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M. Kachelrieß¹, E. Resconi² and S. Schönert^{3,4}

¹ TH Division, CERN, CH-1211 Geneva 23

²Dipartimento di Fisica, Universitá di Genova, Via Dodecaneso, 33, I–16146 Genova

^{3*}Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D–69117 Heidelberg

⁴Research Center for Cosmic Neutrinos, Institute for Cosmic Ray Research, University of Tokyo

Discussions of lepton mixing and oscillations consider generally only flavor oscillations of neutrinos and neglect the accompanying charged leptons. In cases of experimental interest like pion or nuclear beta decay an oscillation pattern is expected indeed only for neutrinos if only one of the two produced particles is observed. We argue that flavor oscillations of neutrinos without detecting the accompanying lepton is a peculiarity of the two-particle states $|l\nu\rangle$ produced in pion or nuclear beta decay. Generally, an oscillation pattern is only found if both particles are detected. We discuss in a pedagogical way how this distinction of the neutrinos arises, although on the level of the Lagrangian lepton mixing does not single them out against charged leptons. As examples, we discuss the difference between the state $|l\nu\rangle$ produced by the decay of real W boson and a W originating from pion decay.

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I. INTRODUCTION

The Standard Model (SM) of particle physics has proven to be a firm basis on which all our knowledge of this field rests since its construction 30 years ago [1]. Precision tests performed in the last decade demonstrated in particular that it is also correct at the quantum level. Novel phenomena such as neutrino masses or supersymmetric particles, which cannot be accommodated within the SM, should not be thought to contradict it, but rather to guide us as to new physics beyond it. Nowadays, this hunt for new phenomena is the main topic in particle physics. In contrast to the search for supersymmetry, for which there is no positive signal so far¹, there is mounting experimental evidence for neutrino oscillations. On the one hand, there are five solar neutrino experiments using different techniques that see a deficit in the solar neutrino flux [4]. Although this deficit could have its origin in principle also in non-standard solar- or nuclear physics, it can be shown that these explanations are experimentally excluded [5]. On the other hand, the case for an oscillation solution to the atmospheric neutrino deficit has become recently even stronger in the general perception. This is largely because both the zenith-angle distribution and the dependence of the ratio ν_e/ν_μ as function of the ratio (oscillation length)/(neutrino energy) found by the Superkamiokande collaboration support the neutrino oscillation hypothesis [6].

Most aspects of neutrino oscillations have been dis-

cussed extensively in the literature. The usual derivation, presented e.g. in Ref. [7], of the probability P that a relativistic neutrino with momentum p and flavor α has the flavor β after the time t,

$$P_{\nu_{\alpha} \to \nu_{\beta}} = \left| \sum_{l} U_{\alpha l}^{(\nu)} U_{\beta l}^{(\nu)*} \exp\left(-\mathrm{i}E_{l}t\right) \right|^{2}, \tag{1}$$

uses only basic facts of quantum mechanics. Here, $E_l = \sqrt{m_l^2 + p^2}$ is the energy of the neutrino mass eigenstate l and the matrix U is commonly chosen to represent the unitary transformation matrix between weak and mass eigenstates of the neutrino. In spite (or, perhaps, because) of the simplicity of its derivation, Eq. (1) raises several conceptual questions. The most prominent ones are if for the neutrino mass eigenstates a definite energy or momentum should be used, under which conditions the use of wave packets with smeared energy and/or momentum is necessary, the problem of coherence, and the connection between the quantum mechanical treatment and quantum field theory [8].

In this article we want to discuss a more basic question, namely why mixing in the lepton sector reveals itself experimentally in flavor oscillations of neutrinos but not of charged leptons. This question is motivated by the following simple fact: At the level of the Lagrangian describing the charged-current interactions of leptons,

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{i,j} \bar{l}_{L,i} \gamma^{\mu} V_{ij} \nu_{L,j} W_{\mu}^{-} + \text{h.c.}, \qquad (2)$$

^{*}Permanent address

¹We should however mention that the annual modulation signal seen by the DAMA experiment can be consistently explained as the scattering of supersymmetric dark matter particles on their target nucleons [2]. Also the experimental evidence for a light Higgs found at LEPII points towards low-energy supersymmetry [3].

the mixing $V=U^{(\mathrm{ch})\dagger}U^{(\nu)}$ between charged leptons and neutrinos has two different sources: It could be ascribed either completely to mixing in the neutrino $(V=U^{(\nu)})$ or in the charged lepton sector $(V=U^{(\mathrm{ch})\dagger})$, or most probably to some superposition of both. Physical results do not depend on the particular decomposition of V in the Standard Model with lepton mixing². Thus, knowing only the charged-current Lagrangian, one would not expect any fundamental difference between neutrinos and charged leptons in oscillation experiments. Main purpose of this article is to clarify the exact reason for this difference.

Another formulation of this question is to ask for which conditions it is allowed to neglect the charged lepton produced together with the neutrino in a two-particle state, e.g., in the decay of a pion or a real W boson. We will show that there is a crucial difference between these two cases. While in the first one the charged lepton plays only a "spectator rôle" and can be neglected to a good approximation, in the second case the two-particle state has to be considered.

Furthermore, we shall address the question whether charged leptons can show an oscillation pattern. Usually it is argued that their oscillation frequencies are too high to be observable and moreover, that the coherence of wave packets corresponding to different mass eigenstates is lost under experimental conditions.

In general, an oscillation pattern is observed if 1) the distance between the source and the detector is smaller than the coherence length $l_{\rm coh}$ and 2) the size of the source and of the detector are smaller than the oscillation length $l_{\rm osc}$. In analogy to neutrino oscillations [9], the coherence length for charged leptons can be written as

$$l_{\rm coh} = \sigma \, \frac{2E^2}{\Delta m^2} \,, \tag{3}$$

with the width of the wave packet σ , the energy E and mass difference $\Delta m^2 = m_i^2 - m_j^2$ of the charged leptons. As an example we estimate $l_{\rm coh}$ for pion decay in flight with an energy of $E_\pi \approx 40$ GeV. Using $\sigma \approx \gamma \tau$ and the pion half life $\tau = 2.6 \times 10^{-8}$ s one obtains $l_{\rm coh} = \mathcal{O}(10^8 \,\mathrm{m})$. The oscillation length $l_{\rm osc} = 4\pi E/\Delta m^2$ would amount to $\mathcal{O}(10^{-11} \,\mathrm{m})$. Consequently, the coherence condition could be met experimentally while the oscillation pattern would be smeared out. If this were the complete argument, one still could hope to derive information about V measuring only charged leptons. We will show however, that it is necessary to measure the accompanying neutrino simultaneously in order to obtain any information on V.

II. MIXING IN THE LEPTON SECTOR

Let us first recall how fermion mass matrices are diagonalized. Generally, the mass terms in the Lagrangian are given by

$$\mathcal{L}_{\text{mass}} = -\sum_{\alpha,\beta} \bar{\nu}_{L,\alpha} M_{\alpha\beta}^{(\nu)} \nu_{R,\beta} - \sum_{\alpha,\beta} \bar{l}_{L,\alpha} M_{\alpha\beta}^{(\text{ch})} l_{R,\beta} + \text{h.c.},$$
(4)

and the mass matrices $M_{\alpha\beta}$ are not hermitian or even diagonal in the basis of the weak eigenstates. (We denote the weak eigenstates $\nu_{\alpha} = \{\nu_e, \nu_{\mu}, \nu_{\tau}\}$ and $l_{\alpha} = \{e, \mu, \tau\}$ by greek indices $\alpha, \beta \dots$, and the mass eigenstates by latin indices i, j, \dots Furthermore, we have assumed that the neutrinos have only Dirac mass terms to simplify the formulas). Since the mass matrices are not hermitian, they cannot be diagonalized by a simple unitary transformation. However, arbitrary mass matrices can be diagonalized by a biunitary transformation [10],

$$M_{\text{diag}}^{(\nu)} = U^{(\nu)\dagger} M^{(\nu)} T^{(\nu)} ,$$
 (5)

where $U^{(\nu)}U^{(\nu)\dagger} = T^{(\nu)}T^{(\nu)\dagger} = 1$. Then, the connection between weak and mass eigenstates is given by

$$\nu_{R,\alpha} = \sum_{i} T_{\alpha i}^{(\nu)} \nu_{R,i} , \qquad \nu_{L,\alpha} = \sum_{i} U_{\alpha i}^{(\nu)} \nu_{L,i} , \quad (6)$$

and similar equations hold for the charged leptons.

Inserting the transformations of Eq. (6) into the charged current Lagrangian of the SM,

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{\alpha} \bar{l}_{L,\alpha} \gamma^{\mu} \nu_{L,\alpha} W_{\mu}^{-} + \text{h.c.}, \qquad (7)$$

results in

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{i,j} \bar{l}_{L,i} \gamma^{\mu} V_{ij} \nu_{L,j} W_{\mu}^{-} + \text{h.c.},$$
 (8)

where we introduced the analogue of the CKM matrix in the lepton sector [11], $V = U^{(\mathrm{ch})\dagger}U^{(\nu)}$. Since the charged current interaction involves only left-chiral fields of both charged leptons and neutrinos, the product of the two mixing matrices of the right-handed leptons, $T^{(\mathrm{ch})\dagger}T^{(\nu)}$, is unobservable.

In the case of massless neutrinos we can choose the neutrino mass eigenstates arbitrarily. In particular, we

²We call SM with lepton mixing any model that allows non-zero neutrino masses but reproduces otherwise in the low-energy limit the SM. This means in particular that we do not consider sterile neutrinos.

can set $U^{(\nu)} = U^{(\mathrm{ch})}$ for any given $U^{(\mathrm{ch})}$, hereby rotating away the mixing. This shows that neutrino masses are a necessary condition for non-trivial consequences of mixing in the lepton sector.

Finally, we recall that there are no flavor-changing neutral currents within the standard model: The neutral current Lagrangian is diagonal both in the weak eigenbasis,

$$\mathcal{L}_{NC} = -\frac{g}{2\cos\theta_W} \sum_{\alpha} [\bar{\nu}_{L,\alpha} \gamma_{\mu} \nu_{L,\alpha} + \bar{l}_{\alpha} (g_V + g_A \gamma_5) \gamma_{\mu} l_{\alpha}] Z^{\mu}$$

(9)

and in the mass basis

$$\mathcal{L}_{NC} = -\frac{g}{2\cos\theta_W} \sum_{i} [\bar{\nu}_{L,i}\gamma_{\mu}\nu_{L,i} + \bar{l}_i(g_V + g_A\gamma_5)\gamma_{\mu}l_i]Z^{\mu}$$

(10)

due to the unitarity conditions, $U^{(\nu)\dagger}U^{(\nu)}=\mathbf{1}$ and $U^{(\mathrm{ch})\dagger}U^{(\mathrm{ch})}=\mathbf{1}$.

III. PION DECAY AND LEPTON MIXING

Many neutrino oscillations experiments use as source for the initial lepton-neutrino state charged pions. In the SM without lepton mixing, a tree-level calculation gives for the ratio R of $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$ decay rates

$$R = \frac{\Gamma(\pi \to e\nu_e)}{\Gamma(\pi \to \mu\nu_\mu)} = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} \approx 1.28 \times 10^{-4} \,.$$
(11)

Since angular momentum conservation in the pion rest frame requires a helicity flip of the lepton, the S-matrix elements of these decays are proportional to the lepton masses m_{α} and, therefore, the branching into electrons is suppressed. Hence, the two-particle state $|l^+\nu\rangle_{\pi}$ created by a decaying positively charged pion is given by

$$|l^+\nu\rangle_{\pi} = \frac{1}{\sqrt{N}} \sum_{\alpha=e,\mu} m_{\alpha} (1 - m_{\alpha}^2/m_{\pi}^2) |l_{\alpha}^+\nu_{\alpha}\rangle, \qquad (12)$$

where N is a normalization constant. We have included the phase space factor $1-m_{\alpha}^2/m_{\pi}^2$ into Eq. (12) because we assume that the state $|l^+\nu\rangle_{\pi}$ lives for a macroscopic time between its creation and detection. Therefore, both the lepton and the neutrino are approximately on their mass-shell.

Let us now examine what are the necessary changes if we want to account for lepton mixing. Inserting Eq. (6) into Eq. (12) we obtain

$$|l^+\nu(t)\rangle_{\pi} = \frac{1}{\sqrt{N}} \sum_{i=1}^2 \sum_{j=1}^3 a_i V_{ij} |l_i^+\nu_j\rangle e^{-\mathrm{i}(E_i + E_j)t}, \quad (13)$$

where $a_i = m_i (1 - m_i^2 / m_{\pi}^2)$.

We are ordering the mass eigenstates according to the value of m_i , i.e. $m_{l_1} < m_{l_2} < m_{l_3}$. Thus the state l_3 cannot be populated in pion decay and, therefore, is omitted in the summation. Furthermore, we have assumed that all three neutrinos masses are extremely small compared to the electron mass as it is suggested by the currently favored interpretation of neutrino oscillation experiments and cosmology. Therefore, we could omit safely new terms in the S-matrix element proportional to m_{ν_i} , that in principle change the branching ratio Eq. (11) from its SM value [12]. Note that the relative phases V_{ij} of the different components $|l_i^+\nu_j\rangle$ are fixed by the Lagrangian, while the a_i are real numbers. The time evolution of $|l^+\nu(t)\rangle_{\pi}$ is trivial, because we have expressed $|l^+\nu(t)\rangle_{\pi}$ as a sum over mass eigenstates.

In Eq. (13), we have not displayed explicitly the finite lifetimes $\tau = 1/\Gamma$ of the states l_2 and $\nu_{2,3}$, because this point is not essential for our discussion. However, the finite lifetimes can be restored treating the energy as a complex number, $E = (m^2 + p^2) - i\Gamma/2$ and noting that the decay products of $|l^+\nu\rangle_{\pi}$ do not interfere with it.

Apart from the large difference between the lifetime of l_2 and of $\nu_{2,3}$, there is another, more important, distinction between neutrinos and charged leptons. If one decomposes $|l^+\nu\rangle_{\pi}$ explicitly into its basis states, then

$$|l^{+}\nu\rangle_{\pi} = \frac{1}{\sqrt{N}} \sum_{\alpha=1}^{3} \left(\sum_{i=1}^{2} a_{i} U_{\alpha i}^{(\text{ch})*} |l_{i}^{+}\rangle \right) \otimes \left(\sum_{j=1}^{3} U_{\alpha j}^{(\nu)} |\nu_{j}\rangle \right) .$$

$$(14)$$

Defining a new basis appropriate for $|l^+\nu\rangle_{\pi}$ by

$$|l^{+}\nu\rangle_{\pi} = \sum_{\alpha=1}^{3} |l_{\alpha}^{+}\rangle_{\pi} \otimes |\nu_{\alpha}\rangle_{\pi}$$
 (15)

and comparing with Eq. (14), it follows that the neutrino state $|\nu_{\alpha}\rangle_{\pi}$ produced in pion decay is just a usual weak eigenstate, $|\nu_{\alpha}\rangle_{\pi} = \sum_{j=1}^{3} U_{\alpha j}^{(\nu)} |\nu_{j}\rangle = |\nu_{\alpha}\rangle$. By contrast, the charged lepton state is $|l_{\alpha}^{+}\rangle_{\pi} = \sum_{i=1}^{2} a_{i}/\sqrt{N} U_{\alpha i}^{(\mathrm{ch})*} |l_{i}^{+}\rangle \neq |l_{\alpha}^{+}\rangle$. We will see below that it is the presence of the prefactors $a_{1} \neq a_{2}$ which allows the observation of neutrino oscillations in pion decay without detecting the charged lepton. Note however also that $|l_{\alpha}^{+}\rangle_{\pi} \neq |l_{\alpha}^{+}\rangle$ even for $a_{1} = a_{2}$, because the component l_{3} is missing.

Using $a_2 \gg a_1$, we can approximate $|l^+\nu(t)\rangle_{\pi}$ as

$$|l^+\nu(t)\rangle_{\pi} \approx \sum_{j=1}^3 V_{2j} |l_2^+\nu_j\rangle e^{-\mathrm{i}(E_2+E_j)t}$$
 (16)

with $a_2/\sqrt{N} \approx 1$. Clearly, one obtains in this approximation only neutrino oscillations, because the charged

lepton is in a pure mass eigenstate. Choosing furthermore $V = U^{(\nu)}$, we obtain the state normally considered as initial state in pion decay,

$$|l^{+}\nu\rangle_{\pi} \approx \sum_{\alpha=1}^{3} \sum_{j=1}^{3} \delta_{2\alpha} U_{\alpha j}^{(\nu)} |l_{2}^{+}\nu_{j}\rangle = |l_{2}^{+}\nu_{\mu}\rangle.$$
 (17)

The approximation $a_2 \gg a_1$ which is widely used in textbooks is numerically well justified. However, its use obscures the fact that even for the choice $V = U^{(\nu)}$, i.e. identifying mass and flavor eigenstates of the charged leptons, the charged lepton is nevertheless produced in a mixed state.

Let us now discuss different measurements of the exact two-particle state, Eq. (13). Since we are only interested in flavor oscillations, we do not consider possible momentum measurements of the two particles. Then, a measurement of the state $|l^+\nu(t)\rangle_{\pi}$ is complete if at time t the quantum numbers i or α of both the neutrino and the charged lepton are determined. In the case that only one quantum number is observed, the probability P(l) of this measurement is obtained by summing over the quantum number of the unobserved particle, symbolically $P(l) = \sum_{l'} P(l, l')$.

To begin with, we recall the case normally treated in the literature, namely that the neutrino flavor is detected while the lepton is not observed. In a first try, we associate the probability $P(l_k, \nu_\alpha) = |\langle l_k \nu_\alpha | l^+ \nu(t) \rangle_\pi|^2$ to the measurement of the lepton mass eigenstate k and the neutrino flavor eigenstate α at time t. This would result in

$$\langle l_k \nu_\alpha | l^+ \nu(t) \rangle_\pi = \frac{a_k}{\sqrt{N}} \sum_{l=1}^3 V_{kl} U_{\alpha l}^{(\nu)*} e^{-i(E_k + E_l)t}, \quad (18)$$

i.e. in an amplitude which does not only depend on V but also on the neutrino mixing matrix $U^{(\nu)}$. However, in practice one cannot observe the flavor of a neutrino directly. Instead, the flavor of the neutrino is determined looking at the mass eigenstates of the charged lepton l' produced in a secondary charged current reaction, cf. Fig. 1.

Therefore, we should calculate

$$\langle l_k l'_m | \hat{H}_{CC}(t) | l^+ \nu(t) \rangle_{\pi} = \frac{a_k}{\sqrt{N}} \sum_{l=1}^3 V_{kl} V_{ml}^* e^{-i(E_k + E_l)t},$$
(19)

where the action of \hat{H}_{CC} destroys at time t a neutrino ν_{β} and creates a superposition of mass eigenstates of charged leptons $l'_m = U_{m\beta}^{\rm (ch)} l_{\beta}$. Here, \hat{H}_{CC} denotes the second quantized Hamiltonian of the usual charged-current interaction. The corresponding probability to measure the primary lepton l_k from the pion decay and the secondary lepton l'_m produced by the neutrino is

$$P(l_k, l'_m) = \frac{a_k^2}{N} \left\{ \sum_{l=1}^3 |V_{kl}|^2 |V_{ml}|^2 + 2 \sum_{n>l}^3 |V_{kl}V_{ml}^*V_{kn}^*V_{mn}| \cos\left[(E_l - E_n)t + \xi_{klmn}\right] \right\}, \tag{20}$$

where $\xi_{klmn} = \arg(V_{kl}V_{ml}^*V_{kn}^*V_{mn})$. If both charged leptons are observed, the probability (20) shows clearly an oscillatory behavior.

In the case that only one of the two leptons is observed, the result is completely different depending on if the primary l_k or the secondary l_m' (as indicator for the neutrino flavor) is observed. In the first case, summing over m, we obtain

$$P(l_k) = \sum_{m} P(l_k, l'_m) = \frac{a_k^2}{N}$$
 (21)

using the unitarity of V, i.e. $\sum_k V_{ik} V_{jk}^* = \delta_{ij}$. In the second case, we cannot make use of these unitarity relations because the prefactors a_k depend on the summation index k. However, in the limit $a_2 \gg a_1$, the result simplifies and we obtain the well-known neutrino oscillation formula

$$P(l'_m) = \sum_k P(l_k, l'_m) \approx \left| \sum_l V_{2l} V_{ml}^* \exp(-iE_l t) \right|^2.$$
 (22)

Let us now comment shortly on the reason for this asymmetry. A flavor oscillation experiment measures the correlation between the flavor or mass quantum numbers of two particles. In the case of pion decay, we can use our knowledge about the initial state created in the decay as replacement for an actual measurement of the primary lepton. However, the kinematic of the decay does not give us any useful information about the neutrino state: all three flavor eigenstates are with equal probability produced. Therefore, the neutrino has to be measured via its secondary lepton to obtain an oscillation pattern.

From this discussion, it should became clear that the difference between charged leptons and neutrinos in $|l^+\nu\rangle_\pi$ is rather specific to the process considered: The V–A structure of the weak current together with angular momentum conservation forbid the decay of the spinless pion into two massless spin 1/2 particles. Therefore, the pion decay rate is proportional to the fermion masses m^2 . Moreover, it is used that the neutrino masses can be neglected compared to the masses of the charged leptons. Therefore, the fact that we have no a-priori information about the neutrino but about the charged lepton is specific to pion decay.

Next, we want to discuss if it is possible to measure not the flavor of a neutrino but its mass without measuring the lepton. A possible way to do this is to use Cherenkov or transition radiation of the neutrino [13]. In vacuum, neutrinos can interact with real photons whose squared four-momentum q^2 is zero only via their electromagnetic dipole or transition moments. In a medium, the photon acquires a more complicated dispersion relation $(q^2 \neq 0)$ and, therefore, neutrinos can emit real photons without decaying, $\nu_i \rightarrow \nu_i + \gamma$. At least in principle, it is possible to reconstruct the mass of the radiating neutrino from the spectra of emitted photons.

The probability to find the neutrino in the mass eigenstate l without measuring the charged lepton is

$$P(\nu_l) = \sum_{k=1}^{2} P(l_k, \nu_l) = \frac{1}{N} \sum_{k=1}^{2} a_k^2 |V_{kl}|^2 \approx \frac{a_2^2}{N} |V_{2l}|^2.$$
 (23)

In this case, we have the interesting result that $P(\nu_l)$ depends on the mixing matrix V but does not show an oscillatory behavior. Compared to a flavor measurement which introduces an additional summation $\sum_j V_{jl}$ in the probability amplitude, the phase of the probability amplitude of a mass measurement is constant and, consequently, no oscillation pattern can arise. On the other hand, $P(\nu_l)$ depends on V because of the factors a_k . Consequently, it is possible to extract information on the mixing matrix V measuring the mass eigenstate of the neutrino.

Finally, we comment briefly on charged leptonneutrino states created in nuclear beta decay. Due to the low nuclear energies involved, only the state l_1 is produced and, therefore, the charged lepton is not only approximately but exactly in a pure state.

IV. W DECAY AND LEPTON MIXING

We discuss now the evolution of the two-particle state $|l\nu\rangle$ created by a decaying real W boson. The W boson is a spin-1 particle and, therefore, can decay into two massless fermions. Neglecting small corrections of $\mathcal{O}(m^2/m_W^2)$, the state produced is equally populated for all three generations. Thus, while the state

$$|l^+\nu\rangle_W = \frac{1}{\sqrt{3}} \sum_{\alpha=1}^3 |l_\alpha^+\nu_\alpha\rangle \tag{24}$$

is produced in the SM without lepton mixing by a decaying W^+ , the state

$$|l^{+}\nu\rangle_{W} = \frac{1}{\sqrt{3}} \sum_{i,j=1}^{3} V_{ij} |l_{i}^{+}\nu_{j}\rangle$$
 (25)

is created with mixing.

We can repeat now the discussion of different measurements similar to the case of $|l^+\nu\rangle_\pi$. The only change necessary is the replacement of a_k/\sqrt{N} by $1/\sqrt{3}$. Hence, the probability to find the primary lepton in a mass eigenstate k and the secondary lepton in l'_m becomes

$$P(l_k, l'_m) = \frac{1}{3} \left\{ \sum_{l=1}^3 |V_{kl}|^2 |V_{ml}|^2 + 2 \sum_{n>l}^3 |V_{kl} V_{ml}^* V_{kn}^* V_{mn}| \cos \left[(E_l - E_n)t + \xi_{klmn} \right] \right\}.$$
 (26)

In contrast to Eq. (20), the probability is now symmetric in l_k and l'_m . In particular, the oscillation pattern vanishes now in both cases as long as only one particle is observed. Only when both the primary and the secondary lepton are observed, an oscillation pattern according to Eq. (26) is observed.

We note that the same observation was made in Ref. [14]. There, the neutrino state produced in the decay of a real Z was examined. The authors of [14] showed that also in this case neutrino flavor oscillations can be observed, although the neutrinos are produced by neutral-current interaction. Moreover, they showed that it is necessary to measure both neutrinos in order to observe a oscillation pattern. Thus, their results are in line with our findings presented above.

V. CONCLUSION

Flavor oscillations are observed by the detection of correlations between two states. In an ideal experiment, the composition of both states is measured. In experiments which use nuclear beta decay to produce the initial charged lepton–neutrino state the energy available is limited to nuclear energies. Only the state $|l_1\nu_j\rangle$ can be populated thus making a measurement of the charged lepton obsolete.

Experiments in which pion decay create the initial lepton-neutrino state, one exploits the known branching ratios into the different states $|l_i\nu_i\rangle$ as a substitute for the measurement of the charged lepton. These branching ratios differ only for different charged lepton mass eigenstates, but are the same for different neutrino states. Therefore, the knowledge of the branching ratios "replaces" only a measurement of the state of the charged lepton, and the measurement of the neutrino state is necessary to obtain information about lepton mixing. In contrast, the lepton-neutrino states produced in the decay of real W Bosons are symmetrical in their branching ratios. If experiments were carried out with such initial states, both charged lepton and neutrino would be required to be measured in order to observe flavor oscillations. In summary, the specific nature of the initial state used in oscillation experiments explains the distinguished rôle of neutrinos compared to charged leptons.

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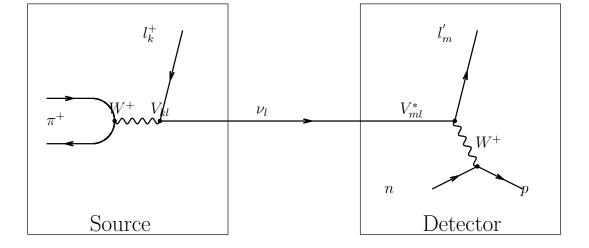


FIG. 1. Production of a superposition of neutrino mass eigenstates ν_l in pion decay and subsequent detection of the neutrino flavour via the secondary lepton l_m' .