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The exact democratic structure for the quark mass matrix, resulting from the action of the family symmetry group $A_{3L} \times A_{3R}$, is broken by the vaccum expectation values of heavy singlet fields appearing in non renormalizable dimension 6 operators. Within this specific context of breaking of the family symmetry we formulate a very simple ansatz which leads to correct quark masses and mixings.

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Introduction

One of the outstanding problems in particle physics is the problem of the fermion masses and mixings. In the standard model (SM), which, most likely, is an effective theory at low energy, these physical quantities are computed from the Yukawa couplings. With regard to the quarks, one can have, in principle, for the 3 families of the up and down sector, 18 complex Yukawa couplings. This gives us a total of 36 parameters from which one has to extract the 10 physical quantities: 6 quark masses, 3 mixing angles and a CP violating complex phase.

To reduce this large amount of parameters, or even to find possible relations between the quark masses and mixings [1], one is lead to seek, e.g., for new symmetries which act among the family structure [2]. Another approach is to postulate, ab initio, ansätze for the Yukawa couplings which lead to phenomenological viable patterns [3] [4] [5] [9]. The hope is to find some hint about a symmetry principle behind the mechanism of fermion mass generation. In the literature, there are, grosso modo, two classes of ansätze. Those which are formulated in a "heavy" weak basis [3] [4], where one of the Yukawa couplings of each sector is much larger then the other, and ansätze which are formulated in the "democratic" weak basis [5] [9], and where all Yukawa couplings of each sector are almost equal to each other.

In this paper, we present a very simple but phenomenological correct pattern within the democratic weak basis. In our approach, the exact democratic structure is generated through the action of the family symmetry group $A_{3L} \times A_{3R}$, where $A_3 \subset S_3$ is the subgroup of even permutations. This group is then broken by the vacuum expectation values (v.e.v.) of heavy singlet fields appearing only in non renormalizable dimension 6 operators. The idea is, therefore, that the exact democratic structure is broken by contributions from higher order operators arrizing in the scenario (which will not be discussed here) of some unified theory at a large scale $M = M_{GUT} - M_{Pl}$ [6]. Within this specific context of breaking of the $A_{3L} \times A_{3R}$ family symmetry we formulate a very simple ansatz which leads to correct quark masses and mixings.

General framework

As known, the discrete family symmetry $A_{3L} \times A_{3R}$ generates (and not necessarily $S_{3L} \times S_{3R}$ as one often finds) the democratic mass matrix:

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

Our model consists of the usual $SU(2)_L \times U(1)_Y$ SM Higgs doublet ϕ, the left-handed quark doublets L_i and right-handed singlets R_i (which here represent either the right handed up quarks u_{R_i} or the right handed down quarks d_{R_i}). Both L_i and R_i transform trivially with respect to the A_3 family symmetry, i.e., the family indices transform as

$$(1) = e \ ; \quad (123) = a \ ; \quad (132) = b \tag{2}$$

A_3 is isomorf to Z_3. This can be easily checked from its multiplication table: $a^2 = b$ and $a\ b = e$, which leads to $a^3 = b^3 = e$ (and also $b^2 = a$). Then, the lowest dimension mass term in the Lagrangean which is invariant under this independent interchange of the left and right-handed fields is

$$\lambda\ (\overline{L_1} + \overline{L_2} + \overline{L_3})\ \phi\ (R_1 + R_2 + R_3) \tag{3}$$

and one gets the democratic mass matrix.

In order to change the democratic structure, we introduce now two independent Higgs (A_3 family) triplets X_i and Y_i: one transforming (in the same way and) together with the left-handed and the other with the right-handed fields. Under $SU(2)_L \times U(1)_Y$ they are singlets. One can form three independent A_3 invariant combinations:

$$
\begin{aligned}
Z_1 &= a_1\ b_1 + a_2\ b_2 + a_3\ b_3 \\
Z_2 &= a_1\ b_3 + a_2\ b_1 + a_3\ b_2 \\
Z_3 &= a_1\ b_2 + a_2\ b_3 + a_3\ b_1
\end{aligned}
\tag{4}
$$

where the (a_i, b_i) either stand for the independent A_3 partners $(\overline{L_i}, X_i)$ or for the (R_i, Y_i). The next to lowest dimension (and non-renormalizable) mass terms, are, e.g., combinations like $(\overline{L_1}X_1 + \overline{L_2}X_2 + \overline{L_3}X_3)\ \phi\ (R_1Y_2 + R_2Y_3 + R_3Y_1)$. An extra Z_2 symmetry is needed to avoid the combinations $\overline{L}X\phi R$ or $\overline{L}\phi Y R$. Please notice also that the exact democratic structure appearing in the Lagrangean, as a result of the combination in Eq. (3) is in fact invariant under $A_{3L} \times A_{3u_R} \times A_{3d_R}$, because it is possible to transform the right-handed up quark fields independently from the right-handed down quark fields. However, with the introduction of the new singlets fields X_i and Y_i, this larger symmetry is no longer valid as the Y_i fields require that the u_{R_i} and d_{R_i} transform simultaneously. Thus, here, we have an exact $A_{3L} \times A_{3R}$ family symmetry.

The whole mass term in the Lagrangean, including the lowest and the relevant next to lowest order dimension mass operator, will be

$$
\lambda\ (\overline{L_1} + \overline{L_2} + \overline{L_3})\ \phi\ (R_1 + R_2 + R_3) + \lambda_{mk}\ \frac{Z_m^{(\overline{L},X)}}{M}\ \ \phi\ \frac{Z_k^{(R,Y)}}{M} \tag{5}
$$

where the Z_k were defined in Eq.(5), e.g., $Z_2^{(\overline{L},X)} = (\overline{L_1}X_3 + \overline{L_2}X_1 + \overline{L_3}X_2)$, and where M is the heavy mass where the large scale structure of the unified theory becomes apparent. The A_3 symmetry of the singlet fields is broken when they acquire the following v.e.v.'s [7]:

$$
\begin{aligned}
(< X_1 >, < X_2 >, < X_3 >) &= (0, 0, V_X) \\
(< Y_1 >, < Y_2 >, < Y_3 >) &= (0, 0, V_Y)
\end{aligned}
\tag{6}
$$

The quark mass matrix, thus obtained, for each sector, will then be of the form:

$$
M^\circ = \lambda v \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right) \tag{7}
$$

where $a_{ij} = (\lambda_{ij}/\lambda)\ (V_X V_Y\ /\ M^2)$ and which is, as one can clearly see, not democratic any more. In fact, all family symmetries have been broken. The heavy singlets get their v.e.v.'s at a scale which, at least, should be smaller than the mass scale M. Thus the a_{ij} are smaller than 1. Because of the large scale M, the other dimension 6 operators involving only quark field combinations should be even (much) smaller, as the v.e.v's from the heavy singlets do not contribute to these terms.

The ansatz

Within this type of democracy breaking context, we shall consider the specific case where, compared to the three parameters (a_{13}, a_{31}, a_{32}), all other a_{ij} are small. This is a natural limit, in the sense that we are not demanding any special relations between the a_{ij} like, e.g., in the ansatz of Fritzsch [8] or the cases classified by Ramond Roberts and Ross [3] where $M_{12}^\circ = M_{21}^\circ$ and $M_{23}^\circ = M_{32}^\circ$. Taking the limit where the small $a_{ij} \to 0$ we obtain the following (dimensionless) asymmetric mass matrix,

$$
M = \begin{bmatrix} 1 & 1 & 1 + a_{13} \\ 1 & 1 & 1 \\ 1 + a_{31} & 1 + a_{32} & 1 \end{bmatrix} \tag{8}
$$

Parametrizing a_{31}, a_{32} and a_{13} as follows,

$$
\begin{aligned}
a_{31} &= q\ e^{i\ \alpha} + r\ e^{i\ \beta} \\
a_{32} &= q\ e^{i\ \alpha} \\
a_{13} &= r\ e^{i\ \beta}\ (1 + \varepsilon\ e^{i\ \gamma})
\end{aligned}
\tag{9}
$$

does not add anything to our ansatz, as a_{31}, a_{32} and a_{13} remain independent. However, it is very useful to study the phenomenological implications of Eq. (8). To do this, we shall first concentrate on a simplification of Eq. (8). As an example, we take the case where $\varepsilon = 0$ and $\alpha, \beta = \pi/2$. One gets,

$$M[\begin{smallmatrix}\varepsilon=0,\\ \alpha,\beta=\pi/2\end{smallmatrix}] \approx \begin{bmatrix} 1 & 1 & e^{i\ r} \\ 1 & 1 & 1 \\ e^{i\ (q+r)} & e^{i\ q} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & e^{i\ (q+r)} \\ 1 & 1 & e^{i\ q} \\ e^{i\ (q+r)} & e^{i\ q} & e^{i\ q} \end{bmatrix} \cdot K_R \tag{10}$$

where we have used the approximation $1 + i\ x \approx e^{ix}$. The unitary matrix $K_R = \text{diag}\ (1, 1, e^{-i\ q})$ is non-relevant and can be absorbed in a transformation of the right-handed quark fields. The mass matrix on the right-hand side of Eq. (10) is exactly one of the familiar symmetric cases described in the USY hypothesis of Ref. [9] with two dimensionless parameters. Obviously, the diagonalization matrix elements, such as U_{12} and U_{23}, depend on these. In a first order approximation, it was found that $U_{12} = (\sqrt{3}/2)\ (r/q)$ and $U_{23} = (2\sqrt{2}/9)\ q$. Since q and r depend on the mass ratios through the (approximate) relations, $q = (9/2)(m_2/m_3)$ and $r = 3(3m_1m_2)^{1/2}/m_3$, the phenomenological formulas $U_{12} = (m_1/m_2)^{1/2}$and $U_{23} = \sqrt{2}(m_2/m_3)$ are obtained [5] [9]. Notice the precise (and peculiar) cancellation of the numerical factors.

Let us now present an analysis of the general mass matrix in Eq. (8). We shall assume that $\varepsilon = o(m_2/m_3) \ll 1$. This is rather a special choice in parameter space, i.e., it is not natural (in the sense explained above), because in that case $a_{31} - a_{32} \approx a_{13}$; it is a choice motivated by predictability. We shall not go into the details of solving the characteristic equations, which involve the mass ratios of the quarks of the physical relevant square mass matrix; that was done in Ref. [9]. Defining $H = \ M\ M^\dagger/t$, where $t = tr(H)$ is such that $tr(H) \equiv 1$, one obtains eigenvalues that respect exact, $\lambda_1 + \lambda_2 + \lambda_3 = 1$, and approximate relations:

$$\lambda_1 = \tfrac{m_1^2}{m_3^2}\ ; \qquad \lambda_2 = \tfrac{m_2^2}{m_3^2}\ ; \qquad \lambda_3 = 1 \tag{11}$$

From the characteristic equations one finds, in first order, approximate values for $q = (9/2)(m_2/m_3)$ and $r = 3(3m_1m_2)^{1/2}/m_3$. Then, using an iteration method starting with these initial approximate values, one finds expressions for q and r as series in mass ratios.

$$\begin{aligned} r &= \tfrac{3\sqrt{3m_1m_2}}{m_3} \cdot \left[1 + \tfrac{3}{2}\left(\tfrac{m_2}{m_3}\right)\cos(\alpha) - \tfrac{1}{2}\varepsilon\cos(\gamma) + \ldots\right] \\ q &= \tfrac{9}{2}\ \tfrac{m_2}{m_3} \cdot \left[1 - \sqrt{\tfrac{4m_1}{3m_2}}\cos(\alpha-\beta) + \ldots\right] \end{aligned} \tag{12}$$

The phases α , β and the ε are free parameters; they are not determined by the mass ratios. We shall come to this later.

After introducing these relations into the square mass matrix H, one computes the eigenvectors, also as a series in the mass ratios. The diagonalization matrix U is calculated in the heavy weak basis. In this weak basis all matrix elements of are small except H_{33}, and only the relevant contributions of H_u and H_d to V_{CKM} are present. Thus the irrelevant parts, which cancel out in the Cabibbo-Kobayashi-Maskawa matrix (product),

$$V_{CKM} = U_u^\dagger \cdot U_d \tag{13}$$

are absent. In this way, both U_u and U_d are both near $\mathbb{1}$. The heavy weak basis is defined in the following way,

$$\begin{aligned} H_u &\longrightarrow H_u^{\text{Heavy}} = F^\dagger \cdot H_u \cdot F \\ H_d &\longrightarrow H_d^{\text{Heavy}} = F^\dagger \cdot H_d \cdot F \end{aligned} \qquad ; \qquad F = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \tag{14}$$

One finds for the diagonalization matrix elements U_{12} and U_{13},

$$\begin{aligned} |U_{12}| &= \sqrt{\tfrac{m_1}{m_2}}\left[1 - \tfrac{m_1}{2m_2} + \tfrac{m_2}{m_3}\cos\alpha + \tfrac{\varepsilon}{2}\cos\gamma + \ldots\right] \\ |U_{13}| &= \tfrac{1}{\sqrt{2}}\tfrac{\sqrt{m_1m_2}}{m_3}\left[1 - \tfrac{m_2}{2m_3}\cos\alpha + \tfrac{\varepsilon}{2}\cos\gamma + \ldots\right] \end{aligned} \tag{15}$$

where the next to leading order terms are of small influence. For the elements U_{23} and U_{31} one obtains next to leading order terms which are somewhat larger,

$$\begin{aligned} |U_{23}| &= \sqrt{2}\tfrac{m_2}{m_3}\left[1 - \sqrt{\tfrac{3m_1}{4m_2}}\cos(\alpha-\beta) + \ldots\right] \\ |U_{31}| &= \tfrac{3}{\sqrt{2}}\tfrac{\sqrt{m_1m_2}}{m_3}\left[1 - \sqrt{\tfrac{m_1}{3m_2}}\cos(\alpha-\beta) + \ldots\right] \end{aligned} \tag{16}$$

Approximate relations hold

$$|U_{13}| = \frac{1}{2}\,|U_{23}\,U_{12}| \quad ; \qquad |U_{31}| = 3\,|U_{13}| \tag{17}$$

CP violation and a numerical example

In this section we describe the CP violation and the masses and mixings of a numerical example of the ansatz in Eq. (8). We find that CP violation is mainly restricted by the range which, within our framework, is possible to have for the up quark mass m_u.

It is clear that, on the one hand, for general mass matrices $M_{u,d}$ of type Eq. (8), the CP violation depends, crucially, on the complex phases $\alpha_{u,d}$ and $\beta_{u,d}$, which are free parameters, independent of the mass ratios, and which for a specific numerical (ansatz) example still have to be fixed. Obviously, for $\alpha, \beta = k\pi$, there is no CP violation. On the other hand, if M_u and M_d are real, we find for the V_{CKM} matrix element

$$|V_{us}| = \left|\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}\right| \tag{18}$$

where the $\pm$ sign depends on the relative signs of r and q (i.e., if $\alpha, \beta = k\pi$) for the up and down sector. Combining the experimental limits on m_d/m_s, m_s and m_c, one can only accommodate the experimental value of $|V_{us}| = 0.2196(23)$ [10] in Eq. (18) if one takes a very small value for $m_u \leq 1\ MeV$ or even $m_u = 0$. However, when $\alpha, \beta \neq k\pi$, the $\pm$ sign in Eq. (18) is replaced by a complex phase factor such that

$$|V_{us}| = \left|\sqrt{\frac{m_d}{m_s}} + e^{i\delta}\ \sqrt{\frac{m_u}{m_c}}\right| \tag{19}$$

and it is possible to accommodate a larger value for m_u [11]. Clearly for our ansatz, CP violation is closely related to this problem, i.e., it depends also on the α's and β's and subsequently on δ which is a function of these. Numerically, we have found that CP violation, given by $|J_{CP}| = |Im(V_{us}V_{cb}V^\star_{cs}V^\star_{ub})|$, is large when also δ mod π is large. Thus, a larger value for m_u can only be accommodated if one takes values for $\alpha, \beta \neq 0$ mod π such that δ mod π is large and this results in a large value for the CP violation parameter (and vice versa). In order to find (ansatz) examples with sufficient large CP violation, it is useful to have an expression for δ.

Let us compute δ in a first order approximation. Writing the eigenvalue equation of each quark sector as $H = U \cdot D \cdot U^\dagger$, where H is given in the heavy basis of Eq. (14) and $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ contains the eigenvalues of H, one obtains (using the unitarity of U), the exact relations

$$\begin{array}{l} (\lambda_2 - \lambda_1)\ U_{12}U^\star_{32} + (\lambda_3 - \lambda_1)\ U_{13}U^\star_{33} = H_{13} \\ (\lambda_2 - \lambda_1)\ U_{22}U^\star_{32} + (\lambda_3 - \lambda_1)\ U_{23}U^\star_{33} = H_{23} \\ (\lambda_2 - \lambda_1)\ U_{12}U^\star_{22} + (\lambda_3 - \lambda_1)\ U_{13}U^\star_{23} = H_{12} \end{array} \tag{20}$$

Using Eqs. (11, 15, 16) and choosing U_{33} real (this is always possible), we find from the first two equations that the complex phases of U_{13} and U_{23} are approximately equal to those of H_{13} respectively H_{23}. Computing H in the heavy basis with the parametrization of Eq. (9), one finds (for α and β not too close to $k\pi$) in a first order approximation

$$\begin{array}{lll} H_{13} = \frac{1}{3\sqrt{6}}\ \ r\ e^{i\beta}\ ; & H_{23} = \frac{2\sqrt{2}}{9}\ \ q\ e^{i\alpha} & \Longrightarrow \\ & & \\ U_{13} = |U_{13}|\ e^{i\beta}\ \ ; & U_{23} = |U_{23}|\ e^{i\alpha} & \end{array} \tag{21}$$

In addition, from Eqs. (11, 15, 16) one obtains $\lambda_2/\lambda_3 = |U_{13}U^\star_{23}|/|U_{12}U^\star_{22}|$. Thus

$$|\lambda_2 U_{12}U^*_{22}| = |\lambda_3 U_{13}U^\star_{23}| = \frac{m_1 m_2}{m_3^2}\ \sqrt{\frac{m_2}{m_1}} \tag{22}$$

holds and because $H_{12} = -r^2/18\sqrt{3} = -\sqrt{3}m_1m_2/2m_3^2$ is smaller than this (in absolute value), we may conclude from the third relation in Eq. (20) that, aside from a factor π

$$\arg(U_{12}U^*_{22}) = \arg(U_{13}U^\star_{23}) \tag{23}$$

Unitarity also tells us that, in this approximation, $\arg(U_{11}U^*_{21}) = \arg(U_{12}U^\star_{22})$. Finally, putting together all these phase relations for the up and down sector, we get (aside from any factors π)

$$\delta = (\alpha_d - \beta_d) - (\alpha_u - \beta_u) \tag{24}$$

With this expression, we can now choose suitable combinations for $\alpha_{u,d}$ and $\beta_{u,d}$ to have a large δ (mod π) in order to account for suitable large values for CP and m_u.

Next we give a numerical example,where we take $\varepsilon_{u,d} = 0$ and the simplest combinations for $\alpha_{u,d}$ and $\beta_{u,d}$ to obtain a large δ. The mass matrices of both sectors are (as explained) of type

$$M = c \begin{bmatrix} 1 & 1 & 1 + r\ e^{i\beta} \\ 1 & 1 & 1 \\ 1 + q\ e^{i\alpha} + r\ e^{i\beta} & 1 + q\ e^{i\alpha} & 1 \end{bmatrix} \tag{25}$$

Example with $\delta = -\pi/3$, where $\alpha_d = \alpha_u = \beta_u = 0$ and only $\beta_d = \pi/3$ (extra π factors are put in as signs in the q's and r's) and

$$\begin{array}{ll} r_d = -3.259 \times 10^{-2} & r_u = 9.368 \times 10^{-3} \\ q_d = 0.1254 & q_u = 1.463 \times 10^{-2} \\ c_d = 2\ GeV & c_u = 133\ GeV \end{array} \tag{26}$$

which at 1 GeV correspond to,

$$\begin{array}{ll} m_d = 7.39\ MeV & m_u = 3.73\ MeV \\ m_s = 186\ MeV & m_c = 1.38\ GeV \\ m_b = 6.2\ GeV & m_t = 400\ GeV \end{array}$$

give

$$|V_{CKM}| = \begin{bmatrix} 0.9748 & 0.2229 & 0.0037 \\ 0.2225 & 0.9740 & 0.0414 \\ 0.0124 & 0.0397 & 0.9991 \end{bmatrix} \quad ; \quad \frac{|V_{ub}|}{|V_{cs}|} = 0.0896 \tag{27}$$

and $|J_{CP}| = 1.8 \times 10^{-5}$. To obtain a large value for $|J_{CP}|$ one would expect that a value for $\delta = \pm\pi/2$ would be more suitable. However, J_{CP} depends also on other order contributions which are of significant importance. Numerically, we have found that $\delta = \pm\pi/3$ gives the largest values for $|J_{CP}|$.

Concluding remarks

We have shown that the exact democratic structure for the quark mass matrices, resulting from the action of the family symmetry group $A_{3L} \times A_{3R}$, can be totally broken by the effects of non renormalizable dimension 6 operators adding a small perturbation to this structure. Within this context, we formulate a unique ansatz: one of the simplest deviations from democracy, requiring a minimum of parameters, and which predicts the well known phenomenological mixings in terms of quark mass ratios. We have also shown that CP violation is determined by a simple combination of complex phases of these parameters. A numerical ansatz-example is given in good agreement with experiment.

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