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Analysis of Couplings with Large Tensor Representations in SO(2N) and Proton Decay

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Abstract

We develop techniques for the analysis of SO(2N) invariant couplings which allow a full exhibition of the SU(N) invariant content of the spinor and tensor representations. The technique utilizes a basis consisting of a specific set of reducible SU(N) tensors in terms of which the SO(2N) invariant couplings have a simple expansion. The technique is specially useful for couplings involving large tensor representations. We exhibit the technique by performing a complete determination of the trilinear couplings in the superpotential for the case of SO(10) involving the 16 plet of matter, i.e., we give a full determination of the $16-16-10_s$, $16-16-120_a$ and $16-16-1\overline{2}6_s$ couplings. The possible role of large tensor representations in the generation of quark lepton textures is discussed. It is shown that the couplings involving $1\overline{2}6$ dimensional representation generate extra zeros in the Higgs triplet textures which can lead to an enhancement of the proton decay lifetime by a factor of 10^3. These results also have implications for neutrino mass textures.

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1 Introduction

The group SO(10) is one of the candidates for a theory of grand unification and has come under increasing scrutiny since the early work of Ref.[1] because of its desirable features such as the unification of one generation of quarks and leptons in a single multiplet and a relatively natural way in which the doublet-triplet splitting can be achieved in the model[2]. On the technical side the introduction of the oscillator technique[3, 4, 5] in SO(10) analyses has proven useful. However, SO(10) matter interactions may involve large tensor representations, i.e., 120, $1\bar{2}6$ and 210. Specifically the representations 120 and $1\bar{2}6$ have already surfaced in the analyses of quark, charged lepton and neutrino mass textures[6, 7, 8, 9, 10]. However, a full analysis of the couplings of such large representations such as $16-16-120$ and $16-16-1\bar{2}6$ does not exist in the literature in any explicit form. We develop here a systematic approach that enables one to carry out a full computation of such couplings with relative ease. We then illustrate our technique by giving a complete analysis of the trilinear superpotential with the 16 plet of matter. Since $16\times 16 = 10_s + 120_a + 126_s$ we give an explicit computation of the $16-16-10_s$, $16-16-120_a$, and $16-16-1\bar{2}6_s$ couplings.

Our technique is a natural extension of the work of Refs.[3, 4] which introduced the oscillator expansion in the analysis of SO(2N) interactions [One may also use completely group theoretic methods to compute the couplings as done in E_6 model building analysis of Ref.[11]. Our technique is field theoretic and more straightforward.]. We briefly review this analysis first. In the oscillator technique of Refs.[3] one defines a set of N operators b_i (i=1,...,N) obeying the anti-commutation rules

$$\{b_i, b_j^\dagger\} = \delta_{ij}; \quad \{b_i, b_j\} = 0 \tag{1}$$

and represents the set of 2N operators Γ_μ ($\mu = 1, 2, .., 2N$) by

$$\Gamma_{2i} = (b_i + b_i^\dagger); \quad \Gamma_{2i-1} = -i(b_i - b_i^\dagger) \tag{2}$$

where Γ_μ satisfy a rank 2N Clifford algebra $\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}$. The group SO(2N) has a 2^N dimensional spinor representation ψ. This representation can be split into 2^{N-1} dimensional representation under the action of the chirality operator so that

$$\psi_\pm = \frac{1}{2}(1 \pm \Gamma_0)\psi, \quad \Gamma_0 = i^N \Gamma_1 \Gamma_2 ... \Gamma_{2N} \tag{3}$$

where $\psi_\pm$ are each 2^{N-1} dimensional. In the analysis of SO(2N) interactions one encounters couplings of the type $\tilde{\psi} B \Gamma_\mu .. \Gamma_\sigma \psi \phi_{\mu..\sigma}$ where $B(=\prod_{\mu=odd}\Gamma_\mu)$ is an

SO(2N) charge conjugation matrix. We wish to develop here a simple technique for the explicit evaluation of the couplings in terms of the physical degrees of freedom even for the case when the tensor representation that couples has a large dimensionality.

2 The Basic Theorem

We begin with the observation that the natural basis for the expansion of the SO(2N) vertex is in terms of a specific set of SU(N) reducible tensors which we define below. We introduce the notation $\phi_{c_i} = \phi_{2i} + i\phi_{2i-1}$ and $\phi_{\bar{c}_i} = \phi_{2i} - i\phi_{2i-1}$ which can be extended immediately to define the quantity $\phi_{c_ic_j\bar{c}_k..}$ with an arbitrary number of unbarred and barred indices where each c index can be expanded out so that $\phi_{c_ic_j\bar{c}_k..} = \phi_{2ic_j\bar{c}_k...} + i\phi_{2i-1c_j\bar{c}_k..}$, $\phi_{c_ic_j\bar{c}_k..} = \phi_{c_ic_j2k...} - i\phi_{c_ic_j2k-1...}$,etc. Thus, for example, the quantity $\phi_{c_ic_j\bar{c}_k...c_N}$ is a sum of 2^N terms gotten by expanding all the c indices. $\phi_{c_ic_j\bar{c}_k...c_n}$ is completely anti-symmetric in the interchange of its c indices whether unbarred or barred. We now make the observation that the object $\phi_{c_ic_j\bar{c}_k...c_n}$ transforms like a reducible representation of SU(N). Thus if we are able to compute the SO(2N) invariant couplings in terms of these reducible tensors of SU(N) then there remains only the further step of decomposing the reducible tensors into their irreducible parts. Finally, one can take the result obtained in terms of the SU(N) irreducible representations and expand out in terms of the particles of the model.

The result essential to our analysis is the theorem that the quantity $\Gamma_\mu\Gamma_\nu\Gamma_\lambda..\Gamma_\sigma$ $\phi_{\mu\nu\lambda..\sigma}$ can be expanded in the following form

$$\Gamma_\mu\Gamma_\nu\Gamma_\lambda..\Gamma_\sigma\phi_{\mu\nu\lambda..\sigma} = b_i^\dagger b_j^\dagger b_k^\dagger..b_n^\dagger\phi_{c_ic_jc_k...c_n} + (b_ib_j^\dagger b_k^\dagger..b_n^\dagger\phi_{\bar{c}_ic_jc_k...c_n} + \ perms)$$
$$+(b_ib_jb_k^\dagger..b_n^\dagger\phi_{\bar{c}_i\bar{c}_jc_k..c_n} + \ perms) + ... + (b_ib_jb_k..b_{n-1}b_n^\dagger\phi_{\bar{c}_i\bar{c}_j\bar{c}_k..\bar{c}_{n-1}c_n} + \ perms) +$$
$$+b_ib_jb_k.....b_n\phi_{\bar{c}_i\bar{c}_j\bar{c}_k...\bar{c}_n} \quad (4)$$

Eq.(4) is the basic result we need in the analysis of the SO(2N) invariant couplings. It is found convenient to arrange the right hand side of Eq.(4) in a normal ordered form by which we mean that all the b's are either to the right or to the left and all the $b^\dagger$'s are either to the left or to the right using strictly the anti-commutation relations on the b's and $b^\dagger$'s of Eq.(1). When a pair of $b^\dagger$ and b have a summed index such as $b_n^\dagger b_n$ we will move them together either to the left or to the right. After normal ordering one decomposes $\phi_{c_ic_j\bar{c}_k..c_n}$ into its SU(5) irreducible components.

The final step consists of carrying out the process of removing all the b and $b^\dagger$ using the anti-commutation relation Eq.(1) along with the condition $b_i|0>=0$ which allows us to compute the couplings in the SU(N) invariant decomposition.

3 The 120 plet and the $1\bar{2}6$ plet tensor couplings

The above procedure makes it straightforward to analyze the SO(2N) invariant couplings involving large tensor representations. As an illustration of our technique we give a complete determination of the superpotential at the trilinear level involving two spinor representations which consists of the couplings $16 \times 16 \times 10$, $16 \times 16 \times 120$ and $16 \times 16 \times 1\bar{2}6$. We begin by computing the $16 \times 16 \times 10$ coupling which is given by

$$f_{ab}\tilde{\psi}_a B\Gamma_\mu \psi_b \phi_\mu \tag{5}$$

where a, b are the generation indices. Following the procedure of sec.2 we decompose the vertex so that

$$\Gamma_\mu \phi_\mu = b_i \phi_{\bar{c}_i} + b_i^\dagger \phi_{c_i} \tag{6}$$

In Eq.(6) the tensors are already in their irreducible form and one can identify ϕ_{c_i} with the 5 plet of Higgs and $\phi_{\bar{c}_i}$ with the $\bar{5}$ plet of Higgs. To normalize the tensors we define $H_{1i} = \frac{1}{\sqrt{2}}\phi_{\bar{c}_i}$, $H_2^i = \frac{1}{\sqrt{2}}\phi_{c_i}$, so that the kinetic energy $-\partial_\alpha \phi_\mu \partial^\alpha \phi_\mu^\dagger$ of the tensor ϕ_μ takes the form $-\partial_\alpha H_{1i}\partial^\alpha H_{1i}^\dagger - \partial_\alpha H_2^i \partial^\alpha H_2^{i\dagger}$. For the computation of the superpotential we need to expand the 16 plet spinor representation ψ_+ in its oscillator modes

$$|\psi_+>=|0>M_0 + \frac{1}{2}b_i^\dagger b_j^\dagger|0>M^{ij} + \frac{1}{24}\epsilon^{ijklm}b_i^\dagger b_j^\dagger b_k^\dagger b_l^\dagger|0>M_m' \tag{7}$$

so that ψ_+ contains $1_M + 10_M + \bar{5}_M$ in its SU(5) decomposition. Using the above we compute the $16-16-10$ couplings and find

$$W^{(10)} = (2\sqrt{2}i)f_{ab}^{(+)}(M_a^{ij}M_{ib}'H_{1j} - M_{0a}M_{ib}'H_2^i + \frac{1}{8}\epsilon_{ijklm}M_a^{ij}M_b^{kl}H_2^m) \tag{8}$$

where $f_{ab}^{(\pm)}$ are defined by

$$f_{ab}^{(\pm)} = \frac{1}{2}(f_{ab} \pm f_{ba}) \tag{9}$$

and $f_{ab}^{(\pm)}$ are symmetric (antisymmetric) under the interchange of generation indices a and b. As expected the 16-16-10 couplings given by Eq.(8) are correctly symmetric in the generation indices. We note that the couplings have the SU(5) invariant structure consisting of $1_M - \bar{5}_M - 5_H$, $10_M - \bar{5}_M - \bar{5}_H$ and $10_M - 10_M - 5_H$.

Next we discuss the 16-16-120 coupling which is given by

$$\frac{1}{3!}f_{ab}\tilde{\psi}_a B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\psi_b\phi_{\mu\nu\lambda} \tag{10}$$

We expand the vertex using Eq.(4) and find

$$\Gamma_\mu\Gamma_\nu\Gamma_\lambda\phi_{\mu\nu\lambda} = b_ib_jb_k\phi_{\bar{c}_i\bar{c}_j\bar{c}_k} + b_i^\dagger b_j^\dagger b_k^\dagger\phi_{c_ic_jc_k} + 3(b_i^\dagger b_jb_k\phi_{c_i\bar{c}_j\bar{c}_k} + b_i^\dagger b_j^\dagger b_k\phi_{c_ic_j\bar{c}_k}) \\ +(3b_i\phi_{\bar{c}_nc_n\bar{c}_i} + 3b_i^\dagger\phi_{\bar{c}_nc_nc_i}) \tag{11}$$

The 120 plet of SO(10) has the SU(5) decomposition $120 = 5 + \bar{5} + 10 + \bar{10} + 45 + \bar{45}$. Eq.(11) can be decomposed in terms of these irreducible SU(5) tensors as explained in the appendix. A straightforward computation using Eq.(11)[2] and the normalization of Eq.(23) in the appendix gives

$$W^{(120)} = i\frac{2}{\sqrt{3}}f_{ab}^{(-)}(2M_{0a}M_{ib}h^i + M_a^{ij}M_{0b}h_{ij} + M_{ia}M_{jb}h^{ij} \\ -M_a^{ij}M_{ib}h_j + M_{ia}M_b^{jk}h_{jk}^i - \frac{1}{4}\epsilon_{ijklm}M_a^{ij}M_b^{mn}h_n^{kl}) \tag{12}$$

The front factor $f_{ab}^{(-)}$ in Eq.(12) exhibits correctly the anti-symmetry in the generation indices. Further, the couplings have the $1_M - \bar{5}_M - 5_H$, $1_M - 10_M - \bar{10}_H$, $\bar{5}_M - \bar{5}_M - 10_H$, $10_M - \bar{5}_M - \bar{5}_H$, $\bar{5}_M - 10_M - \bar{45}_H$ and $10_M - 10_M - 45_H$ SU(5) invariant structures.

We now turn to the most difficult of the three cases, i.e., the $16 - 16 - \bar{126}$ coupling which is given by

$$\frac{1}{5!}f_{ab}\tilde{\psi}_a B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\rho\Gamma_\sigma\psi_b\Delta_{\mu\nu\lambda\rho\sigma} \tag{13}$$

where $\Delta_{\mu\nu\lambda\rho\sigma}$ is 252 dimensional and can be decomposed so that $\Delta_{\mu\nu\lambda\rho\sigma}=\bar{\phi}_{\mu\nu\lambda\rho\sigma}+\phi_{\mu\nu\lambda\rho\sigma}$, where[7]

$$\begin{pmatrix} \bar{\phi}_{\mu\nu\lambda\rho\sigma} \\ \phi_{\mu\nu\lambda\rho\sigma} \end{pmatrix} = \frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta}\delta_{\rho\gamma}\delta_{\lambda\delta}\delta_{\sigma\theta} \pm \frac{i}{5!}\epsilon_{\mu\nu\rho\lambda\sigma\alpha\beta\gamma\delta\theta})\Delta_{\alpha\beta\gamma\delta\theta} \tag{14}$$

and where the $\bar{\phi}_{\mu\nu\lambda\rho\sigma}$ is the $1\bar{2}6$ plet and $\phi_{\mu\nu\lambda\rho\sigma}$ is the 126 plet representation. It is only the $1\bar{2}6$ that couples in Eq.(13) with the 16 plet spinors. However, for the reduction of the SO(10) vertex it is more convenient initially to work with the full 252 dimensional tensor and in the final computation only the $1\bar{2}6$ couplings will survive. We begin by expanding $\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\rho\Gamma_\sigma\Delta_{\mu\nu\lambda\rho\sigma}$ using Eq.(4) following steps similar

[2]The symmetrical arrangement in the first brace of Eq.(11) is necessary for achieving an automatic anti-symmetry in the generation indices for the SU(5) $10_M - 10_M - 45_H$ coupling.

to the previous case using normal ordering and further decomposing the tensors into their irreducible components. The $1\bar{2}6$ and the 126 dimensional representations break into the SU(5) irreducible parts as $1\bar{2}6 = 1 + 5 + 1\bar{0} + 15 + 4\bar{5} + 50$ and $126 = 1 + \bar{5} + 10 + 1\bar{5} + 45 + 5\bar{0}$. The details of the decomposition are given in the appendix. Using Eq.(25) in the appendix a straightforward analysis gives

$$W^{(1\bar{2}6)} = if^{(+)}_{ab}\frac{\sqrt{2}}{\sqrt{15}}[-\sqrt{2}M_{0a}M_{0b}h - \sqrt{3}M_{0a}M'_{ib}h^i + M_{0a}M^{ij}_b h_{ij} \\ -\frac{1}{8\sqrt{3}}M^{ij}_a M^{kl}_b h^m \epsilon_{ijklm} - h^{ij}_S M'_{ia}M'_{jb} + M^{ij}_a M'_{bk}h^k_{ij}] \tag{15}$$

where we have expressed the result in terms of fields h^i, h_{ij} etc with normalizations given by Eq.(27). As in the 10 plet tensor case the couplings are symmetric under the interchange of generation indices. Further, the $16 - 16 - 1\bar{2}6$ coupling has the SU(5) structure consisting of $1_M - 1_M - 1_H$, $1_M - \bar{5}_M - 5_H$, $1_M - 10_M - 1\bar{0}_H$, $10_M - 10_M - 5_H$, $\bar{5}_M - \bar{5}_M - 15_H$ and $10_M - \bar{5}_M - 4\bar{5}_H$. A similar analysis can be carried out for the tensor couplings involving $1\bar{6} - 16$ which includes $1\bar{6} - 16 - 45$ and $1\bar{6} - 16 - 210$ couplings. These will be discussed elsewhere.

4 Large representations, textures and proton lifetime

Proton decay is an important signal for grand unification and detailed analyses for the proton lifetime exist in SU(5) models[12, 13, 14] and in SO(10) models[15, 16]. In this section we discuss the possibility that couplings in the superpotential that involve large representations can drastically change the Higgs triplet textures and affect proton decay in a very significant manner. These results are of significance in view of the recent data from SuperKamiokande which has significantly improved the limit on the proton decay mode $p \to \nu + K^+$. Thus the most recent limit from SuperKamiokande gives $\tau/B(p \to \bar{\nu} + K^+) > 1.9 \times 10^{33}$ yr[17]. At the same time there is a new lattice gauge evaluation of the three quark matrix elements α and β of the nucleon wave function[18] (where α and β are defined by[19] $\epsilon_{abc} < 0|\epsilon_{\alpha\beta}d^{\alpha}_{aR}u^{\beta}_{bR}u^{\gamma}_{cL}|p >= \alpha u^{\gamma}_L$ and $\epsilon_{abc} < 0|\epsilon_{\alpha\beta}d^{\alpha}_{aL}u^{\beta}_{bL}u^{\gamma}_{cL}|p >= \beta u^{\gamma}_L$). Previous evaluations of these quantities have varied over a wide range from $\beta = 0.003GeV^3$[20] to $\beta = 0.03GeV^3$[19, 21] while recent p decay analyses have often used the lattice gauge evaluation of Gavela et.al.[22] which gives $\beta = (5.6 \pm 0.5) \times 10^{-3}GeV^3$. The more recent evaluation of Ref.[18] gives $\alpha = -0.015(1)GeV^3$ and

$\beta = 0.014(1) GeV^3$ which is a factor of about two and a half times larger than the evaluation of Ref.[22]. The new experimental limit on the proton decay lifetime[17] combined with the new lattice gauge evaluations have begun to constrain the SUSY GUT models prompting some reanalyses[23, 24]. In this context the enhancement of the proton lifetime by textures is of interest. To make this idea more concrete we define textures in the low energy theory in the quark lepton sector of the theory just below the GUT scale as follows

$$\begin{aligned} W_Y = -M_H H_{1t} H_{2t} + (l A^E e^c h_1 + q A^D d^c h_1 + h_2 u^c A^U q) \\ +(q B^E l H_{1t} + \epsilon_{abc} H_{1ta} d^c_b B^D u^c_c + H_{2ta} u^c_a B^U e^c + \epsilon_{abc} H^a_{2t} u_b C^U d_c) \end{aligned} \tag{16}$$

where A^E, A^D and A^U are the textures in the Higgs doublet sector and B^E, B^D, B^U and C^U are the textures in the Higgs triplet sector. A classification of the possible textures in the Higgs doublet sector is given in Refs.[25, 26]. For our purpose here we adopt the textures in the Higgs doublet sector in the form

$$A^E = \begin{pmatrix} 0 & f & 0 \\ f & -3e & 0 \\ 0 & 0 & d \end{pmatrix}, A^D = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & e & 0 \\ 0 & 0 & d \end{pmatrix}, A^U = \begin{pmatrix} 0 & c & 0 \\ c & 0 & b \\ 0 & b & a \end{pmatrix} \tag{17}$$

As is well known[25] the appearance of -3 vs 1 in the 22 element of A^E vs A^D is one of the important ingredients in achieving the desired quark and lepton mass hierarchy and may provide an insight into the nature of the fundamental coupling. Now the textures in the Higgs triplet sector are generally different than those in the Higgs doublet sector and they are sensitively dependent on the nature of GUT and Planck scale physics[27]. The current experimental constraints on the proton lifetime leads us to conjecture that the Higgs triplet sector contains additional texture zeros over and above the texture zeros that appear in the Higgs doublet sector and the coupling of the $1\bar{2}6$ tensor field plays an important role in this regard. In the following we shall assume CP invariance and set the phases to zero in Eq.(17). Since in this case the textures of Eq.(17) are symmetric it is only the 10 plet and the $1\bar{2}6$ plet of Higgs couplings that enter in the analysis and the 120 plet couplings do not. To exhibit the above phenomenon more concretely we consider on phenomenological grounds a superpotential in the Yukawa sector of the following type

$$\begin{aligned} W_Y = f^{(0)}_{ij}(Y,M)\psi_i\psi_j\phi^{(0)}_{1\bar{2}6} + f^{(d)}_{12}(Y,M)\psi_1\psi_2\phi^{(1)}_{10} + f^{(u)}_{12}(Y,M)\psi_1\psi_2\phi^{(2)}_{10} \\ +f^{(d)}_{22}(Y,M)\psi_2\psi_2\phi^{(1)}_{1\bar{2}6} + f^{(u)}_{23}(Y,M)\psi_2\psi_3\phi^{(2)}_{1\bar{2}6} + f^{(d)}_{33}(Y,M)\psi_3\psi_3\phi^{(1)}_{10} \\ +f^{(u)}_{33}(Y,M)\psi_3\psi_3\phi^{(2)}_{1\bar{2}6} \end{aligned} \tag{18}$$

where M is a superheavy scale and $f_{ij}^{(d)}(Y,M)$ and $f_{ij}^{(u)}(Y,M)$ are functions of a set of scalar fields Y which develop VEVs and the appropriate factors of $< Y^n\phi > /M^n$ generate the right sizes. The model of Eq.(18) is of the generic type discussed in refs.[7, 10]. We do not go into detail here regarding the symmetry breaking mechanism, the doublet-triplet splitting and the mass generation for the pseudo-goldstone bosons. All of these topics have been dealt with at some length in the previous literature[2, 7, 8, 9]. Further, while models with large representations are not asymptotically free and lead rapidly to non-perturbative physics above the unification scale, the effective theories below the unification scale gotten by integration over the heavy modes are nonetheless perfectly normal and thus such theories are acceptable unified theories. For our purpose here we assume a pattern of VEV formation for the neutral components of the Higgs so that $< \phi^{(0)}_{1\bar{2}6} >$ develops a VEV along the SU(5) singlet direction (this corresponds to h in Eq.(15) developing a VEV), $< \phi^{(1)}_{10} >$ develops a VEV in the $\bar{5}$ plet of SU(5) direction (this corresponds to H_1 developing a VEV in Eq.(8)), $< \phi^{(2)}_{10} >$ develops a VEV in the 5 plet direction (this corresponds to H_2 developing a VEV in Eq.(8)), $< \phi^{(1)}_{1\bar{2}6} >$ develops a VEV in the direction of $\bar{45}$ plet of Higgs (this corresponds to h^k_{ij} in Eq.(15) developing a VEV), and $< \phi^{(2)}_{1\bar{2}6} >$ develops a VEV in the direction of 5 plet of Higgs (this correspons to h^i in Eq.(15) developing a VEV). It is the VEV of the 45 plet that leads to -3 and 1 factors in A^E vs A^D. The superpotential of Eq.(18) with the above VEV alignments then leads automatically to the textures in the Higgs doublet sector of Eq.(17). One may now compute the textures in the Higgs triplet sector that result from superpotential of Eq.(18). One finds

$$B^E = \begin{pmatrix} 0 & f & 0 \\ f & 0 & 0 \\ 0 & 0 & d \end{pmatrix}, B^U = \begin{pmatrix} 0 & c & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

and $B^D = B^E$ and $C^U = B^U$. We note the existence of the additional zeros in B^E and B^D relative to A^E and A^D and in B^U and C^U relative to A^U. We shall show shortly that the existence of the additional zeros in B^E, B^D,B^U and C^U increases in a very significant manner the proton decay lifetime. Before we discuss this enhancement in greater detail, we wish to discuss the origin of the additional zeros. It is easy to see that a coupling of the matter sector with the $1\bar{2}6$ of Higgs which contributes a non-vanishing element in the Higgs doublet sector produces a vanishing contribution in the lepton and baryon number violating dimension five operator or equivalently generates a corresponding zero in the texture in the Higgs triplet sector. The reason for this is rather straightforward. While one also needs a 126 plet of Higgs to cancel the D term generated by the VEV of the $1\bar{2}6$ of Higgs, the 126 plet of Higgs has no coupling with the ordinary 16 plet of matter. Since the only bilinear with the $1\bar{2}6$ in the superpotential is of the form $126 \times 1\bar{2}6$ (i.e., one cannot write a $(126)^2$ term in the superpotential) one finds that no lepton and baryon number violating dimension five operators arise as a consequence of integrating out the 126 and $1\bar{2}6$ of Higgs which effectively corresponds to a texture zero in the Higgs triplet sector. Of course, the extra zeros in the Higgs triplet sector could also arise from accidental cancellations. However, the group theoretic origin is more appealing.

The extra zeros in the textures in the Higgs triplet sector lead to a substantial enhancement of the proton decay lifetime. To see their effect we begin by integrating out the Higgs triplet field in Eq.(16) which generates lepton and baryon number violating dimension five operators with the chiral structure LLLL and RRRR. Of these the LLLL operator involves the textures B^E and C^U while the RRRR operator involves the textures B^D and B^U. Since the number of extra zeros in B^E and B^D are the same and the same holds for B^U and C^U, it suffices to discuss only one of these operators, and in the following we focus on the LLLL dimension five operator. Here the texture zero in the 22 element of B^E suppresses the $\bar{\nu}_\mu K^+$ decay mode of the proton by a factor m_d/m_s making the $\bar{\nu}_\tau K^+$ the dominant mode. Since the decay channel $\bar{\nu}_\mu K^+$ is highly suppressed (while the decay channel $\bar{\nu}_e K^+$ which is normally suppressed remains suppressed) we estimate that there is an over all suppression in all the neutrino decay channels from the extra zero in B^E to be about a factor of about 2. The texture zeros in C^U lead roughly to a replacement of m_c by $\sqrt{m_u m_c}$ and thus lead to a suppression of the proton decay lifetime roughly by m_u/m_c. A similar suppression holds for decays via the RRRR dimension five operator. Including the suppression from both B^E and C^U and using $m_c = 1.35$ GeV and the up quark mass in the range 1-5 MeV, one finds that the texture zeros can lead to an enhancement of the proton decay lifetime in the $\bar{\nu} K^+$ mode by a factor of $(1.5 \pm 1) \times 10^3$ over the minimal SU(5) model. Such lifetimes fall in the interesting range for the next generation of proton decay experiments. The texture effects are generic and similar effects are expected in other decay modes as well. The superpotential of Eq.(18) also generates a Dirac neutrino mass matrix which is given by

$$M_{\nu LR} = \begin{pmatrix} 0 & c & 0 \\ c & 0 & -3b \\ 0 & -3b & -3a \end{pmatrix} h_2 \tag{20}$$

However, a full analysis of neutrino masses requires a model for the Majorana mass matrix M_{Maj} to generate a see-saw mechanism[28] so that $M_{\nu LL} = -M_{\nu LR}^T M_{Maj}^{-1} M_{\nu LR}$ where the mass scale associated with M_{Maj} is much larger than the mass scale that appears in $M_{\nu LR}$. M_{Maj} depends on $f_{ij}^{(0)}$ in Eq.(18) which are in general additional arbitrary parameters. [For a sample of recent analyses on neutrino masses in SO(10) see Ref.[10, 29] and for a review and a classification of models of neutrino masses see Ref.[30]]. The appearance of M_{Maj} in the analysis of neutrino masses with additional arbitrary parameters and a scale much larger than the mass scale that appears in $M_{\nu LR}$ implies that there is not a rigid relationship between proton decay and neutrino masses in SO(10). Nonetheless, it is interesting to investigate the correlation that exists between these two important phenomena in specific models. A more extensive analysis of this topic involving the detailed coupling structure of Eqs.(8), (12) and (15) as well as other texture possibilities in the Higgs doublet sector[26] will be discussed elsewhere.

In conclusion, we have developed a technique which allows the explicit computation of the SO(2N) invariant couplings in terms of SU(N) invariant couplings. The technique is specially useful in the analysis of couplings involving large tensor representations. We have illustrated the technique by carrying out a complete analysis of the SO(10) invariant superpotential at the trilinear level involving interactions of matter with Higgs which consists of the $16_a 16_b 10_H$, $16_a 16_b 120_H$, and the $16_a 16_b \overline{126}_H$ couplings. The technique can be used with relative ease to compute other couplings involving large tensor representations such as $\overline{16} - 16 - 210$. We note that the decomposition of SO(10) into multiplets of SU(5) is merely a convenient device for expanding the SO(10) interaction in a compact form and does not necessarily imply a preference for the symmetry breaking pattern. Indeed one can compute the SO(10) interactions using the technique used here and then use any symmetry breaking scheme one wishes to get to the low energy theory. We also discussed in this paper the phenomena that the coupling of the $\overline{126}$ with matter leads to extra zeros in the Higgs triplet sector and the existence of such zeros can enhance the proton decay lifetime by as much as 10^3. Thus $\overline{126}$ couplings might help relieve the constraint on SUSY GUT models because of the recent SuperKamiokande data and improved lattice gauge calculations of α and β. The coupling involving large tensor representation given in sec.3 also have implications for neutrino mass textures. Finally, the technique discussed here is easily extendible to models with SO(2N+1) invariance. [Note added: The enhancement of the proton lifetime by a factor of $O(10^3)$ discussed here may be needed to overcome the light sparticle spectrum (see e.g., U. Chattopadhyay and P. Nath, hep-ph/01021577) implied by the Brookhaven g-2 experiment, H.N. Brown et.al, hep-ex/0102017.]

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Appendix: Details of 120 plet and $1\bar{2}6$ plet couplings and normalizations
We discuss now the details of the 120 plet and $1\bar{2}6$ plet couplings. For the tensors that appears in the 120 plet coupling, can be decomposed in their irreducible forms by the decomposition

$$\phi_{c_i c_j \bar{c}_k} = f^{ij}_k + \frac{1}{4}(\delta^i_k f^j - \delta^j_k f^i), \quad \phi_{c_i \bar{c}_j \bar{c}_k} = f^i_{jk} + \frac{1}{4}(\delta^i_j f_k - \delta^i_k f_j)$$
$$\phi_{c_i c_j c_k} = \epsilon^{ijklm} f_{lm}, \quad \phi_{\bar{c}_i \bar{c}_j \bar{c}_k} = \epsilon_{ijklm} f^{lm}, \quad \phi_{\bar{c}_n c_n c_i} = f^i, \phi_{\bar{c}_n c_n \bar{c}_i} = f_i \tag{21}$$

where f^{ij}_k and f^i_{jk} are traceless and are the 45 plet and the $\bar{4}5$ plet representations of SU(5). The irreducible tensors f^i, f^{ij} etc are not yet properly normalized. To normalize them we make the following redefinition of fields

$$f^i = \frac{4}{\sqrt{3}} h^i, \quad f^{ij} = \frac{1}{\sqrt{3}} h^{ij}, \quad f^{ij}_k = \frac{2}{\sqrt{3}} h^{ij}_k$$
$$f_i = \frac{4}{\sqrt{3}} h_i, \quad f_{ij} = \frac{1}{\sqrt{3}} h_{ij}, \quad f^i_{jk} = \frac{2}{\sqrt{3}} h^i_{jk} \tag{22}$$

In terms of the redefined fields the kinetic energy term for the 120 multiplet which is given by $-\partial_\alpha \phi_{\mu\nu\lambda}\, \partial^\alpha \phi^\dagger_{\mu\nu\lambda}$ takes on the form

$$L^{(120)}_{kin} = -(\frac{1}{2}\partial_\alpha h^{ij} \partial^\alpha h^{ij\dagger} + \frac{1}{2}\partial_\alpha h_{ij} \partial^\alpha h^\dagger_{ij} + \frac{1}{2}\partial_\alpha h^{ij}_k \partial^\alpha h^{ij\dagger}_k$$
$$+\frac{1}{2}\partial_\alpha h^i_{jk} \partial^\alpha h^{i\dagger}_{jk} + \partial_\alpha h^i \partial^\alpha h^{i\dagger} + \partial_\alpha h_i \partial^\alpha h^\dagger_i) \tag{23}$$

where the factors of 1/2 are to account for (ij) permutations.

Next we discuss details of the $1\bar{2}6$ plet couplings. Here the reducible tensors that enter in the expansion of the vertex are $\Delta_{c_ic_jc_kc_l\bar{c}_m}$, $\Delta_{c_ic_jc_k\bar{c}_l\bar{c}_m}$ etc. These can be decomposed into their irreducible parts as follows

$$\begin{aligned}
\Delta_{c_ic_jc_kc_l\bar{c}_m} &= f^{ijkl}_m + \frac{1}{2}(\delta^i_m f^{jkl} - \delta^j_m f^{ikl} + \delta^k_m f^{ijl} - \delta^l_m f^{ijk}) \\
\Delta_{c_ic_jc_k\bar{c}_l\bar{c}_m} &= f^{ijk}_{lm} + \frac{1}{2}(\delta^i_l f^{jk}_m - \delta^j_l f^{ik}_m + \delta^k_l f^{ij}_m - \delta^i_m f^{jk}_l + \delta^j_m f^{ik}_l - \delta^k_m f^{ij}_l) \\
&+ \frac{1}{12}(\delta^i_l\delta^j_m f^k - \delta^j_l\delta^i_m f^k - \delta^i_l\delta^k_m f^j + \delta^k_l\delta^i_m f^j + \delta^j_l\delta^k_m f^i - \delta^k_l\delta^j_m f^i) \\
\Delta_{c_ic_j\bar{c}_k\bar{c}_l\bar{c}_m} &= f^{ij}_{klm} + \frac{1}{2}(\delta^i_k f^j_{lm} - \delta^i_l f^j_{km} + \delta^i_m f^j_{kl} - \delta^j_k f^i_{lm} + \delta^j_l f^i_{km} - \delta^j_m f^i_{kl}) \\
&+ \frac{1}{12}(\delta^i_k\delta^j_l f_m - \delta^i_k\delta^j_m f_l - \delta^i_l\delta^j_k f_m + \delta^i_l\delta^j_m f_l + \delta^i_m\delta^j_k f_l - \delta^i_m\delta^j_l f_k) \\
\Delta_{c_i\bar{c}_j\bar{c}_k\bar{c}_l\bar{c}_m} &= f^i_{jklm} + \frac{1}{2}(\delta^i_j f_{klm} - \delta^i_k f_{jlm} + \delta^i_l f_{jkm} - \delta^i_m f_{jkl}) \\
\Delta_{c_ic_jc_kc_lc_m} &= \epsilon^{ijklm} f, \quad \Delta_{\bar{c}_i\bar{c}_j\bar{c}_k\bar{c}_l\bar{c}_m} = \epsilon_{ijklm}\bar{f} \qquad (24)
\end{aligned}$$

In terms of the irreducible tensors the vertex that enters in Eq.(13) can be decomposed as follows

$$\begin{aligned}
\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\rho\Gamma_\sigma\Delta_{\mu\nu\lambda\rho\sigma} &= (\epsilon_{ijklm}b_ib_jb_kb_lb_m f + \epsilon^{ijklm}b^\dagger_i b^\dagger_j b^\dagger_k b^\dagger_l b^\dagger_m \bar{f}) + \\
+(15b^\dagger_i f^i - 20b^\dagger_i b^\dagger_n b_n f^i &+ 5b^\dagger_i b^\dagger_n b_n b^\dagger_m b_m f^i + 15b_i f_i - 20b^\dagger_n b_n b_i f_i + 5b^\dagger_n b_n b^\dagger_m b_m b_i f_i) \\
&+10(b^\dagger_i b^\dagger_j b^\dagger_k f^{ijk} - b^\dagger_i b^\dagger_j b^\dagger_k b^\dagger_n b_n f^{ijk} + b_ib_jb_k f_{ijk} - b^\dagger_n b_n b_ib_jb_k f_{ijk}) \\
&+(60b^\dagger_i b^\dagger_j b_k f^{ij}_k - 30b^\dagger_i b^\dagger_j b_k b^\dagger_n b_n f^{ij}_k + 60b^\dagger_i b_jb_k f^i_{jk} - 30b^\dagger_n b_n b^\dagger_i b_jb_k f^i_{jk}) \\
+(5b^\dagger_i b^\dagger_j b^\dagger_k b^\dagger_l b_m f^{ijkl}_m &+ 5b^\dagger_i b_jb_kb_lb_m f^i_{jklm}) + (10b^\dagger_i b^\dagger_j b^\dagger_k b_lb_m f^{ijk}_{lm} + 10b^\dagger_i b^\dagger_j b_kb_lb_m f^{ij}_{klm}) \quad (25)
\end{aligned}$$

The fields that appear above are not yet properly normalized. To normalize the fields we carry out a field redefinition so that

$$\begin{aligned}
f = \frac{2}{\sqrt{15}}h, \quad f^i = \frac{4\sqrt{2}}{\sqrt{5}}h^i, \quad f^{ijk} = \frac{\sqrt{2}}{\sqrt{15}}\epsilon^{ijklm}h_{lm} \\
f^i_{jklm} = \frac{\sqrt{2}}{\sqrt{15}}\epsilon_{jklmn}h^{(S)ni}, \quad f^i_{jk} = \frac{2\sqrt{2}}{\sqrt{15}}h^i_{jk}, \quad f^{ijk}_{lm} = \frac{2}{\sqrt{15}}h^{ijk}_{lm}, \qquad (26)
\end{aligned}$$

The kinetic energy for the $1\bar{2}6$ plet field $-\partial_\alpha\phi_{\mu\nu\lambda\rho\sigma}\partial^\alpha\phi^\dagger_{\mu\nu\lambda\rho\sigma}$ in terms of the normalized fields is then given by

$$\begin{aligned}
L^{(1\bar{2}6)}_{kin} = -(\partial_\alpha h\partial^\alpha h^\dagger + \partial_\alpha h^i\partial^\alpha h^{i\dagger} + \frac{1}{2}\partial_\alpha h_{ij}\partial^\alpha h^\dagger_{ij} + \frac{1}{2}\partial_\alpha h^{(S)ij}\partial^\alpha h^{(S)ij\dagger} \\
+\frac{1}{2}\partial_\alpha h^i_{jk}\partial^\alpha h^{i\dagger}_{jk} + \frac{1}{3!2}\partial_\alpha h^{ijk}_{lm}\partial^\alpha h^{ijk\dagger}_{lm}) \qquad (27)
\end{aligned}$$

where the numerical factors are to account for the permutations.

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