Supersymmetry and the Anomalous Anomalous Magnetic Moment of the Muon

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The recently reported measurement of the muon's anomalous magnetic moment differs from the standard model prediction by 2.6σ. We examine the implications of this discrepancy for supersymmetry. Deviations of the reported magnitude are generic in supersymmetric theories. Based on the new result, we derive model-independent upper bounds on the masses of observable supersymmetric particles. We also examine several model frameworks. The sign of the reported deviation is as predicted in many simple models, but disfavors anomaly-mediated supersymmetry breaking.

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Measurements of spin magnetic dipole moments have a rich history as harbingers of profound progress in particle physics. In the leptonic sector, the electron's gyromagnetic ratio $g_e \approx 2$ pointed the way toward Dirac's theory of the electron. Later, the electron's anomalous magnetic moment $a_e \equiv (g_e - 2)/2 \approx \alpha/2\pi$ played an important role in the development of quantum electrodynamics and renormalization. Since then, increasingly precise measurements have become sensitive both to very high order effects in quantum electrodynamics and to hadronic processes, and the consistency of experiment and theory has stringently tested these sectors of the standard model.

Very recently, the Muon $(g-2)$ Collaboration has reported a measurement of the muon's anomalous magnetic moment, which, for the first time, is sensitive to contributions comparable to those of the weak interactions [1]. (See Tables I and II.) The new Brookhaven E821 result is

$$a_\mu^{\rm exp} = 11\ 659\ 202\,(14)\,(6) \times 10^{-10}\ (1.3\ {\rm ppm})\ , \quad (1)$$

where the first uncertainty is statistical and the second systematic. Combining experimental and theoretical uncertainties in quadrature, the new world average differs from the standard model prediction by 2.6σ [1]:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (42 \pm 16) \times 10^{-10}\ . \quad (2)$$

Although of unprecedented precision, the new result is based on a well-tested method used in several previous measurements. Polarized positive muons are circulated in a uniform magnetic field. They then decay to positrons, which are emitted preferentially in the direction of the muon's spin. By analyzing the number of energetic positrons detected at positions around the storage ring, the muon's spin precession frequency and anomalous magnetic moment are determined. The new result is based solely on 1999 data. Analysis of the 2000 data is underway, with an expected error of $\sim 7 \times 10^{-10}$ (0.6 ppm), and the final goal is an uncertainty of 4×10^{-10} (0.35 ppm) [6].

The standard model prediction has been greatly refined in recent years. The current theoretical status is reviewed in Ref. [5] and summarized in Table II. The uncertainty is dominated by the hadronic vacuum polarization contribution that enters at 2-loops. This is expected to be reduced by recent measurements of $\sigma(e^+e^- \to$ hadrons) at center-of-mass energies $\sqrt{s} \sim 1$ GeV. Thus, although the statistical significance of the present deviation leaves open the possibility of agreement between experiment and the standard model, the prospects for a definitive resolution are bright. If the current deviation remains after close scrutiny and the expected improvements, the anomalous value of a_μ will become unambiguous.

In this study, we consider the recent measurement of a_μ to be a signal of physics beyond the standard model. In particular, we consider its implications for supersymmetric theories. Supersymmetry is motivated by many independent considerations, ranging from the gauge hierarchy problem to the unification of gauge couplings to the necessity of non-baryonic dark matter, all of which require supersymmetric particles to have weak scale masses. Deviations in a_μ with the reported magnitude are therefore generic in supersymmetry, as we will see. In addition, in contrast to other low-energy probes, a_μ is both flavor- and CP-conserving. Thus, while the impact of supersymmetry on other observables can be highly suppressed by scalar degeneracy or small CP-violating phases in simple models, supersymmetric contributions to a_μ cannot be. In this sense, the anomalous magnetic moment of the muon is a uniquely robust probe of supersymmetry, and an anomaly in a_μ is a natural place for the effects of supersymmetry to appear.

The anomalous magnetic moment of the muon is the coefficient of the operator $a_\mu \frac{e}{4m_\mu}\, \bar{\mu}\sigma^{mn}\mu\, F_{mn}$, where $\sigma^{mn} = \frac{i}{2}\,[\gamma^m, \gamma^n]$. The supersymmetric contribution, $a_\mu^{\rm SUSY}$, is dominated by well-known neutralino-smuon and chargino-sneutrino diagrams [7]. In the absence of significant slepton flavor violation, these diagrams are completely determined by only seven supersymme-

TABLE I. Recent measurements of $a_\mu \times 10^{10}$ and the cumulative world average.

Data Set	Result		World Average
CERN77 [2]	11 659 230 (85)	(7 ppm)	—
BNL97 [3]	11 659 250 (150)	(13 ppm)	11 659 235 (73)
BNL98 [4]	11 659 191 (59)	(5 ppm)	11 659 205 (46)
BNL99 [1]	11 659 202 (14) (6)	(1.3 ppm)	11 659 203 (15)

TABLE II. Contributions to the standard model prediction for $a_\mu \times 10^{10}$. (See Ref. [5] and references therein.)

Standard model source	Contribution
QED (up to 5-loops)	11 658 470.6 (0.3)
Hadronic vac. polarization (2-loop piece)	692.4 (6.2)
Hadronic vac. polarization (3-loop piece)	−10.0 (0.6)
Hadronic light-by-light	−8.5 (2.5)
Weak interactions (up to 2-loops)	15.2 (0.4)
Total	11 659 159.7 (6.7)

try parameters: M_1, M_2, μ, $\tan\beta$, $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$, and A_μ. The first four enter through the chargino and neutralino masses: M_1, M_2, and μ are the U(1) gaugino, SU(2) gaugino, and Higgsino mass parameters, respectively, and $\tan\beta = \langle H_u^0\rangle / \langle H_d^0\rangle$ governs gaugino-Higgsino mixing. The last five determine the slepton masses, where $m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_R}$ are the SU(2) doublet and singlet slepton masses, respectively, and the combination $m_\mu(A_\mu - \mu\tan\beta)$ mixes left- and right-handed smuons. Our sign conventions are as in Ref. [8].

The qualitative features of the supersymmetric contributions are most transparent in the mass insertion approximation. The structure of the magnetic dipole moment operator requires a left-right transition along the lepton-slepton line. In the interaction basis, this transition may occur through a mass insertion in an external muon line, at a Higgsino vertex, or through a left-right mass insertion in the smuon propagator. The last two contributions are proportional to the muon Yukawa coupling and so may be enhanced by $\tan\beta$. For large and moderate $\tan\beta$, it is not hard to show that the supersymmetric contributions in the mass insertion approximation are all of the form

$$\frac{g_i^2}{16\pi^2} m_\mu^2 \, \mu M_i \tan\beta \, F \ , \tag{3}$$

where $i = 1, 2$, and F is a function of superparticle masses, with $F \propto M_{\rm SUSY}^{-4}$ in the large mass limit [9].

Equation (3) implies $a_\mu^{\rm SUSY}/a_e^{\rm SUSY} \sim m_\mu^2/m_e^2 \approx 4 \times 10^4$; a_μ is therefore far more sensitive to supersymmetric effects than a_e, despite the fact that the latter is 350 times better measured. Also, for $M_2/M_1 > 0$, although the contributions of Eq. (3) may destructively interfere, typically $\text{sign}(a_\mu^{\rm SUSY}) = \text{sign}(\mu M_{1,2})$; we have found exceptions only rarely in highly model-independent scans. Finally, the parameter $\tan\beta$ is expected to be in the range $2.5 \lesssim \tan\beta \lesssim 50$, where the lower limit is from Higgs boson searches, and the upper limit follows from requiring a perturbative bottom quark Yukawa coupling up to $\sim 10^{16}$ GeV. Supersymmetric contributions may therefore be greatly enhanced by large $\tan\beta$.

To determine the possible values of $a_\mu^{\rm SUSY}$ without model-dependent biases, we have calculated $a_\mu^{\rm SUSY}$ in a series of high statistics scans of parameter space. We use exact mass eigenstate expressions for $a_\mu^{\rm SUSY}$. Our calculations agree with Refs. [9–11] and cancel the corresponding standard model diagrams in the supersymmetric limit [12]. We require chargino masses above 103 GeV and smuon masses above 95 GeV [13]. We also assume that the lightest supersymmetric particle (LSP) is stable, as in gravity-mediated theories, and require it to be neutral. Finally, we record the mass and identity of the lightest observable supersymmetric particle (LOSP) for each scan point. Given the assumption of a stable LSP, the LOSP is the 2nd lightest supersymmetric particle, or the 3rd if the two lightest are a neutralino and the sneutrino.

We begin by scanning over the parameters M_2, μ, $m_{\tilde{\mu}_L}$, and $m_{\tilde{\mu}_R}$, assuming gaugino mass unification $M_1 = M_2/2$, $A_\mu = 0$, and $\tan\beta = 50$. The free parameters take values up to 2.5 TeV. The resulting values in the $(M_{\rm LOSP}, a_\mu^{\rm SUSY})$ plane are given by the points in Fig. 1. We then relax the gaugino mass relation and consider both positive and negative values of M_2/M_1. The resulting values are bounded by the solid curve. As can be seen, and as verified by high statistics sampling targeting the border area, the assumption of gaugino mass unification has no appreciable impact on the envelope curve. Finally, we allow any A_μ in the interval $[-100 \text{ TeV}, 100 \text{ TeV}]$. The resulting sample is extremely model-independent, and is bounded by the dashed contour of Fig. 1. Note that the envelope contours scale linearly with $\tan\beta$ to excellent approximation.

From Fig. 1 we see that the measured deviation in a_μ is in the range accessible to supersymmetric theories and is easily explained by supersymmetric effects.

The anomaly in a_μ also has strong implications for the superpartner spectrum. Among the most important is that at least two superpartners cannot decouple if supersymmetry is to explain the deviation, and one of these must be charged and so observable at colliders. This is reflected in the upper bound on $M_{\rm LOSP}$ for non-vanishing $a_\mu^{\rm SUSY}$. The large value of $\tan\beta$ is chosen to allow the largest possible $M_{\rm LOSP}$. The solid contour is parametrized by

$$\frac{a_\mu^{\rm SUSY}}{42 \times 10^{-10}} = \frac{\tan\beta}{50} \left(\frac{390 \text{ GeV}}{M_{\rm LOSP}^{\rm max}} \right)^2 \ . \tag{4}$$

If $a_\mu^{\rm SUSY}$ is required to be within 1σ (2σ) of the measured deviation, at least one observable superpartner must be lighter than 490 GeV (800 GeV).

In Fig. 2 we repeat the above analysis, but for the

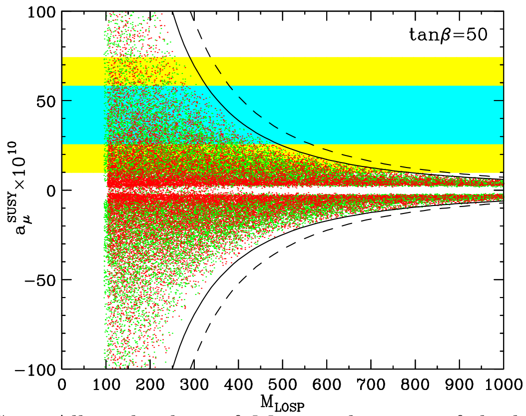

FIG. 1. Allowed values of $M_{\rm LOSP}$, the mass of the lightest observable supersymmetric particle, and $a_\mu^{\rm SUSY}$ from a scan of parameter space with $M_2 = 2M_1$, $A_\mu = 0$, and $\tan\beta = 50$. Green (red) points have smuons (charginos/neutralinos) as the LOSP. The 1σ and 2σ allowed $a_\mu^{\rm SUSY}$ ranges are indicated. The allowed $a_\mu^{\rm SUSY}$ scale linearly with $\tan\beta$. Relaxing the relation $M_2 = 2M_1$ leads to the solid envelope curve, and further allowing arbitrary A_μ leads to the dashed curve. A stable LSP is assumed.

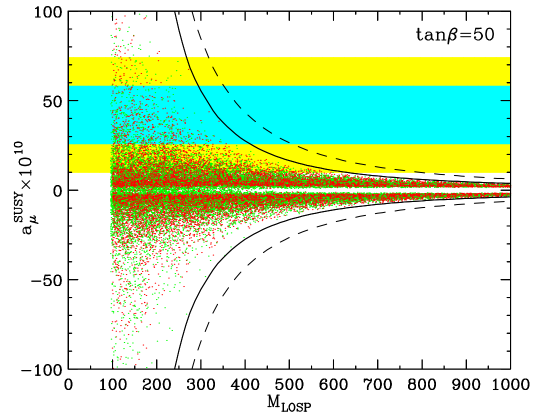

FIG. 2. As in Fig. 1, but assuming a visibly decaying LSP.

case where the LSP decays visibly in collider detectors, as in models with low-scale supersymmetry breaking or R-parity violating interactions. In this case, the LOSP is the LSP. We relax the requirement of a neutral LSP, and require slepton masses above 95 GeV and neutralino masses above 99 GeV [13]. The results are given in Fig. 2. For this case, the solid envelope curve is parametrized by

$$\frac{a_\mu^{\rm SUSY}}{42\times10^{-10}} = \frac{\tan\beta}{50}\left[\left(\frac{300\ {\rm GeV}}{M_{\rm LOSP}^{\rm max}}\right)^2 + \left(\frac{230\ {\rm GeV}}{M_{\rm LOSP}^{\rm max}}\right)^4\right] , \quad (5)$$

and the 1σ (2σ) bound is $M_{\rm LOSP} < 410$ GeV (640 GeV).

These model-independent upper bounds have many implications. They improve the prospects for observation of weakly-interacting superpartners at the Tevatron and LHC. They also impact linear colliders, where the study of supersymmetry requires $\sqrt{s} > 2M_{\rm LOSP}$ (with the possible exception of associated neutralino production in stable LSP scenarios). Finally, these bounds provide fresh impetus for searches for lepton flavor violation, which is also mediated by sleptons, charginos, and neutralinos.

We now turn to specific models. The supersymmetric contributions to a_μ have been discussed in various supergravity theories [7], and more recently in models of gauge-mediated [10,14] and anomaly-mediated supersymmetry breaking [8,15].

We first consider the framework of minimal supergravity, in which the entire weak scale superparticle spectrum is fixed by four continuous parameters and one binary choice: m_0, $M_{1/2}$, A_0, $\tan\beta$, and ${\rm sign}\,(\mu)$, where the first three are the universal scalar, gaugino, and trilinear coupling masses at the grand unified theory (GUT) scale $M_{\rm GUT} \simeq 2\times10^{16}$ GeV. We relate these to weak scale parameters through two-loop renormalization group equations [16] with one-loop threshold corrections and calculate all superpartner masses to one-loop [17]. Electroweak symmetry is broken radiatively with a full one-loop analysis, which determines $|\mu|$.

In minimal supergravity, many potential low-energy effects are eliminated by scalar degeneracy. However, $a_\mu^{\rm SUSY}$ is not suppressed in this way and may be large. In this framework, ${\rm sign}\,(a_\mu^{\rm SUSY}) = {\rm sign}\,(\mu M_{1,2})$. As is well-known, however, the sign of μ also enters in the supersymmetric contributions to $B \to X_s\gamma$. Current constraints on $B \to X_s\gamma$ require $\mu M_3 > 0$ if $\tan\beta$ is large. In minimal supergravity, then, gaugino mass unification implies that a large discrepancy in a_μ is only possible for $a_\mu^{\rm SUSY} > 0$, in accord with the new measurement.

In Fig. 3, the 2σ allowed region for $a_\mu^{\rm SUSY}$ is plotted for $\mu > 0$. Several important constraints are also included: bounds on the neutralino relic density, the Higgs boson mass limit $m_h > 113.5$ GeV, and the 2σ constraint $2.18\times10^{-4} < B(B\to X_s\gamma) < 4.10\times10^{-4}$.

For moderate $\tan\beta$, the region preferred by $a_\mu^{\rm SUSY}$ is at low m_0. This region is consistent with the requirement of supersymmetric dark matter, and, intriguingly, is roughly that obtained in no-scale supergravity [18] and minimal gaugino-mediated [19] models. For large $\tan\beta$, the preferred area extends to large $M_{1/2}$ and $m_0 > 1$ TeV. Again there is significant overlap with a region with desirable relic density. The cosmologically preferred regions of minimal supergravity are probed by many pre-LHC experiments [20]. Here we note only that the sign of μ preferred by a_μ implies destructive interference in the leptonic decays of the second lightest neutralino, and so the Tevatron search for trileptons is ineffective for 200 GeV $< m_0 <$ 400 GeV and moderate $\tan\beta$ [21].

We close by considering the framework of anomaly-mediated supersymmetry breaking [22]. One of the most robust and striking predictions of this framework is that the gaugino masses are proportional to the corresponding beta function coefficients, and so $M_{1,2}M_3 < 0$. Consistency with the $B \to X_s\gamma$ constraint then implies that only negative $a_\mu^{\rm SUSY}$ may have large magnitude, in

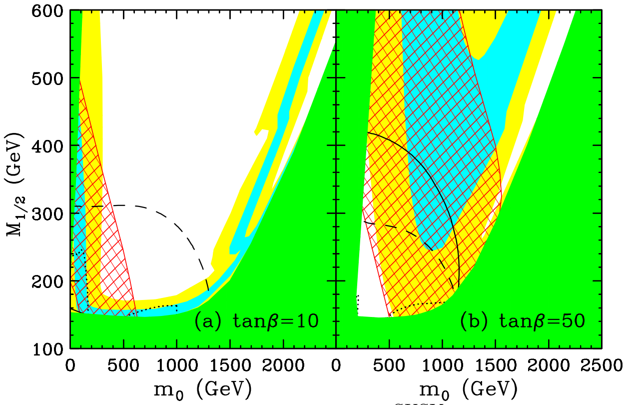

FIG. 3. The 2σ allowed region for $a_\mu^{\rm SUSY}$ (hatched) in minimal supergravity, for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan\beta$. The green regions are excluded by the requirement of a neutral LSP (left) and by the chargino mass limit of 103 GeV (bottom and right), and the blue (yellow) region has LSP relic density $0.1 \le \Omega h^2 \le 0.3$ ($0.025 \le \Omega h^2 \le 1$). The area below the solid (dashed) contour is excluded by $B \to X_s\gamma$ (the Higgs boson mass), and the regions probed by the tri-lepton search at Tevatron Run II are below the dotted black contours. (See text.)

contrast to the case of conventional supergravity theories [8,15].

In Fig. 4 we investigate how large a positive $a_\mu^{\rm SUSY}$ may be in the minimal anomaly-mediated model. This model is parametrized by $M_{\rm aux}$, m_0, $\tan\beta$, and sign(μ), where $M_{\rm aux}$ determines the scale of the anomaly-mediated soft terms, and m_0 is a universal scalar mass introduced to remove tachyonic sleptons. To get $a_\mu^{\rm SUSY} > 0$, we choose $\mu M_{1,2} > 0$. We see, however, that the constraint from $B \to X_s\gamma$ is severe, as this sign of μ implies a constructive contribution from charginos to $B \to X_s\gamma$ in anomaly mediation. Even allowing a 1σ deviation in a_μ, we have checked that for all $\tan\beta$, it is barely possible to obtain 2σ consistency with the $B \to X_s\gamma$ constraint, and minimal anomaly mediation is therefore disfavored. The dependence of this argument on the characteristic gaugino mass relations of anomaly mediation suggests that similar conclusions will remain valid beyond the minimal model.

In conclusion, the recently reported deviation in a_μ is easily accommodated in supersymmetric models. Its value provides *model-independent* upper bounds on masses of observable superpartners and already serves to discriminate between well-motivated models. We await the expected improved measurements with great anticipation.

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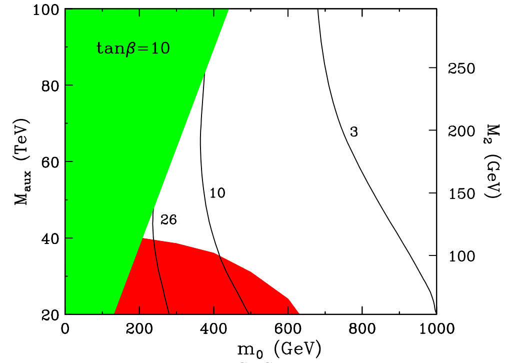

FIG. 4. Contours of $a_\mu^{\rm SUSY} \times 10^{10}$ in the minimal anomaly-mediated model, for $\tan\beta = 10$ and $\mu > 0$. The green region is excluded by $m_{\tilde\tau} > 72$ GeV, and the red region is excluded at 2σ by $B \to X_s\gamma < 4.10 \times 10^{-4}$.

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