# Bloch-Nordsieck Violation in Spontaneously Broken Abelian Theories *

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We point out that, in a spontaneously broken $U(1)$ gauge theory, inclusive processes, whose primary particles are mass eigenstates that do not coincide with the gauge eigenstates, are not free of infrared logarithms. The charge mixing allowed by symmetry breaking and the ensuing Bloch-Nordsieck violation are here analyzed in a few relevant cases and in particular for processes initiated by longitudinal gauge bosons. Of particular interest is the example of weak hypercharge in the Standard Model where, in addition, left-right mixing effects arise in transversely polarized fermion beams.

The planning of TeV scale accelerators has brought attention to the fact that the Standard Model, at energies larger than the weak scale, shows enhanced double log corrections [1] of infrared origin, even in inclusive observables. Such enhancements, involving the effective coupling $(\alpha_W/4\pi)\log^2(s/M_W^2)$, signal a lack of compensation of virtual corrections with real emission in the $M_W^2 \ll s$ limit, due to the non abelian (weak isospin) charges of the accelerator beams. In other words, the Bloch-Nordsieck (B-N) cancellation theorem [2], valid in QED, is here violated.

The key point which invalidates the B-N cancellation is the fact that gauge boson emission off one incoming beam state changes it into another state of the same gauge multiplet (e.g., a neutrino for an incoming electron) and the latter happens to have a different cross section off the other beam. As a consequence, virtual corrections are unable to cancel this contribution except on the average, i.e., by summing over all possible beams in the multiplet.

It is usually thought that such phenomenon cannot occur in the abelian case, because initial states (the mass eigenstates) are charge eigenstates, which do not change during the (neutral) gauge boson emission, so that the real-virtual cancellation is valid.

In this note we point out that, in case of spontaneous symmetry breaking, the B-N theorem is violated in *abelian theories* too. The point is that, in a broken theory, mass eigenstates can be mixed charge states, so that soft boson emission is off diagonal. For instance, if a normal Higgs mechanism [3] is assumed, longitudinal gauge bosons can occur as (massive) initial states which act as mixed charge states and interact with the (similarly mixed) Higgs boson. As a consequence, longitudinal and Higgs bosons are interchanged during soft emission, and the basic noncancellation mechanism is again at work, as in the non abelian case illustrated before.

In order to understand this point, let us recall the structure of soft interactions accompanying a hard process of type $\{\alpha_I p_I\} \to \{\alpha_F p_F\}$, where $I$=1,2, $F$=1,2,...,n, and $p$'s and $\alpha$'s denote momenta and charge states of initial and final asymptotic states, which are mass eigenstates. The corresponding S-matrix is an operator in the soft Hilbert space and a matrix in the hard labels, with form [4,5]

$$S = \mathcal{U}^F_{\alpha_F\alpha'_F}(a_s, a_s^\dagger) \quad S^H_{\alpha'_F\alpha'_I}(p_F, p_I) \quad \mathcal{U}^I_{\alpha'_I\alpha_I}(a_s, a_s^\dagger) \tag{1}$$

where $\mathcal{U}^F$ and $\mathcal{U}^I$ are unitary coherent state operators, functionals of the soft emission operators $a_s, a_s^\dagger$. They take in the abelian case a simple eikonal form [4] and are *diagonal* with respect to charge eigenstates, i.e., they have a well defined form for each energetic particle of well defined charge.

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An inclusive observable is obtained by squaring and summing eq.(1) over soft final states. In this procedure, the coherent state $\mathcal{U}^F$ cancels out by unitarity, and we are left with the overlap matrix

$$\mathcal{O}_{\beta_I \alpha_I} = {}_S\langle 0|\mathcal{U}^{I\dagger}_{\beta_I \beta'_I}(S^{H\dagger}S^H)_{\beta'_I \alpha'_I}\mathcal{U}^I_{\alpha'_I \alpha_I}|0\rangle_S \tag{2}$$

where an average over the state with no soft quanta is made in the initial state. We also refer to $\mathcal{O}^H = S^{H\dagger}S^H$ as the hard overlap matrix, and we allow in general $\beta_I \neq \alpha_I$, even if a cross section with initial charge state $\alpha_I$ is diagonal, i.e., $\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I \alpha_I}$ (no sum over $\alpha_I$).

The abelian Bloch-Nordsieck cancellation theorem is valid if the initial mass eigenstates are also charge eigenstates. In fact, in such case $\mathcal{U}^I$ is diagonal with respect to the labels $\alpha_I = (\alpha_1, \alpha_2)$ which represent definite charges, i.e.,

$$\mathcal{U}^I_{\alpha'_I \alpha_I} = \delta_{\alpha'_I \alpha_I}\,\mathcal{U}^{\alpha_I p_I}, \qquad \mathcal{U}^{\alpha_I p_I} \equiv \Pi_{i=1,2}\,\mathcal{U}^{\alpha_i p_i}(a_s, a_s^\dagger) \tag{3}$$

Therefore, the inclusive cross section becomes, by eq.(2)

$$\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I \alpha_I} = {}_S\langle 0|\mathcal{U}^{\alpha_I p_I\dagger}\,\mathcal{O}^H_{\alpha_I \alpha_I}\,\mathcal{U}^{\alpha_I p_I}|0\rangle_S \tag{4}$$

where $\alpha_I$, in both $\mathcal{U}$ and $\mathcal{U}^\dagger$, is now the same set of labels (with no sum). Since soft operators only occur in the $\mathcal{U}$'s, the latter commute with $\mathcal{O}^H$, and soft enhancements cancel out by unitarity, in a trivial way.

The above reasoning fails in the non abelian case, because both $\mathcal{U}$ and $\mathcal{O}^H$ are (non commuting) matrices in a non abelian charge multiplet, and one is unable to use the unitarity sum. But it fails in the abelian case too, if the initial states *are not* charge eigenstates, as allowed by symmetry breaking. In such a case, the coherent states are not diagonal in the initial labels $\alpha_I$, and normally do not commute with the hard overlap matrix $\mathcal{O}^H$. More precisely, by introducing the mixing matrix $\mathcal{M}_{A\alpha}$ and the overlap matrix $\mathcal{O}_{AB}$ in the charge eigenstates basis $\{A\}$, we obtain:

$$\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I \alpha_I} = \sum_{A,B} \mathcal{M}^\dagger_{\alpha_I B}\,\mathcal{O}_{BA}\,\mathcal{M}_{A\alpha_I} \tag{5}$$

While soft enhancements cancel out by eq.(4) in the diagonal terms $\mathcal{O}_{AA}$ of the sum (5), they are non vanishing in the off diagonal ones $\mathcal{O}_{AB}$, $(A \neq B)$, which are induced by the mixing, so that the BN theorem is violated.

We shall illustrate the features above in the example of the longitudinal sector of the $U(1)$ Higgs model [3].

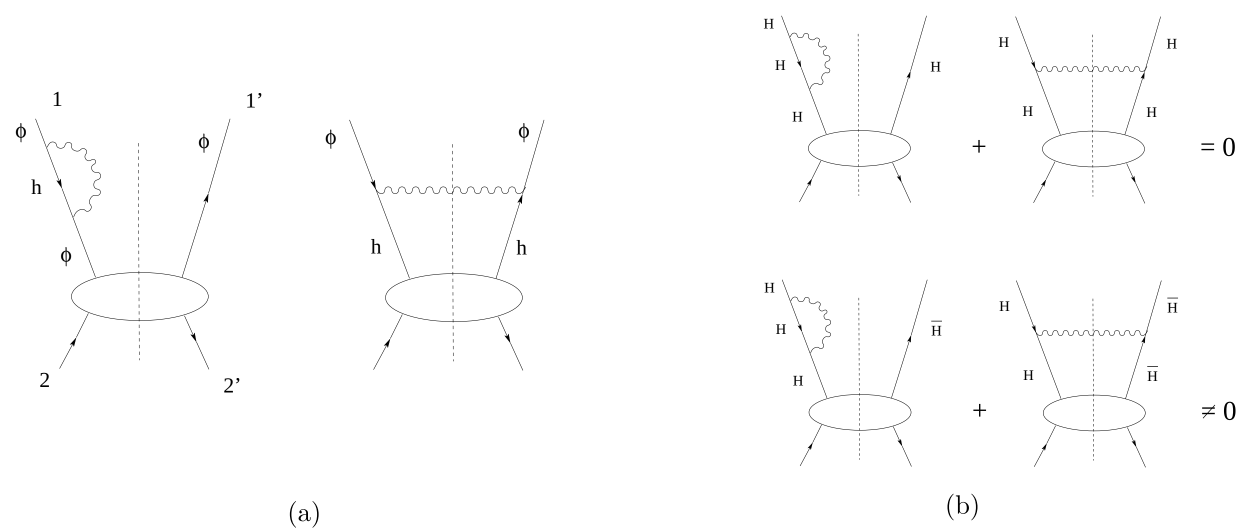


FIG. 1. Picture of radiative corrections to the overlap matrix in (a) the mass eigenstates basis and (b) the charge eigenstates basis, where off-diagonal matrix elements occur.

The Lagrangian in a 't Hooft gauge is

$$\mathcal{L} = (D_\mu\Phi)^+\,D^\mu\Phi - V[\Phi^+\Phi] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\,\zeta}(\partial_\mu A^\mu - \zeta\,M\,\Phi)^2 + \bar{\Psi}(i\not{D} - m)\Psi \tag{6}$$

where $\Phi - v/\sqrt{2} = H = (h + i\phi)/\sqrt{2}$ is the Higgs field, $V$ is the potential, $M$ is the gauge boson mass, $\zeta$ is the gauge parameter, and charged fermions of mass $m$ have been introduced. We also take $M_H \simeq M$ as $h$ field mass, the case $M_H \gg M$ being discussed in [6].

The states we consider are the Higgs boson $h$ and the longitudinal boson $A^L_\mu \equiv L$ as prepared, e.g., by coupling to initial charges in a boson fusion process. Longitudinal amplitudes are related to the Goldstone boson ones by the equivalence theorem [7]

$$\frac{p^\mu}{M}\,\mathcal{M}_\mu(p;...) = i\mathcal{M}(\phi(p);...), \qquad (p^2 \simeq M^2) \tag{7}$$

where the remaining amplitude labels are understood. For this reason, the soft emission properties in the $L/h$ sector are determined by the current $h(x)\,\overleftrightarrow{\partial_\mu}\,\phi(x)$ of the scalar sector.

At leading double log level, the emission of a soft gauge boson off an energetic longitudinal boson $\epsilon^L_\mu(p) \sim p_\mu/M + O(M/E)$ changes it into a Higgs boson, and all subsequent interactions are described by the eikonal current [5,6]

$$J^\mu_{\alpha\beta}(k) = e\,\frac{p^\mu}{p\cdot k}\,q_{\alpha\beta} \qquad (\alpha,\beta = \phi, h) \tag{8}$$

where $q = \tau_2$ is just a Pauli matrix connecting the $L/\phi$ and $h$ indices. The peculiarity of eq.(8) is that it is off diagonal, as expected from the fact that mass eigenstates are the mixed charge states $h = 1/\sqrt{2}(H + H^\dagger)$, $\phi = -i/\sqrt{2}(H - H^\dagger)$.

The actual evaluation of double logs in eq. (4) is simplified by the remark that the eikonal current (8) is conserved in the fixed angle, high energy regime $s \gg M^2$ that we are investigating. This means that, by applying the current $J^\mu$ to states in the overlap matrix $\langle\beta_1\beta_2|\mathcal{O}|\alpha_1\alpha_2\rangle$, we have

$$k^\mu J_\mu(k)\,\mathcal{O} = \sum_i q_i\,\mathcal{O} = (q_1 + q_2 - q_{1'} - q_{2'})\mathcal{O} = 0 \tag{9}$$

where the sum runs over all legs of the overlap matrix, as depicted in Fig.1(a). Furthermore, if we like, we can diagonalize each leg charge $q_i$ by reverting to the complex field $H$ ($H^\dagger$) with charge $q = 1 (q = -1)$.

By using charge conservation for $s \gg M^2$, the total eikonal current occurring in the coherent states of eq.(2) can be expressed in the simple form

$$J^\mu = e\sum_i q_i \frac{p^\mu_i}{p_i k} = e\,Q\,(\frac{p^\mu_1}{p_1 k} - \frac{p^\mu_2}{p_2 k}) \tag{10}$$

where $Q = q_1 - q_{1'} = q_{2'} - q_2$ is the total t-channel charge. Therefore the eikonal radiation factor, involving the squared eikonal current $J^\mu J_\mu$, becomes:

$$-Q^2\,\frac{e^2}{8\pi^3}\int \frac{d^3k}{2\,\omega_k}\frac{p_1\cdot p_2}{p_1\cdot k\;p_2\cdot k} \equiv -Q^2\,\mathcal{L} \tag{11}$$

where $\mathcal{L} = (\alpha/4\pi)log^2(s/M^2)$ is the effective double log coupling mentioned above. The structure of radiative corrections is then the one depicted in Fig.1(a), where for each power of $\alpha$ the operator $-Q^2 = -(q_1 - q'_1)^2 = -2(1 - q_1 q'_1)$ is applied. While the virtual corrections are diagonal, the term $q_1 q'_1$, corresponding to real emission, exchanges the $h$ and $\phi$ indices on both legs, as anticipated before. If we fix $\alpha_2 = \beta_2 = L$, and we define $\sigma_\alpha = \sigma_{\alpha L}$, ($\alpha = L, h = \phi, h$), the action of $q_1 q'_1$ on the $\alpha$ indices is that of a $\tau_1$ Pauli matrix. Therefore, by restoring the full radiation factor, we find

$$\sigma_\alpha = \left(e^{-2\mathcal{L}(1-\tau_1)}\right)_{\alpha\beta}\sigma^H_\beta \tag{12}$$

where the $\sigma^H$'s are the hard (tree level) cross sections. The final result (12) is easily recast in the diagonal form

$$\sigma_{LL} + \sigma_{hL} = \sigma^H_{LL} + \sigma^H_{hL}, \qquad \sigma_{LL} - \sigma_{hL} = (\sigma^H_{LL} - \sigma^H_{hL})e^{-4\mathcal{L}} \tag{13}$$

This means that the average cross section has no radiative corrections, while the difference is suppressed by the form factor corresponding to t-channel charge $Q^2 = 4$. Therefore, at infinite energy, radiative corrections equalize the longitudinal and Higgs cross sections.

The occurrence of the t-channel charge $Q^2 = 4$ is related to the basic fact that $h$ and $L$ are not charge eigenstates, due to symmetry breaking at low energies. In fact, by rewriting the cross sections in term of the charge eigenstates $H$ and $H^\dagger$, and by using charge conjugation invariance we find

$$\begin{aligned}\sigma_{LL} = \sigma_{hh} = \tfrac{1}{2}(\sigma_{HH} + \sigma_{H\bar{H}}) + Re\mathcal{O}(H\bar{H} \to \bar{H}H) \\ \sigma_{Lh} = \tfrac{1}{2}(\sigma_{HH} + \sigma_{H\bar{H}}) - Re\mathcal{O}(H\bar{H} \to \bar{H}H)\end{aligned} \tag{14}$$

where, as in eq. (5) we notice the occurrence of the off diagonal overlap matrix elements $\mathcal{O}$ and $\mathcal{O}^\dagger$, corresponding to the values $Q_{tot} = q_1 - q_1' = \pm 2$ of the total charge in the t-channel (Fig.1(b)). While the diagonal terms $\sigma_{HH}$ and $\sigma_{H\bar{H}}$ correspond to $Q = 0$ and have no form factor, the off diagonal ones are suppressed by the form factor with $Q^2 = 4$, already found before. Therefore, from eq.(14) we find the expressions

$$\begin{aligned}\sigma_{LL}(s) = \sigma_{hh}(s) = \tfrac{1}{2}(\sigma^H_{LL} + \sigma^H_{Lh}) + \tfrac{1}{2}(\sigma^H_{LL} - \sigma^H_{Lh})\; e^{-4\,\mathcal{L}} \\ \sigma_{Lh}(s) = \tfrac{1}{2}(\sigma^H_{LL} + \sigma^H_{Lh}) - \tfrac{1}{2}(\sigma^H_{LL} - \sigma^H_{Lh})\; e^{-4\,\mathcal{L}}\end{aligned} \tag{15}$$

which are equivalent to eq.(13). The derivation based on eq.(14) makes it clear that this phenomenon is not limited to longitudinal and Higgs states, but applies to any mixed charge states which are allowed by symmetry breaking.

A final comment is about longitudinal couplings to external charges, e.g. fermions of mass $m$, which make the above effect observable. It is known that, by fermion current conservation, longitudinal polarizations are suppressed by a factor $M^2/k_T^2$ with respect to transverse ones, where $k_T^2$ denotes the boson transverse momentum, related to its virtuality. However, if $M \gg m$, then the longitudinal $k_T^2$ distribution is dominated by $k_T^2 = O(M^2)$, yielding a cross section of the same order as the transverse one [8]. The situation changes in the limit of vanishing symmetry breaking parameter. In fact if $M \ll m$, the longitudinal $k_T^2$ distribution is cut off by $m^2$, rather than $M^2$, thus yielding a cross section of relative order $M^2/m^2$ which vanishes, eventually. Therefore, in the vanishing $M/m$ limit, gauge symmetry and BN theorem are recovered at the same time.

The U(1) Higgs model just discussed is a prototype. A slightly more complicated example, which is relevant for planned accelerators, is electroweak theory itself. Here the gauge group is $SU(2)_L \otimes U(1)_Y$, and important B-N violating corrections are found in the longitudinal sector [6] of both non abelian and *abelian* type. The latter survive in the formal limit of vanishing isospin coupling and have a structure similar to the one illustrated here.

An additional peculiarity of the standard model is that, because of the chiral nature of the gauge group, massive *fermions* of mass $m$ are themselves a superposition of left and right states of different weak hypercharge (and isospin). Therefore, by the general argument of eqs.(5) and (15), *abelian* double logs are expected for fermion beams also. If initial beams are longitudinally polarized, the left/right mixing is small at high energies, so that the corresponding off-diagonal overlap is suppressed by a factor $m^2/s$, and was not explicitly considered before [1]. *Transverse* polarizations, however, are a superposition of left and right states with comparable weights: mixing is therefore maximal, as in the longitudinal boson case considered so far. The corresponding off-diagonal overlap provides the azimuthal dependence [9] of the inclusive cross section at tree level, and is then affected at higher orders by the appropriate double log form factor (carrying t-channel quantum numbers $Y = t_L = 1/2$ in the present case). Polarized beam effects provide thus another instance in which infrared enhancements related to mixing are to be investigated.

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