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Large-N_c meson theory

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Abstract

We derive an effective Lagrangian for meson fields. This is done in the light-cone gauge for two-dimensional large-N_c QCD by using the bilocal auxiliary field method. The auxiliary fields are bilocal on light-cone space and their Fourier transformation determines the parton momentum distribution. As the first test of our method, the 't Hooft equation is derived from the effective Lagrangian.

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1 Introduction

QCD successfully describes the dynamics of quarks and gluons in the perturbative regime. However the dynamics of mesons and baryons are not yet understood from the first principle. Although we believe that the meson is the bound state of quarks and the numerical simulation based on lattice QCD is successful, it still is a difficult task to clarify this connection analytically. Large-N_c QCD has played an important role on our understanding of QCD [1–3]. Especially in two dimensions, that is, in the 't Hooft model, a bound-state equation is obtained. This is the well-known 't Hooft equation [1, 2], and it is derived from a ladder Bethe–Salpeter equation. This equation can be solved by several methods, e.g., the discretized light-cone quantization [4, 5], the variational method [6], and the Multhopp technique [7]. These calculations give us various information about the meson wave function and the bound state mass. Our problem is how we can understand this equation in the Lagrangian level and, more, how we can deduce the Lagrangian corresponding to this equation from QCD.

In this paper, we derive an effective Lagrangian of large-N_c QCD$_2$. The derivation of the Lagrangian is based on the bilocal auxiliary field method [8–10], and then the effective Lagrangian contains a meson field as a bilocal field. At first, we take the light-cone gauge $A^+ = 0$. The remaining gauge field A^- induces non-local four-fermi interaction with light-cone distance r^- at equal light-cone time x^+. In this interaction term, we introduce the bilocal auxiliary fields using the path-integral, and this field corresponds to the meson. The Fourier transformation of the bilocal auxiliary fields determines the parton momentum distribution function. From the effective Lagrangian, we obtain the 't Hooft equation for the bilocal auxiliary fields.

The paper is organized as follows. In section 2, we briefly review QCD$_2$ with the light-cone gauge. In section 3, we derive the large-N_c effective Lagrangian, and we decompose the auxiliary fields into the vacuum expectation value and its fluctuation field. This vacuum expectation value of the auxiliary fields is obtained by the effective potential. In section 4, we discuss the effective potential and the vacuum expectation value, which is obtained from the gap equation and determines the dynamical quark mass. In section 5, we derive the 't Hooft equation from our effective Lagrangian. We conclude the paper in section 6, and we summarize our discussion.

2 SU(N_c) QCD

An SU(N_c) gauge theory is defined by the Lagrangian

$$\mathcal{L} = \overline{q}_i(i\gamma^\mu D_\mu - m_i)\, q_i - \frac{1}{4}\, F_{a\mu\nu}F_a^{\mu\nu}, \tag{1}$$

where i denotes the flavour ($i = 1, \cdots, N_f$) and a denotes the adjoint representation of colour ($a = 1, \cdots, N_c^2 - 1$). $F_{a\mu\nu}$ is the field-strength tensor $F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g f_{abc} A_{b\mu} A_{c\nu}$, and the covariant derivative is defined as $D_\mu = \partial_\mu + igt^a A_{a\mu}$. The fermion field q_i is the fundamental representation of SU(N_c), and in two dimensions this field is a two-component spinor. The light-cone coordinate is defined by $x^+ = x^0 + x^3,\ x^- = x^0 - x^3$. We employ a specific representation $\gamma^\pm = \sigma_1 \pm i\sigma_2$. In this representation, the spinor is given by $q^T = (q_-,\ q_+)$, where the lower component (upper component) q_+ (q_-) is a projection of $\Lambda^\pm =$

$\frac{1}{4}\gamma^{\mp}\gamma^{\pm}$. The light-cone gauge is given by $A^+ = 0$. In this gauge there are no ghosts, and the remaining gauge field is only A^-. Then the Lagrangian is given by

$$\mathcal{L} = q_+^{i\dagger}\, i\partial^-\, q_+^i + q_-^{i\dagger}\, i\partial^+\, q_-^i - m_i \left(q_+^{i\dagger}\, q_-^i + q_-^{i\dagger}\, q_+^i\right) - g q_+^{i\dagger}\, t^a A_a^-\, q_+^i + \frac{1}{8}\left(\partial^+ A_a^-\right)^2 . \tag{2}$$

Integrating over A^-, this gauge field induces a non-local four-fermi interaction:

$$\mathcal{L} = q_+^{i\dagger}\, i\partial^-\, q_+^i + q_-^{i\dagger}\, i\partial^+\, q_-^i - m_i \left(q_+^{i\dagger}\, q_-^i + q_-^{i\dagger}\, q_+^i\right) + 2g^2 \left(q_+^{i\dagger}\, t^a\, q_+^i\right) \frac{1}{\partial^{+2}} \left(q_+^{j\dagger}\, t^a\, q_+^j\right), \tag{3}$$

where the inverse operator $\frac{1}{\partial^{+2}}$ is defined by [11, 12]

$$\frac{1}{\partial^{+2}} q_+(x^-) = \frac{1}{8}\int_{-\infty}^{\infty} dy^- |x^- - y^-| q_+(y^-). \tag{4}$$

The equation of motion for $q_+^{i\dagger}$ is given as $q_-^i = \frac{m_i}{i\partial^+} q_+^i$, and then the field q_-^i is the constrained field because at any x^+ it is determined by q_+^i. The action is given in terms of the one-component spinor q_+^i as:

$$S = \frac{1}{2}\int dx^- dx^+\, q_+^{i\dagger} \left(i\partial^- - \frac{m_i^2}{i\partial^+}\right) q_+^i + \frac{g^2}{8}\int dx^- dx^+ dy^- \left(q_+^{i\dagger}(x^-, x^+)\, t^a\, q_+^i(x^-, x^+)\right) \times |x^- - y^-| \left(q_+^{j\dagger}(y^-, x^+)\, t^a\, q_+^j(y^-, x^+)\right). \tag{5}$$

The non-local four-fermi interaction with linear potential induces the bound state of quark and antiquark.

3 The large-N_c effective action with the auxiliary fields

We expand the Lagrangian by $1/N_c$, with $g^2 N_c$ fixed. We discuss the theory in the leading order of $1/N_c$ expansion. At first we rearrange the non-local four-fermi interaction by using the identity $(t_a)_{\alpha\beta}(t_b)_{\gamma\delta} = -\frac{1}{2N_c}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$. In the leading order only the colour singlet part of the quark pair contributes. Then the effective action in the large-N_c limit is given by

$$S^{\mathrm{Large}N_c} = \frac{1}{2}\int dx^- dx^+ q_+^{\dagger i} \left(i\partial^- - \frac{m^2}{i\partial^+}\right) q_+^i - \frac{g^2}{16}\int dx^- dx^+ dy^- \left(q_+^{\dagger i}(x) q_+^j(y)\right) |x^- - y^-| \left(q_+^{\dagger j}(y) q_+^i(x)\right), \tag{6}$$

where the quark pair in the interaction term is a colour singlet, and this quark pair is non-local in the light-cone space, that is $q_+^{\dagger i}(x) q_+^j(y) \equiv q_{+\alpha}^{\dagger i}(x^-, x^+)\; q_{+\alpha}^j(y^-, x^+)$. Here we ignore next-to-leading terms. The partition function of this theory is written as

$$Z^{\mathrm{Large}N_c} = \int \prod_i [dq_+^{\dagger i}][dq_+^i] \exp\left\{iS^{\mathrm{Large}N_c}\right\}. \tag{7}$$

In order to treat the singlet of quark pair as a dynamical variable, we introduce bilocal auxiliary fields. After integrating over quark fields, the bilocal auxiliary field corresponds to the singlet of the quark pair. At first we multiply the partition function by a constant factor of δ-functions:

$$Z^{\mathrm{Large}N_c} = \int \prod_{i,j} [dq_+^{\dagger i}][dq_+^i][ds^{ij}] \exp\left\{iS^{\mathrm{Large}N_c}\right\} \prod_{i,j} \delta\left(s^{ij} - q_+^{\dagger j}(x)q_+^i(y)\right), \tag{8}$$

where we introduce the auxiliary fields s^{ij}, which correspond to $q_+^{\dagger j}(x)q_+^i(y)$. These δ-functions are defined with the additional bilocal fields σ:

$$\begin{aligned}
&\prod_{i,j} \delta\left(s^{ij}(y,x) - q_+^{\dagger j}(x)q_+^i(y)\right) \\
&\qquad \propto \int \prod_{i,j} [d\sigma_{ij}] \; \exp\left\{\frac{i}{2}\int dx^- dx^+ dy^- \; \sigma_{ji}(x^-, y^-; x^+)\right. \\
&\qquad\qquad\qquad \left. \times (s^{ij}(y,x) - q_+^{\dagger i}(x)q_+^j(y))\right\}.
\end{aligned} \tag{9}$$

The partition function with the bilocal auxiliary fields is given by

$$Z^{\mathrm{Large}N_c} = \int \prod_{i,j} [dq_+^{\dagger i}][dq_+^i][ds^{ij}][d\sigma_{ij}] \exp\left\{iS^{\mathrm{Large}N_c} + iS^{\mathrm{Aux}}\right\}, \tag{10}$$

where the action S^{Aux} is written as

$$S^{\mathrm{Aux}} = \frac{1}{2}\int dx^- dx^+ dy^- \; \sigma_{ji}(x^-, y^-; x^+)\left(s^{ij}(y,x) - q_+^{\dagger j}(x)q_+^i(y)\right). \tag{11}$$

In order to consider the correspondence between the bilocal fields σ_{ij} and quarks q_+^i, varying $Z^{\mathrm{Large}N_c}$ by $s^{ij}(x,y)$, we get the relation

$$\langle \sigma_{ij}(x^-, y^-; x^+)\rangle = \langle \frac{g^2}{8}|x^- - y^-| q_+^{\dagger j}(y)q_+^i(x)\rangle. \tag{12}$$

This equation tells us that the auxiliary fields are of order g^2 and that these fields correspond to quark pairs with gluons; they would then be identified as mesonic dynamical variables. This will be mentioned later. Integrating over auxiliary fields s^{ij} and quarks q_+^i, the Lagrangian is given by

$$\begin{aligned}
S^{\mathrm{Large}N_c} &= -iN_c \ln \mathrm{Det}\; M(x,y) \\
&\quad + \int dx^- dx^+ dy^- \sigma_{ij}(y^-, x^-; x^+) \frac{1}{g^2|x^- - y^-|}\sigma_{ji}(x^-, y^-; x^+),
\end{aligned} \tag{13}$$

where the matrix M is defined by

$$M(x,y) \equiv \left[\; \delta_{ji}\left(i\partial^- - \frac{m^2}{i\partial^+}\right) \; \delta^2(x-y) - \sigma_{ji}(x,y) \;\right], \tag{14}$$

with

$$\sigma_{ij}(x,y) \equiv \sigma_{ij}(x^-, y^-; x^+)\delta(x^+ - y^+). \tag{15}$$

The vacuum expectation value of these bilocal auxiliary fields σ_{ij} is obtained by the effective potential. From the charge conservation, the diagonal part of σ_{ij} takes non-zero vacuum expectation value, and the other is zero:

$$\langle\sigma_{ij}(x^-,y^-;x^+)\rangle = \begin{cases} v_i\ (x^- - y^-) & \text{if} \quad i=j, \\ 0 & \text{if} \quad i\neq j. \end{cases} \tag{16}$$

The bilocal auxiliary fields are expanded into the vacuum expectation values and the fluctuation,

$$\sigma_{ij}(x^-,y^-;x^+) \longrightarrow v_i\ (x^- - y^-)\ \delta_{ij} + \sigma_{ij}(x^-,y^-;x^+). \tag{17}$$

Finally we arrive at the large-N_c effective action:

$$S^{\mathrm{Large}N_c} = S^{(0)} + S^{(2)}, \tag{18}$$

where

$$\begin{aligned} S^{(0)} &= -iN_c \mathrm{TrLn}\ S_F^{-1}(y-z) \\ &\qquad + \int dx^- dx^+ dy^- \frac{v_i(y^- - x^-)v_i(x^- - y^-)}{g^2|x^- - y^-|}, \end{aligned} \tag{19}$$

$$\begin{aligned} S^{(2)} &= \frac{iN_c}{2}\mathrm{Tr}[S_F(z-u)\Sigma(u,x)]^2 \\ &\qquad + \int dx^- dx^+ dy^- \frac{\sigma_{ij}(y^-,x^-;x^+)\sigma_{ji}(x^-,y^-;x^+)}{g^2|x^- - y^-|}. \end{aligned} \tag{20}$$

Here the subscript of S denotes the degree of σ. The first-degree action $S^{(1)}$ vanishes if we use the gap equation of $v_i^\dagger$, which will be discussed in the next section. In this Lagrangian, the matrices S_F^{-1} and Σ are defined by

$$S_F^{-1}(x-y) = \mathrm{diag}\left(\left(i\partial^- - \frac{m_i^2}{i\partial^+}\right)\ \delta^2(x-y) - v_i\ (x^- - y^-)\delta(x^+ - y^+)\right), \tag{21}$$

and

$$\Sigma(x,y) = \Big(\ \sigma_{ij}(x,y)\ \Big). \tag{22}$$

Here we ignore the higher-order terms of the $1/N_c$ expansion in Eq. (18).

4 The effective potential and the gap equation

In this section, we discuss the behaviour of the vacuum expectation value of the bilocal auxiliary field by considering the leading order $S^{(0)}$ solely. We give the momentum space description of $S^{(0)}$. Then we can easily extract the space-time volume from the effective action. The first term in Eq. (19) is

$$\begin{aligned} &\mathrm{TrLn}\ S_F^{-1}(y-x) \\ &= \int dx^+ dx^- dy^+ dy^-\ \delta^2(y-x) \int \frac{d^2p}{(4\pi)^2}\ e^{ip\cdot(y-x)} \ln\det S_F^{-1}(p) \\ &= \delta^2(0)\int dp^+ dp^-\ \sum_i \ln\left(p^- - \frac{m_i^2}{p^+} - v_i(p^+)\right), \end{aligned} \tag{23}$$

where $S_F^{-1}(p)$ and $v(p^+)$ are the Fourier transformation of Eq. (21) and $v_i(x^-)$:

$$
\begin{aligned}
S_F^{-1}(p) &= \mathrm{diag}\left(p^- - \frac{m_i^2}{p^+} - v_i(p^+)\right), && (24)\\
v_i(x^-) &= \int \frac{dp^+}{4\pi} e^{-\frac{i}{2}p^+x^-} v_i(p^+). && (25)
\end{aligned}
$$

The second term in Eq. (19) is

$$
\begin{aligned}
&\int dx^+ dx^- dy^- \frac{v_i(y^- - x^-)v_i(x^- - y^-)}{g^2|x^- - y^-|} \\
&= 4\pi\delta(0) \int dx^- dy^- \frac{v_i(y^- - x^-)v_i(x^- - y^-)}{g^2|x^- - y^-|} \\
&= \delta^2(0) \int dp^+ dk^+ v_i(p^+) G(p^+ - k^+) v_i(k^+), \qquad (26)
\end{aligned}
$$

where we define the kernel G:

$$
\frac{1}{g^2|x^-|} = \int \frac{dk^+}{4\pi} e^{-\frac{i}{2}k^+x^-} G(k^+). \qquad (27)
$$

The effective potential is defined by $S^{(0)} = -\delta^2(0)V_{\text{eff}}$, then

$$
\begin{aligned}
V_{\text{eff}} &= iN_c \sum_i \int dp^+ dp^- \ln\left(p^- - \frac{m_i^2}{p^+} - v_i(p^+)\right) \\
&\quad - \int dp^+ dk^+ v_i(p^+) G(p^+ - k^+) v_i(k^+). \qquad (28)
\end{aligned}
$$

The behaviour of the vacuum expectation value $v_i(p^+)$ is determined by the gap equation, which is given by varying the effective potential with respect to $v_i(p^+)$:

$$
\begin{aligned}
\frac{\delta V_{\text{eff}}}{\delta v_i(p^+)} &= -iN_c \int dp^- \frac{1}{p^- - \frac{m_i^2}{p^+} - v_i(p^+) + i\epsilon\ \mathrm{sgn}(p^+)} \\
&\quad -2 \int dk^+ G(p^+ - k^+) v_i(k^+) \\
&= 0. \qquad (29)
\end{aligned}
$$

Multiplying these equations by the inverse of $G(p^+)$, the gap equation is written as

$$
v_i(p^+) = \frac{iN_c g^2}{4\pi^2} \int dk^+ dk^- \frac{\mathcal{P}}{(p^+ - k^+)^2} \frac{1}{k^- - \frac{m_i^2}{k^+} - v_i(k^+) + i\epsilon\ \mathrm{sgn}(k^+)}. \qquad (30)
$$

The symbol $\mathcal{P}$ means the principal value of $(p^+ - k^+)^{-2}$. This equation is equivalent to the Schwinger–Dyson equation. The vacuum expectation value gives the self-energy of the quark and the dynamical quark mass. Performing the integration in Eq. (30), we arrive at $v_i(p^+) = -\frac{N_c g^2}{2\pi}\,\frac{1}{p^+}$. However we will not use this explicit expression in the derivation of the 't Hooft equation discussed in the next section.

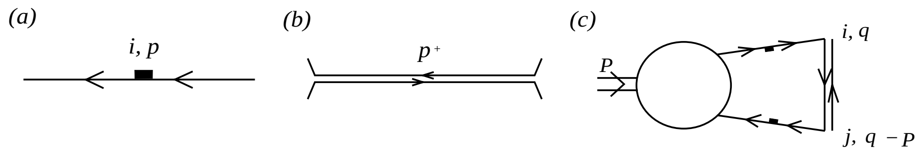

Figure 1: Feynman rule: (a) quark propagator $S_F^i(p) = \left(p^- - \frac{m_i^2}{p^+} - v_i(p^+) + i\epsilon \,\mathrm{sgn}(p^+)\right)^{-1}$, which is the diagonal part of $S_F(p)$, (b) gluon propagator, with vertices $G^{-1}(p^+) = -8g^2 \frac{\mathcal{P}}{p^{+2}}$; and (c) bilocal field $\sigma_{ij}(q^+, q^+ - P^+ : P^-)$.

5 The 't Hooft equation

Our bilocal auxiliary field is supposed to be the mesonic field in the large-N_c theory. In two dimensional large-N_c QCD, there is a well-known bound-state equation called the 't Hooft equation. If our bilocal auxiliary field is a bound state, it should satisfy this equation. This equation was first derived in [1], and the mesonic field was introduced as a solution of the ladder Bethe–Salpeter equation. As we will show, our effective theory contains the bilocal field as a meson, which satisfies the 't Hooft equation. In this section we derive the 't Hooft equation from our effective theory. At first we give the definition of the Fourier transformation, which, for $\sigma_{ij}(x^-, y^-; x^+)$, is defined by

$$\sigma_{ij}(x^-, y^-; x^+) = \int \frac{dP^-}{4\pi}\frac{dp^+}{4\pi}\frac{dk^+}{4\pi} e^{-\frac{i}{2}P^- x^+} e^{-\frac{i}{2}(p^+ x^- - k^+ y^-)}\ \sigma_{ij}(p^+, k^+; P^-), \tag{31}$$

or in the variables $p^+ \to q^+$ and $k^+ \to q^+ - P^+$,

$$\sigma_{ij}(x^-, y^-; x^+) = \int \frac{dP^-}{4\pi}\frac{dq^+}{4\pi}\frac{dP^+}{4\pi} e^{-\frac{i}{2}P^- x^+} e^{-\frac{i}{2}(q^+(x^- - y^-) + P^+ y^-)} \times \sigma_{ij}(q^+, q^+ - P^+; P^-), \tag{32}$$

where this P^+ is the total momentum and q^+ is the momentum carried by the quark (see Fig. 1). The quark propagator is given by

$$\begin{aligned} S_F(p) &= \int dx^+ dx^- e^{ip\cdot x} S_F(x) \\ &= \mathrm{diag}\left(\frac{1}{p^- - \frac{m_i^2}{p^+} - v_i(p^+) + i\epsilon\ \mathrm{sgn}(p^+)}\right). \end{aligned} \tag{33}$$

The Feynman rule is shown in Fig. 1. The momentum-space description of the action (20) is given by

$$\begin{aligned} S^{(2)} = {} & \frac{iN_c}{2}\mathrm{tr}\int \frac{dP^- dP^+ dq^- dq^+}{(4\pi)^4}\ S_F(q^+, q^-)\ \Sigma(q^+, q^+ - P^+; P^-) \\ & \times S_F(q^+ - P^+, q^- - P^-)\ \Sigma(q^+ - P^+, q^+; -P^-) \\ & + \mathrm{tr}\int \frac{dP^- dP^+ dq'^+ dq^+}{(4\pi)^4}\ \sigma(q'^+, q'^+ - P^+; P^-) \\ & \times G(q'^+ - q^+)\ \sigma(q^+ - P^+, q^+; -P^-). \end{aligned} \tag{34}$$

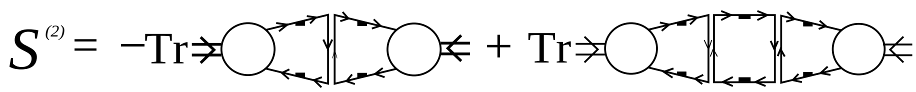

Figure 2: Diagrammatic representation of the effective action (34).

This action is depicted in Fig. 2. The Bethe–Salpeter equation is derived by varying the $S^{(2)}$ with respect to the bilocal field $\sigma_{ji}(q^+ - P^+, q^+; -P^-)$

$$
\begin{aligned}
&(4\pi)^4 \frac{\delta S^{(2)}}{\delta\sigma_{ji}(q^+ - P^+, q^+; -P^-)} \\
&\quad = iN_c \int dq^- S_F^i(q^+, q^-)\sigma_{ij}(q^+, q^+ - P^+; P^-) S_F^j(q^+ - P^+, q^- - P^-) \\
&\qquad +2\int dq'^+ G(q^+ - q'^+)\sigma_{ij}(q'^+, q'^+ - P^+; P^-) \\
&\quad = 0.
\end{aligned} \tag{35}
$$

This equation is depicted in Fig. 3. We multiply the Bethe–Salpeter equation by the kernel G^{-1}, which is written as a simple form:

$$
\begin{aligned}
&\sigma_{ij}(q^+, q^+ - P^+; P^-) \\
&\quad = \frac{iN_c g^2}{4\pi^2}\int dk^- dk^+ \frac{\mathcal{P}}{(q^+ - k^+)^2} \\
&\qquad \times \frac{1}{k^- - \frac{m_i^2}{k^+} - v_i(k^+) + i\epsilon\ \mathrm{sgn}(k^+)}\sigma_{ij}(k^+, k^+ - P^+; P^-) \\
&\qquad \times \frac{1}{k^- - P^- - \frac{m_j^2}{k^+ - P^+} - v_j(k^+ - P^+) + i\epsilon\ \mathrm{sgn}(k^+ - P^+)}.
\end{aligned} \tag{36}
$$

In Eq. (36) the variable k^- is only included in the quark propagators, and we can easily perform the integration over k^-. If the condition $\mathrm{sgn}(k^+) = -\mathrm{sgn}(k^+ - P^+)$ is satisfied, the integration over k^- gives the finite contribution. If P^+ is positive[4], the integration region of k^+ is restricted as follows:

$$
0 \ < \ k^+ \ < \ P^+. \tag{37}
$$

Then the Bethe–Salpeter equation is written as

$$
\begin{aligned}
&\sigma_{ij}(q^+, q^+ - P^+; P^-) \\
&\quad = -\frac{N_c g^2}{2\pi}\int_0^{P^+} dk^+ \frac{\mathcal{P}}{(q^+ - k^+)^2} \frac{\sigma_{ij}(k^+, k^+ - P^+; P^-)}{P^- - (\frac{m_i^2}{k^+} + \frac{m_j^2}{P^+ - k^+}) - v_i(k^+) + v_j(k^+ - P^+)}.
\end{aligned} \tag{38}
$$

We define the field ψ_{ij} by the field σ_{ij} and two quark propagators:

[4] In spite of the sign of P^+, we arrive at the same expression Eq.(38). In the light-cone quantization, P^+ is positive, because of the dispersion relation and the positive-definite light-cone energy.

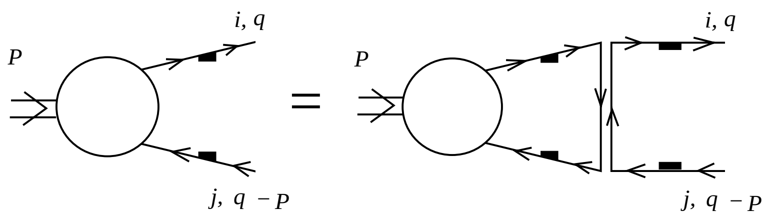

Figure 3: Diagrammatic representation of Eq. (35).

$$\psi_{ij}(q^+, q^+ - P^+; P^-) \equiv \frac{\sigma_{ij}(q^+, q^+ - P^+; P^-)}{P^- - (\frac{m_i^2}{q^+} + \frac{m_j^2}{P^+ - q^+}) - v_i(q^+) + v_j(q^+ - P^+)}. \tag{39}$$

This field ψ_{ij} is depicted in Fig. 4. Using the field ψ_{ij}, the Bethe–Salpeter equation is rewritten as

$$\begin{aligned}\left[P^- - \left(\frac{m_i^2}{q^+} + \frac{m_j^2}{P^+ - q^+}\right) - v_i(q^+) + v_j(q^+ - P^+)\right] \psi_{ij}(q^+, q^+ - P^+; P^-) \\ = -\frac{N_c g^2}{2\pi} \mathcal{P} \int_0^{P^+} dk^+ \, \frac{\psi_{ij}(k^+, k^+ - P^+; P^-)}{(q^+ - k^+)^2}. \end{aligned} \tag{40}$$

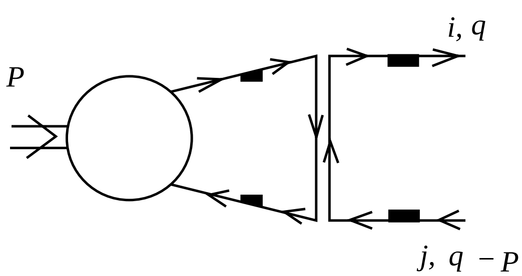

Figure 4: Diagrammatic representation of $\psi_{ij}(q^+, q^+ - P^+; P^-)$.

The bound-state mass M and the momentum fraction variable x are defined as

$$M^2 \equiv P^+ P^-, \ x \equiv \frac{q^+}{P^+}. \tag{41}$$

The field ψ_{ij} is rewritten in terms of the momentum fraction x,

$$\psi_{ij}(xP^+) = \psi_{ij}(q^+, q^+ - P^+; P^-). \tag{42}$$

Using these variables, the Bethe–Salpeter equation is written as

$$\begin{aligned}\left[M^2 - \left(\frac{m_i^2}{x} + \frac{m_j^2}{1-x}\right) - P^+ \ v_i(q^+) + P^+ \ v_j(q^+ - P^+)\right] \psi_{ij}(xP^+) \\ = -\frac{N_c g^2}{2\pi} \mathcal{P} \int_0^1 dy \frac{\psi_{ij}(yP^+)}{(x-y)^2}. \end{aligned} \tag{43}$$

The difference between the two vacuum expectation values is easily calculated by using the gap equation Eq. (30),

$$v_i(q^+) - v_j(q^+ - P^+) = \frac{N_c g^2}{2\pi} \int_0^{P^+} dk^+ \frac{\mathcal{P}}{(q^+ - k^+)^2}. \tag{44}$$

At last, the 't Hooft equation is derived:

$$\left[M^2 - \left(\frac{m_i^2}{x} + \frac{m_j^2}{1-x} \right) \right] \psi_{ij}(xP^+) = -\frac{N_c g^2}{2\pi} \mathcal{P} \int_0^1 dy \frac{\psi_{ij}(yP^+) - \psi_{ij}(xP^+)}{(x-y)^2}. \qquad (45)$$

In our large-N_c effective theory, this equation is derived using a bilocal auxiliary field. This shows that the auxiliary field is a collective field, corresponding to the bound state.

6 Summary

We construct the large-N_c QCD_2 effective theory of the bilocal auxiliary fields. In this theory we introduce the bilocal auxiliary field as a colour singlet, which is bilocal in the light-cone space and local in the light-cone time. This non-locality means the distance between the constituent quarks. From the consideration of the 't Hooft equation, we identify the bilocal field as the parton momentum distribution function, or the meson field. Original derivation of the 't Hooft equation was based on the ladder Bethe–Salpeter equation, and the meson field is introduced as the solution of this equation. From our point of view, there is a meson theory that is reduced from large-N_c QCD_2, and from this theory we derive the 't Hooft equation. In our approach the vacuum expectation value of the bilocal field gives the dynamical quark mass, and then it corresponds to the self-energy of the quark. We may suppose the result of the gap equation contradicts the triviality of the light-cone vacuum. In this respect, our auxiliary field is bilocal and indeed has a vacuum expectation value. The result implies that the light-cone vacuum is not trivial.

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