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SU(5) Grand Unification in Extra Dimensions and Proton Decay

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Abstract

We analyse proton decay in the context of simple supersymmetric SU(5) grand unified models with an extra compact spatial dimension described by the orbifold $S^1/(Z_2 \times Z_2')$. Gauge and Higgs degrees of freedom live in the bulk, while matter fields can only live at the fixed point branes. We present an extended discussion of matter interactions on the brane. We show that proton decay is naturally suppressed or even forbidden by suitable implementations of the parity symmetries on the brane. The corresponding mechanism does not affect the SU(5) description of fermion masses also including the neutrino sector, where Majorana mass terms remain allowed.

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1 Introduction

The idea that all particle interactions merge into a unified theory at very high energies is so attractive that this concept has become widely accepted by now. The quantitative success of coupling unification in Supersymmetric (SUSY) Grand Unified Theories (GUT's) has added much support to this idea. The recent developments on neutrino oscillations, pointing to lepton number violation at large scales, have further strengthened the general confidence. However the actual realization of this idea is not precisely defined. Recently it has been observed that the GUT gauge symmetry could be actually realized in 5 (or more) space-time dimensions and broken down to the the Standard Model (SM) by compactification [1]. In particular a model with N=2 Supersymmetry (SUSY) and gauge SU(5) in 5 dimensions has been proposed [2] where the GUT symmetry is broken by compactification on $S^1/(Z_2 \times Z_2')$ down to a N=1 SUSY-extended version of the SM on a 4-dimensional brane. In this model many good properties of GUT's, like coupling unification and charge quantization are maintained while some unsatisfactory properties of the conventional breaking mechanism, like doublet-triplet splitting, are avoided. In this note we elaborate further on this class of models. We differ from ref. [2] in the form of the interactions on the 4-dimensional brane. As a consequence we not only avoid the problem of the doublet-triplet splitting but also directly suppress or even forbid proton decay, since the conventional Higgsino and gauge boson exchange amplitudes are absent, as a consequence of $Z_2 \times Z_2'$ parity assignments on matter fields on the brane. Most good predictions of SUSY SU(5) are thus maintained without unnatural fine tunings as needed in the minimal model (for a realistic conventional model, as opposed to a minimal model see ref. [3]). We find that the relations among fermion masses implied by the minimal model, for example $m_b = m_\tau$ at M_{GUT}, are preserved in our version of the model, although the Yukawa interactions are not fully SU(5) symmetric. The mechanism that forbids proton decay still allows Majorana mass terms for neutrinos so that the good SU(5) potentiality for the description of neutrino masses and mixing is preserved. In the following we first summarise the model of ref. [2], we then introduce our different implementations of the boundary conditions and finally we discuss the physical implications of the model and draw our conclusion.

2 The Model

Following ref. [2] we consider a 5-dimensional space-time factorised into a product of the ordinary 4-dimensional space-time M^4 and of the orbifold $S^1/(Z_2 \times Z_2')$, with coordinates x^μ, $(\mu = 0, 1, 2, 3)$ and $y = x^5$. The orbifold S^1/Z_2 is obtained by dividing the circle S^1 of radius R $(1/R \sim M_{GUT})$ with a Z_2 transformation $y \to -y$. To obtain the orbifold $S^1/(Z_2 \times Z_2')$ we divide S^1/Z_2 by Z_2' which act as $y' \to -y'$, with $y' = y + \pi R/2$. There are two 4-dimensional branes at the fixed points $y = 0$, and at $y = \pi R/2$ (the brane at $y = -\pi R$ is identified with that at $y = 0$ and those at $y = \pm\pi R/2$ are also identified). For a generic field $\phi(x^\mu, y)$ living in

[1]Grand unified supersymmetric models in six dimensions, with the grand unified scale related to the compactification scale were also proposed by Fayet [1].

the 5-dimensional bulk the Z_2 and Z_2' parities P and P' are defined by

$$\begin{aligned} \phi(x^\mu, y) \quad &\to \phi(x^\mu, -y) = P\phi(x^\mu, y) \quad , \\ \phi(x^\mu, y') \quad &\to \phi(x^\mu, -y') = P'\phi(x^\mu, y') \quad . \end{aligned} \tag{1}$$

Denoting by $\phi_{\pm\pm}$ the fields with $(P, P') = (\pm, \pm)$ we have the y-Fourier expansions:

$$\begin{aligned} \phi_{++}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x^\mu) \cos \frac{2ny}{R} \quad , \\ \phi_{+-}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x^\mu) \cos \frac{(2n+1)y}{R} \quad , \\ \phi_{-+}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{R} \quad , \\ \phi_{--}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R} \quad . \end{aligned} \tag{2}$$

where n is a non negative integer and the Fourier component fields $\phi_{++}^{(2n)}$ acquire a mass $2n/R$, $\phi_{+-}^{(2n+1)}$ and $\phi_{-+}^{(2n+1)}$ take a mass $(2n+1)/R$ and $\phi_{--}^{(2n+2)}$ acquire a mass $(2n+2)/R$, upon compactification. Only ϕ_{++} have massless components and only ϕ_{++} and ϕ_{+-} can exist on the $y = 0$ brane. The fields ϕ_{++} and ϕ_{-+} are non-vanishing on the $y = \pi R/2$ brane, while ϕ_{--} vanishes on both branes.

(P, P')	field	mass
$(+,+)$	A^a_μ, λ^{2a}_W, H^D_u, H^D_d	$\frac{2n}{R}$
$(+,-)$	$A^{\hat{a}}_\mu$, $\lambda^{2\hat{a}}_W$, H^T_u, H^T_d	$\frac{(2n+1)}{R}$
$(-,+)$	$A^{\hat{a}}_5$, $\Sigma^{\hat{a}}$, $\lambda^{1\hat{a}}_W$, $\hat{H}^T_u$, $\hat{H}^T_d$	$\frac{(2n+1)}{R}$
$(-,-)$	A^a_5, Σ^a, λ^{1a}_W, $\hat{H}^D_u$, $\hat{H}^D_d$	$\frac{(2n+2)}{R}$

Table 1. Parity assignment and masses ($n \geq 0$) of fields in the vector and Higgs supermultiplets.

Our aim is to reproduce most of the good predictions of a minimal (N=1) SUSY SU(5) GUT. For this purpose we start from a 5-dimensional theory invariant under N=2 SUSY, gauge SU(5) and $Z_2 \times Z_2'$ parities. The parities are assigned in such a way that compactification reduces N=2 to N=1 SUSY and breaks SU(5) down to the SM gauge group SU(3) $\times$ SU(2) $\times$ U(1). Leaving aside for the moment quarks and leptons and their SUSY partners, the 5-dimensional theory contains the following N=2 SUSY multiplets of fields. First, a vector multiplet (with vector bosons A_M, $M = 0, 1, 2, 3, 5$, two bispinors λ^i_W, $i = 1, 2$, and a real scalar Σ, each of them transforming as a 24 representation of SU(5). Then there are two hypermultiplets H^s $(s = 1, 2)$ that are equivalent to four sets of N=1 chiral multiplets. Two of them are the ordinary H_5 and $H_{\bar{5}}$ supermultiplets (which include the scalar Higgs doublets H^D_u and H^D_d and the corresponding scalar triplets H^T_u and H^T_d); the remaining two are denoted by $\hat{H}_5$ and $\hat{H}_{\bar{5}}$ and will be called mirror. The parity P, P' assignments are the same as in ref. [2] and are given in table 1.

Here the index a $(\hat{a})$ labels the unbroken (broken) SU(5) generators T^a $(T^{\hat{a}})$, H stands for the whole chiral multiplet of given quantum numbers. The parity P breaks N=2 SUSY down to N=1 and would allow complete SU(5) massless supermultiplets, contained in the first two rows of table 1. The additional parity P' respects the surviving N=1 SUSY and breaks SU(5) down to the standard model gauge group. Note that the derivative ∂_5 transforms as $(-, -)$. The $(+, +)$ fields, which remain massless and do not vanish on both branes are the gauge and Higgs multiplets of the low energy Minimal SUSY Model (MSSM). The bulk 5-dimensional Lagrangian is exactly as in ref. [2] and we do not reproduce it here.

If we expand the bulk Lagrangian around the local minimum $A_5 = \Sigma = 0$ of the scalar potential, then the mass terms are only those provided by the derivatives $\partial\phi/\partial y$ of the fields ϕ. The resulting spectrum is the one shown in table 1. An Higgs mechanism occurs at each level $n \neq 0$. The vector bosons $A^{a(0)}_\mu$ remains massless and, together with the gauginos $\lambda^{2a(0)}_W$, form a vector multiplet of N=1. The vector bosons $A^{a(2n)}_\mu$ $(n > 0)$ eat the Goldstone bosons $A^{a(2n)}_5$, acquire a mass $2n/R$ and, together with $\Sigma^{a(2n)}$, $\lambda^{1a(2n)}_W$ and $\lambda^{2a(2n)}_W$, form a massive vector multiplet of N=1. Likewise, full N=1 massive vector multiplets are reproduced by $\Sigma^{\hat{a}(2n+1)}$, $\lambda^{1\hat{a}(2n+1)}_W$, $\lambda^{2\hat{a}(2n+1)}_W$ and by $A^{\hat{a}(2n+1)}_\mu$ that acquire a mass $(2n + 1)/R$ by eating the Goldstone bosons $A^{\hat{a}(2n+1)}_5$.

Note that mass terms for the Higgs hypermultiplets are allowed by N=2 SUSY [4], but they are not compatible with the P parity assignment of the fermion fields: the Lagrangian would contain both $\tilde{H}_5 \Sigma \tilde{\hat{H}}_{\bar{5}}$ and $\tilde{H}_5 \tilde{\hat{H}}_{\bar{5}}$ terms, where a tilde denotes the fermion component of the corresponding chiral multiplet. But the two sets of terms cannot be both allowed given the negative P parity of Σ, which is uniquely determined by the gauge sector alone. Therefore the P parity symmetry forbids a mass term for any hypermultiplet. As a consequence, the zero modes of the doublets $H^D_{u,d}$ cannot receive a bulk mass contribution and are automatically split from the $H^T_{u,d}$ modes.

3 Matter Fields

We now discuss the introduction of quark and lepton matter. In this sector our formulation of the model is different than in ref. [2]. We first observe that it is not possible that the matter fields propagate and interact in the bulk. To introduce quark and lepton fields in the bulk they should enter in the Lagrangian in a N=2 SUSY, gauge SU(5) and $Z_2 \times Z_2'$ invariant form. Thus we should define N=2 hypermultiplets $\chi_{\bar{5}}$, χ_{10} (and possibly χ_1). Each N=2 hypermultiplet χ contains a N=1 chiral multiplet ψ, ϕ of fermions and scalars and another chiral multiplet $\hat{\psi}$, $\hat{\phi}$. The ordinary quark and lepton supermultiplets must remain massless and must be non-vanishing on branes. Thus the parity assignments must be $(+,+)$ for all SU(5) indices of $\psi_{\bar{5}}$, $\phi_{\bar{5}}$, ψ_{10}, ϕ_{10} (and possibly of ψ_1, ϕ_1). Thus, contrary to what is the case for the vector and Higgs hypermultiplets, for the $\bar{5}$ and 10 chiral supermultiplets the whole SU(5) representations have the same parity transformation. Due to this fact, it is straightforward to prove that it is impossible to construct gauge SU(5) and $Z_2 \times Z_2'$ invariant interactions of matter fields with the gauge and Higgs supermultiplets in the bulk. For example, this can be seen by inspecting the term $\psi_{\bar{5}} \Sigma \hat{\psi}_5$. (In N=2 SUSY all interactions are gauge interactions and this term is part of them.) If we decompose the pentaplets in their doublet (D) and triplet (T) components, this term can be symbolically written as:

$$\psi^T \Sigma^a \hat{\psi}^T + \psi^D \Sigma^a \hat{\psi}^D + \psi^T \Sigma^{\hat{a}} \hat{\psi}^D + \psi^D \Sigma^{\hat{a}} \hat{\psi}^T \qquad . \tag{3}$$

Positive P' for both matter fields ψ^T and ψ^D cannot be obtained whatever P' parity we may assign to the mirror fields $\hat{\psi}^T$ and $\hat{\psi}^D$. Hence the matter fields have necessarily to be introduced only on the fixed point branes.

Interaction terms between bulk and brane fields have been discussed in ref. [5, 6] and explicitly worked out for SM matter in ref. [7]. An important point is the symmetry that should be respected by these interactions. To analyse this in detail, it is instructive to build the theory in two steps, by first constructing the orbifold S^1/Z^2 and by subsequently performing the orbifold identification required by Z_2'. The first Z_2 leaves both the SU(5) gauge symmetry and a residual N=1 supersymmetry unbroken. Matter interactions, located at the fixed points $(0, \pi R)$, are allowed to respect these symmetries [5, 6, 7, 8, 9]. In particular we can introduce SU(5) invariant interactions between matter and gauge fields and also SU(5) invariant Yukawa couplings on the branes at $y = (0, \pi R)$. No relation between the interactions on the two different branes is required at this stage. In the following we take the point of view that on the $y = (0, \pi R)$ branes matter must transform according to complete representations of SU(5) and all interactions will have SU(5) symmetric couplings. When the y integration is performed the breaking of SU(5) will be generated by imposing P' invariance (*i.e.* terms that are odd under $y = 0 \leftrightarrow y = \pi R$ disappear), as we now illustrate. When we identify y' and $-y'$ and we enforce the Z_2' discrete symmetry, we are no longer allowed to consider the two fixed points $y = (0, \pi R)$ separately. Indeed they are now related and we expect that interaction terms on one brane are not independent from those on the other brane. Consider for instance the gauge couplings induced on the brane at $y = 0$ by the Kahler potential:

$$K = 10^\dagger e^V 10 + \bar{5}^\dagger e^V \bar{5} \qquad . \tag{4}$$

Here $10 \equiv (Q, U^c, E^c)$ and $\bar{5} \equiv (L, D^c)$ are N=1 chiral multiplets, $V \equiv (A_\mu, \lambda^2_W)$ is an N=1 vector multiplet, with obvious meaning of the notation. Each chiral multiplet $\Phi_M \equiv (\phi_M, \psi_M)$, $M = (Q, L, U^c, D^c, E^c)$, contains a complex scalar ϕ_M and a two-component fermion ψ_M. The corresponding fermion interactions, in a compact notation, are given by:

$$\mathcal{L}_g = \mathcal{L}_g^a + \mathcal{L}_g^{\hat{a}} \quad , \tag{5}$$

$$\mathcal{L}_g^a = \sum_M \overline{\psi_M} \bar{\sigma}^\mu T^a \psi_M A^a_\mu \quad , \tag{6}$$

$$\mathcal{L}_g^{\hat{a}} = \overline{\psi_Q} \bar{\sigma}^\mu T^{\hat{a}} \psi_{U^c} A^{\hat{a}}_\mu + \overline{\psi_Q} \bar{\sigma}^\mu T^{\hat{a}} \psi_{E^c} A^{\hat{a}}_\mu + \overline{\psi_L} \bar{\sigma}^\mu T^{\hat{a}} \psi_{D^c} A^{\hat{a}}_\mu + \text{h.c.} \quad . \tag{7}$$

where $\bar{\sigma}_\mu = (1, -\sigma^k)$ and σ^k are the Pauli matrices. Before the Z'_2 orbifolding, the gauge symmetry on the $y = 0$ brane is the largest possible one, compatible with the full SU(5) group, in agreement with the fact that the Z_2 orbifold leaves SU(5) invariant. If we now perform a P' parity transformation, the fixed point $y = 0$ is mapped in $y = \pi R$ and, to obtain a Z'_2 invariant action, we should put on the brane at $y = \pi R$ the same gauge interactions of eq. (6,7). We obtain the effective four-dimensional Lagrangian $\mathcal{L}_g^{(4)}$:

$$\mathcal{L}_g^{(4)} = \int dy \, [\delta(y) + \delta(-y + \pi R)] \, \mathcal{L}_g(y) \quad . \tag{8}$$

At this point we have different possibilities, depending on the P' parity of matter fields, that specifies how a matter field on the brane at $y = 0$ is related to the same field on the brane at $y = \pi R$. For generic P' parity assignments SU(5) is broken as a result of the integration in eq. (8). In particular this is the case if all matter fields are assumed to be invariant under Z'_2. Then the terms in eq. (6) are invariant under $y' \to -y'$, that is $y \to -y + \pi R$, whereas those in eq. (7) change sign. After integrating over y the terms of eq. (7) cancel and we are left with gauge interactions of matter with the vector bosons of SU(3) × SU(2)× U(1). The SU(5) invariance has been lost. Only the invariance under the subgroup left unbroken by Z'_2 survives.

Alternatively we can try to preserve SU(5) by assigning suitable non-trivial parities under Z'_2. In other words we allow for a change of sign between a matter field on the brane at $y = 0$ and the same field on the brane at $y = \pi R$. In this way we can preserve SU(5) invariant gauge interactions on the branes at $y = (0, \pi R)$. Indeed, the P' parity transformation does not destroy the SU(5) algebra. It effectively induces a redefinition of the generators according to $(T^a, T^{\hat{a}}) \to (T^{a'}, T^{\hat{a}'}) = (T^a, -T^{\hat{a}})$. The old generators T and the new ones T' satisfy the same algebra. Therefore, if we assign opposite P' parities to Q and U^c, E^c and similarly for L and D^c, the gauge interactions of eq. (6,7) are Z'_2 invariant and, after the integration over y, we obtain full SU(5) four-dimensional gauge interactions for matter fields. This possibility can be realized for the gauge couplings in eqs. (6,7), but we shall see that it does not lead to a viable alternative for Yukawa couplings.

This discussion shows that the theory is not completely defined unless we specify the transformation properties of matter fields at $y = (0, \pi R)$ under Z'_2. Having matter fields invariant under Z'_2 represents just one of the possible choices, not necessarily the most interesting or the most realistic one [2]. It is also clear that in general SU(5) invariance of brane interactions

[2] Likewise, when considering matter located at $y = \pm\pi R/2$, we must define the Z_2 parities of the corresponding fields.

is not guaranteed. It depends on how the matter fields transform under Z_2'. For generic P' parities of matter multiplets only the invariance under the group unbroken by the compactification, SU(3) $\times$ SU(2)$\times$ U(1), is recovered. Notice that in both of the cases discussed above the $Z_2 \times Z_2'$ parity, that plays an essential role in the orbifold construction, is conserved.

We now analyse some of the possible parity assignments for the matter fields and discuss the related physics, focusing on the features of proton decay. We will show that along these lines we can construct a model where not only the doublet-triplet splitting problem is solved but also proton decay is drastically suppressed or even forbidden in a natural way.

4 Matter interactions on the branes

We now introduce Yukawa couplings on the branes. We start from the interactions induced on the brane at y by the SU(5) invariant superpotential [3]:

$$w(y) = y_u \; 10 \; 10 \; H_5 + y_d \; 10 \; \bar{5} \; H_{\bar{5}} + y_R \; 10 \; \bar{5} \; \bar{5} + ... \tag{9}$$

where $y_{u,d,R}$ are coupling constants and dots denote terms depending on mirror Higgs multiplets. The usual decomposition holds:

$$10 \; 10 \; H_5 = Q U^c H_u^D + \frac{1}{2} Q Q H_u^T + U^c E^c H_u^T \quad , \tag{10}$$

$$10 \; \bar{5} \; H_{\bar{5}} = Q D^c H_d^D + L E^c H_d^D + Q L H_d^T + U^c D^c H_d^T \quad , \tag{11}$$

$$10 \; \bar{5} \; \bar{5} = Q L D^c + E^c L L + U^c D^c D^c \quad . \tag{12}$$

Yukawa couplings invariant under Z_2' are obtained by requiring the same superpotential w on the branes at $y = (0, \pi R)$:

$$w^{(4)} = \int dy \, [\delta(y) + \delta(-y + \pi R)] \, w(y) \quad . \tag{13}$$

We first discuss the possibility of preserving full SU(5) symmetry on the branes. It is easy to see that this does not lead to a realistic model. As we have seen before, the only assignments of P' parities to matter compatible with SU(5) symmetry of the gauge couplings are $(Q, L, U^c, D^c, E^c) \sim \pm(+,+,-,-,-)$ and $(Q, L, U^c, D^c, E^c) \sim \pm(+,-,-,+,-)$. The first possibility however leads to P'-odd Yukawa couplings that cancel after integrating over y the two contributions in eq. (13). In the second case, only the term $10 \; \bar{5} \; H_{\bar{5}}$ is P'-even and survives in the four-dimensional Lagrangian. The other two interactions, $10 \; 10 \; H_5$ and $10 \; \bar{5} \; \bar{5}$ are P'-odd and eventually cancel. Thus, masses for up type quarks cannot be generated in this model. As a consequence, we must abandon the possibility of SU(5)-invariant brane interactions and look for a more realistic choice of P' parity for matter.

[3]The Lagrangian terms are derived from the superpotential w with the procedure described in [5, 6, 7], entailing the inclusion of appropriate couplings between brane fields and the ∂_5 derivative of bulk fields.

As we saw in the case of gauge interactions, for generic parity assignments of matter fields, the four-dimensional interactions on the fixed point branes at $y = (0, \pi R)$ only respect the residual symmetries after compactification, that is N=1 SUSY, gauge SU(3) $\times$ SU(2) $\times$ U(1) and $Z_2 \times Z_2'$ parity. In general on the branes also interactions involving ∂_5 appear. At first we omit these derivative terms and we will comment on them later in the following. We also do not include all higher dimensional operators at this stage. We start by observing that the (P, P') parities of the matter supermultiplets Q, U^c, D^c, L and E^c should allow at least the superpotential terms that provide masses to the observed fermions (neglecting neutrino masses for the time being):

$$w_{mass} = QU^c H_u^D + QD^c H_d^D + LE^c H_d^D \quad . \tag{14}$$

Recalling that H_u^D and H_d^D have parities $(+,+)$ it is clear that Q, U^c and D^c should have equal parities and similarly for L and E^c. The points $y = (0, \pi R)$ are kept fixed by a Z_2 transformation. It is then natural to choose positive P parity for all matter fields located on these branes. With this choice of parities that guarantee non-vanishing fermion masses, we now examine the features of proton decay. It is well known that in the minimal SUSY SU(5) model the exchange of coloured Higgsinos can induce proton decay at a rate which is difficult to reconcile with present experimental limits [10]. The superpotential terms describing the corresponding amplitudes are:

$$H_u^T H_d^T \quad , \tag{15}$$

together with

$$\begin{array}{lll} QQH_u^T & , & U^c D^c H_d^T \quad , \\ QLH_d^T & , & U^c E^c H_u^T \quad . \end{array} \tag{16}$$

In minimal SUSY SU(5) the B/L-violating dimension 5 operators [4] $[QQQL]_F$ and $[U^cU^cD^cE^c]_F$ arise from the trilinear terms of eq. (16) in the low-energy limit, after integration over the heavy supermultiplets $H_{u,d}^T$ whose mixing is provided by (15). In the model under discussion the bilinear brane couplings $H_u^T H_d^T$ and $\hat{H}_u^T \hat{H}_d^T$ are allowed by the parity symmetry. On the contrary, the couplings in the first line of eq. (16) have always parity $(+,-)$ and drop from the final four-dimensional action. A similar consideration holds for the analogous couplings obtained with the replacements $H_{u,d}^T \to \hat{H}_{u,d}^T$. Note that ∂_5 derivatives being $(-,-)$ cannot rescue these couplings. Terms in ∂_5 of $(-,-)$ fields can appear on the brane but they do not have important consequences because the lightest $(-,-)$ states are heavy. Therefore, while the $H_{u,d}^T$, $\hat{H}_{u,d}^T$ masses are of order M_{GUT} as in the conventional model, nevertheless their allowed interactions with matter are not sufficient to give rise to the dangerous dimension five $[QQQL]_F$ and $[U^cU^cD^cE^c]_F$ operators.

While the dimension 5 operators $[QQQL]_F$, $[U^cU^cD^cE^c]_F$ cannot arise from tree-level bulk field exchange, they may be already present on the brane as non-renormalizable interactions induced, for instance, by new physics at the Planck scale. On dimensional grounds, these interactions would lead to unacceptably fast proton decay [11]. It is interesting to note that

[4] As usual we denote by the suffix F (D) the integration over $d^2\theta$ $(d^2\theta d^2\bar{\theta})$ in superspace.

these operators, can be directly forbidden by a suitable parity assignment, for instance by taking, at $y = (0, \pi R)$:

$$(Q, U^c, D^c) \sim (+,+) \quad ; \qquad (L, E^c) \sim (+,-) \quad . \tag{17}$$

This is an assignment that we consider in the following as a particularly interesting example. With this choice proton decay is actually forbidden.

We see from eq. (7) that a non-vanishing coupling $\overline{\psi_Q}\bar{\sigma}^\mu T^{\hat{a}}\psi_{U^c}A_\mu^{\hat{a}}$ is a necessary condition for proton decay to occur through gauge vector boson exchange. However the requirement of non-vanishing Yukawa couplings forces $QU^{c\dagger}$ to be $(+,+)$, while $A_\mu^{\hat{a}}$ is $(+,-)$. Therefore the coupling $\overline{\psi_Q}\bar{\sigma}^\mu T^{\hat{a}}\psi_{U^c}A_\mu^{\hat{a}}$ drops from the total four-dimensional action. This forbids the occurrence of dimension six operators $[QQU^{c\dagger}E^{c\dagger}]_D$ and $[U^{c\dagger}D^{c\dagger}QL]_D$, for any parity choice of matter fields that guarantees the invariance of the superpotential w_{mass}. In particular, such operators are also directly forbidden by our example of parity assignment given in eq. (17).

To avoid fast proton decay we should also dispose of at least some of the terms

$$QD^cL \quad , \qquad LE^cL \quad , \qquad U^cD^cD^c \quad . \tag{18}$$

For this purpose, in the conventional approach, one imposes the usual R-parity, as an additional symmetry. In our case we immediately see that the first two vertices are P'-odd according to the parity assignment in eq. (17). The remaining one cannot by itself lead to proton decay.

From eq. (13) we see that the parity assignment in eq. (17) gives rise to the following four-dimensional superpotential:

$$\begin{aligned} w^{(4)} \;&=\; 2\int dy\delta(y)\Big[y_d\,(QD^cH_d^D + LE^cH_d^D + QLH_d^T) + \\ &\qquad y_u\,(QU^cH_u^D + U^cE^cH_u^T) + y_R\,U^cD^cD^c\Big] \quad . \end{aligned} \tag{19}$$

In particular, specific properties of SU(5) Yukawa couplings are maintained, like the minimal SU(5) relation $m_d = m_e^T$. Considering now the neutrino mass sector, for each family, we introduce an SU(5)-singlet right-handed neutrino field ν^c on the branes $y = (0, \pi R)$ and attribute to it the same intrinsic parities as for L and E^c, for example $(+,-)$ in the case of eq. (17). Then the Yukawa interaction term $L\nu^cH_u^D + D^c\nu^cH_u^T$, (that leads to Dirac mass terms after electroweak symmetry breaking) and the ν^c Majorana mass terms $M_R\nu^c\nu^c$ are both allowed by parities. The mass M_R is naturally of order M_{GUT} as the ν^c Majorana mass is compatible with the low energy symmetry. Thus the usual see-saw mechanism remains viable in this model. Similarly the higher dimensional operator $(\lambda/M_L)LH_d^DLH_d^D$, which leads to light neutrino Majorana mass terms upon electroweak symmetry breaking, is also allowed, with a mass M_L naturally of order $M_{GUT} - M_{Planck}$. Thus we see that the mechanisms that suppress of even forbid proton decay leave the good qualitative features of the neutrino mass sector unaltered. Moreover, the relation between left-handed mixings for charged leptons and right-handed mixings for down quarks, which plays an important role in reproducing large mixings for neutrinos [12], is preserved as a consequence of $m_d = m_e^T$.

Summarising, we have the following interesting properties of the interactions on the branes at $y = (0, \pi R)$ [5]. First of all, any parity assignment of matter fields that allows non-vanishing fermion masses forbids tree-level Higgsino or gauge boson exchange amplitudes for proton decay. Proton decay can be actually forbidden by a suitable set of parity assignments, as those given in (17). The qualitative good features induced by lepton number non-conservation in the neutrino mass sector are preserved. The doublet-triplet splitting problem is solved, as observed in ref. [2]. The gauge couplings on the brane of the matter fields are those induced by $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge invariance. The gauge coupling unification and charge quantisation constraints are valid on the brane because they are guaranteed by the fact that gauge multiplets interact in the bulk. The Yukawa interactions that provide fermion masses after electroweak symmetry breaking satisfy the minimal SU(5) symmetry relations. Thus mass relations like $m_b = m_\tau$ at M_{GUT} are preserved in this approach. A μ term can technically be implemented on the brane as it is compatible with all the unbroken symmetries. The smallness of the μ term is a naturalness problem that has no solution in this context.

5 Conclusion

The beauty and elegance of grand unification clash with their specific realization in the context of conventional models in four space-time dimensions. Realistic models with natural doublet-triplet splitting and proton decay compatible with the present experimental limits are rather complicated [3]. We find very attractive that in a simple SU(5) model, with a compact fifth dimension, gauge symmetry breaking and doublet-triplet splitting are accomplished without the need of a baroque Higgs sector [2]. The gauge symmetry is broken by the action of a Z_2' discrete symmetry and the inverse radius of the fifth dimension provides the grand unified mass scale. In this note we have analysed in detail matter interactions in this class of models. We have shown that matter fields should necessarily live on the branes, where Yukawa couplings are localised. Next we have shown that, when considering matter interactions located at the points invariant under Z_2 (Z_2'), the theory is fully defined only when the transformation properties of matter under Z_2' (Z_2) are specified. Four-dimensional matter couplings invariant under the full SU(5) group are in principle allowed, but they lead to vanishing up-type quark masses and are thus ruled out. As a consequence, the interactions only respects $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. Proton decay cannot proceed via tree-level Higgsino or gauge boson exchange, for any matter parity compatible with non-vanishing fermion masses. Appropriate parity assignments to matter fields can also forbid four and five dimensional B/L violating operators, thus playing the role of R parity symmetry. Fermion mass relations of minimal SU(5), like $m_b = m_\tau$, are preserved. Both Dirac and Majorana neutrino masses can be included and see-saw masses for light neutrinos

[5] Matter fields and interactions can also be confined on the branes located at $y = (\pm\pi R/2)$. Those branes are only relevant after the Z_2' compactification and in this case the assumption of starting from SU(5) invariant interactions is unjustified. Nevertheless the tree-level suppression of proton decay is still valid. The fields $A_\mu^{\hat{a}}$ and $H_{u,d}^T$ all vanish on these branes and any interaction involving these fields is automatically absent. Also the interactions involving $\hat{H}_{u,d}^T$ are not dangerous for proton decay. Indeed these fields are P-odd and the couplings $QQ\hat{H}_u^T$, $U^cD^c\hat{H}_d^T$ again cancel if non-zero fermion masses can be generated.

can be reproduced. A large mixing for neutrinos can still arise from a large mixing between right-handed quarks, thanks to the relation $m_e = m_d^T$. Gauge coupling unification and charge quantization are guaranteed by the SU(5) bulk symmetry.

In the models discussed here the absence of tree-level amplitudes leading to proton decay appears deeply entangled with the orbifold construction and it can provide an explanation to the present negative experimental results. Proton decay could still proceed through higher dimensional non-renormalizable operators whose suppression is not a model independent feature, but a particular parity assignment of matter fields can forbid proton decay at all. The idea of forbidding proton decay by a suitable discrete symmetry is not new [13], but the physical origin of the relevant parity is particularly clear in the present context.

Of course there is a long way to the construction of a realistic model. In this note we have left aside some crucial issues like the breaking of the residual N=1 SUSY, realistic fermion masses (including neutrinos) and threshold corrections to gauge coupling unification. However we find very encouraging that at the simplest level the main problems that plague minimal versions of grand unified theories, like the doublet-triplet splitting and fast proton decay, can be overcome rather easily.

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