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RESUMMED COEFFICIENT FUNCTION FOR THE SHAPE FUNCTION

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Abstract

We present a leading evaluation of the resummed coefficient function for the shape function. It is also shown that the coefficient function is short-distance-dominated. Our results allow relating the shape function computed on the lattice to the physical QCD distributions.

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1 Introduction

In this note we present a leading evaluation of the coefficient function of the shape function [?]–[?]. The latter is also called structure function of the heavy flavours. The coefficient function allows relating the shape function — computed with a non-perturbative technique such as lattice QCD — to distributions in semi-inclusive heavy-flavour decays. We consider in particular the processes

$$B \to X_s + \gamma \tag{1}$$

and

$$B \to X_u + l + \nu \tag{2}$$

in the hard limit

$$Q \gg \Lambda. \tag{3}$$

The quantity Q is the hard scale of these time-like processes: $Q \equiv 2\mathcal{E}$, where $\mathcal{E}$ is the hadronic energy of the state X and Λ is the QCD scale[1]. For a B-meson at rest, $v = (1; 0, 0, 0)$, and the jet X flying along the minus direction ($-z$ axis), the shape function is defined as

$$\varphi\left(k_+\right) \equiv \langle B\left(v\right)|h_v^{\dagger}\delta\left(k_+ - iD_+\right)h_v|B\left(v\right)\rangle. \tag{4}$$

This represents the probability that the b-quark in the B-meson has momenta

$$p_b = m_B v + k' \tag{5}$$

with any transverse and minus components and with given plus component

$$k'_+ = k_+. \tag{6}$$

The static field $h_v\left(x\right)$ is related to the Dirac field of the beauty quark $b\left(x\right)$ by

$$b\left(x\right) = e^{-im_B v\cdot x}h_v\left(x\right) + O\left(1 m_B\right). \tag{7}$$

With the shape function, the decays (1) and (2) are related to their respective quark-level processes

$$b \to \widehat{X}_s + \gamma \tag{8}$$

and

$$b \to \widehat{X}_u + l + \nu, \tag{9}$$

[1]For the rare decay (1), one can actually set $Q = m_B$.

where the b-quark has the momentum (5) with the distribution (4). It can be shown that the invariant mass m of the state $\widehat{X}$ is related to the virtuality of the b-quark by

$$k_+ = -m^2 Q. \tag{10}$$

The shape function is a non-perturbative distribution — analogous to parton distribution functions — and it describes the slice of the semi-inclusive region[2] in which

$$m^2 \sim \Lambda\, Q,$$

i.e.

$$|k_+| \sim \Lambda, \qquad \text{or} \qquad z \sim 1 - \Lambda Q, \tag{11}$$

where

$$z \equiv 1 - m^2 Q^2. \tag{12}$$

Because of ultraviolet divergences affecting its matrix elements, the shape function has a dependence on the ultraviolet cut-off or renormalization point μ :

$$\varphi\left(k_+\right) = \varphi\left(k_+;\mu\right), \tag{13}$$

and is related to a physical QCD distribution by means of a coefficient function by (cf. eq. (**??**))

$$\varphi\left(k_+;Q\right) = \int dk'_+ \, C\left(k_+ - k'_+;Q,\mu\right)\, \varphi\left(k'_+;\mu\right). \tag{14}$$

The QCD distribution does not depend on μ :

$$dd\mu\varphi\left(k_+;Q\right) = 0, \tag{15}$$

while the shape function does not depend on Q. As a consequence, the coefficient function depends on both Q and μ.

The shape function can be computed with a non-perturbative technique or extracted from experimental data. If it is computed inside a field-theory model — such as lattice QCD — its expression will exhibit a μ-dependence that cancels against that of the coefficient function. If it is instead computed inside a phenomenological model — such as a quark model — the situation is less transparent. The μ-independence is not "automatic" and one has to specify the value of μ appropriate for the model. Some care is needed also in extracting the shape function from the experimental data, in order to avoid double counting of perturbative corrections. A factorization scheme must be defined and the coefficient functions for the various processes all have to be computed in the same scheme[3]. In particular, if a branching MonteCarlo is used for the analysis, the perturbative corrections generated by the program must be subtracted.

The coefficient function is obtained by evaluating in leading approximation both φ and φ and inserting their expression in eq. (14). Since the coefficient function is expected to be a short-distance quantity, we compute the QCD distribution and the shape function in PT for an on-shell b-quark ($k' = 0$). This expectation will be verified *a posteriori.*

[2] This region is also called threshold region, large-x region, radiation-inhibited region and Sudakov region.

[3] The situation is analogous to usual hard processes, where various factorization schemes for the parton distribution functions are defined: DIS, $\overline{\text{MS}}$, etc.

2 The QCD distribution

The (perturbative) long-distance effects occurring in (1) and (2) can be factorized in the function

$$\mathbf{f}\left(z\right)=\delta\left(1-z-0\right)-A_{1}\alpha_{S}\left(\log\left[1-z\right]1-z\right)_{+}, \tag{16}$$

where

$$A_{1}=C_{F}\pi \tag{17}$$

and $C_F=\left(N_c^2-1\right)/\left(2N_c\right)=4/3$. The plus-distribution is defined as usual as

$$P\left(z\right)_{+}\equiv P\left(z\right)\;-\;\delta\left(1-z-0\right)\int_{0}^{1}dy\,P\left(y\right). \tag{18}$$

The integrated or cumulative distribution is defined as

$$\mathbf{F}\left(z\right)\equiv\int_{z}^{1}dz'\,\mathbf{f}\left(z'\right). \tag{19}$$

Inserting expression (16) in this, one obtains the well-known double logarithm:

$$\mathbf{F}\left(z\right)=1-A_{1}\alpha_{S}2\log^{2}\left(1-z\right). \tag{20}$$

The cumulative distribution satisfies the normalization condition $\mathbf{F}\left(0\right)=1$. Multiple soft-gluon emission exponentiates the one-loop distribution, so that

$$\mathbf{F}\left(z\right)=\exp\left[-A_{1}\alpha_{S}2\log^{2}\left(1-z\right)\right]. \tag{21}$$

For further improvement, it is convenient to write the function $\mathbf{f}\left(z\right)$ in an "unintegrated" form, as

$$\mathbf{f}\left(z\right)=\delta\left(1-z\right)+A_{1}\alpha_{S}\int_{0}^{1}d\epsilon\epsilon\int_{0}^{1}dtt\left[\delta\left(1-z-\epsilon t\right)-\delta\left(1-z\right)\right], \tag{22}$$

where we have defined the unitary energy and angular variables

$$\epsilon\equiv EQ\qquad\text{and}\qquad t\equiv1-\cos\theta2. \tag{23}$$

The quantity E is two times the energy of the soft gluon, $E=2E_g$, and θ is the emission angle. Leading logarithmic corrections are included replacing the bare coupling with the running coupling evaluated at the gluon transverse momentum squared [**?**]:

$$\alpha_{S}\rightarrow\alpha_{S}\left(l_{\perp}^{2}\right), \tag{24}$$