge field. So to have a unique world-line gauge field, and consequently common charge variables $T^a_{\rm cl}(\tau)$, we have the condition $\dot{\delta}\xi_{a\mu} A^\mu_a + \dot{\xi}_{a\mu}\delta\xi^\rho_a \partial_\rho A^\mu_a = 0$, for each $a$. Therefore we can define

$$D_\tau \equiv \partial_\tau - \{A_\tau,\ \}, \tag{33}$$

with $A_\tau \equiv \dot{\xi}_\mu A^\mu_{\rm cl}(\xi,\tau)$. Thus all of the charges can be taken equal, satisfying $D_\tau T^a_{\rm cl}(\tau)=0$. Then we define $X^\mu(\tau)=\xi^\mu_a(\tau)T^a_{\rm cl}(\tau)$, and accordingly we have

$$\begin{aligned} mD_\tau D_\tau X^\mu &= m\ddot{\xi}^\mu_a T^a_{\rm cl}, \\ mD_\tau D_\tau X^\mu &= F^{\mu\nu}_{\rm cl}(\xi_a,\tau)\dot{\xi}_{a\nu}T^a_{\rm cl}(\tau), \\ mD_\tau D_\tau X^\mu &= \Big(F^{\mu\nu}_b(\xi_a)T^b_{\rm cl}(\tau)\Big)\dot{\xi}_{a\nu}T^a_{\rm cl}(\tau), \end{aligned} \tag{34}$$

where in the right-hand side the sums on $a$ and $b$ are recalled. The dependence of field strength on $\xi_a$ prevents us to do the sums on $a$ and $b$ independently, to get $\dot{\xi}_{a\nu}T^a_{\rm cl}(\tau)=D_\tau X_\nu$. Like the case for a single particle, we can define the map between fields $F^{\mu\nu}_{\rm cl}(\xi_a,\tau)$ and $\hat{F}^{\mu\nu}(X,\tau)$ such that the expression in the right-hand side appears as following

$$\Big(F^{\mu\nu}_b(\xi_a)T^b_{\rm cl}(\tau)\Big)\dot{\xi}_{a\nu}T^a_{\rm cl}(\tau)\equiv \hat{F}^{\mu\nu}(X,\tau)D_\tau X_\nu. \tag{35}$$

The map can also be presented in components with group indices. Finally one concludes with equations

$$mD_\tau D_\tau X^\mu = \hat{F}^{\mu\nu}(X,\tau)D_\tau X_\nu. \tag{36}$$

All the relations we had in above, rather than matrices, were about numbers unfortunately! It is because that in Wong's theory also the charge variables are treated with their classical analogues. It may be possible to find a semi-classical version of the problem, assuming space-time motion classically, while the isotopic spin $T^a$'s remain matrix variables, as they should be as group generators; similar to the situation we have for ordinary spin in Stern-Gerlach experiment. Then, a relation between the semi-classical dynamics of charged particles and D0-brane dynamics will appear very interesting. It is remain for future progresses to know more about both D0-brane dynamics and charged particle dynamics in Yang-Mills background.

All the above can be considered in non-relativistic limit, though the case needs more study, may be defined by [6]

$$D_\tau X^0 \simeq 1, \quad \dot{\xi}^i_a \ll 1. \tag{37}$$

In this limit we have $A_\tau(t)=A_{0a}(t,\xi^i)T^a_{\rm cl}(t)+O(\dot{\xi}^i)$, which means that the world-line gauge field equals effectively to zero component of gauge field, which for very small velocities may be assumed to be function of $t$ only.

[6]For the case concerning more than one massless particle, the more systematic way can be going to the light-cone gauge to recover non-relativistic dynamics in the transverse directions; see Appen. of [18].

In the above we saw the important role of a map between the Yang-Mills fields, and fields which depend on $X^\mu$ variables. In [13] a map between field configurations of non-commutative and ordinary gauge theories is introduced, which preserves the gauge equivalence relation. It is emphasized that the map is not an isomorphism between the gauge groups. It will be interesting to study the properties of the map between non-Abelian gauge theory and gauge theory associated with matrix coordinates of D0-branes; on one side the quantum theory of matrix fields, and on the other side the quantum mechanics of matrix coordinates. Since in this case we have matrices on both sides, it may be possible to find an isomorphism between all objects involving in the two theories, *i.e.*, dynamical variables and transformation parameters.

We have seen how the $U(1)$ symmetry of the 1-form RR field $C^{(1)}_\mu(x^\nu)$ can show a $U(N)$ structure inside the bound states. The $U(N)$ structure in bound states is a consequence of the fact that the correct dynamical variables of the bound states are $N \times N$ hermitian matrices, rather than numbers. From the String Theory point of view, this is possible by taking into account degrees of freedom corresponding to $N$ copies of the $U(1)$ structure, together with $N^2 - N$ additional ones coming from the dynamics of strings stretched between charged particles. Here each D0-brane carry $1/N$ fraction of the bound state total charge. This is an example of a mechanism that how small (fractional) charges of an Abelian symmetry can form a bound state with an internal non-Abelian symmetry; a mechanism which may be called "non-Abelian from fractional Abelian charges."

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