Models for RHIC and LHC: New Developments

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We outline inconsistencies in presently used models for high energy nuclear scattering, which make their application quite unreliable. Many "successes" are essentially based on an artificial freedom of parameters, which does not exist when the models are constructed properly.

The problem is the fact that any multiple scattering theory requires an appropriate treatment of the energy sharing between the individual interactions, which is technically very difficult to implement. Lacking a satisfying solution to this problem, it has been simply ignored.

We introduce a fully self-consistent formulation of the multiple-scattering scheme. Inclusion of soft and hard components – very crucial at high energies – appears in a "natural way", providing a smooth transition from soft to hard physics.

We can show that the effect of appropriately considering energy conservation has a big influence on the results, and MUST therefore be included in any serious calculation.

1. Open Problems

With the start of the RHIC program to investigate nucleus-nucleus collisions at very high energies, there is an increasing need of computational tools in order to provide a clear interpretation of the data. The situation is not satisfactory in the sense that there exists a nice theory (QCD) but we are not able to treat nuclear collisions strictly within this framework, and on the other hand there are simple models, which can be applied easily but which have no solid theoretical basis. A good compromise is provided by effective theories, which are not derived from first principles, but which are nevertheless self-consistent and calculable. A candidate seems to be the Gribov-Regge approach, and – being formally quite similar – the eikonalized parton model. Here, however, some inconsistencies occur, which we are going to discuss in the following, before we provide a solution to the problem.

Gribov-Regge theory [1,2] is by construction a multiple scattering theory. The elementary interactions are realized by complex objects called "Pomerons", who's precise nature is not known, and which are therefore simply parameterized, with a couple of parameters to be determined by experiment [3]. Even in hadron-hadron scattering, several of these Pomerons are exchanged in parallel (the cross section for exchanging a given number of Pomerons is called"topological cross section"). Simple formulas can be derived for the (topological) cross sections, expressed in terms of the Pomeron parameters.

In order to calculate exclusive particle production, one needs to know how to share the energy between the individual elementary interactions in case of multiple scattering. We do not want to discuss the different recipes used to do the energy sharing (in particular in Monte Carlo applications). The point is, whatever procedure is used, this is not taken into account in the calculation of cross sections discussed above [4],[5]. So, actually, one is using two different models for cross section calculations and for treating particle production. Taking energy conservation into account in exactly the same way will modify the (topological) cross section results considerably.

Being another popular approach, the parton model [6] amounts to presenting the partons of projectile and target by momentum distribution functions, f_i and f_j, and calculating inclusive cross sections for the production of parton jets as a convolution of these distribution functions with the elementary parton-parton cross section $d\hat{\sigma}_{ij}/dp_\perp^2$, where i, j represent parton flavors. This simple factorization formula is the result of cancelations of complicated diagrams and hides therefore the complicated multiple scattering structure of the reaction, which is finally recovered via some unitarization procedure. The latter one makes the approach formally equivalent to the Gribov-Regge one and one therefore encounters the same conceptual problems (see above).

2. A Solution: Parton-based Gribov-Regge Theory

As a solution of the above-mentioned problems, we present a new approach which we call "Parton-based Gribov-Regge Theory": we have a consistent treatment for calculating (topological) cross sections and particle production considering energy conservation in both cases; in addition, we introduce hard processes in a natural way.

The basic guideline of our approach is theoretical consistency. We cannot derive everything from first principles, but we use rigorously the language of field theory to make sure not to violate basic laws of physics, which is easily done in more phenomenological treatments (see discussion above).

Let us consider nucleus-nucleus (AB) scattering. In the Glauber-Gribov approach [7,2], the nucleus-nucleus scattering amplitude is defined by the sum of contributions of diagrams, corresponding to multiple elementary scattering processes between parton constituents of projectile and target nucleons. These elementary scatterings are the sum of soft, semi-hard, and hard contributions: $T_{2\rightarrow 2} = T_{\mathrm{soft}} + T_{\mathrm{semi}} + T_{\mathrm{hard}}$. A corresponding relation holds for the inelastic amplitude $T_{2\rightarrow X}$. A cut elementary diagram will be graphically represented by a vertical dashed line, whereas the elastic amplitude by an unbroken line:

$$= T_{2\rightarrow 2}, \qquad\qquad = \textstyle\sum_X (T_{2\rightarrow X})(T_{2\rightarrow X})^*.$$

This is very handy for treating the nuclear scattering model. We define the model via the elastic scattering amplitude $T_{AB\rightarrow AB}$ which is assumed to consist of purely parallel elementary interactions between partonic constituents. The amplitude is therefore a sum of such terms. One has to be careful about energy conservation: all the partonic constituents leaving a nucleon have to share the momentum of the nucleon. So in the explicit formula one has an integration over momentum fractions of the partons, taking care of momentum conservation. Having defined elastic scattering, inelastic scattering and particle production is practically given, if one employs a quantum mechanically self-consistent picture. Let us now consider inelastic scattering: one has of course the same parallel structure, just some of the elementary interactions may be inelastic, some elastic. The inelastic amplitude being a sum over many terms – $T_{AB\to X} = \sum_i T^{(i)}_{AB\to X}$ – has to be squared and summed over final states in order to get the inelastic cross section, which provides interference terms $\sum_X (T^{(i)}_{AB\to X})(T^{(j)}_{AB\to X})^*$, which can be conveniently expressed in terms of the cut and uncut elementary diagrams, as shown in fig. 1. So we are doing nothing more than following basic rules of quantum mechanics. Of course a diagram with

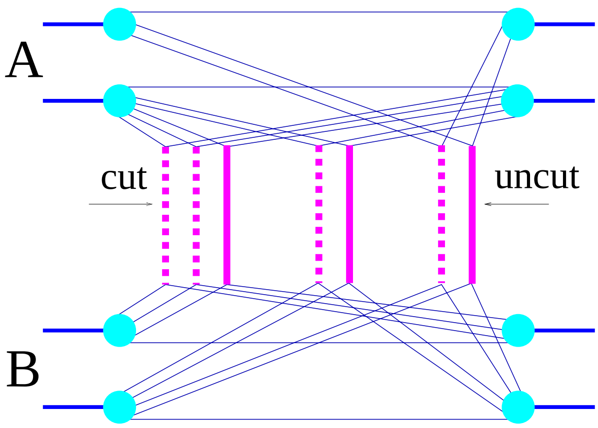

Figure 1. Typical interference term contributing to the squared inelastic amplitude.

3 inelastic elementary interactions does not interfere with the one with 300, because the final states are different. So it makes sense to define classes K of interference terms (cut diagrams) contributing to the same final state, as all diagrams with a certain number of inelastic interactions and with fixed momentum fractions of the corresponding partonic constituents. One then sums over all terms within each class K, and obtains for the inelastic cross section

$$\sigma_{AB}(s) = \int d^2b \sum_K \Omega^{(s,b)}(K)$$

where we use the symbolic notation $d^2b = \int d^2b_0 \int d^{2A}b_A\, \rho(b_A) \int d^{2B}b_B\, \rho(b_B)$ which means integration over impact parameter b_0 and in addition averaging over nuclear coordinates for projectile and target. The variable K is characterized by AB numbers m_k representing the number of cut elementary diagrams for each possible pair of nucleons and all the momentum fractions x^+ and x^- of all these elementary interactions (so K is a partly discrete and partly continuous variable, and $\sum$ is meant to represent $\sum\int$). This is the really new and very important feature of our approach: we keep explicitly the dependence on the longitudinal momenta, assuring energy conservation at any level of our calculation.

The calculation of Ω actually very difficult and technical, but it can be done and we refer the interested reader to the literature [3].

The quantity $\Omega^{(s,b)}(K)$ can now be interpreted as the probability to produce a configuration K at given s and b. So we have a solid basis for applying Monte Carlo techniques: one generates configurations K according to the probability distribution Ω and one may then calculate mean values of observables by averaging Monte Carlo samples. The problem is that Ω represents a very high dimensional probability distribution, and it is not obvious how to deal with it. We decided to develop powerful Markov chain techniques [8] in order to avoid to introduce additional approximations.

3. Summary

We provide a new formulation of the multiple scattering mechanism in nucleus-nucleus scattering, where the basic guideline is theoretical consistency. We avoid in this way many of the problems encountered in present day models. We also introduce the necessary numerical techniques to apply the formalism in order to perform practical calculations.

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