Indirect Determination of the Vertex and Angles of the Unitarity Triangle

SALVATORE MELE[*]

CERN, EP Division, CH1211, Genève 23, Switzerland[†]

The values of the elements of the Cabibbo-Kobayashi-Maskawa matrix are constrained by direct and indirect measurements. A fit to experimental data and theory calculations allows the indirect determination of the vertex and angles of the unitarity triangle as:

$$\rho = 0.18 \pm 0.07 \quad \eta = 0.35 \pm 0.05$$

$$\sin 2\alpha = 0.14^{+0.25}_{-0.38} \quad \sin 2\beta = 0.73 \pm 0.07 \quad \gamma = 63^{+8}_{-11} \text{ degrees.}$$

Information is derived on the presence of CP violation in the matrix, on non-perturbative QCD parameters and on the B_s^0 oscillation frequency.

Presented at the

5th International Symposium on Radiative Corrections
(RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000

---

[*]e-mail: Salvatore.Mele@cern.ch.
[†]On leave of absence from INFN Sezione di Napoli, 80125, Napoli, Italy.

1 Introduction

In the Standard Model of the electroweak interactions [1–3], a 3×3 unitary matrix describes the mixing of the quark mass eigenstates into the weak interaction ones. This matrix is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4,5], and can be written in terms of just four real parameters [6]:

$$V_{\text{CKM}} = \begin{pmatrix} V_{\text{ud}} & V_{\text{us}} & V_{\text{ub}} \\ V_{\text{cd}} & V_{\text{cs}} & V_{\text{cb}} \\ V_{\text{td}} & V_{\text{ts}} & V_{\text{tb}} \end{pmatrix} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (1)$$

A, ρ and η are of order unity and λ is the sine of the Cabibbo angle. The parameter η is the complex phase of the matrix, directly related to the violation of the CP symmetry in the weak interactions. The measurement of the parameters of the CKM matrix is of fundamental importance for both the description of the weak interaction of quarks and to shed light on the mechanism of CP violation.

The parameters A and λ are known with an accuracy of a few percent and this work concentrates on the indirect determination of ρ and η. This is also described as the study of the vertex or the angles of a triangle in the $\rho - \eta$ plane, whose other two vertices are located in (0,0) and (1,0). This triangle, called the unitarity triangle, is depicted in Figure 1. This study follows the same procedure as a previous publication [7] with an update of the input parameters, as described in the following.

A large number of physical processes are parametrised in terms of the values of the elements of the CKM matrix. Among them, four present the largest sensitivity to ρ and η, given the knowledge of the involved theoretical and experimental quantities. These processes are discussed in the following and then used in a fit to derive ρ and η. Conclusions are then drawn from the results of this fit.
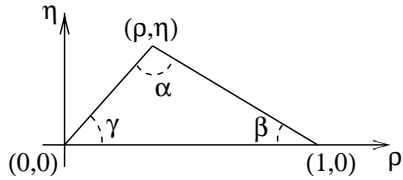

Figure 1: The unitarity triangle.

2 Constraints

2.1 λ and A

The value of the sine of the Cabibbo angle is measured as [8]:

$$\lambda = 0.2196 \pm 0.0023.$$

The study of inclusive semileptonic B decays by the CLEO [9] and the LEP [10] experiments yields information on the value of $|V_{cb}|$. Further constraints are derived from the study of the $B^0 \to D^{*+}\ell\nu$ decay, both at the $\Upsilon(4S)$ [11] and at the Z pole [10]. From these measurements, a value $|V_{cb}| = (40.9 \pm 1.9) \times 10^{-3}$ is extracted, which yields:

$$A = \frac{|V_{cb}|}{\lambda^2} = 0.83 \pm 0.04.$$

2.2 CP Violation for Neutral Kaons

The mass eigenstates of the neutral kaons can be written as $|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$ and $|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$. The relation $p \neq q$ implies the violation of the CP symmetry that, in the Wu-Yang phase convention [12], is described by the parameter ϵ_K defined as:

$$\frac{p}{q} = \frac{1 + \epsilon_K}{1 - \epsilon_K}.$$

The precise measurements of the $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$ decay rates imply [8]:

$$|\epsilon_K| = (2.280 \pm 0.019) \times 10^{-3}.$$

The relation of $|\epsilon_K|$ to the CKM matrix parameters is [13,14]:

$$\begin{aligned}
|\epsilon_K| &= \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K} B_K \left(A^2\lambda^6\eta\right) \\
&\times \left[y_c\left(\eta_{ct}f_3(y_c, y_t) - \eta_{cc}\right) + \eta_{tt}y_t f_2(y_t)A^2\lambda^4\left(1 - \rho\right)\right].
\end{aligned} \tag{2}$$

The functions f_3 and f_2 of the variables $y_t = m_t^2/m_W^2$ and $y_c = m_c^2/m_W^2$ are given in Reference [15]. The measured value of the top quark mass, 174.3 ± 5.1 GeV [8], is scaled as proposed in Reference [16], giving:

$$\overline{m_t}(m_t) = 167.3 \pm 5.2 \text{ GeV},$$

while the mass of the charm quark is chosen as [8]:

$$\overline{m_c}(m_c) = 1.25 \pm 0.10 \text{ GeV}.$$

The calculated QCD corrections in Equation (2) are described by the consistent set of parameters [16–19]:

$$\eta_{cc} = 1.38 \pm 0.53, \quad \eta_{tt} = 0.574 \pm 0.004, \quad \eta_{ct} = 0.47 \pm 0.04.$$

Non-perturbative QCD contributions to this process are affected by a large uncertainty and are summarised by the "bag" parameter B_K, chosen as [20]:

$$B_K = 0.87 \pm 0.14.$$

The other physical constants appearing in Equation (2) are reported in Table 1. The measurement of $|\epsilon_K|$ constrains the vertex of the unitarity triangle onto an hyperbola in the the $\rho - \eta$ plane.

Recent measurements of direct CP violation in the neutral kaon sector from the KTEV [21] and NA48 [22] experiments confirm the previous NA31 result [23]. These measurements could result in a lower bound to η. Nonetheless they are not used to constrain the CKM matrix owing to the large uncertainties that affect the corresponding theoretical calculations [24,25].

2.3 Oscillations of B_d^0 Mesons

The behaviour of neutral mesons containing a b quark depends on the the mass difference between the heavy and light mass eigenstates, B_H and B_L. These are different from the CP eigenstates B_d^0 and $\overline{B}_d^0$. The mass difference, $\Delta m_d = m_{B_H} - m_{B_L}$, is measured [26] at LEP, the $\Upsilon(4S)$ and the TEVATRON by means of the study of the oscillations of one CP eigenstate into the other. A recent average is:

$$\Delta m_d = 0.487 \pm 0.014 \, \text{ps}^{-1}.$$

Recent results from the Babar [27] and Belle [28] collaborations are not yet included in this average with which they are statistically comparable. The value of Δm_d is related to the CKM parameters as:

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 m_B \left(f_{B_d} \sqrt{B_{B_d}} \right)^2 \eta_B y_t f_2(y_t) A^2 \lambda^6 \left[(1 - \rho)^2 + \eta^2 \right]. \tag{3}$$

The calculated QCD correction η_B amounts to [16–19]:

$$\eta_B = 0.55 \pm 0.01,$$

while non-perturbative QCD contributions are summarised by [29]:

$$f_{B_d} \sqrt{B_{B_d}} = 0.206 \pm 0.029 \ \text{GeV}.$$

The vertex of the unitarity triangle is constrained by Δm_d onto a circle in the $\rho - \eta$ plane, with centre in $(1, 0)$.

2.4 Oscillations of B_s^0 Mesons

The B_s^0 mesons are predicted to mix like the B_d^0 mesons, but their larger mass difference, Δm_s, results into faster oscillations. These have eluded direct observation and the current 95% Confidence Level (CL) lower limit on Δm_s from the LEP, SLD and CDF collaborations is [26]:

$$\Delta m_s > 14.9 \, \text{ps}^{-1} \ \ (95\% \, \text{CL}).$$

3

The experiments, once combined, are sensitive to values of Δm_s up to $17.9\,\mathrm{ps}^{-1}$ and a $2.5\,\sigma$ indication for the observation of B_s^0 oscillations is observed around $\Delta m_s = 17.7\,\mathrm{ps}^{-1}$.

The expression for Δm_s as a function of the CKM parameters is similar to that for Δm_d, and taking their ratio, it follows:

$$\Delta m_s = \Delta m_d \frac{1}{\lambda^2} \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{1}{(1-\rho)^2 + \eta^2}. \tag{4}$$

All the theoretical parameters and their uncertainties are included in the quantity ξ, known as [29]:

$$\xi = \frac{f_{B_d}\sqrt{B_{B_d}}}{f_{B_s}\sqrt{B_{B_s}}} = 1.16 \pm 0.07.$$

The lower limit on Δm_s constrains the vertex of the unitarity triangle in a circle in the $\rho - \eta$ plane with centre in $(1,0)$.

2.5 Charmless Semileptonic b Decays

The constraints described so far suffer from the uncertainties in non-perturbative QCD quantities entering their expressions. The determination of either $|V_{ub}|$ or the ratio $|V_{ub}|/|V_{cb}|$ constitutes a constraint free from these uncertainties as:

$$|V_{ub}|/|V_{cb}| = \lambda\sqrt{\rho^2 + \eta^2}. \tag{5}$$

The CLEO collaboration has measured this ratio by means of the endpoint of inclusive [30] charmless semileptonic B decays as: $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$. The ALEPH [31], DELPHI [32] and L3 [33] collaborations have measured at LEP the inclusive charmless semileptonic branching fraction of beauty hadrons; these are averaged [10] as:

$$|V_{ub}| = (4.13\,^{+0.63}_{-0.75}) \times 10^{-3}.$$

Using the quoted value of $|V_{cb}|$, a combination with the CLEO measurement yields:

$$|V_{ub}|/|V_{cb}| = 0.089 \pm 0.010.$$

This constraint is represented by a circle in the $\rho - \eta$ plane with centre in $(0,0)$, shown in Figure 2, that also presents all the other constraints described so far.

3 The fit

The ρ and η parameters are determined from a fit to the constraints described above. The experimental and theoretical quantities appearing in the formulae (2), (3),
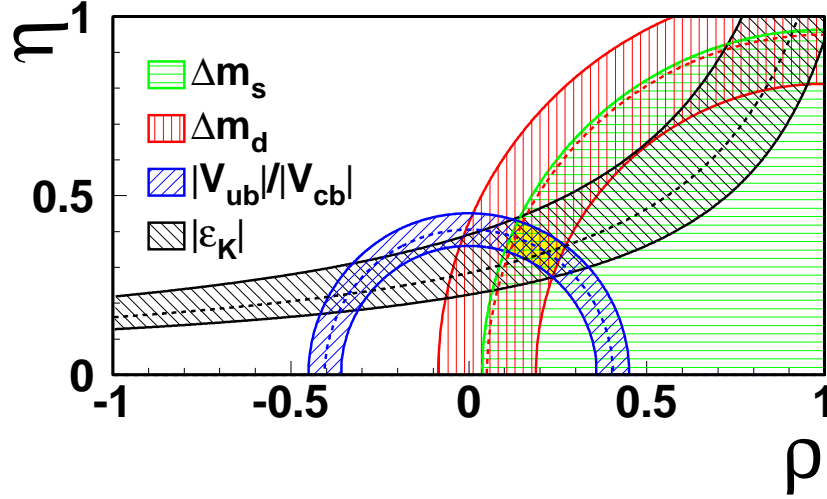
Figure 2: Constraints in the $\rho - \eta$ plane. B_s^0 oscillations are reported as a 95% CL limit, while the other constraints represent a $\pm 1\sigma$ variation of the experimental and theoretical parameters entering in the formulae in the text. Central values are indicated by the dashed lines. A darker area shows the overlap among the constraints.

Table 1: Physical constants and parameters of the fit. The values not discussed in the text follow from Reference [8].

λ	=	$0.2196(23)$	A	=	$0.83(4)$
G_F	=	$1.16639(1) \times 10^{-5} \, \text{GeV}^{-2}$	η_{ct}	=	$0.47(4)$
f_K	=	$0.1598(15) \, \text{GeV}$	η_{cc}	=	$1.38(53)$
Δm_K	=	$0.5304(14) \times 10^{-2} \, \text{ps}^{-1}$	$\overline{m_c}(m_c)$	=	$1.25(10) \, \text{GeV}$
m_K	=	$0.497672(31) \, \text{GeV}$	$\overline{m_t}(m_t)$	=	$167.3(5.2) \, \text{GeV}$
m_W	=	$80.419(38) \, \text{GeV}$	$f_{B_d}\sqrt{B_{B_d}}$	=	$0.206(29) \, \text{GeV}$
m_{B_d}	=	$5.2792(18) \, \text{GeV}$	B_K	=	$0.87(14)$
m_{B_s}	=	$5.3692(20) \, \text{GeV}$	ξ	=	$1.16(7)$
m_B	=	$5.290(2) \, \text{GeV}$	$|\epsilon_K|$	=	$2.280(19) \times 10^{-3}$
η_B	=	$0.55(1)$	Δm_d	=	$0.487(14) \, \text{ps}^{-1}$
η_{tt}	=	$0.574(4)$	$|V_{ub}|/|V_{cb}|$	=	$0.089(10)$

(4) and (5) are divided in two classes. Those whose uncertainties are below 2% are fixed to their central value as listed in the left half of Table 1. The quantities affected by a larger uncertainty and $|\epsilon_K|$ are considered as additional parameters of the fit,

constraining their values to the estimates summarised in the right half of Table 1. The following expression is then minimised:

$$\chi^2 = \frac{\left(\widehat{A} - A\right)^2}{\sigma_A^2} + \frac{\left(\widehat{m_c} - m_c\right)^2}{\sigma_{m_c}^2} + \frac{\left(\widehat{m_t} - m_t\right)^2}{\sigma_{m_t}^2} + \frac{\left(\widehat{B_K} - B_K\right)^2}{\sigma_{B_K}^2} + \frac{\left(\widehat{\eta_{cc}} - \eta_{cc}\right)^2}{\sigma_{\eta_{cc}}^2} +$$

$$\frac{\left(\widehat{\eta_{ct}} - \eta_{ct}\right)^2}{\sigma_{\eta_{ct}}^2} + \frac{\left(\widehat{f_{B_d}\sqrt{B_{B_d}}} - f_{B_d}\sqrt{B_{B_d}}\right)^2}{\sigma_{f_{B_d}\sqrt{B_{B_d}}}^2} + \frac{\left(\widehat{\xi} - \xi\right)^2}{\sigma_{\xi}^2} + \frac{\left(\widehat{\frac{|V_{ub}|}{|V_{cb}|}} - \frac{|V_{ub}|}{|V_{cb}|}\right)^2}{\sigma_{\frac{|V_{ub}|}{|V_{cb}|}}^2} +$$

$$\frac{\left(\widehat{|\epsilon_K|} - |\epsilon_K|\right)^2}{\sigma_{|\epsilon_K|}^2} + \frac{\left(\widehat{\Delta m_d} - \Delta m_d\right)^2}{\sigma_{\Delta m_d}^2} + \chi^2\left(\mathcal{A}\left(\Delta m_s\right), \sigma_{\mathcal{A}}\left(\Delta m_s\right)\right).$$

The symbols with a hat represent the reference values and the corresponding σ denote their uncertainties. The free parameters of the fit are ρ, η, A, m_c, m_t, B_K, η_{ct}, η_{cc}, $f_{B_d}\sqrt{B_{B_d}}$ and ξ, used to calculate the values of $|\epsilon_K|$, Δm_d, Δm_s and $|V_{ub}|\,|V_{cb}|$ by means of the formulae (2), (3), (4) and (5).

As Δm_s is not yet measured, its experimental information has to be included in the χ^2 following a different approach [7]. The results of the search for B_s^0 oscillations are combined [26] in terms of the oscillation amplitude $\mathcal{A}$ [34], a parameter that is zero in the absence of any signal and compatible with one otherwise, as expressed by the oscillation probability P:

$$P\left[B_s^0 \to (B_s^0, \overline{B}_s^0)\right] = \frac{1}{2\tau_s}e^{-t/\tau_s}\left(1 \pm \mathcal{A}\cos\Delta m_s\right).$$

The results of different experiments are combined in terms of $\mathcal{A}\left(\Delta m_s\right)$ and of its uncertainty $\sigma_{\mathcal{A}}\left(\Delta m_s\right)$. The 95% CL limit on Δm_s is the value for which the area above one of the Gaussian distribution with mean $\mathcal{A}\left(\Delta m_s\right)$ and variance $\sigma_{\mathcal{A}}^2\left(\Delta m_s\right)$ equals 5% of its total area. In the fit, each value taken by the parameters ρ, η and ξ is converted into a value of Δm_s by means of formula (4). A value of the CL for the oscillation hypothesis is then calculated by integrating the Gaussian distribution with mean $\mathcal{A}\left(\Delta m_s\right)$ and variance $\sigma_{\mathcal{A}}^2\left(\Delta m_s\right)$. The value $\chi^2\left(\mathcal{A}\left(\Delta m_s\right), \sigma_{\mathcal{A}}\left(\Delta m_s\right)\right)$ of a χ^2 distribution with one degree of freedom corresponding to this CL is then calculated and finally added to the χ^2 of the fit.

The fit indicates the following values for the ρ and η parameters:

$$\rho = 0.18 \pm 0.07 \quad \eta = 0.35 \pm 0.05$$

$$0.05 < \rho < 0.30 \quad 0.26 < \eta < 0.44 \ (95\%\text{CL}).$$

No large change in these results is observed if the theory contribution constraints are removed from the χ^2 and a flat distribution within the uncertainties is used in their

place. Figure 3 presents the confidence regions for the vertex of the unitarity triangle. The value of the angles of the unitarity triangle are determined as as:

$$\sin 2\alpha = 0.14^{+0.25}_{-0.38} \quad \sin 2\beta = 0.73 \pm 0.07 \quad \gamma = 63^{+\ 8}_{-11} \text{ degrees.}$$

$$-0.77 < \sin 2\alpha < 0.50 \quad 0.59 < \sin 2\beta < 0.87 \quad 44° < \gamma < 82° \ (95\%\text{CL})$$

The angles α and β are reported in terms of the functions $\sin 2\alpha$ and $\sin 2\beta$, to which the studies of the CP symmetry usually refer.
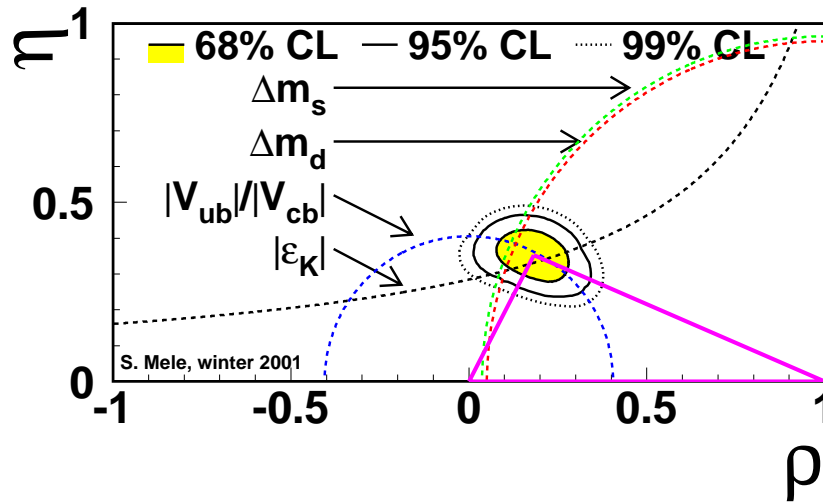

Figure 3: The favoured unitarity triangle and the confidence regions for its vertex; the Δm_s limit and the central values of the other constraints are also shown.

Several direct measurements of $\sin 2\beta$ were recently reported, as listed in Table 2. Their average, $\sin 2\beta^{\text{exp.}} = 0.48 \pm 0.16$, is lower but in agreement with the present estimate, that does not make use of this direct information.

4   Consequences of the fit

A strong experimental evidence for CP violation in the CKM matrix, described by values of its complex phase, η, different from zero, comes from the neutral kaon system. It is of interest [39] to investigate whether processes other than kaon physics predict a value of η compatible with zero or not. Figure 4 presents the results of a fit from which the information from the kaon system is removed.. The presence of a CP violating phase in the matrix, *i.e.* its complex nature is strongly favoured.

7

Table 2: Measurements of $\sin 2\beta$ and their average, compared with the fit result.

Aleph [35]	$0.93^{+0.64\ +0.36}_{-0.88\ -0.24}$
BaBar [36]	$0.34 \pm 0.20 \pm 0.05$
Belle [37]	$0.58^{+0.32\ +0.09}_{-0.34\ -0.10}$
CDF [38]	$0.79^{+0.41}_{-0.44}$
Average	0.48 ± 0.16
This fit	0.73 ± 0.07

The values of the parameters B_K and $f_{B_d}\sqrt{B_{B_d}}$ that describe non-perturbative QCD effects can be estimated by removing their constraint from the fit. This procedure yields:

$$\rho = 0.17 \pm 0.07 \quad \eta = 0.38^{+0.05}_{-0.06} \quad B_K = 0.76^{+0.21}_{-0.15}$$

in the first case and in the second:

$$\rho = 0.21^{+0.07}_{-0.08} \quad \eta = 0.34 \pm 0.05 \quad f_{B_d}\sqrt{B_{B_d}} = 0.227^{+0.019}_{-0.015} \text{ GeV}.$$

The fit indicates a value of B_K with an uncertainty larger than the input one, yet
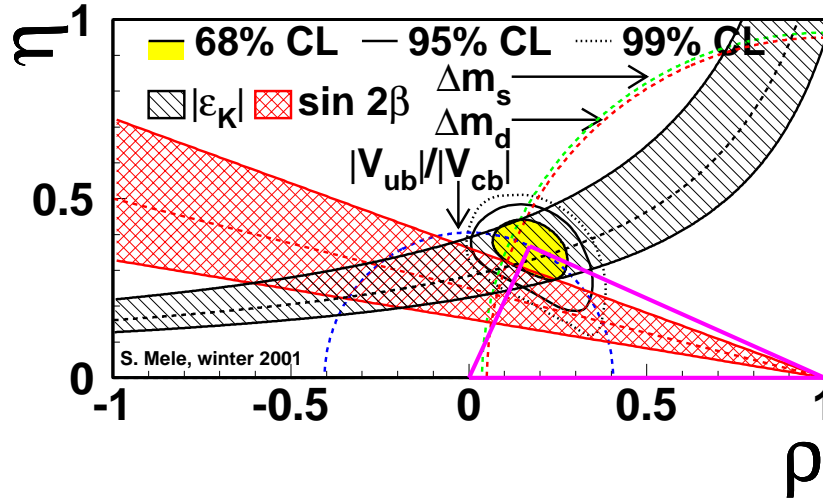

Figure 4: The favoured unitarity triangle and the confidence regions for its vertex, no information from the kaon system is used in the fit. The experimental measurements of CP violation in the neutral kaon and neutral b meson systems are superimposed.

8

of similar magnitude. The value of $f_{B_d}\sqrt{B_{B_d}}$ is found to be well in agreement with the predicted one with a smaller uncertainty, this implies that the high experimental precision of the Δm_d constraint is not fully exploited by the fit, limited by the uncertainty on the $f_{B_d}\sqrt{B_{B_d}}$ parameter.

The Δm_s constraint heavily affects the ρ uncertainty. Indeed, a fit that does not make use of the Δm_s information results in:

$$\rho = 0.09^{+0.10}_{-0.15} \quad \eta = 0.40 \pm 0.05.$$

This fit is used to estimate the favoured values of Δm_s as:

$$\Delta m_s = 14.0^{+3.4}_{-3.3}\,\mathrm{ps}^{-1}$$
$$7.4\,\mathrm{ps}^{-1} < \Delta m_s < 21.0\,\mathrm{ps}^{-1} \quad (95\%\mathrm{CL}).$$

5 Conclusions

The measurements of $|\epsilon_K|$, Δm_d and $|\mathrm{V_{ub}}|$, together with the lower limit on Δm_s, effectively constrain the CKM matrix: from a fit to the experimental results and theory parameters, the vertex and the angles of the unitarity triangle are determined as:

$$\rho = 0.18 \pm 0.07 \quad \eta = 0.35 \pm 0.05$$
$$\sin 2\alpha = 0.14^{+0.25}_{-0.38} \quad \sin 2\beta = 0.73 \pm 0.07 \quad \gamma = 63^{+\,8}_{-11}\,\text{degrees}.$$

These results are in agreement with those of recent similar analyses [40–44].

A coherent picture of the current understanding of the CKM matrix is presented by a fit that does not use any constraint from the kaon system. Its results are displayed in Figure 4. The favoured region for the vertex of the unitarity triangle corresponds to that experimentally indicated by the measurement of the CP violation in the neutral kaon system and overlaps with the one indicated by the recent measurements of $\sin 2\beta$.

Acknowledgments

9

References

[1] S. L. Glashow, Nucl. Phys. **22** (1961) 579.

[2] A. Salam, *Originally printed in *Svartholm: Elementary Particle Theory, Proceedings Of The Nobel Symposium Held 1968 At Lerum, Sweden*, Stockholm 1968, 367-377.*

[3] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.

[4] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.

[5] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[6] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.

[7] S. Mele, Phys. Rev. D **59** (1999) 113011 [hep-ph/9810333].

[8] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15** (2000) 1.

[9] B. Barish *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 1570.

[10] D. Abbaneo *et al.*, hep-ex/0009052.

[11] D. Cinabro [CLEO Collaboration], hep-ex/0009045.

[12] T. T. Wu and C. N. Yang, Phys. Rev. Lett. **13** (1964) 380.

[13] A. J. Buras, W. Slominski and H. Steger, Nucl. Phys. B **238** (1984) 529.

[14] A. J. Buras, W. Slominski and H. Steger, Nucl. Phys. B **245** (1984) 369.

[15] A. Ali and D. London, Z. Phys. C **65** (1995) 431 [hep-ph/9408332].

[16] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.

[17] S. Herrlich and U. Nierste, Nucl. Phys. B **419** (1994) 292 [hep-ph/9310311].

[18] S. Herrlich and U. Nierste, Phys. Rev. D **52** (1995) 6505 [hep-ph/9507262].

[19] S. Herrlich and U. Nierste, Nucl. Phys. B **476** (1996) 27 [hep-ph/9604330].

[20] Y. Kuramashi, Nucl. Phys. Proc. Suppl. **83** (2000) 24 [hep-lat/9910032].

[21] A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **83** (1999) 22 [hep-ex/9905060].

[22] V. Fanti *et al.* [NA48 Collaboration], Phys. Lett. B **465** (1999) 335 [hep-ex/9909022].

[23] G. D. Barr *et al.* [NA31 Collaboration], Phys. Lett. B **317** (1993) 233.

[24] S. Bertolini, hep-ph/0101212.

[25] A. J. Buras, hep-ph/9806471.

[26] A. Stocchi, hep-ph/0010222.

[27] C. Touramanis [BaBar Collaboration], hep-ex/0101035.

[28] K. Abe *et al.* [Belle Collaboration], hep-ex/0011090.

[29] D. Becirevic, D. Meloni, A. Retico, V. Gimenez, L. Giusti, V. Lubicz and G. Martinelli, hep-lat/0002025.

[30] J. Bartelt *et al.* [CLEO Collaboration], Phys. Rev. Lett. **71** (1993) 4111.

[31] R. Barate *et al.* [ALEPH Collaboration], Eur. Phys. J. C **6** (1999) 555.

[32] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **478** (2000) 14.

[33] M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **436** (1998) 174.

[34] H. G. Moser and A. Roussarie, Nucl. Instrum. Meth. A **384** (1997) 491.

[35] R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. B **492** (2000) 259 [hep-ex/0009058].

[36] B. Aubert *et al.* [BABAR Collaboration], hep-ex/0102030.

[37] A. Abashian *et al.* [BELLE Collaboration], hep-ex/0102018.

[38] T. Affolder *et al.* [CDF Collaboration], Phys. Rev. D **61** (2000) 072005 [hep-ex/9909003].

[39] R. Barbieri, L. Hall, A. Stocchi and N. Weiner, Phys. Lett. B **425** (1998) 119 [hep-ph/9712252].

[40] M. Bargiotti *et al.*, Riv. Nuovo Cim. **23N3** (2000) 1 [hep-ph/0001293].

[41] P. Faccioli, hep-ph/0011269.

[42] A. Ali and D. London, hep-ph/0012155.

[43] M. Ciuchini *et al.*, hep-ph/0012308.

[44] A. J. Buras, hep-ph/0101336.