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Combining the Muon Anomalous Magnetic Moment with other Constraints on the CMSSM

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Abstract

We combine the constraint suggested by the recent BNL E821 measurement of the anomalous magnetic moment of the muon on the parameter space of the constrained MSSM (CMSSM) with those provided previously by LEP, the measured rate of $b \to s\gamma$ decay and the cosmological relic density $\Omega_\chi h^2$. Our treatment of $\Omega_\chi h^2$ includes carefully the direct-channel Higgs poles in annihilation of pairs of neutralinos χ and a complete analysis of $\chi - \tilde{\ell}$ coannihilation. We find excellent consistency between all the constraints for $\tan\beta \gtrsim 10$ and $\mu > 0$, for restricted ranges of the CMSSM parameters m_0 and $m_{1/2}$. All the preferred CMSSM parameter space is within reach of the LHC, but may not be accessible to the Tevatron collider, or to a first-generation e^+e^- linear collider with centre-of-mass energy below 1.2 TeV.

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The recent BNL E821 measurement [1] of the anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, may indeed be a harbinger of new physics [2] beyond the Standard Model: $\delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (43 \pm 16) \times 10^{-10}$. The largest error in the Standard Model prediction is that due to the hadronic contributions, principally vacuum polarization diagrams, with the most important uncertainty being that in the low-energy region around the ρ peak. The value of these hadronic contributions [3] used in the E821 paper [1] does not include the latest data from Novosibirsk [4], Beijing [5] and CLEO [6], but these are unlikely [7] to change the overall picture: we recall that the hadronic error $\lesssim 7 \times 10^{-10}$ is much smaller than the apparent discrepancy and the experimental error. Advocates of new physics beyond the Standard Model may therefore be encouraged. However, we recall that the $Z \to \bar{b}b$ branching ratio was once thought to show a bigger discrepancy with the Standard Model, and we also caution that the 2.6 σ significance of the muon anomaly is formally less than the preliminary 2.9 σ significance of the LEP Higgs 'signal' [8].

A priori, the BNL measurement favours new physics at the TeV scale, and we consider the best motivated candidate to be supersymmetry. Even before the hierarchy motivation for supersymmetry emerged, the potential interest of a_μ was mentioned, and a pilot calculation performed [9]. Soon after the realization that supersymmetry could alleviate the hierarchy problem, the first 'modern' calculations of supersymmetric contributions to a_μ were published [10, 11, 12, 13]. These were followed by more complete calculations [14, 15, 16, 17] including the mixing expected for neutralinos, charginos and smuons. In particular, it was noted in [17] that some contributions are enhanced at large $\tan\beta$. The supersymmetric calculations we use in this paper are taken from [18] - for other recent calculations, see [19], and we include the leading two-loop electroweak correction factor [20]. For some time, it has been emphasized [17, 21] that the BNL experiment would be sensitive to a large range of the parameter space of the constrained minimal supersymmetric extension of the Standard Model (CMSSM) with universal soft superymmetry-breaking parameters at the input GUT scale, determining in particular the sign of the Higgs mixing parameter μ [17, 21]. Combining these calculations with the BNL measurement, $\mu > 0$ is favoured, along with values of $\tan\beta$ that are not very small.

The constraints from the E821 experiment are particularly interesting when combined with the information from LEP [8], the measured value of the $b \to s\gamma$ decay rate [22] and restrictions on cold dark matter imposed by astrophysics and cosmology, assuming that the lightest supersymmetric particle (LSP) is the lightest neutralino χ [23], and that R parity is conserved. Several combinations of these other constraints have been made by us [24, 25, 26, 27, 28, 29] and others [30, 31], before the advent of the E821 result.

We draw particular attention to a recent combined analysis [28] of these constraints at large $\tan\beta > 20$, which benefited from recently available $b \to s\gamma$ calculations [32] and made new calculations at large $\tan\beta$ of the the relic density $\Omega_\chi h^2$. We found [28] two important effects on the calculation of $\Omega_\chi h^2$, due to improvements of previous calculations of $\chi - \tilde{\ell}$ coannihilations and direct-channel $\chi\chi$ annihilations through the heavier neutral MSSM Higgs bosons H and A. Both of these effects extended the region of CMSSM parameter space consistent with cosmology out to values of the universal soft supersymmetry-breaking mass parameters $m_0, m_{1/2}$ that were larger than at $\tan\beta \lesssim 20$. As a result, the discovery of sparticles at the LHC could not be 'guaranteed' in the CMSSM at large $\tan\beta$, unlike the case when $\tan\beta \leq 20$ [25, 26]. Since the recent BNL measurement favours qualitatively values of $\tan\beta$ that are not small, as does the LEP Higgs 'signal', and since the $b \to s\gamma$ constraint also begins to bite at large $\tan\beta$ even for $\mu > 0$, it is important to understand the interplay of all these constraints.

We find good compatibility between all these constraints for $\tan\beta \gtrsim 10$. Even if one generously allows a 2-σ downward fluctuation in the E821 discrepancy, one finds interesting upper bounds on m_0 and $m_{1/2}$ that effectively extend the previous 'guarantee' of CMSSM discovery at the LHC to large values of $\tan\beta$. However, no such 'guarantee' can be offered to a linear e^+e^- collider (LC) with centre-of-mass energy below 1.2 TeV. We discuss the uncertainties in our analysis associated with A_0, m_b and m_t [1].

As already mentioned, our analysis is based on the one-loop calculations of [18]. In relating the masses of the sparticles appearing in the loops to the basic CMSSM soft supersymmetry-breaking parameters $m_0, m_{1/2}$, we incorporate the one-loop corrections for charginos and neutralinos. We also incorporate the leading two-loop electroweak correction factor $(1 - (4\alpha/\pi)\ln(\tilde{m}/m_\mu))$ [20], where $\tilde{m}$ is a sparticle mass. We set the trilinear soft supersymmetry-breaking parameter $A_0 = 0$ as a default, but we also discuss the consequences of varying it over the range $-2m_{1/2} \leq A_0 \leq 2m_{1/2}$. The b and t quark masses enter in our m_h, RGE and relic annihilation calculations. We use as defaults $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ GeV and the pole mass $m_t = 175$ GeV, commenting later on the changes as these mass parameters vary over our allowed ranges ± 0.25 [28, 33], ± 5 GeV.

In our subsequent discussion, we consider the 2-σ range $75 \times 10^{-10} \geq \delta a_\mu \geq 11 \times 10^{-10}$ to be allowed by the E821 measurement [1], with the 1-σ range $59 \times 10^{-10} \geq \delta a_\mu \geq 27 \times 10^{-10}$ preferred. We interpret 75×10^{-10} as a hard upper limit on δa_μ, but models yielding $\delta a_\mu < 11 \times 10^{-10}$ should perhaps not be completely excluded yet. We note that a large amount of

[1] Several other papers on the supersymmetric interpretation of the BNL measurement have already appeared during the last few days, and we comment on them at the end of this paper.

extra data have already been taken by E821, and that the present uncertainty will soon be reduced, which might have dramatic consequences.

The LEP lower limit on the mass of the Higgs boson is $m_h > 113.5$ GeV, and the possible signal corresponds to $m_h = 115^{+1.3}_{-0.7}$ GeV [8]. This lower limit applies in the CMSSM, because the ZZh coupling is unsuppressed relative to the Standard Model ZZH coupling, unlike in general mixing scenarios possible in the MSSM. In the following, we display the range 113 GeV $< m_h <$ 117 GeV. Given the uncertainties in the Higgs mass calculations [34], choices of the MSSM parameters that yield slightly lower values of m_h might be acceptable, whereas values larger than 117 Gev are certainly allowed if one discards the LEP 'signal'.

For $b \to s\gamma$, we allow parameter choices that, after including the theoretical errors due to the scale and model dependences, may fall within the 95% confidence level range $2.33\times 10^{-4} < \mathcal{B}(b \to s < \gamma) < 4.15 \times 10^{-4}$. For the cosmological relic density, we allow the range $0.1 \leq \Omega_\chi h^2 \leq 0.3$: the lower bound is optional, as there may be other sources of dark matter, but the upper bound cannot be relaxed significantly.

We display in Fig. 1 the $(m_0, m_{1/2})$ planes for representative choices of $\tan\beta$, assuming $\mu > 0$ and $A_0 = 0$. The regions allowed by the E821 measurement of a_μ at the 2-σ level are (pink) shaded with solid black line boundaries. Also shown as black dashed lines are the regions favoured by a_μ at the 1-σ level. We display the $m_h = 113, 117$ GeV mass contours as (red) dash-dotted lines, the dark (green) shaded regions are excluded by $b \to s\gamma$, the darker (red) shaded regions are excluded because the lightest supersymmetric particle is the lighter $\tilde{\tau}$, and the light (turquoise) regions are where $0.1 \leq \Omega_\chi h^2 \leq 0.3$. In panel (a) for $\tan\beta = 10$, where there is no relevant constraint from $b \to s\gamma$, we also show as a dashed line the lower limit $m_{\chi^\pm} > 104$ GeV. This excludes the tail of the cosmological region at large m_0 and small $m_{1/2}$, where there is rapid annihilation through the h pole. For clarity, this chargino mass contour is not shown in the other panels, but its effect is similar [2].

We observe that there is *remarkable* consistency between the constraints from a_μ, m_h, $b \to s\gamma$ and cosmology for $\tan\beta \gtrsim 10$, as also seen in panels (b, c, d) of Fig. 1. Even given the uncertainties in the calculation of m_h, it is difficult to maintain consistency between a_μ and m_h for smaller values of $\tan\beta$. When $\tan\beta \sim 10$, the other constraints are consistent with cosmology only for $m_0 \sim 100$ GeV, increasing gradually to $m_0 \sim 170$ GeV for $\tan\beta \sim 30$, as seen in panel (b) of Fig. 1. The favoured range of m_0 increases further as $\tan\beta$ increases, as a result of the cosmological constraint and the appearance, in particular, of the rapid $\chi\chi \to A, H$ annihilation process visible in panels (c, d) for $\tan\beta = 50, 55$, respectively,

[2] We do not show the slepton mass constraint from LEP, which is weaker than the Higgs constraint in the CMSSM, and is weaker than the upper limit $\delta a_\mu < 75 \times 10^{-10}$.

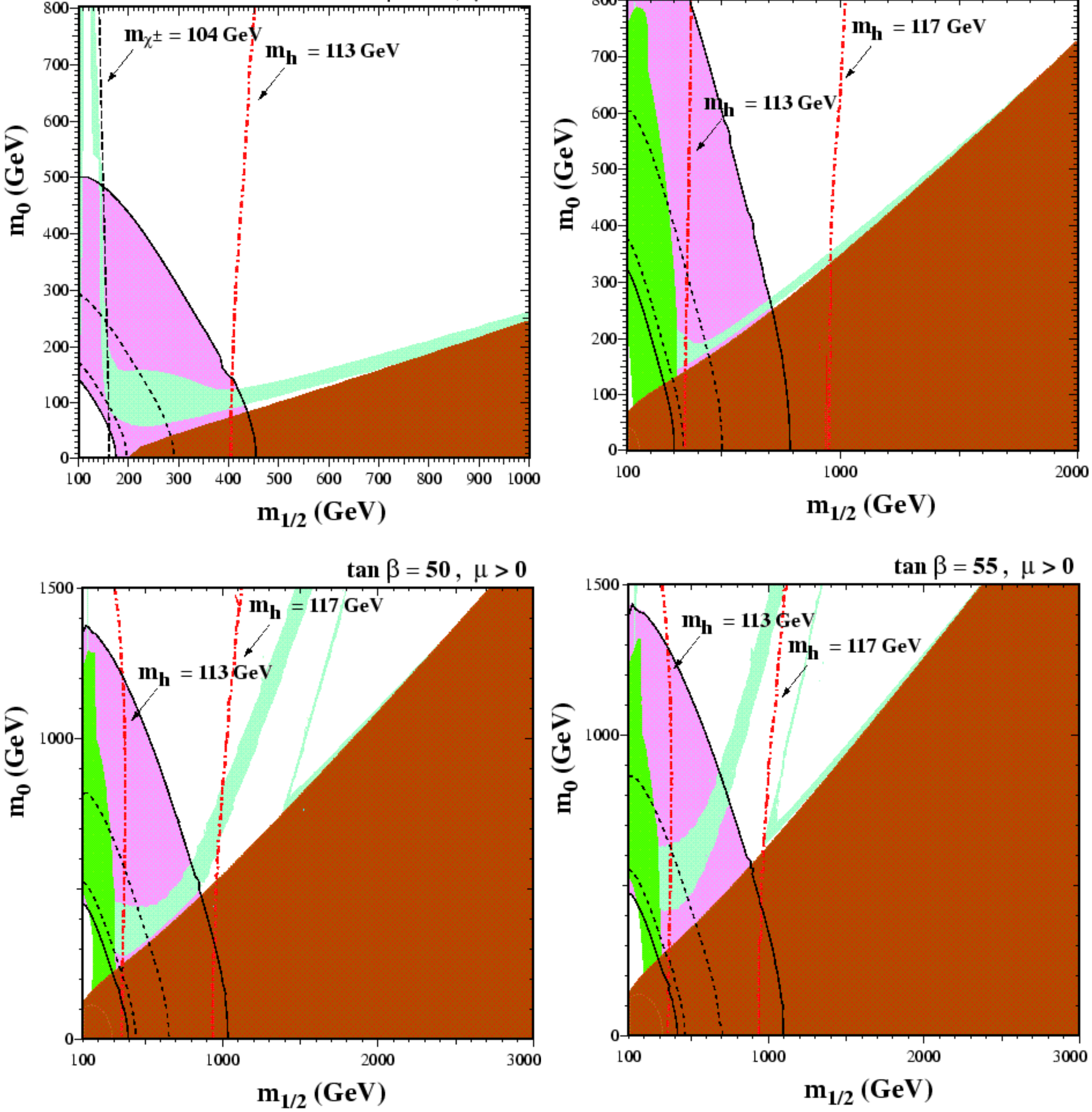

Figure 1: *The* $(m_{1/2}, m_0)$ *planes for* $\mu > 0$ *and* $\tan\beta =$ *(a) 10, (b) 30, (c) 50 and (d) 55, found assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The near-vertical (red) dot-dashed lines are the contours* $m_h = 113, 117$ *GeV, and the near-vertical (black) dashed line in panel (a) is the contour* $m_{\chi^\pm} = 104$ *GeV. The medium (dark green) shaded regions are excluded by* $b \to s\gamma$*. The light (turquoise) shaded areas are the cosmologically preferred regions with* $0.1 \leq \Omega_\chi h^2 \leq 0.3$*. In the dark (brick red) shaded regions, the LSP is the charged* $\tilde{\tau}_1$*, so this region is excluded. The regions allowed by the E821 measurement of* a_μ *at the 2-*σ *level are shaded (pink) and bounded by solid black lines, with dashed lines indicating the 1-*σ *ranges.*

which allows $m_0 \lesssim 800$ GeV. The allowed ranges of $m_{1/2}$ also increase as $\tan\beta$ increases to ~ 50, where $m_{1/2} \lesssim 900$ GeV is allowed, falling slightly when $\tan\beta = 55$ because of the rapid $\chi\chi \to A, H$ annihilation [3].

The allowed ranges of $m_{1/2}$ are, however, much restricted if one uses the 1-σ range for a_μ, with the maximum value being $\lesssim 500$ GeV. Indeed, combining all constraints and the 1-σ range for a_μ, we find quite small allowed regions of the $(m_{1/2}, m_0)$ plane centred on: $\sim (250, 100)$ GeV for $\tan\beta = 10$, $\sim (350, 170)$ GeV for $\tan\beta = 30$, $\sim (400, 350)$ GeV for $\tan\beta = 50$, and $\sim (400, 500)$ GeV for $\tan\beta = 55$ [4]. Typical sparticle masses corresponding to these choices are given in the Table. Comparing with the CMSSM physics reach for Run II of the Fermilab Tevatron collider, we see that the trilepton signature may be visible over some fraction of the allowed region of the $(m_{1/2}, m_0)$ plane for $\tan\beta = 10$, but not for the larger values of $\tan\beta$ studied here and in [35].

$\tan\beta$	$m_{1/2}$	m_0	m_χ	$m_{\chi^\pm}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{e}_1}$	$m_{\tilde{t}_1}$	$m_{\tilde{q}}$	$m_{\tilde{g}}$	m_h	m_A	$\delta a_\mu \times 10^{10}$
10	250	100	99	180	135	145	385	535	580	110	380	30
30	350	170	145	270	170	220	540	735	790	113	475	42
50	400	350	170	315	240	385	635	875	895	114	460	40
55	400	500	170	315	315	525	665	940	895	114	450	34

Table 1: *Typical sparticle masses for points in the $(m_{1/2}, m_0)$ plane consistent with the 1-σ range in a_μ and other phenomenological and cosmological constraints. For $\tilde{\tau}$, $\tilde{e}$, and $\tilde{t}$, the mass corresponds to the lightest eigenstate, which is mostly right-handed, and $m_{\tilde{q}}$ corresponds to an average slight squark mass.*

As for the LHC, we have shown earlier [25] that, in the absence of the LEP Higgs 'signal' and the E821 value of a_μ, cosmology would allow $m_{1/2} \lesssim 1400$ GeV for $\tan\beta \leq 20$ in the coannihilation region. Studies by members of the CMS Collaboration have shown that at least some sparticles would be detectable at the LHC throughout this cosmological region [36]. More recently, however, it has been shown [28] that the maximum value of $m_{1/2}$ increases to 1700 (2200) GeV for $\tan\beta = 30(50)$, and that the 'funnel' of parameters allowed by rapid $\chi\chi \to A, H$ annihilation also extends out to large m_0 and $m_{1/2}$. These extensions of the CMSSM parameter space allowed by cosmology raised the spectre that the LHC might miss supersymmetry.

[3] Generically, we do not find consistent electroweak vacua for significantly larger choices of $\tan\beta$, with our default values of m_t, m_b and A_0.

[4] Note that, for $\tan\beta = 10$, we must relax the LEP Higgs constraint to remain compatible with the 1-σ range for a_μ.

This is no longer a concern if the E821 lower limit $\delta a_\mu > 11 \times 10^{-10}$ is confirmed. Fig. 2 shows the upper limits on $m_{1/2}$ obtained as functions of $\tan\beta$ by combining cosmology with E821 or with the upper limit $m_h < 117$ GeV suggested by the possible LEP Higgs 'signal'. The a_μ constraint is somewhat stronger, but either would bring supersymmetry back within the range of the LHC. We also show in Fig. 2 the upper limits on m_0 imposed by cosmology alone and in association with the m_h or a_μ constraints. The rapid rise in the upper limit to m_0 from cosmology at large $\tan\beta$ is due to the appearance of the annihilation poles seen in panels (c) and (d) of Fig. 1. Below $\tan\beta \simeq 10$, no independent limit on m_0 is provided by the upper bound $m_h < 117 GeV$. For $\tan\beta > 10$, the limit on m_0 is strengthened gradually as the $m_h = 117$ GeV contour slides down the coannihilation region, until this is offset by the shift in the cosmological region to higher m_0 as $\tan\beta$ is further increased. Eventually, for very large $\tan\beta \gtrsim 45$, the annihilation poles again allow very large values of m_0. In contrast, the lower limit from a_μ always imposes a significant upper bound on m_0. We conclude that the LHC will find supersymmetry, if the CMSSM is correct and the E821 lower limit holds up.

It has been commented previously [37] that although discovery of the CMSSM could be 'guaranteed' at the LHC if $\tan\beta \leq 20$, there was no such 'guarantee' for a first-generation linear e^+e^- collider such as TESLA or the NLC with a centre-of-mass energy below 1.25 TeV, because of the extension of the cosmologically allowed region by coannihilation [5]. Since the E821 lower limit $\delta a_\mu > 11 \times 10^{-10}$ excludes the 'tail' of the coannihilation region, the concern that such a first-generation linear e^+e^- collider might miss supersymmetry is diminished. At the boundary of the region allowed by E821, the lightest detectable supersymmetric particle is the lighter stau $\tilde{\tau}_1$. For our default choices $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$, $m_t = 175$ GeV and $A_0 = 0$ as in Fig. 2, and the test values $\tan\beta = 10, 30, 50, 55$, we find that $m_{\tilde{\tau}_1} \lesssim 190, 320, 420, 580$ GeV. We therefore conclude that a first-generation linear e^+e^- collider with centre-of-mass energy above 1.2 TeV would be 'guaranteed' to find supersymmetry within our CMSSM framework. A machine with centre-of-mass energy above 800 GeV would be similarly 'guaranteed' to find supersymmetry if $\tan\beta \lesssim 45$.

Fig. 3 for $\tan\beta = 50, \mu > 0$ and $A_0 = 0$ illustrates the effects of varying $m_b(m_b)^{\overline{MS}}_{SM}$ between (a) 4.0 and (b) 4.5 GeV, and of varying m_t between (c) 170 and (d) 180 GeV. As one would expect, the changes in the δa_μ constraint are minor, being essentially associated with changes in the RGE and vacuum analysis. Also as expected, the m_h contours and the $b \to s\gamma$ constraint are rather similar in panels (a) and (b): the important change as one varies m_b is in the cosmological constraint. In particular, the rapid $\chi\chi \to A, H$ annihilation

[5] There was also no such 'guarantee' in the focus-point region [38].

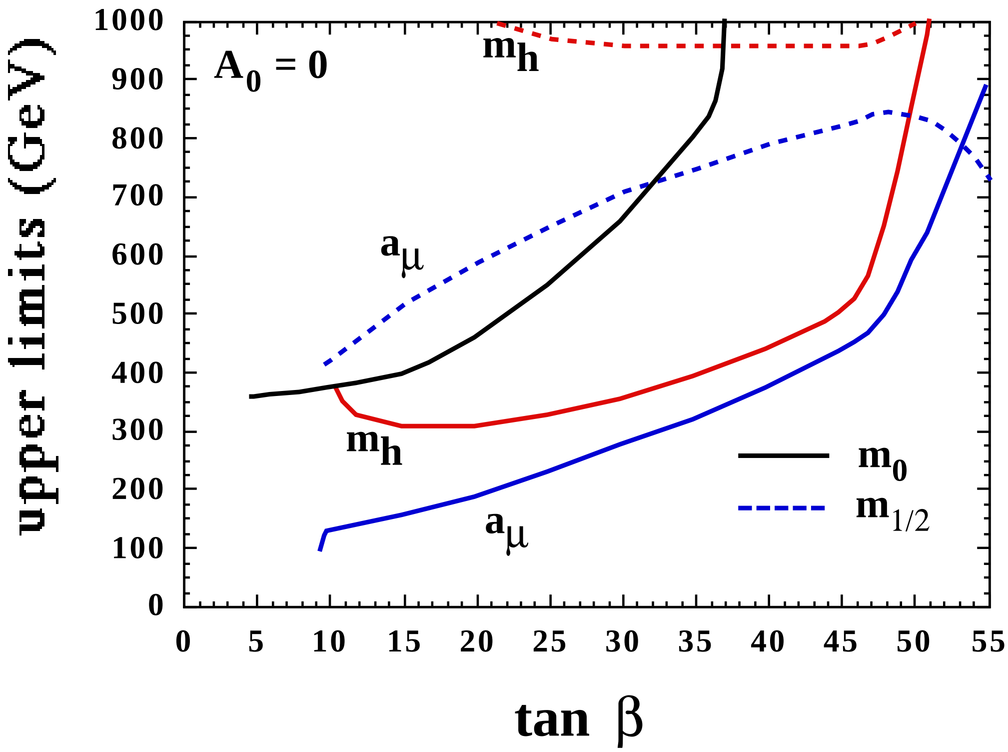

Figure 2: *Upper limits on* $m_{1/2}$ *and* m_0 *obtained as functions of* $\tan\beta$ *for* $\mu > 0$*, assuming* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$*,* $m_t = 175$ *GeV and* $A_0 = 0$*. We show the upper limits on* $m_{1/2}$ *obtained by combining cosmology with the LEP Higgs 'signal' and the E821 lower limit on* δa_μ*, and the upper limits on* m_0 *imposed by cosmology alone and in association with either* a_μ *or the LEP Higgs 'signal'.*

'funnel' moves to lower $m_{1/2}$ as m_b increases, reducing the combined upper limit on $m_{1/2}$ when $m_b(m_b)^{\overline{MS}}_{SM} = 4.5$ GeV, and increasing the upper limit on m_0. However, our overall conclusions on the observability of the CMSSM at different colliders are unchanged. As seen in panels (c) and (d), the main effects of varying m_t are to move the m_h contours and the allowed cosmological region [6]. As a result, the lower bound on $\tan\beta$ is relaxed for $m_t = 180$ GeV. However, the effects on the bounds on $m_{1/2}$ and m_0 in Fig. 2 are again relatively minor. We do not display the effects of varying $-2 \times m_{1/2} \leq A_0 \leq 2 \times m_{1/2}$: the main changes are in the allowed cosmological region, whose sensitivity to input assumptions were commented on previously [28], but the effects on the bounds on $m_{1/2}$ and m_0 in Fig. 2

[6]Note also the black region in panel (c) of Fig. 3, which is where we find no consistent electroweak vacuum. There are similar but smaller regions for larger m_t, that are not shown. The size of this forbidden region is quite sensitive to the treatment of m_t, a topic we leave for another occasion.

are again not very important, though the lower bound on $\tan\beta$ may again be relaxed.

In conclusion: we have combined the E821 δa_μ constraint with other constraints on the CMSSM, including the LEP 'signal' for the Higgs boson, $b \to s\gamma$ and the favoured range of the cosmological relic density $\Omega_\chi h^2$. We find a high degree of consistency for $\tan\beta \gtrsim 10$, and interesting upper bounds on $m_{1/2}$ and m_0. There is a corner of parameter space where the Fermilab Tevatron collider may find supersymmetry. On the other hand, discovery of supersymmetry is 'guaranteed' at the LHC, within our stated theoretical assumptions. The E821 δa_μ constraint increases the chance that a first-generation e^+e^- linear collider will find supersymmetry, though there is no 'guarantee' unless its centre-of-mass energy exceeds 1.2 GeV.

The E821 measurement of δa_μ provides an important constraint on the CMSSM, and may already be the most promising positive evidence for it. With the prospects of a significant reduction in the E821 error bar in the near future, we may be living in exciting times for supersymmetry.

Finally, for completeness, we comment on recent papers related to ours.

Ref. [39] analyzes the BNL measurement but not the other constraints discussed here. It is suggested that sparticles may be produced at the Tevatron in Run II, but their observability is not discussed. In this regard, as discussed above, we are not very encouraged by previous studies [35].

The most complete previous supersymmetric interpretation of the BNL measurement is given in [40], which also includes some discussions of the LEP Higgs, $b \to s\gamma$ and cosmological constraints. The main difference between that work and ours is in the cosmology: the regions of CMSSM parameter space given in [40] do not include all the coannihilation region, that extends in our calculations [28] up to $m_{1/2} = 1400(2200)$ GeV for $0.1 \leq \Omega_\chi h^2 \leq 0.3$ for $\tan\beta = 10(50)$. Conversely, the 'focus point' region, that is now disallowed by the BNL and other constraints, is beyond the domains of the $(m_{1/2}, m_0)$ plane that we plot. Our analysis for $\tan\beta = 50$ also differs in the treatment of the direct-channel A, H poles, that are responsible for a large allowed region for $\Omega_\chi h^2$ in the analysis of [40], but lead to narrow funnels in our analysis [28].

The prospects for dark matter detection in the light of the BNL measurement are discussed in [41], where LEP, $b \to s\gamma$ and $\Omega_\chi h^2$ are also taken into account. The current LEP constraint $m_h \geq 113.5$ GeV was not used. This applies in the generic MSSM for $\tan\beta \lesssim 10$ and in the CMSSM at essentially all $\tan\beta$, excludes a large range of $m_{1/2}$ and requires $m_\chi \gtrsim 150$ GeV [28]. We are not in a position to compare treatments of $b \to s\gamma$. The

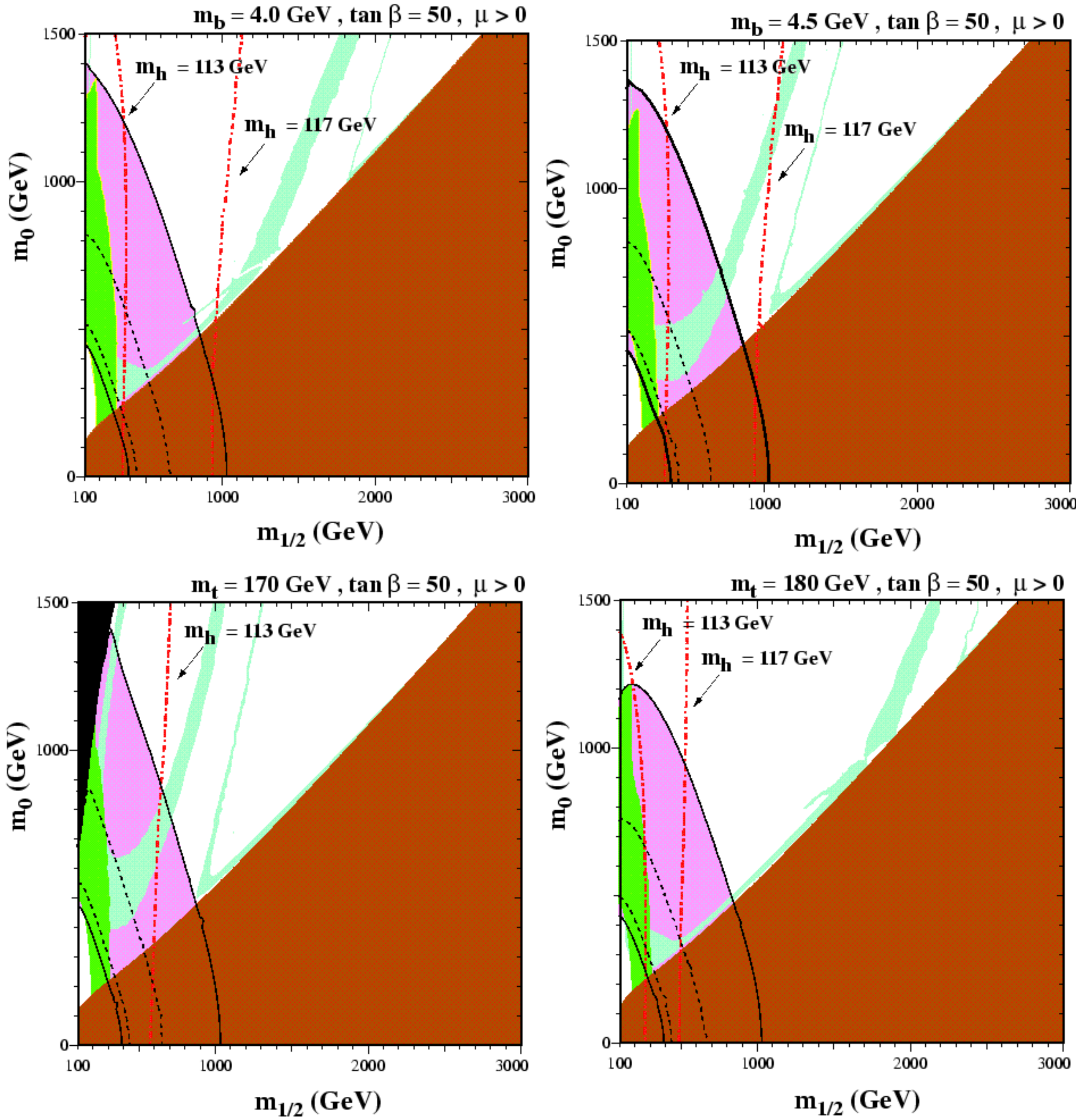

Figure 3: *Comparison between the* $(m_{1/2}, m_0)$ *planes for* $\tan\beta = 50, \mu > 0$ *and* $A_0 = 0$*, with different values of other input parameters. Panels (a) and (b) are for* $m_t = 175$ *GeV,* $m_b(m_b)^{\overline{MS}}_{SM} = 4.0$ *and* 4.5 *GeV, respectively. Panels (c) and (d) are for* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV and* $m_t = 170$ *and* 180 *GeV, respectively.*

analysis of $\Omega_\chi h^2$ in [41] is based on DarkSUSY [42], which does not include all the effects at large $\tan\beta$ that we discussed in [28] and here.

We use the same formulae [18] as [43] to implement the BNL constraints on m_0 and $m_{1/2}$, with which we agree quite closely. Ref. [43] also discusses the LEP Higgs 'signal' and makes qualitative comments on supersymmetric dark matter, but does not discuss $b \to s\gamma$.

Ref. [44] discusses implications for the unconstrained MSSM and for gauge-mediated models of supersymmetry breaking.

Ref. [45] discusses the supersymmetric interpretation of δa_μ in relation to models of neutrino masses and the observability of $\mu \to e\gamma$ decay.

We note that anomaly-mediated supersymmetry breaking is considered in [40, 43]. We also note that non-supersymmetric interpretations of the E821 result are discussed in [46].

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