Non-BPS Instability in Heterotic M-theory

Ph. Brax[1]

Theoretical Physics Division, CERN
CH-1211 Geneva 23[2]

Abstract

We study the warped geometry of heterotic M-Theory in five dimensions where five-branes are included in the bulk. Five-branes wrapping holomorphic curves lead to BPS configurations where the junction conditions are automatically satisfied. We consider five-branes wrapped around non-supersymmetric cycles and show that the configuration is unstable. We describe explicitly the resulting time-dependent geometry where the bulk five-branes move towards the Horova-Witten boundary walls. The five-branes collide with the boundary walls in a finite time resulting in the restoration of supersymmetry.

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[1]email: philippe.brax@cern.ch
[2]On leave of absence from Service de Physique Théorique, CEA-Saclay F-91191 Gif/Yvette Cedex, France

1 Introduction

Five dimensional models with warped geometries have played a significant role in high energy physics and cosmology during the past few years. Many puzzles in four dimensions can been tackled by extending space-time to five dimensions[1]. In most settings the fifth dimension is either an interval in models inspired by the Horava-Witten theory[2] or a half-line with an AdS geometry for the Randall-Sundrum proposal[3]. The phenomenological applications are numerous ranging from the hierarchy problem[4] to the cosmological constant problem[5].

Another guise of five dimensional models has been motivated by the AdS/CFT correspondence(for a review see [6]). The search for a supersymmetric version of the RS scenario has led to the concept of supergravity in singular spaces[7, 8]. A particularly interesting class of supergravity models coupled to boundaries emerges from the compactification of M-theory on Calabi-Yau three-folds[9]. In five dimensions this leads to gauged supergravity theories with background fluxes. The origin of the background fluxes springs from the modification of the Bianchi identities due to the Horava-Witten boundaries and the inclusion of five-branes in the bulk[10]. The fluxes lead to the existence of potential terms for the vector multiplets.

The five dimensional solutions of the supergravity equations of motion have been widely studied (for a review see [11]). In particular one finds BPS configurations preserving $N = 1$ supersymmetry in four dimensions. It also conspicuous to find cosmological solutions with a time dependent background [12]. These solutions should have some relevance to the physics of the early universe.

Recently non-BPS configurations have been extensively studied both from a theoretical and phenomenological point of view[13, 14] and might eventually lead to a better understanding of the origin of supersymmetry breaking. One of the purpose of the present letter is to consider non-BPS configurations in the context of the strongly coupled heterotic string theory[15]. In particular we shall be concerned with unstable configurations resulting from the presence of five-branes wrapping non-supersymmetric cycles in the bulk.

In a first section we recall the necessary ingredients of five dimensional supergravity and its link to heterotic M-theory. We describe the BPS situation with five-branes wrapping supersymmetric cycles. We pay particular attention to the boundary conditions and show that the BPS property guarantees that the junction conditions are automatically satisfied. We then break supersymmetry by wrapping five-branes around non-supersymmetric cycles. After recalling the topological features of such configurations, we show that the modified boundary action on the five-branes leads to an instability. We find explicit solutions of the equations of motion where the five-branes move towards the boundary walls. The collision occurs within a time proportional to the fifth-dimension radius.

2 Bulk Five-Branes and BPS Configurations

We compactify M-theory[2] on a Calabi-Yau three-fold with background fluxes switched on[9]. The compactification of eleven dimensional supergravity on a Calabi-Yau threefold with Hodge numbers $(h^{(1,1)}, h^{(2,1)})$ leads to an N=2 D=5 supergravity theory with $h^{(1,1)} - 1$ vector multiplets and $h^{(2,1)} + 1$ hypermultiplets. In particular there is always one universal hypermultiplet containing the volume of the Calabi-Yau manifold Y. The other hypermultiplets belonging to a quaternionic moduli space will not play a role here. Switching on background fluxes leads

to a potential for the vector multiplets. The resulting supergravity is gauged with the axion field charged under a $U(1)_R$ symmetry. One can introduce five-branes spread along four of the five non-compact directions and two of the internal dimensions. For five-branes wrapping holomorphic curves the resulting configuration is BPS and breaks half of the supersymmetries.

Let us now be a bit more specific. The compactifying manifold Y lives along the x_A, $A = 5 \ldots 10$, coordinates while space time is along x_i, $i = 0 \ldots 3$, and x_5. The x_5-axis is a Z2 orbifold under $x_5 \to -x_5$ that we identify with the interval $[0, +\pi\rho]$. We also include N five-branes M_5^i extended along the four non-compact dimensions x_i and wrapped around holomorphic curves Σ^i in Y. The Bianchi identity for the four form G_4 of eleven dimensional supergravity is modified due to the presence of the five-brane

$$dG_4 = 4\sqrt{2}(\frac{\kappa}{4\pi})^{2/3}[\delta_{M_5} - \delta_0 dx_5 \wedge J^{(1)} - \delta_{\pi\rho} dx_5 \wedge J^{(2)}] \tag{1}$$

with

$$J^{(i)} = \frac{1}{16\pi^2}(\mathrm{Tr}(F^{(i)} \wedge F^{(i)}) - \frac{1}{2}\mathrm{Tr}(R \wedge R)). \tag{2}$$

Due to supersymmetry preservation each of the Horava-Witten planes carries a holomorphic vector bundle characterized by the two forms $F^{(i)}$, $i = 1, 2$. We will choose $F^{(2)} = 0$ and consider the first plane at $x_5 = 0$ as our brane-world. The fundamental class δ_{M_5} is defined by

$$\int_{M_5} f = \int_{M_{11}} f \wedge \delta_{M_5} \tag{3}$$

and can be written as

$$\delta_{M_5} = \sum_{i=1}^{N} \delta_{x_0^i} dx_5 \wedge \delta_{\Sigma^i} \tag{4}$$

where x_0^i are the coordinates of the five-branes. In appropriate units δ_{Σ^i} has dimension three in eleven dimensional Planck units.

Due to the compactness of the x_5 direction there is a topological condition to be satisfied

$$\delta_\Sigma \equiv \frac{1}{16\pi^2}(\mathrm{Tr}(F^{(1)} \wedge F^{(1)}) - \mathrm{Tr}(R \wedge R)) \tag{5}$$

as cohomology classes where $\delta_\Sigma = \sum_{i=1}^{N} \delta_{\Sigma^i}$. This explicitly determines the homology class of the two dimensional surface Σ.

An explicit solution to the Bianchi identity is then

$$G_4 = 2\sqrt{2}(\frac{\kappa}{4\pi})^{2/3} \sum_{i=0}^{N} \epsilon_{x_0^i} \delta_{\Sigma^i} \tag{6}$$

where $\epsilon_{x_0^i}$ jumps from -1 to 1 at x_0^i and we have used the notation

$$\delta_{\Sigma^0} = -\frac{1}{16\pi^2}(\mathrm{Tr}(F^{(1)} \wedge F^{(1)}) - \frac{1}{2}\mathrm{Tr}(R \wedge R)). \tag{7}$$

In each interval separating the i-th and the $(i+1)$-th five-branes there are background magnetic charges defined by

$$\alpha_I^i = \frac{\sqrt{2}\epsilon_S}{\rho} \int_{C_I} \sum_{j=0}^{i} \delta_{\Sigma^j} \tag{8}$$

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where the four-manifolds C_I are Poincaré duals to the the ω_I's forming a basis of the $h^{(1,1)}$ holomorphic two-forms. We have introduced the expansion parameter $\epsilon_S = (\frac{\kappa}{4\pi})^{2/3}\frac{2\pi\rho}{v^{2/3}}$ and v is the volume of Y. The effective action obtained by substituting in the eleven dimensional supergravity action depends crucially on these magnetic charges.

The vector multiplets follow from the expansion of the Kahler form ω

$$\omega = t^I \omega_I. \tag{9}$$

The volume modulus of the Calabi-Yau manifold is given by

$$\mathcal{V} = \frac{1}{6}\int_Y \omega \wedge \omega \wedge \omega. \tag{10}$$

Defining

$$t^I = \mathcal{V}^{1/3}X^I \tag{11}$$

the scalars in the vector multiplets parameterize the solutions of

$$C_{IJK}X^I X^J X^K = 6 \tag{12}$$

where C_{IJK} are the intersection numbers $\int_Y \omega_I \wedge \omega_J \wedge \omega_K$. The volume modulus belongs to the universal hypermultiplet. The low energy bosonic action takes the form of a non-linear sigma model with the metric defined by

$$G_{IJ} = -\frac{1}{2}C_{IJK}X^K + \frac{1}{8}(C_{ILM}X^L X^M)(C_{JPQ}X^P X^Q). \tag{13}$$

In the Einstein frame $ds_E^2 = \mathcal{V}^{2/3}ds_{str}^2$ the action for the i-th interval reads[9]

$$S_{bulk}^i = -\frac{1}{2\kappa_5^2}\int d^5 x\sqrt{-g_E^{(5)}}(R + G_{IJ}\partial_\mu X^I \partial^\mu X^J - \frac{1}{2\mathcal{V}^2}(\partial\mathcal{V})^2 - \frac{1}{2\mathcal{V}^2}\alpha_I^i \alpha_J^i G^{IJ}(X)). \tag{14}$$

The last term is the potential for the scalars in the vector multiplets.

Let us now consider the boundary actions. The boundary wall action reads [9]

$$S_B^0 = \frac{\sqrt{2}}{\kappa_5^2}\int d^4 x\sqrt{-g_E^{(4)}}\frac{\alpha_I^0 X^I}{\mathcal{V}} \tag{15}$$

in the Einstein frame. The action on the three branes resulting from the bulk five-branes depends on the nature of the surface Σ. For BPS configurations the BPS bound for five-branes is saturated implying the equality between the central charge and the five-brane tension. This leads to the action for each five-brane

$$S_B^i = T_5 \int\sqrt{-g_{str}^{(4)}}\mathrm{Vol}(\Sigma^i) \tag{16}$$

in the string frame. Now using[16]

$$T_5 = \frac{2\pi}{(4\pi\kappa)^{2/3}} \tag{17}$$

and

$$\mathrm{Vol}(\Sigma^i) = v^{1/3}\int_{\Sigma^i}\omega \tag{18}$$

one obtains the boundary action

$$S_B^i = \frac{1}{\sqrt{2}\kappa_5^2}\int d^4 x\sqrt{-g_E^{(4)}}\frac{[\alpha_I^i]X^I}{\mathcal{V}} \tag{19}$$

with $[\alpha_I^i] = \alpha_I^i - \alpha_I^{(i-1)}$. Notice that this boundary action has the same functional form as the boundary wall action up to a factor of two. In the following section we will concentrate on the solutions of the equations of motions with a particular emphasis on the junction conditions.

3   Non-BPS Configurations

Before describing the non-BPS configurations we analyse the solutions of the equations of motions in the BPS case. Let us concentrate on the following warped geometry

$$ds^2 = e^{2A(x_5)}dx^2_{//} + dx^2_5 \tag{20}$$

and consider x_5-dependent fields only. From the action one can read off the junction conditions at the origin

$$[\frac{d\mathcal{V}}{dx_5}]_0 = 2\sqrt{2}(\alpha^0.X)|_0$$

$$[\frac{dA}{dx_5}]_0 = \frac{\sqrt{2}}{3}\frac{(\alpha^0.X)}{\mathcal{V}}|_0$$

$$[\frac{dX_I}{dx_5}]_0 = \frac{\sqrt{2}}{\mathcal{V}}(\alpha^0_I - \frac{2}{3}(\alpha^0.X)X_I)|_0 \tag{21}$$

where in the last equation we have used a Lagrange multiplier to impose the constraint (12). Similarly the junction conditions at the one of the bulk five-branes read

$$[\frac{d\mathcal{V}}{dx_5}]_{x^i_0} = \sqrt{2}([\alpha^i].X)|_{x^i_0}$$

$$[\frac{dA}{dx_5}]_{x^i_0} = \frac{1}{3\sqrt{2}}\frac{([\alpha^i].X)}{\mathcal{V}}|_{x^i_0}$$

$$[\frac{dX_I}{dx_5}]_{x^i_0} = \frac{1}{\sqrt{2}\mathcal{V}}([\alpha^i_I] - \frac{2}{3}([\alpha^i].X)X_I)|_{x^i_0} \tag{22}$$

where the latter differ from the former by a factor of two. Due to the $Z2$ action one has $[f]_0 = 2f|_0$ relating the jump at the origin to twice the value at the origin.

One of the features of BPS configurations is that the junctions conditions are automatically satisfied. This can be seen from the BPS equations deduced from the fermionic supersymmetry variations

$$\frac{d\mathcal{V}}{dx_5} = \sqrt{2}(\alpha^i.X)$$

$$\frac{dA}{dx_5} = \frac{1}{3\sqrt{2}}\frac{(\alpha^i.X)}{\mathcal{V}}$$

$$\frac{dX_I}{dx_5} = \frac{1}{\sqrt{2}\mathcal{V}}(\alpha^i_I - \frac{2}{3}(\alpha^i.X)X_I) \tag{23}$$

in each interval. Combining these equations one gets[9]

$$\frac{d(\mathcal{V}^{1/3}X_I)}{dx_5} = \mathcal{V}^{-2/3}\frac{\alpha^i_I}{\sqrt{2}} \tag{24}$$

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from which we deduce that in each interval

$$C_{IJK}t^J t^K = 2\sqrt{2}\mathcal{V}^{1/3}\alpha_I^i y + C_I \tag{25}$$

where we have introduced $dy = \mathcal{V}^{-2/3}dx_5$. Inverting (25) for t^I gives

$$\mathcal{V} = \frac{1}{6}C_{IJK}t^I t^J t^K \tag{26}$$

and the metric

$$ds^2 = \mathcal{V}^{1/3}dx_{//}^2 + \mathcal{V}^{4/3}dy^2. \tag{27}$$

The position of the five-branes is not constrained reflecting the no-force condition.

The non-BPS configurations appear when the cohomology class δ_Σ given by (5) is not effective, i.e. when the expansion

$$\delta_\Sigma = \sum_{i=1}^{M} a_i \delta_{C^i}, \tag{28}$$

in terms of the classes δ_{C^i} of holomorphic curves C^i, contains both positive and negative integers. This implies that there is no holomorphic curve whose class coincides with δ_Σ. This topological characterization of non-supersymmetric cycle has several physical consequences. On the one hand the no-force condition between BPS five-branes is no longer valid. Writing

$$\delta_\Sigma = [A] - [B] \tag{29}$$

where $[A]$ and $[B]$ are effective cycles, i.e. there exist holomorphic curves A (resp. B) whose classes are $[A]$ (resp. $[B]$), one expects that separating the five-branes wrapped around A from the anti-five-branes wrapped around B is energetically disfavoured. Therefore we will consider that the five-branes coincide and wrap a single surface in the class δ_Σ. Within the homology class dual to δ_Σ we consider the surface S whose volume is minimum. As the brane tension is minimal this configuration is stable for a given Calabi-Yau manifold Y, i.e. the five brane wrapped around S gives rise to a stable non-BPS brane. The BPS bound states that the tension of the five brane wrapped around S is bounded from below by the central charge

$$|Q| = T_5 v^{1/3} \int d^4x \sqrt{-g_{str}^{(4)}} \ |\int_\Sigma \omega|. \tag{30}$$

Equality would imply that S is a calibrated surface[19, 20], i.e. a holomorphic curve, realizing a BPS configuration. Such stable non-BPS branes are sensitive to deformations of the Calabi-Yau manifold Y[21].

Let us denote by $T > 1$ the ratio beween the tension and the central charge. This leads to the boundary action

$$S_B^i = \frac{1}{\sqrt{2}\kappa_5^2} \int d^4x \sqrt{-g_E^{(4)}} \frac{[\alpha_I]X^I}{\mathcal{V}} T \tag{31}$$

with

$$\alpha_I = \frac{\sqrt{2}\epsilon_S}{\rho} \int_{C_I} \delta_\Sigma. \tag{32}$$

The effect of supersymmetry breaking is to modify the boundary action. In particular the bulk equations of motion are still the same as before supersymmetry breaking. Therefore the solutions of the bulk equations of motion are not modified by the supersymmetry breaking

mechanism. We first assume that T is independent of $\mathcal{V}$ and X^I. The only effect of supersymmetry breaking is to alter the boundary conditions

$$[\partial_n \mathcal{V}]_{x_0} = \sqrt{2} T([\alpha].X)|_{x_0}$$

$$[\partial_n A]_{x_0} = \frac{T}{3\sqrt{2}} \frac{([\alpha].X)}{\mathcal{V}}|_{x_0}$$

$$[\partial_n X_I]_{x_0} = \frac{T}{\sqrt{2}\mathcal{V}}([\alpha_I] - \frac{2}{3}([\alpha].X)X_I)|_{x_0}$$

(33)

where ∂_n is the normal derivative. The boundary conditions deduced from the bulk solutions do not match with the boundary conditions arising from the non-BPS brane action. Such a discrepancy has already been analysed in the context of supergravity in singular spaces [17]. In particular we expect that the presence of a non-BPS brane destabilizes the vacuum. The perfect balance between the gravitational and scalar forces disappears and the non-BPS brane moves towards the boundary walls.

We can generate time-dependent conformally flat solutions from the static solutions[17, 18]. This is most easily achieved by using a boost along the x_5 direction. To do so we first introduce conformal coordinates so that the metric becomes

$$ds_5^2 = a^2(u)(dx^2 + du^2).$$

(34)

where

$$a^2 = \mathcal{V}^{1/3}, \quad du = \mathcal{V}^{1/2}dy.$$

(35)

Under a boost and a rescaling the new solutions of the bulk equations of motions are

$$\tilde{A}(u, \eta) = A(u + h\eta, \frac{\alpha_I^i}{\sqrt{1 - h^2}})$$

$$\tilde{\mathcal{V}}(u, \eta) = \mathcal{V}(u + h\eta, \frac{\alpha_I^i}{\sqrt{1 - h^2}})$$

$$\tilde{X}_I(u, \eta) = X_I(u + h\eta, \frac{\alpha_I^i}{\sqrt{1 - h^2}})$$

(36)

where η is the conformal time. We have displayed the explicit dependence on the magnetic charges α_I^i. One can now use the BPS equations satisfied by $(A, \mathcal{V}, X_I)$ to deduce that

$$\partial_n \tilde{\mathcal{V}} = \frac{\sqrt{2}}{\sqrt{1 - h^2}}(\alpha^i.\tilde{X})$$

$$\partial_n \tilde{A} = \frac{1}{3\sqrt{2}\sqrt{1 - h^2}} \frac{(\alpha^i.\tilde{X})}{\tilde{\mathcal{V}}}$$

$$\partial_n \tilde{X}_I = \frac{1}{\sqrt{2}\sqrt{1 - h^2}} \frac{1}{\tilde{\mathcal{V}}} (\alpha_I^i - \frac{2}{3}(\alpha^i.\tilde{X})\tilde{X}_I)$$

(37)

in each of the two intervals. Evaluating the jumps at u_0, the fixed coordinate of the non-BPS brane, and using (37) one finds that the boundary conditions are automatically satisfied

provided that

$$h = \pm \frac{\sqrt{T^2 - 1}}{T}. \tag{38}$$

Notice that this requires $T \geq 1$. This is exactly the BPS bound with a static solution only in the BPS case.

After applying the boost the two boundary walls are moving and the non-BPS brane is static. By reverting to the original coordinates one finds that the solution describes a moving non-BPS brane surrounded by two static boundary walls. As the bulk equations of motions are not modified by the presence of the non-BPS brane the boundary conditions at the two boundary walls are automatically satisfied. The moving non-BPS brane eventually hits the boundary walls in a finite time determined by the speed h. For a generic supersymmetry breaking parameter the life-time of the non-BPS brane is of the order of the size of the fifth dimension.

When the supersymmetry breaking T depends on $\mathcal{V}$ and X^I, the bulk equations are still satisfied but the boundary conditions cannot be simply fulfilled by applying a boost in the fifth direction. In [15] the case where supersymmetry is broken in this fashion by one of the boundary walls was considered. It has been shown that static solutions would require a fine-tuning of the radius of the extra dimension and of the magnetic charges. The latter being unlikely because this requires tuning continuous and discrete variables. From our point of view the reason for the non-existence of static solution follows from the absence of balance between the forces on the non-BPS brane. It seems likely that an appropriate change of variables performed on the bulk solutions will imply the matching of the boundary conditions in this more general setting. The time dependence of the resulting solution would describe the motion of the non-BPS brane towards the boundary walls.

Let us now briefly discuss the fate of the non-BPS brane after hitting one of the boundary walls. First of all the case of BPS five branes merging with the boundary walls has been extensively studied[22]. In particular one finds small instanton transitions where the gauge bundle is modified. In the non-BPS case the flux conservation condition indicates that supersymmetry configurations such as the standard embedding case can spontaneously appear after the collision. This leads to a restoration of supersymmetry.

4   Conclusion

We have described the sharp difference between BPS and non-BPS configurations in heterotic M-Theory. In particular we have explicitly shown that the absence of balance between the gravitational and scalar forces leads to the motion of the non-BPS five-branes towards the boundary walls. Due to the finite size of the fifth dimension the life-time of the non-BPS configuration is measured in units of the eleven dimensional Planck length. Nevertheless one may use the resulting configuration in a phenomenological way by considering that the size of the extra dimension is large. Indeed the BPS condition on the boundary walls guarantees the absence of a radion potential and therefore allows to consider an arbitrarily large extra dimension. Moreover by tuning the supersymmetry breaking scale one may consider the speed of the extra dimension to be sufficiently small to allow for an adiabatic treatment. The resulting scenario might be useful in order to study the supersymmetry breaking induced by non-BPS branes in brane-world models.

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