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Analysis Of Shielding Charged Particle Beams By Thin Conductors

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Abstract

We present an analysis of shielding of electro-magnetic fields excited by beams of charged particles surrounded by thin conducting layers or metal stripes inside an external structure of finite length. The ability of shielding by a layer thinner than the skin depth is explained and expressions for the impedance are derived. A previous result[1] showing preferential penetration through the shielding layer at the resonant frequencies of the surrounding structure is verified, and extended to include finite resistivity of the outer structure. Integration over the spectrum of the beam bunch shows that penetration is (nearly) independent of the quality factors of the resonances. The transition of these results to those for a geometry of infinite length requires numerical evaluation.

1 Introduction

The shielding of rf fields emanating from coaxial cables was already treated in the literature more than 50 years ago[2]. A large number of publications followed afterwards, even a whole journal issue was devoted to this subject[3]. Already then it was known that conducting layers of a thickness less than the skin depth could reduce the outside field strength sufficiently to avoid "cross talk" between adjacent cables. Also the wall penetration of rf fields excited by charged particle beams has been analyzed in a number of publications[4, 5, 6]. In most of these papers rotationally symmetric

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structures or concentric wire cages of infinite extent had been assumed for simplicity, as will also be done here except for the wire cage.

The importance of the *finite length* of structures surrounding a thin conducting shield has been recognized when investigating the field penetration into the LHC kickers[7]. An analysis of such a geometry has recently been made[1] and showed that rf fields will penetrate through a thin shielding layer preferentially at the resonant frequencies of the cavity formed by the surrounding structure. However, that analysis did not include a finite resistivity of the structure material which will be treated here.

In addition, we will discuss also the effects of shielding by conducting stripes or wire cages, which are often preferred to a continuous metal layer in order to reduce eddy current losses in rapid cycling synchrotrons or pulsed devices like kickers. The results of a number of bench measurements on such structures have been published[7, 8] as well as recent measurements with beam[9] .

2 Shielding by a thin conducting cylinder

For the calculation of the longitudinal coupling impedance we take as source field a narrow ring of charge Q with radius a, traveling with velocity $v = \beta c$ along the z-axis inside a circular cylindrical screen of inside radius b and thickness τ, surrounded by a concentric vacuum chamber of radius d .

In the frequency domain with $k = \omega/v$, this corresponds to a charge and current density given by

$$\begin{aligned} \rho^{(s)}(r,z) &= \frac{Q}{2\pi a v}\delta(r-a)\mathrm{e}^{-jkz} , \\ J_z^{(s)}(r,z) = v\rho^{(s)}(r,z) &= \frac{Q}{2\pi a}\delta(r-a)\mathrm{e}^{-jkz} , \end{aligned} \tag{2.1}$$

The (Fourier transforms of the) electro-magnetic (EM) field components generated by this source in free space outside the beam, $r \geq a$, are then given by

$$\begin{aligned} E_z^{(s)}(r,z) &= j\omega\mu_0 \frac{QI_0(\nu a)}{2\pi\beta^2\gamma^2} K_0(\nu r)\mathrm{e}^{-jkz}, \\ Z_0 H_\theta^{(s)}(r,z) &= -\omega\mu_0 \frac{QI_0(\nu a)}{2\pi\beta\gamma} K_1(\nu r)\mathrm{e}^{-jkz}, \\ E_r^{(s)}(r,z) &= -\omega\mu_0 \frac{QI_0(\nu a)}{2\pi\beta^2\gamma} K_1(\nu r)\mathrm{e}^{-jkz}, \end{aligned} \tag{2.2}$$

where μ_0 is the free space permeability, $k_0 = \omega/c = \beta k$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ is the relativistic energy factor, $\nu = k/\gamma$ the radial propagation constant in vacuum, and I_n, K_n are modified Bessel functions of the first and second kind, order n, regular at $r = 0$ and at $r = \infty$, respectively.

The fields given by Eqs. (2.2) can be thought of as a wave moving in the outward radial direction. As discussed in the preceding section, we investigate a thin conducting cylinder, extending from $r = b$ to $r = b + \tau$, which is used to isolate the beam and the region $r > b + \tau$ from one another. We shall assume that $\tau \ll b$ and $\delta \ll b$, where $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the *skin depth* in a metal with conductivity σ and permeability μ. At the moment, we make no assumptions about the relative size of τ and δ.

Due to the presence of the conducting layer, the EM field components in Eqs. (2.2) must be revised to include the reflected wave. In the region $a < r \leq b$, the components required for matching are

$$\begin{aligned} E_z(r,z) &= A[K_0(\nu r) + \alpha I_0(\nu r)]\mathrm{e}^{-jkz} \\ Z_0 H_\theta(r,z) &= -j\beta\gamma A[K_1(\nu r) - \alpha I_1(\nu r)]\mathrm{e}^{-jkz} \end{aligned} \tag{2.3}$$

where α is a not yet determined reflection coefficient, and

$$A = j\omega \frac{\mu_0 Q I_0(\nu a)}{2\pi\beta^2\gamma^2}. \tag{2.4}$$

The radially outgoing wave for $r \geq (b + \tau)$ is

$$\begin{aligned} E_z(r,z) &= A\mathcal{T} K_0(\nu r)\mathrm{e}^{-jkz}, \\ Z_0 H_\theta(r,z) &= -j\beta\gamma A\mathcal{T} K_1(\nu r)\mathrm{e}^{-jkz}, \end{aligned} \tag{2.5}$$

where $\mathcal{T}$ is a *transmission coefficient*. For $b \leq r \leq b + \tau$, inside the metal with a conductivity $\sigma \gg \omega\varepsilon_0$, the radial propagation constant becomes approximately $\nu_c = (1 + j)/\delta$ and hence $|\nu_c b| \gg 1$. With the large argument approximations for modified Bessel functions and $H_\theta \cong (\sigma/\nu_c^2)(\partial E_z/\partial r)$ we get

$$\begin{aligned} E_z(r,z) &\cong A\left[B\mathrm{e}^{(1+j)(r-b)/\delta} + C\mathrm{e}^{-(1+j)(r-b)/\delta}\right]\mathrm{e}^{-jkz}, \\ H_\theta(r,z) &\cong \frac{\sigma\delta A}{1+j}\left[B\mathrm{e}^{(1+j)(r-b)/\delta} - C\mathrm{e}^{-(1+j)(r-b)/\delta}\right]\mathrm{e}^{-jkz} \end{aligned} \tag{2.6}$$

when $\tau \ll b$. The coefficients α, $\mathcal{T}$ and the amplitude factors B, C can be determined by requiring continuity of E_z and H_θ at $r = b$ and $r = b + \tau$.

After considerable algebra the transmission coefficient is found to be

$$\mathcal{T} = \frac{1}{\cosh\left[(1+j)\tau/\delta\right] + D\sinh\left[(1+j)\tau/\delta\right]} \approx \frac{1}{1+(1+j)D\tau/\delta} \tag{2.7}$$

where the approximation is valid for $\tau \ll \delta$. With the abbreviation

$$\xi = -(1+j)\frac{j\beta\gamma}{Z_0\sigma\delta} = \frac{1-j}{2}\beta^2\gamma k\delta \tag{2.8}$$

the parameter D can be written

$$D = -\frac{\xi I_1 K_1 + I_0 K_0/\xi}{I_1 K_0 + I_0 K_1} \tag{2.9}$$

where $I_n = I_n(\nu b)$ while $K_n = K_n(\nu(b+\tau))$. Because $\tau \ll b$, all Bessel functions may be evaluated at νb. Then we can use the Wronskian[10] $K_0(x)I_1(x) + I_0(x)K_1(x) = 1/x$ to simplify the denominator of Eq. (2.9):

$$D = -\nu b[\xi I_1 K_1 + I_0 K_0/\xi] \approx -\nu b\left[\frac{\xi}{2} - \frac{\ln(\nu b)}{\xi}\right] . \tag{2.10}$$

Now we approximate $\mathcal{T}$ for $kb \ll \gamma$, i.e. for not too high frequencies or for ultra-relativistic beam energies, Eq. (2.7) becomes

$$\mathcal{T} = \frac{1}{1 - \frac{\beta^2 k^2 b\tau}{2} + \frac{2j}{\beta^2\gamma^2}\frac{b\tau}{\delta^2}\ln\frac{kb}{\gamma}} . \tag{2.11}$$

The conducting shell shields the beam from the region outside when the transmission coefficient is small, $|\mathcal{T}| \ll 1$. We thus get the *shielding condition*

$$\frac{\tau}{\delta} \gg \frac{2\delta}{b}\left[(\beta k\delta)^4 + \left(\frac{2}{\beta\gamma}\right)^4 \ln^2(kb/\gamma)\right]^{-\frac{1}{2}} , \tag{2.12}$$

For $k\delta \ll 2\sqrt{\ln(\nu b)}/\beta^2\gamma$, this condition simplifies to

$$\frac{\tau}{\delta} \gg \frac{\beta^2\gamma^2}{2\ln(kb/\gamma)}\frac{\delta}{b} , \tag{2.13}$$

On the other hand, $k\delta > 2\sqrt{\ln(\nu b)}/\gamma$ for large γ, and the condition becomes $\tau \gg 2/(k_0^2 b)$, independent of the skin depth. Depending on the values of $\beta\gamma$ and kb, shielding can then be achieved with a layer whose thickness is smaller than the skin depth. The ability of a conducting layer thin compared to the skin depth to shield electro-magnetic fields has been known for many years[2, 3]. This astonishing feature can be explained by the fact that **part of the field is reflected by the conducting layer, while the transmitted part undergoes a succession of reflections from both interfaces at $r = b$ and $r = b + \tau$.** When $\tau \to 0$, the accumulated result is complete transmission. However, for finite τ, the successive reflections are shifted in phase and damped so as to lead to the result in Eq. (2.11).

A major simplification of the analysis can be made when $\tau \ll \delta$. Then *the tangential electric field in the conducting layer can be considered to be constant in r, implying a current density σE_z within the conductor*. The discontinuity in the tangential magnetic field through the layer is then given by

$$\delta H_\theta = \tau\sigma E_z. \tag{2.14}$$

In this limit, which is used in the rest of this paper, it is not necessary to consider the variation of E_z or the current density within the conductor.

3 Space charge and resistive wall impedances

We now consider an outer beam pipe of radius d, conductivity σ_d, and skin depth $\delta_d = (2/\omega\mu\sigma_d)^{1/2}$, shielded by a cylindrical layer at $r = b$ of conductivity σ_b, skin depth $\delta_b = (2/\omega\mu\sigma_b)^{1/2}$, and thickness $\tau \ll \delta$ at $r = b$ (see Fig.1). The source fields in Eqs. (2.2) are modified as in Eqs. (2.3) to include both the effects of the conducting layer and the beam pipe. With the still undetermined reflection coefficient α they can be written:

$$\begin{aligned} E_z(r,z) &= A[K_0(\nu r) + \alpha I_0(\nu r)]\mathrm{e}^{-jkz}, \\ Z_0 H_\theta(r,z) &= -j\beta\gamma A[K_1(\nu r) - \alpha I_1(\nu r)]\mathrm{e}^{-jkz} \end{aligned} \tag{3.1}$$

for $r < b$. With the undetermined transmission coefficient p one can write the fields for $r > b$:

$$\begin{aligned} E_z(r,z) &= pA\left[K_0(\nu r) + \alpha_d I_0(\nu r)\right]\mathrm{e}^{-jkz}, \\ Z_0 H_\theta(r,z) &= -j\beta\gamma pA\left[K_1(\nu r) - \alpha_d I_1(\nu r)\right]\mathrm{e}^{-jkz} \end{aligned} \tag{3.2}$$

The second reflection coefficient α_d at the outer layer $r = d$ can be obtained directly by applying the "Leontovich boundary condition" $E_z = -\sqrt{j\omega\mu/\sigma}H_\theta$ there. This yields

$$\alpha_d = -\frac{K_0(\nu d) + rK_1(\nu d)}{I_0(\nu d) - rI_1(\nu d)} = -\frac{K_0(\nu d)}{I_0(\nu d)} - \xi_d\ , \tag{3.3}$$

where $r = (1-j)\beta^2\gamma k\delta/2$. For $|r| \ll 1$, one gets approximately

$$\xi_d = \frac{1-j}{2}\frac{\beta^2\gamma^2}{I_0^2(\nu d)}\frac{\delta_d}{d} \tag{3.4}$$

We require $E_z(r,z)$ to be continuous at $r = b$, which yields one condition for α and p

$$K_0(\nu b) + \alpha I_0(\nu b) = p[K_0(\nu b) + \alpha_d I_0(\nu b)]. \tag{3.5}$$

The change in H_θ at $r = b$ must satisfy Eq. (2.14), leading to the second condition for α and p:

$$p\left[\alpha_d I_1(\nu b) - K_1(\nu b)\right] - \left[\alpha I_1(\nu b) - K_1(\nu b)\right] = \frac{\eta}{\nu b}\left[K_0(\nu b) + \alpha I_0(\nu b)\right], \tag{3.6}$$

where

$$\eta = \frac{2j\tau b}{\beta^2\gamma^2\delta_b^2}. \tag{3.7}$$

The longitudinal impedance is usually defined as the integral over (the Fourier transform of) the axial electric field component along the axis $r = 0$. When it is obtained by integrating at the annular radius $r = a$ this will only suppress the constant term of unity in the g-factor $g = 1 + 2\ln(b/a)$. This term actually reduces to 1/2 if one correctly averages over the beam cross section. Ignoring these small differences, the impedance can be written

$$Z_\parallel(\omega) = -\frac{1}{Q}\int_{-\infty}^{\infty} dz E_z(a,z)\mathrm{e}^{jkz}. \tag{3.8}$$

Solving Eqs. (3.5) and (3.6) for α and p and assuming $k_0 d \ll \beta\gamma$ (low frequency and/or high γ), we thus find for the impedance divided by the azimuthal mode number $n = \omega/\omega_0$

$$\frac{Z_\parallel(\omega)}{nZ_0} \cong -\frac{j}{\beta\gamma^2}\left[\ln\frac{b}{a} + \frac{\ln(d/b) - \xi_d}{1 - \eta[\ln(d/b) - \xi_d]}\right]. \tag{3.9}$$

The impedance $Z_\parallel/n$, computed without low-frequency approximations, is shown in Fig.2. In the absence of a shielding layer $(\eta = 0)$ the impedance in Eq. (3.9) reduces to the standard form of space charge plus resistive wall impedance. The dependence on the beam pipe radius and conductivity becomes negligible when

$$|\eta||\ln(d/b) - \xi_d| \gg 1, \tag{3.10}$$

in which limit $Z_\parallel(\omega) = Z_\parallel^{\rm SC}(\omega) + Z_\parallel^{\rm RW}(\omega)$, with

$$\frac{Z_\parallel^{\rm SC}(\omega)}{nZ_0} = -\frac{j}{\beta\gamma^2}\ln\frac{b}{a} \tag{3.11}$$

and

$$Z_\parallel^{\rm RW}(\omega) = -\frac{\beta\delta_b^2 nZ_0}{2\tau b} = \frac{\beta Z_0}{\omega_0\mu b\tau\sigma_b} = \frac{2\pi R}{2\pi b\tau\sigma_b} = \mathcal{R}_{\rm shield}. \tag{3.12}$$

The rhs of Eq. (3.12) is simply the resistance of the conducting shield in the axial direction. We shall later see that the result $(Z_\parallel^{\rm RW}(\omega) = \mathcal{R}_{\rm shield})$ is also true for a shield of finite length.

For $\delta_d \ll d/\beta^2\gamma^2$ or $|\xi_d| \ll \ln(d/b)$, the shielding condition Eq. (3.10) can be written

$$\frac{\tau}{\delta_b} \gg \frac{\beta^2\gamma^2}{2\ln(d/b)}\frac{\delta_b}{b}\,, \tag{3.13}$$

corresponding to the shielding of the space charge fields. For high energy machines, however, usually $\delta_d \gg d/\beta^2\gamma^2$ or $|\xi_d| \gg \ln(d/b)$, the condition becomes

$$\frac{\tau}{\delta_b} \gg \frac{\delta_b}{b}\cdot\frac{d}{\delta_d}\,, \tag{3.14}$$

corresponding to the shielding of the resistive wall impedance.

4 Shielding by a wire cage

In the previous section, we considered the shielding capability of a thin conducting layer at $r = b$. However, in order to reduce eddy currents due to a rapidly changing magnetic field, one would prefer to shield with N thin wires of conductivity σ, radius r_w, located at $\vec{r_p}$ ($r_p = b$, $\theta_p = 2\pi p/N$), where p goes from 0 to $N-1$ (see Fig.3).

In the absence of any shielding, the EM fields inside a conducting vacuum chamber at $r = d$ can be written, for $a \le r \le d$:

$$\begin{aligned} E_z(r,z) &= AG_0(\nu r)e^{-jkz} \\ Z_0H_\theta(r,z) &= j\beta\gamma AG_0'(\nu r)e^{-jkz}, \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} G_0(\nu r) &= K_0(\nu r) + \alpha_d I_0(\nu r), \\ G_0'(\nu r) &= -K_1(\nu r) + \alpha_d I_1(\nu r), \end{aligned} \tag{4.2}$$

with α_d chosen to satisfy the boundary condition at the beam pipe radius $r = d$, see Eq. (3.3).

We now add the fields which are due to a current I_w in each of the N wires. For this we replace $G_0(\nu r)$ in Eq. (4.1) by

$$G_0(\nu r) + I \sum_{p=0}^{N-1} K_0\left(\nu|\vec{r} - \vec{r_p}|\right), \tag{4.3}$$

where $I = QI_0(\nu a)$ is a dimensionless current. The electric field corresponding to the term proportional to I in Eq. (4.2) does not yet satisfy the required boundary condition at $r = d$. In order to do this, we use the addition theorem[12]

$$K_0\left(\nu|\vec{r} - \vec{r_p}|\right) = \left\{ \begin{array}{ll} \sum_{n=-\infty}^{\infty} I_n(\nu r)K_n(\nu b)\cos n(\theta - \theta_p) & \text{for } r < b \\ \sum_{n=-\infty}^{\infty} K_n(\nu r)I_n(\nu b)\cos n(\theta - \theta_p) & \text{for } r > b \end{array} \right\} \tag{4.4}$$

We may replace $K_n(\nu r)$ in Eq. (4.4) by $G_n(\nu r)$, where

$$G_n(\nu r) = K_n(\nu r) + \alpha_{dn} I_n(\nu r). \tag{4.5}$$

Here α_d in Eq. (3.3) is generalized for the n-th harmonic to

$$\alpha_{dn} = -\frac{K_n(\nu d)}{I_n(\nu d)} - \xi_{dn}, \tag{4.6}$$

with

$$\xi_{dn} = (1 - j)\frac{\beta^2\gamma^2}{2I_n^2(\nu d)}\frac{\delta_d}{d}. \tag{4.7}$$

We furthermore require that the wire current be consistent with the electric field in each wire, leading to

$$I_w = \pi r_w^2 \sigma E_z(b, z, \theta_p). \tag{4.8}$$

After considerable algebra, assuming $kd \ll \beta\gamma$ as for the cylindrical shell, we obtain the impedance in the form analogous to that of Eq. (3.9):

$$\frac{Z_{\|}(\omega)}{nZ_0} = -\frac{j}{\beta\gamma^2}\left[\ln\frac{b}{a} + \frac{(\xi_d - \ln d/b)(1 - \Delta)}{\eta_w(\ln d/b - \xi_d) - (1 - \Delta)}\right]. \tag{4.9}$$

Here

$$\begin{aligned} \eta_w &= \frac{jNr_w^2}{\beta^2\gamma^2\delta_b^2}, \\ \Delta &= \frac{jr_w^2}{\beta^2\gamma^2\delta_d^2}\ln\left[\frac{b}{Nr_w}\left(1 - \frac{b^{2N}}{d^{2N}}\right)\right]. \end{aligned} \tag{4.10}$$

For $r_w \ll \beta\gamma\delta_d$ we can neglect Δ in Eq. (4.9), provided $N \gg 1$ and $d-b > b/N$, i.e. when the radial extent beyond the wires is larger than the spacing between wires. In that case the condition is fulfilled as the harmonics due to the wire periodicity decay rapidly with $|r-b|$. Furthermore

$$\eta_w = \frac{j}{\pi}\frac{\pi N r_w^2}{\beta^2\gamma^2\delta^2} = \frac{j}{\pi}\frac{A_{\rm wires}}{\beta^2\gamma^2\delta^2} \tag{4.11}$$

and

$$\eta = \frac{j2\tau b}{\beta^2\gamma^2\delta^2} = \frac{j}{\pi}\frac{A_{\rm shell}}{\beta^2\gamma^2\delta^2}, \tag{4.12}$$

allowing us to reach the important conclusion that

only the net area of conductors counts for the penetration of fields in a regular array of wires.

The same result is valid for narrow conducting stripes on a thin ceramic cylinder, which is often the most practical implementation of shielding.

5 Cavity of finite length

In analogy to the analysis without a shield[13], we write an integral equation for the longitudinal electric field E_z in a cavity of length g and of outer radius d, coaxial with an infinite beam pipe of radius b (see Fig.4):

$$\begin{aligned} E_z(r,z) &= A\left[K_0(\nu r) - \frac{K_0(\nu b)}{I_0(\nu b)} I_0(\nu r)\right] \mathrm{e}^{-jkz} \\ &+ \int_{-\infty}^{\infty} dq \mathrm{e}^{-jqz} A(q) \frac{J_0(\kappa r)}{J_0(\kappa b)}\ ,\ r \le b, \end{aligned} \tag{5.1}$$

where the propagation constant is

$$\kappa^2 = k^2 - q^2. \tag{5.2}$$

The term proportional to $I_0(\nu r)$ is included so that the first term in brackets vanishes at $r=b$. Then the term including $A(q)$ has a non-vanishing value only for $0<z<g$, where $E_z(b,z) = f(z)$ is different from zero.
The integration contour in the q-plane is taken above (below) the poles where $J_0(\kappa b) = 0$ on the positive (negative) real q-axis to ensure outgoing waves (generated by the cavity) in the beam pipe. A Fourier transform of Eq. (5.1) at $r=b$ leads to

$$A(q) = \frac{1}{2\pi}\int_0^g dz' f(z') \mathrm{e}^{jqz'}. \tag{5.3}$$

Use of Maxwell's equations then leads to

$$Z_0 H_\theta(b,z) = -\frac{j\beta\gamma A}{\nu b I_0(\nu b)} \mathrm{e}^{jkz} - \frac{jk_0 b}{2\pi} \int_0^g dz' f(z') K_p(z-z'), \qquad (5.4)$$

where the pipe kernel $K_p(\zeta)$ can be written as a sum over the zeros of $J_0(p_s)$:

$$K_p(\zeta) = \frac{2\pi j}{b} \sum_{s=1}^{\infty} \frac{\mathrm{e}^{jb_s|\zeta|/b}}{b_s}. \qquad (5.5)$$

Here

$$b_s = \left(k_0^2 b^2 - p_s^2\right)^{1/2} = -j\left(p_s^2 - k_0^2 b^2\right)^{1/2} = -j\beta_s. \qquad (5.6)$$

We now write the magnetic field in the cavity region $b + \tau < r \leq d$ in terms of $f(z)$, which is the electric field at $r = b$, in the presence of a conducting layer at $r = b$ of thickness $\tau \ll \delta$. Thus

$$Z_0 H_\theta(b+\tau, z) = -\frac{jk_0 b}{2\pi} \int_0^g dz' f(z') K_c(z,z') \qquad (5.7)$$

where the cavity kernel $K_c(z,z')$ is given by

$$K_c(z,z') = 4\pi^2 \sum_\ell \frac{h_\ell(z) h_\ell(z')}{k_0^2 - k_\ell^2}\ , \qquad (5.8)$$

where $k_\ell = \omega_\ell/\beta c$ and $h_\ell(z)$ is the normalized magnetic field at $r = b + \tau$ for the mode ℓ in the annular cavity occupying $b + \tau \leq r \leq d$, $0 < z < g$.

We now require that the discontinuity in H_θ across the thin shield satisfy Eq. (2.14). This leads to the integral equation

$$\int_0^g dz' F(z')[K_p(z-z') + K_c(z,z')] = \mathrm{e}^{-jkz} - \frac{4\pi j\tau}{k^2\delta^2 b} F(z) \qquad (5.9)$$

where

$$f(z) = -\frac{jQZ_0}{k_0 b^2 I_0(\nu b)} F(z). \qquad (5.10)$$

Once Eq. (5.9) is solved for $F(z)$, we obtain the cavity impedance

$$\frac{Z_\parallel^{\mathrm{cav}}(\omega)}{Z_0} = \frac{j}{k_0 b^2} \int_0^g dz F(z) \mathrm{e}^{jkz}\ , \qquad (5.11)$$

when we confine our attention to low frequencies where $k_0 \ll \beta\gamma/b$.

The cavity kernels can be evaluated approximately for the case $k_0 g \ll \beta$ and $k(d-b) \ll \beta$. Then they are independent of z and z', and can be written[14]

$$K_c + K_p \cong \frac{2\pi}{k^2 bg(d-b)} + \frac{2\pi j}{b} \sum_{s=1}^{k_0 b/\pi} \frac{1}{b_s} - \frac{2\pi}{b} \sum_{s=kb/\pi}^{b/\pi g} \frac{1}{\beta_s}. \tag{5.12}$$

One can then solve Eq. (5.9) to obtain the cavity *admittance*

$$Y_{\parallel}^{\text{cav}}(\omega) \cong \frac{2\pi k_0 b}{Z_0} \left[\frac{-j}{k_0^2 g(d-b)} + \sum_{s=1}^{k_0 b/\pi} \frac{1}{b_s} + \sum_{s=k_0 b/\pi}^{b/\pi g} \frac{j}{\beta_s} + \frac{2\tau}{k_0^2 g \delta^2} \right]. \tag{5.13}$$

The second and third terms in the bracket come from the pipe kernel. They are independent of the cavity parameters, except for a weak logarithmic dependence on g. The condition for effective shielding is non-dependence on $d-b$ which becomes

$$\frac{\tau}{\delta} \gg \frac{\delta}{2(d-b)}. \tag{5.14}$$

At low frequencies, the impedance (or admittance) is then dominated by the resistance of the shield of length g, namely,

$$Z_{\parallel}^{\text{cav}}(\omega) \cong \mathcal{R}_{\text{shield}} = \frac{g}{2\pi\sigma b\tau}. \tag{5.15}$$

If one chooses to shield with wires of finite length, one can accomplish this using N wires whose total cross sectional area is equal to the cross sectional area of a continuous layer

$$N\pi r_w^2 = 2\pi b\tau, \tag{5.16}$$

as shown in Section 3. In this case, N must be large and the spacing of the wires must be small compared to $d-b$ to achieve effective shielding.

It has been pointed out[15] that other cavity modes will enter into the cavity kernel at higher frequencies, requiring additional contributions to the first term in brackets of Eq. (5.13), which will be proportional to $(\omega^2 - \omega_m^2)^{-1}$ for a cavity mode with frequency $\omega_m/2\pi$. Therefore the conducting layer cannot shield the cavity when ω is close to ω_m. However, for a realistic beam bunch, there is a spread of frequencies. Then only the average value of $[\omega^2 - \omega_m^2(1+1/Q)]^{-1}$ is important, where Q is the quality factor of the resonance. For $Q \gg 1$, the integral becomes independent of Q. Then we obtain the shielding condition

$$\frac{\tau}{\delta} \gg \frac{g\delta}{(d-b)L_{\text{bunch}}} \tag{5.17}$$

where $L_{\rm bunch}$ is the length of the beam bunch.

As g increases, Eq. (5.17) places an ever increasing lower bound on τ/δ, in disagreement with our prediction for infinite g in Eqs. (3.13) and (3.14). However, in that case we would need to solve the integral equation, Eq. (5.9), for $F(z)$ when g is large. We have not been able to do so analytically, but clearly the solution in Eq. (5.13), which applies to the case $g \ll \beta/k$, is no longer expected to be valid.

We also expect Eq. (5.17) to be valid for screening by N wires. In this case we write it in the form

$$A_{\rm wires} \gg \frac{2\pi b g \delta^2}{(d-b)L_{\rm bunch}} \tag{5.18}$$

where we assume that $N \gg 1$ and that the spacing between wires is small compared with $d-b$. In addition these general principles should also apply to the screening of holes by conducting wires.

6 Shielding of transverse fields

It is possible to repeat the foregoing analysis in order to explore shielding of transverse fields. The general discussion of reflection and transmission coefficients in Section 2 also applies to the transverse case, and should lead to the same condition for effective shielding as in Eq. (2.12). In fact a detailed analysis of shielding the transverse space charge impedance for the infinite, perfectly conducting beam pipe confirms this. Unfortunately, the analysis is made more complicated by the need to consider both TE and TM modes in the beam pipe. We plan to present a more detailed discussion of shielding the transverse impedance by a thin conducting layer or a wire cage of finite conductivity in a future paper.

7 Conclusions

Shielding of electro-magnetic fields by thin conducting layers or thin wires inside a vacuum chamber of finite resistivity have been analyzed for both cases of infinite or finite lengths of the layer. Approximate conditions for effective shielding as well as expressions for the longitudinal impedance were derived. It was found that the analysis could be simplified considerably by assuming that the axial electric field is constant across the thin conducting layer, while the magnetic field changes by an amount equal to the current flowing through it.

For shields of finite length, the fields are given by an integral equation, and an approximate expression for the admittance is given which is considerably simpler than the corresponding impedance. However, the transition from the finite to the infinite case could not be done analytically and requires numerical evaluation of the integral equation under conditions when the simplifying assumptions to not apply.

The shielding effect of a layer much thinner than the skin depth is often puzzling; it can be explained by multiple reflections at both surfaces of the layer, taking into account damping and phase shifting of the radial waves inside of it. For the case of conducting wires or strips, it has been found that only the total area is important for shielding, as long as the distance from the shield to the outer wall is large compared to the distance between wires.

References

[1] G. Dôme, CERN SL/AP Beam Physics Note 2 (Nov. 1998);

[2] A. Schelkunoff, Electromagnetic Waves, Van Nostrand 1943;

[3] IEEE Trans. Electro Magnetic Compatibility, vol.10, March 1968;

[4] A. Piwinski, Trans. IEEE-NS 24 (1977) p. 1364 (PAC 1977);

[5] B. Zotter, Particle Accelerators vol.1 (1970) p.311-326;

[6] T. Wang, AIP Conf. Proc.448 (1998) p.286-297;

[7] F. Caspers et al, Proc. PAC 1999 (New York), p. 1408;

[8] L. Walling et al, Nucl. Instr. Meth. A-281 (1989) p.433-447;

[9] D. Brandt et al, CERN LHC Project Report 403 (2000);

[10] I. Abramowitz, I. Stegun, Tables of Functions, Dover 1963;

[11] S. Wolfram, Mathematica Book, Cambridge Univ. Press 1996.

[12] I. Gradshteyn, I. Ryzhik, Table of Integrals, Acad. Press 1965;

[13] R. Gluckstern, Phys. Rev. D 39, 2773 (1989);

[14] R. Gluckstern and F. Neri, Linac Conference 1988, p. 359;

[15] G. Dôme, private communication.

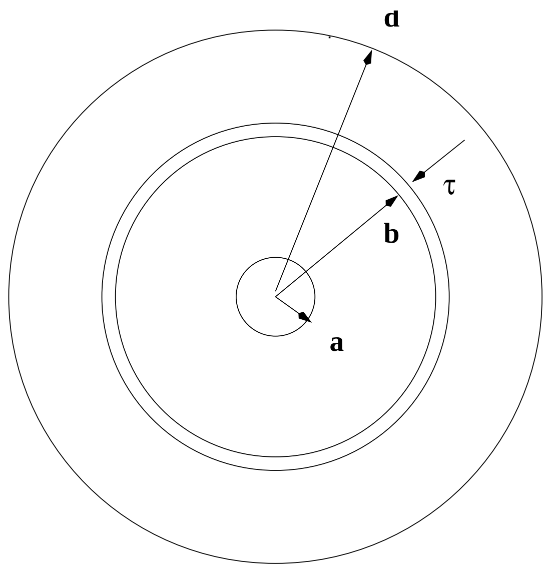

Figure 1: Geometry of a screen in a circular cylindrical vacuum chamber

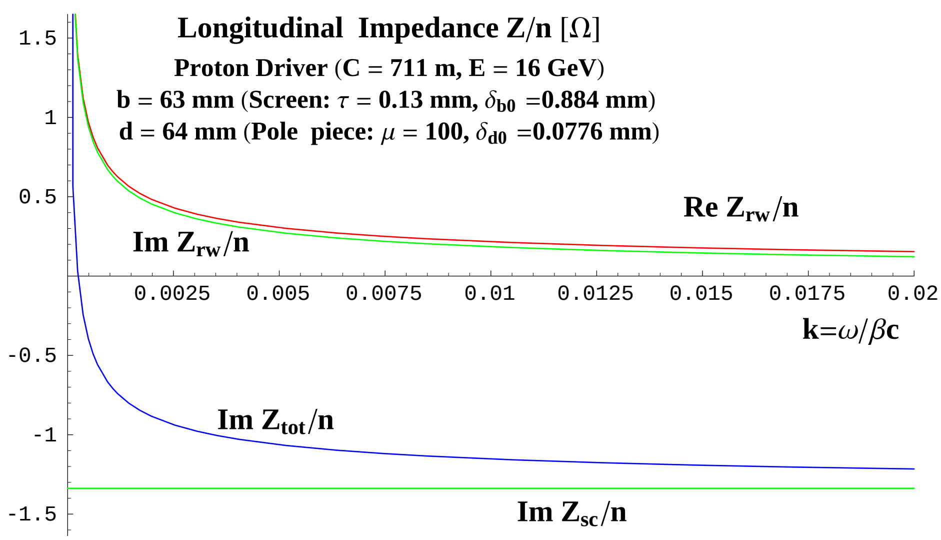

Figure 2: Longitudinal Impedance calculated with Eq.(3.9).

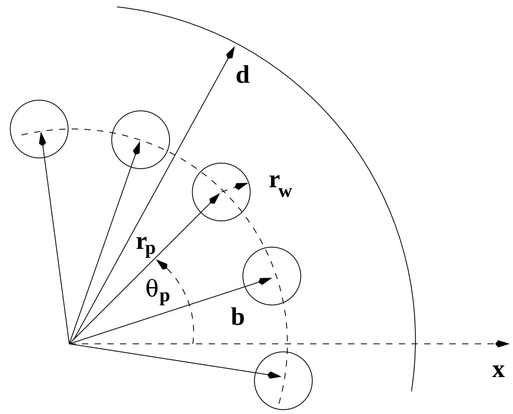

Figure 3: Geometry of wire cage

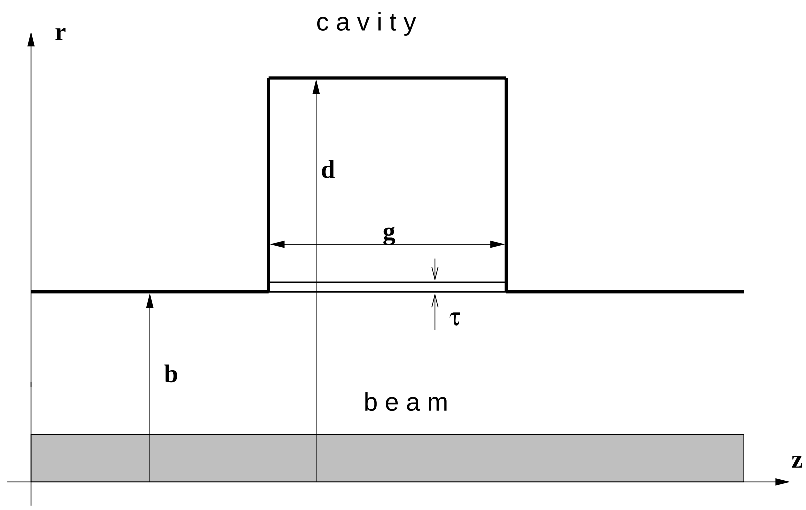

Figure 4: Cavity of finite length