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The Cannonball Model of Gamma Ray Bursts: Lines in the X-Ray Afterglow

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Abstract. Recent observations suggest that gamma-ray bursts (GRBs) and their afterglows are produced by jets of highly relativistic cannonballs (CBs), emitted in supernova explosions. The fully ionized CBs cool to a temperature below 4500 K within a day or two, at which point electron–proton recombination produces an intense Lyman-α emission. The line energy is Doppler-shifted by the CBs' motion to X-ray energies in the observer's frame. The measured line energies, corrected for their cosmological redshift, imply Doppler factors in the range 600 to 1000, consistent with those estimated —in the CB model— from the characteristics of the γ-ray bursts. All other observed properties of the lines are also well described by the CB model. Scattering and self-absorption of the recombination lines within the CB also produce a wide-band flare-up in the GRB afterglow, as the observations indicate. A very specific prediction of the CB model is that the X-ray lines ought to be narrow and move towards lower line energies as they are observed: their current apparently large widths would be the effect of time integration, and/or of the blending of lines from CBs with different Doppler factors.

Key words: supernovae, gamma-ray bursts, X-ray lines

1. Introduction

X-ray lines with energies of a few keV, with very large flux and equivalent width, have been observed in the early-afterglow phase of gamma-ray bursts (GRBs), by the satellites BeppoSAX (GRB 970508, Piro et al. 1998; GRB 000214, Antonelli et al. 2000), ASCA (GRB 970828, Yoshida et al. 1999) and Chandra (GRB 991216, Piro et al. 2000a). In spite of their large intensities, these lines are near the detection limits, making their empirical study difficult, and their interpretation debatable.

X-ray lines of similar energies, emitted by various astrophysical systems, are usually interpreted as Fe emission lines. Iron X-ray lines from a GRB environment have been discussed in the hypernova GRB scenario by Ghisellini et al. (1999) and by Boettcher et al. (1999). But it has been argued (e.g. Lazzati et al. 1999) that the corresponding models cannot produce strong Fe-line emission and that in the usual surrounding of GRB progenitors —star-formation regions— a putative Fe-line emission strong enough to be detectable should last for years, rather than for a day or two, as the observations indicate. These difficulties were claimed not to be present (Lazzati et al. 1999; Vietri et al. 1999; Piro et al. 2000a) if a special geometry of the GRB circumstellar material is assumed: a torus perpendicular to the line of sight, of radius $\sim 10^{16}$ cm and density in excess of $10^{10}\,\mathrm{cm}^{-3}$, made of a few solar masses of iron-rich material. Such a torus might be produced in the supranova model of GRBs (Vietri and Stella 1998), though the model still faces many difficulties (Vietri et al. 2001).

The X-ray lines observed in the afterglows of GRB 970508 and 991216 can, within large errors, be interpreted as Fe lines. In the case of GRB 970828, the observed X-ray line energy (Yoshida el al. 1999) corresponds to $z \sim 0.38$ for a Ly$_\alpha$ line from fully-ionized Fe (or a smaller z for partial ionization), while the measured redshift is $z = 0.957$. If the line is interpreted as the 9.23 keV recombination edge of fully-ionized Fe, the redshift is 0.84 (or smaller for incomplete ionization). But, surprisingly, no K$_\alpha$ line was observed between 3.25 and 3.5 keV, the range, at $z = 0.84$, expected for singly-ionized to fully-ionized Fe. No independent measurement of redshift is available for GRB 000214. All in all, the Fe-line interpretation is not established.

Here we offer an alternative interpretation of the X-ray emission lines in the GRB afterglows, an inevitable sequitur in the cannonball (CB) model of GRBs (Dar and De Rújula 2000a,b). In the CB model, GRBs are produced by highly relativistic "cannonballs", jetted by nascent or dying compact stellar objects. The CBs heat up as they pierce through nearby circumstellar shells, and their radiation is Lorentz-boosted to γ-ray energies by the CBs' fast motion. Later (after a day or two in the observer's frame) the CBs cool sufficiently, by expansion and radiation, for their constituent electrons and protons to recombine, producing a strong Ly$_\alpha$ line emission, Doppler-shifted to X-ray energies in the observer's frame[1]. We show that this interpretation is consistent with the GRB observations and that the CB model can correctly explain all the observed features of the lines. Our main prediction is that the lines should be narrow and move in time from higher to lower frequency, their currently observed large widths being the effect of integrating over this migration in time, or of the blending of the emission from unresolved CBs with different Doppler factors. We also argue that self-absorption and scattering of the recombination lines in the CB produce a multiband flare-up of the afterglow, and we discuss the observational evidence for such a flare in the early afterglow of GRBs.

2. The Cannonball Model of GRBs

2.1. The engine

In the cannonball model of GRBs (Dar and De Rújula 2000a,b), we assume that large and sudden mass-accretion episodes onto compact stellar objects lead to the abrupt ejection of highly relativistic oppositely-directed pairs of CBs, as observed in microquasars (see, for instance, Mirabel and Rodriguez 1994, 1999a,b, Belloni et al. 1997; Rodriguez and Mirabel 1999 and references therein). Such mass-accretion episodes can take place in single-bang supernovae (Shaviv and Dar 1995; Dar 1998; Dar and Plaga 1999; Cen 1999), in double-bang supernovae (De Rújula 1987; Woosley 1993, Woosley and MacFadyen 1999; MacFadyen and Woosley 1999; MacFadyen et al. 1999; Dar and De Rújula 2000a,b), in binaries including a compact object, in mergers of neutron stars with neutron stars or black holes (Paczynski 1986, Goodman et al. 1987), in transitions of neutron stars to hyperon- or quark-stars (Dar 1999; Dar and De Rújula, 2000c), etc.

The correspondance in time and location between GRB 980425 and SN 1998bw establishes their association quite clearly: the chance probability for the coincidence is less than 10^{-4} (e.g. Galama et al. 1998), or much smaller if the revised BeppoSAX position (e.g. Pian et al.1999) is used in the estimate. Assuming that SN 1998bw is close to a standard candle —for SNe associated with GRBs— it is straightforward to derive what its light curve as a function of time and frequency would be, if viewed at a different redshift. Of the sixteen GRBs with known redshift, four or five have a modulation of their afterglow, about one month after the GRB signal, that is very well described by the addition of such a SN 1998bw-like signal. In all other cases there is a good reason why such an effect would be unobservable, e.g. the afterglow or the background galaxy are too luminous, the data on the late afterglow are unavailable, or the measured frequency range in SN 1998bw is insufficient to extrapolate to the required redshift. The conclusion that more than 20% of GRBs are associated with SNe, and perhaps *all* of them, appears to be inescapable (Dar and De Rújula 2000a). This is correct at least for the long-duration GRBs for which it has been possible to establish the existence of an afterglow. It is conceivable that the short-duration GRBs are not associated with (core-collapse) SNe, but with the other stellar transitions referred to in the previous paragraph. In what follows we concentrate on the CB model for GRBs associated with SNe, the ones for which we have enough information to predict the properties of X-ray lines and flares in their afterglow.

In the CB model the jetted CBs would generate GRBs by hitting ejecta, a stellar wind, or an envelope. Each CB corresponds to a single pulse in a GRB light curve. The average observed number of significant GRB pulses is $\mathcal{O}$ (5 to 10), so that a "jet" of CBs would typically carry that many times the energy of a single CB. After a few CBs are emitted by the rather random and currently unpredictable accretion-emission process, the supply of accreting material is exhausted and the γ-ray activity ceases (Dar and De Rújula, 2000a,b).

The kinetic energy of the CBs has been estimated from the assumption that momentum imbalance between the opposite-direction jets in SN events is responsible for the observed large peculiar velocities of the resulting neutron stars: $\mathrm{v_{NS}} \approx 450 \pm 90$ km s^{-1} (Lyne and Lorimer 1994). Such natal kick velocities imply $\mathrm{E_{jet}} \sim 10^{53}$ erg, or $\mathrm{E_{CB}} \sim 10^{52}$ erg for a typical jet consisting of ~ 10 cannonballs (Dar and Plaga 1999, Dar and De Rújula 2000a,b). For this choice, the CB's mass is relatively small: $\sim 1.86\,\mathrm{M_\otimes}$ for the Lorentz factor $\gamma = \mathcal{O}(10^3)$ which we independently estimated from the GRB rate and the GRB-supernova association, from the average fluence per pulse in GRBs observed by BATSE, and from the typical duration of these pulses. We presume the composition of CBs to be "baryonic", as it is in the jets of SS 433, from which Ly_α and Fe K_α lines have been detected (Margon 1984, Kotani et al. 1996), although the violence of the relativistic jetting-process may break many nuclei into their constituents. The baryonic number of the CB is

$$\mathrm{N_b} \simeq \frac{\mathrm{E_{CB}}}{\mathrm{m_p\,c^2}\,\gamma} \simeq 6.7\times10^{51}\left[\frac{\mathrm{E_{CB}}}{10^{52}\,\mathrm{erg}}\right]\left[\frac{10^3}{\gamma}\right]. \tag{1}$$

The collision of the CBs with a SN shell is so violent —at ~ 1 TeV per nucleon— that there is no doubt that, as it exists the shell, the CBs' baryonic number resides in individual protons and neutrons.

2.2. The GRB

As they hit intervening material, such as a massive shell, the CBs heat up. Their radiation is obscured by the shell up to a distance of order one radiation length from the shell's outer surface. As this point is reached, the approximately thermal surface radiation from the CBs —

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[1] Analogous Doppler-shifted Ly_α and K_α recombination lines have been detected from the relativistic jets of SS 433 (e.g. Margon 1984; Kotani et al. 1996).

which continue to travel, expand and cool down— becomes visible. The radiation of a single CB, boosted and forward-collimated by its ultrarelativistic motion, and time-contracted by relativistic aberration (relative to the frame of an observer situated close to the CB's line of flight), appears as a single pulse in a GRB. The observed duration of a single CB pulse is its cooling time after it becomes visible.

Approximately one third of the centre-of-mass energy of a CB's collision with the intervening material is converted in the hadronic cascades within the CB into electromagnetic radiation (via π^0 production and $\pi^0 \to \gamma\gamma$ decay) and two thirds escapes the CB as neutrinos and muons. The enclosed electromagnetic energy heats up the CB to a (proper) temperature in the keV domain. Let $\gamma = \sqrt{1-\beta^2}$ be the bulk Lorentz factor of a CB with velocity $\mathrm{v} = \beta\,\mathrm{c}$. The Doppler factor by which the radiation from its surface is boosted is given by

$$\delta = \frac{1}{\gamma\,(1-\beta\cos\theta)} \approx \frac{2\gamma}{1+\gamma^2\theta^2}\;, \tag{2}$$

where θ is the viewing angle relative to the CB's velocity vector, and the approximation is valid for $\gamma \gg 1$ and $\theta \ll 1$, the case of interest here.

Let the total energy isotropically radiated by a CB's surface, in its rest system, be $\mathrm{E_0}$. This radiation is boosted and collimated by the CB's motion, its time dependence is modified by the observer's time flowing $(1+\mathrm{z})/\delta$ times faster than in the CB's rest system. The fluence seen by an observer at redshift z and luminosity distance $\mathrm{D_L}$ is

$$\frac{\mathrm{d\,F}}{\mathrm{d}\,\Omega} \simeq \frac{\mathrm{E_0}\,(1+\mathrm{z})}{4\,\pi\,\mathrm{D_L^2}}\,\delta^3\,\mathrm{A}(\nu,\mathrm{z})\,, \tag{3}$$

where $\mathrm{A}(\nu,\mathrm{z})$ is the attenuation along the line of sight.

As long as the CB is opaque to its internal radiation, it expands in its rest system at a velocity comparable to the relativistic speed of sound, $\beta_\mathrm{T}\,\mathrm{c}$, with $\beta_\mathrm{T} \lesssim 1/\sqrt{3}$. Because of its relativistic expansion, the possibly distorted shape of a CB —as it is emitted or as it first collides with the SN shell— approaches a spherical shape at a later time; we shall treat CBs as spherical objects and approximate their properties as if they were uniform. An expanding CB cools quasi-adiabatically, so that its temperature decreases as $\mathrm{T} \propto 1/\mathrm{R_{CB}} \propto 1/\mathrm{t}$. When a CB of mass $\mathrm{M_{CB}} \sim 2\,\mathrm{M}_\otimes$ reaches a radius $\mathrm{R_{CB}^{trans}} \simeq [3\,\mathrm{M_{CB}}\,\sigma_\mathrm{T}/(4\,\pi\,\mathrm{m_p})]^{1/2} \sim 3\times10^{13}$ cm (with $\sigma_\mathrm{T} \simeq 0.65\times10^{-24}$ cm^2 the Thomson cross section) it becomes optically thin and its remaining, cooled-down internal radiation escapes. This end to the CB's γ-ray pulse takes place at $\mathrm{t} \simeq (1+\mathrm{z})\,\mathrm{R_{CB}^{trans}}/(\beta_\mathrm{T}\,\mathrm{c}\,\delta) \simeq 3\,(1+\mathrm{z})\,(1/[\sqrt{3}\,\beta_\mathrm{T}])\,(\mathrm{M_{CB}}/\mathrm{M}_\otimes)^{1/2}\,(10^3/\delta)$ seconds in the observer frame. The spectral and temporal properties of GRB pulses are well reproduced by this cannonball model of GRBs, as demonstrated in detail in Dar and De Rújula 2000b.

2.3. The afterglow at X-ray energies

When the radiation enclosed in a CB escapes, its internal radiation pressure drops abruptly and its transverse expansion rate is slowed down by collisions with the interstellar medium. During this phase, the CB cools mainly by emission of bremsstrahlung and synchrotron radiation at an approximate rate (see, for instance, Peebles 1993)

$$\mathrm{L_{brem}} \simeq 1.43\times10^{-27}\,\mathrm{n_e^2}\,\mathrm{T}^{1/2}\,\mathrm{erg\,cm^{-3}\,s^{-1}}\,, \tag{4}$$

where, here and in what follows, $\mathrm{n_e}$ is in c.g.s. units and T is in degrees Kelvin. The above cooling rate corresponds, in the observer's frame, to a cooling time of the CB to temperature T

$$\mathrm{t_{brem}} \simeq \frac{2.9\times10^{11}\,(1+\mathrm{z})\,\mathrm{T}^{1/2}}{\mathrm{n_e}\,\delta}\,\mathrm{s}\,. \tag{5}$$

Let n be the baryon number density in the CB. The fractional ionization, $\mathrm{x} \equiv \mathrm{n_e}/\mathrm{n}$, approximately satisfies the equilibrium Saha equation

$$\frac{\mathrm{x}^2}{1-\mathrm{x}} = \frac{(2\,\pi\,\mathrm{m_e\,c^2\,k\,T})^{3/2}}{\mathrm{n\,h^3\,c^3}}\,\mathrm{e}^{-\mathrm{B/k\,T}}\,, \tag{6}$$

where B = 13.6 eV is the hydrogen binding energy.

Given that the X-ray lines are observed to appear in the afterglow after ~ 1 day, we can use Eqs. 5 and 6 to estimate the electron density at recombination time ($\mathrm{x} \sim 1/2$) to be

$$10^5\,\mathrm{cm^{-3}} < \mathrm{n_e} < 10^6\,\mathrm{cm^{-3}}\,. \tag{7}$$

For electron densities in this range, the exponential term in the Saha equation circumscribes the CB's recombination phase to a temperature around 4500 K. The radius of the CB at peak recombination ($\mathrm{x} = 1/2$) can be estimated from the baryon number of Eq. 1 and the electron density of Eq. 7 to be

$$\mathrm{R_{CB}^{rec}} = \left[\frac{3\,\mathrm{N_b}}{8\,\pi\,\mathrm{n_e}}\right]^{\frac{1}{3}} \sim (1\,\mathrm{to}\,2)\times10^{15}\ \mathrm{cm}, \tag{8}$$

a result that the cubic root makes relatively insensitive to the input parameters. After recombination, bremsstrahlung emission from the CB drops drastically and bremsstrahlung and synchrotron emission from the swept up interstellar medium (ISM) take over, as we proceed to discuss.

Far from their source, the CBs are slowed down by the ISM they sweep, which has been previously ionized by the forward-beamed CB radiation (travelling essentially at v = c, the CB is "catching up" with this radiation, so that the ISM has no time to recombine). As in the jets and lobes of quasars, a fraction of the swept-up ionized particles are "Fermi-accelerated" to cosmic-ray energies and confined to the CB by its turbulent magnetic field, maintained by the same confined cosmic rays (Dar and Plaga 1999). The synchrotron emission from the accelerated electrons, boosted by the relativistic bulk motion of the CB, produces afterglows in all bands between radio and X-rays, collimated within an angle $\sim 1/\gamma(t)$ that widens as the ISM decelerates the CB and its Lorentz factor $\gamma(t)$ diminishes. "Late" X-ray afterglows have been observed up to a week after the burst, after which their flux becomes too weak to be detected (e.g., Costa 1999).

A CB of roughly constant cross section, moving in a previously ionized ISM of roughly constant density, would lose momentum at a roughly constant rate, independently of whether the ISM constituents are rescattered isotropically in the CB's rest frame, or their mass is added to that of the CB. The pace of CB slowdown is $d\gamma/dx = -\gamma^2/x_0$, with $x_0 = M_{CB}/(\pi R_{CB}^2\, n_{\rm ISM}\, m_p)$ and $n_{\rm ISM}$ the number density along the CB trajectory. For $\gamma^2 \gg 1$, the relation between the length of travel dx and the (red-shifted, relativistically aberrant) time of an observer at a small angle θ is $dx = [2\,c\,\gamma^2/(1+\theta^2\,\gamma^2)]\,[dt/(1+z)]$. Inserting this into $d\gamma/dx$ and integrating, we obtain:

$$\frac{1+3\,\theta^2\gamma^2}{3\,\gamma^3} = \frac{1+3\,\theta^2\gamma_0^2}{3\,\gamma_0^3} + \frac{2\,c\,t}{(1+z)\,x_0}\,, \tag{9}$$

where γ_0 is the Lorentz factor of the CB as it exits the SN shell. The real root $\gamma = \gamma(t)$ of the cubic Eq. 9 describes the CB slowdown with observer's time.

In the rest frame of the CB, the particles of the ionized ISM impinge on it with energy $\gamma\, m\, c^2$. If the CB is host to a turbulent magnetic field, the incoming particles are scattered and Fermi-accelerated to a power-law energy distribution, their bremsstrahlung and synchrotron radiation powering the CB's late afterglow. It is natural to assume that in a constant-density ISM a quasi-equilibrium is reached between the number of ISM particles entering and escaping the CB, so that its radiation is approximately constant in the CB's rest frame. This ought to be a good approximation only for frequencies at which the CB is transparent to its own radiation: absorption should be relevant at the lower (radio) end of the observable spectrum. The spectral shape of the emitted synchrotron radiation is $F_0 \propto \nu_0^{-\alpha}$, with $\alpha = (p-1)/2$ and p the spectral index of the electrons. For equilibrium between Fermi acceleration and synchrotron and Compton cooling, $p \approx 3.2$ and $\alpha \approx 1.1$, while for small cooling rates, $p \approx 2.2$ and $\alpha \approx 0.6$ (Dar and De Rújula 2000b), or $p \simeq 1.2$ and $\alpha \simeq 0.1$ if Coulomb losses dominate. At very low radio frequencies, self-absorption becomes important and $\alpha \approx -1/3$ (2.1) for optically thin (thick) CBs. For a detailed modelling of a very similar phenomenon: synchrotron radiation from quasar lobes, see, for instance, Meisenheimer et al. (1989).

The time-dependence of F_0 should be of the form $t^{-\beta}$. For steady emission, i.e. for a small energy-deposition rate by the ISM particles and a correspondingly small emission rate, $\beta \simeq 0$. For an emission rate which is in equilibrium with the energy-deposition rate in the CB rest frame (which is proportional to γ^2), $\beta \simeq 2$. All in all, the CB-model's prediction for the "quasi-steady" state of the CB's evolution —from recombination to a final stopping "Sedov–Taylor" phase— is that the energy density flux in the observer's frame is approximately of the form

$$F[\nu,t] \propto \left[\frac{2\gamma(t)}{1+[\gamma(t)\,\theta]^2}\right]^{3+\alpha+\beta} \nu^{-\alpha}\, A(\nu,z)\,, \tag{10}$$

where $A(\nu,z)$, as in Eq. 3, is the attenuation along the line of sight, and $\gamma(t)$ is the solution to Eq. 9. In Eq. 10 we have used Eq. 2 for δ, to expose how the flux is forward-collimated in a "beaming cone" of opening angle $1/\gamma(t)$.

The temporal behaviour of the GRB afterglow of Eq. 10 depends on the relation between the viewing angle θ and the initial Lorentz factor γ_0. If an observer is initially outside the beaming cone ($\theta\,\gamma_0 > 1$), since $\gamma(t)$ is a decreasing function of time, the afterglow would increase up to a peak time $t = t_p$, for which $\theta\,\gamma(t_p) = 1$, and the beaming cone has widened to the observer's angle. Thereafter the afterglow decreases. This is the behaviour observed, for instance, in GRB 970508 (Dar and De Rújula, 2000a). Beyond the peak, or for observers within the beaming cone from the onset of the burst, the afterglow's flux is a monotonously decreasing function of time. For $\gamma^2\theta^2 \gg 1$ and $\gamma \propto t^{-1/3}$ (the asymptotic behaviour of the solution to Eq. 9), the afterglow declines like

$$F[\nu,t] \propto \left[\frac{(t_p/t)^{1/3}}{1+(t_p/t)^{2/3}}\right]^{3+\alpha+\beta} A(\nu,z)\,, \tag{11}$$

with $0 \le \beta \le 2$ (Dar and De Rújula 2000a,b). For radio afterglows the absorption correction is important, not only in the ISM and the intergalactic medium, but also within the CB itself.

The X-ray lines and multiband flare to be discussed anon are features to be superimposed on the smooth afterglow described by Eq. 11.

3. X-ray lines in GRB afterglows

When the temperature in the CB drops below ~ 5000 K, electrons begin to recombine with protons into hydrogen, causing a strong emission of the Ly_α line (and, possibly, a recombination edge above the Ly_∞ line). These features are Doppler-shifted by the CB's motion to X-ray energies in the observer's frame:

$$E_\alpha \simeq \frac{10.2}{(1+z)}\left[\frac{\delta}{10^3}\right]\ {\rm keV}\,, \tag{12}$$

$$E_\infty \simeq \frac{13.6}{(1+z)}\left[\frac{\delta}{10^3}\right]\ {\rm keV}\,. \tag{13}$$

The total number of these recombination photons is approximately equal to the baryonic number of the CB, given by Eq. 1, so that the photon fluence of these lines (the line fluence) at a luminosity distance D_L is

$$\frac{dN_{\rm lines}}{d\,\Omega} \simeq N_b\,\frac{(1+z)^2\,\delta^2}{4\,\pi\,D_L^2}\,. \tag{14}$$

In the CB model the X-ray lines observed in GRB afterglows are the above Ly_α and/or Ly_∞ lines, not Fe lines or Fe recombination edges from a stationary source.

The redshift, luminosity distance, γ-ray fluence and total γ-ray energy of the GRBs in which lines have been observed are listed in Table I (the quoted "spherical energy" is that deduced under the customary assumption —incorrect in the CB model— that the emission is isotropic). In Table II we report the measured attributes of the X-ray lines and, in Table III, the properties ensuing —in the CB model— from their interpretation as the boosted Ly_α lines of hydrogen recombination. The inferred Doppler factors are consistent with those needed to explain the intensity, energy spectrum and line shapes of the GRB pulses ($\delta \sim 10^3$).

A GRB afterglow can be produced by a single CB, or by the unresolved afterglows of various CBs with different Doppler factors that could also be formed by the merger of CBs with somewhat different velocities. In fact, the 4.4 keV line in the afterglow of GRB 991216 (Piro et al. 2000a) may be either a hydrogen recombination edge from the same CB that produces the 3.49 keV line, or a Ly_α line from a different CB. Similarly, the peak around 2 keV in the spectrum of GRB 990214 (see, for example, Fig. 2. in Antonelli et al. 2000) may be a Doppler-shifted K_α line from another CB with a smaller Doppler factor than that of the CB that produced the 4.7 keV line. Clear assignments will require more precise data.

3.1. Consistency Cheks and Predictions

There are various independent consistency checks and testable predictions of the CB-model's interpretation of the X-ray lines as hydrogen-recombination features that are Doppler-shifted to X-ray energies by the ultrarelativistic motion of the CBs:

a. Time and duration of the line emission: The radiative recombination rate in a hydrogenic plasma to the ground state is $\mathrm{r_{gs}} \approx 2.07\times10^{-11}\,\mathrm{T}^{-1/2}\,\mathrm{n_e\,s^{-1}}$; to any atomic level it is $\mathrm{r_{rec}} \approx 2.52\times10^{-10}\,\mathrm{T}^{-0.7}\,\mathrm{n_e\,s^{-1}}$ (Osterbrock 1989). In the observer's frame the recombination time is

$$\Delta \mathrm{t_{rec}} \approx \frac{4\times10^9\,(1+\mathrm{z})\,\mathrm{T}^{0.7}}{\mathrm{n_e}\,\delta}\ \mathrm{s}\,. \tag{15}$$

At T ~ 4500 K the ratio of $\Delta\mathrm{t_{rec}}$ to the appearance time of the X-ray lines, Eq. 5, is $\Delta\mathrm{t_{rec}}/\mathrm{t_{brem}} \sim 0.07$, insensitive to T and independent of δ, $\mathrm{n_e}$ and z.

The emission time of Ly_α photons, $\Delta\mathrm{t}_{\mathrm{Ly}_\alpha}$, is longer than $\Delta\mathrm{t_{rec}}$, since the CB, at recombination time, is not transparent to them. Their photo-absorption cross section in neutral hydrogen is

$$\sigma_{\mathrm{Ly}_\alpha} \approx \frac{\pi\,\mathrm{e}^2\,\lambda}{\mathrm{m_e\,c^2}}\,\mathrm{f}_{1,2} \simeq 2.1\times10^{-18}\ \mathrm{cm}^2\,, \tag{16}$$

where $\lambda = 1216$ Å, and the oscillator strength for the n = 1 → 2 transition is $\mathrm{f}_{1,2} = 0.194$. At peak recombination ($\mathrm{x} \simeq 1/2$) the optical thickness of the CB to the Ly_α line is

$$\tau = \sigma_{\mathrm{Ly}_\alpha}\,\mathrm{n_e}\,\mathrm{R_{CB}^{rec}} \sim \mathcal{O}\,(10^3)\,, \tag{17}$$

with $\mathrm{R_{CB}^{rec}}$ as in Eq. 8. Since $\tau \gg 1$, the Lyman photons are reabsorbed and re-emitted many times by the recombined hydrogen atoms in the CB before they escape from its surface. The duration of their emission is not the recombination time, but their diffusive residence time. In the observer's frame this time is roughly

$$\begin{aligned}\Delta\mathrm{t}_{\mathrm{Ly}_\alpha} &\simeq (1+\mathrm{z})\,\frac{3\,\mathrm{R_{CB}}\,\tau}{\mathrm{c}\,\delta}\\ &\simeq 10^5\,(1+\mathrm{z})\left[\frac{\mathrm{R_{CB}}}{10^{15}\,\mathrm{cm}}\right]\left[\frac{\tau}{10^3}\right]\left[\frac{10^3}{\delta}\right]\ \mathrm{s}.\end{aligned} \tag{18}$$

This prediction is consistent with the observation that the line emission extended over a day or two in the early afterglows of GRB 970508 (Piro et al. 1999) and GRB 000214 (Antonelli et al. 2000) observed by BeppoSAX, and of GRB 991216 (Piro et al. 2000a) observed by Chandra. It is, however, inconsistent with the brevity ($\Delta_\mathrm{t} \sim 5000\,\mathrm{s}$) of the line emission observed in GRB 970828 (Yoshida et al. 1999), for which we have no a-priori explanation.

The above estimates are valid only for an afterglow made by a single CB (either originally ejected, or the result of mergers). In practice, the afterglows of different CBs in a GRB add up, and cannot be resolved either spatially or temporally. Because of the possibly differing Lorentz and Doppler factors of a succession of CBs, the observed X-ray line emission may extend over a time longer than the expectation of Eq. 18. Comparison with the data is also hampered by the fact that, in some cases, only a lower limit to the duration of the line emission is available. In the case of GRB 991216, for instance, the gamma-ray light curve, as measured by BATSE (see, for instance, Mallozzi 2000) has eight peaks or more, and the line emission was detected by Chandra only during a limited period of 30 ks, beginning 37 hours after the burst (Piro et al. 2000a).

b. Photon fluences: In the three cases for which the redshift to the GRB is known, the observed line energies can be converted into values of the Doppler factor δ using Eq. 12, as we did in Table III. The resulting values of δ agree with those required in the CB model to explain the properties of GRBs and their smooth afterglows. The line fluence of Eq. 14 can be used to estimate the total baryon number in a CB or the ensemble of CBs constituting the jet of a GRB. Since the duration $\Delta\mathrm{t_{obs}}$ of the available observations of X-ray lines around a "central" time $\mathrm{t_{obs}}$ may have been shorter than the actual span $\Delta\mathrm{t}_{\mathrm{Ly}\alpha}$ of the line emission, we can only bound $\mathrm{N_b}$ to an interval corresponding to two extreme assumptions: (a) $\Delta\mathrm{t}_{\mathrm{Ly}\alpha} \sim \Delta\mathrm{t_{obs}}$ and (b) $\Delta\mathrm{t}_{\mathrm{Ly}\alpha} \sim \mathrm{t_{obs}}$. The inferred baryon numbers, listed in Table III, are within the expected range of Eq. 1.

c. Line widths: The thermal broadening of the recombination lines and edge is a rather small effect. But the Doppler factors of the CBs decrease with time during the line emission, owing to the decrease of their Lorentz factors as a result of their deceleration by the ISM. We have seen that sufficiently late into an afterglow $\gamma \propto t^{-1/3}$. Thus, the line energy is expected to shift by an amount

$$\Delta E_L \simeq E_L \, \frac{\Delta t_{obs}}{3 \, t_{obs}} \tag{19}$$

during the observational interval Δt_{obs} around t_{obs}. When integrated over Δt_{obs} to exploit the full statistics, the predicted line shift will appear as a line broadening. The line drifts, ΔE_L, estimated via Eq. 19, are listed in Table III. They are completely consistent with the observed widths, listed in Table II.

A line-broadening may also be caused by the blending of lines from CBs with similar Doppler factors, which is unresolvable in the afterglow phase even if the CBs did not merge into a single CB before e–p recombination. Independently of the origin of the line broadening, the predicted decrease in the line energy with time is a clear fingerprint of the origin of the lines: it may be a decisive signature distinguishing the CB-model's recombination lines from the usually assumed Fe lines.

d. The CB's radius: In Table III we list the radii of the CBs for the different GRBs, approximately one observer day after the γ-ray activity. The figures are deduced from Eq. 8 using the baryon-number values "measured" from the line fluence, via Eq. 14. After one observer day, the CB is already ~ 1 kpc away from the GRB emitter: far from the galactic disk, barring a chance emission in the galactic plane. If a CB continues to expand during the first few observer's weeks with the same mean speed as in the first day or two, as expected for a CB moving in a low-density galactic halo, its radius after a month increases by a factor ~ 30 and should reach a value $R_{CB}[1m] \simeq (3$ to $6) \times 10^{16}$ cm. Indeed, from VLA observations of scintillations (Goodman et al. 1987) in the radio afterglow of GRB 970508 and their disappearance after a month, Taylor et al. (1997) inferred that the linear size, $2\,R_{CB}[1m]$, of its source one month after the burst was $\approx 10^{17}$ cm. This corresponds to $R_{CB}[1m] \sim 5 \times 10^{16}$ cm, in agreement with the expectation.

4. Multiband flare up of the afterglow

The decay of excited hydrogen atoms —produced by recombination or by reabsorption of emission lines— results in a forest of hydrogen emission lines above the Balmer limit. These lines are also shifted by the Doppler factor and the cosmological redshift. This quasi-continuous emission, which accompanies the e–p recombination, appears as a multiband flare in the afterglow, initially in all wavelengths longer than $3.65\,(1+z)\,(10^3/\delta)$ Å. This minimal wavelength increases with time, as δ decreases.

The total energy in the flare is about 50% of the total recombination energy, $E_r \sim N_b\,B/2 \sim 7 \times 10^{40\pm1}$ erg in the CB's rest frame, or $\sim 7 \times 10^{49\pm1}\,(\delta/10^3)^3/(1+z)$ erg equivalent "isotropic" energy in the observer frame. For the only GRB for which a measured total energy in the X-ray afterglow is available (GRB 970508, Piro et al. 1998) the result[2] is $\sim 6.6 \times 10^{49}$ erg, in agreement with the expectation. The total fluence of the forward-collimated and Lorentz-boosted flare of the CB model is given approximately by

$$\frac{d\,F}{d\,\Omega} \simeq \frac{N_b\,B\,(1+z)}{8\,\pi\,D_L^2}\,\delta^3\,A(\nu, z)\,. \tag{20}$$

This recombination flare may explain the ones observed in the afterglow of, for example, GRB 970228 (Masetti et al. 1997); GRB 970508 (Piro et al. 1998); GRB 990123 (Kulkarni et al. 1999); GRB 000301c (Sagar et al. 2000) and perhaps GRB 000926 (Veillet 2000, Piro et al. 2000b,c). The prediction of the spectral and temporal behaviour of the flares requires a detailed modelling of the recombination phase in the expanding and cooling CBs, which we have not performed.

5. Conclusions

We have argued that GRBs and their afterglows may be produced by jets of extremely relativistic cannonballs from the birth or death of compact stellar objects. The cannonballs pierce through a massive shell or other interstellar material, are reheated by their collision with it, and emit highly forward-collimated radiation, which is Doppler-shifted to γ-ray energy. Each cannonball corresponds to an individual pulse in a GRB. The CBs decelerate by sweeping up the ionized interstellar matter in front of them, part of which is accelerated to cosmic-ray energies and emits synchrotron radiation: the afterglow. When the cannonballs cool below 4500 K, electron–proton recombination to hydrogen produces Ly-α lines that are Doppler-shifted to X-ray energies by the CBs' ultrarelativistic motion.

The properties of the X-ray lines discovered so far in the afterglow of four GRBs are completely consistent with this picture. The predicted decrease in the line energy with time is a clear fingerprint of the origin of the lines: it may be a decisive signature distinguishing the CB-model's recombination lines from the usually assumed Fe lines. The Fe-line interpretation has the advantage of implying a redshift value —that can be measured independently— and may or may not strengthen the corresponding hypothesis. It is not inconceivable that the CB model may accommodate Fe lines as well. Indeed, in this model, the GRB emission is not a "delayed" event, as in the "supranova"

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[2] The reported afterglow energy in the 2–10 keV domain is 20% of the 6.6×10^{51} erg GRB energy. The flare energy was $\sim 5\%$ of the integrated X-ray afterglow energy.

model of Vietri and Stella (1998) and Piro et al. (2000a), and Vietri et al. (2001), but occurs only from a few hours to a day after the SN explosion. Very little is empirically known about the emission properties of SNe at such an early time.

Despite their unexpectedly large intensities, the observed lines were near the limit of the BeppoSAX, ASCA and Chandra sensitivities, making the study of their precise properties quite difficult. In particular, the present accuracy is not sufficient to test whether the line energies are redshifted Fe lines (Piro et al. 1999) or Doppler-boosted hydrogen lines. More observations are needed with higher statistics and energy resolution, in order to determine the precise energy, width and temporal evolution of the X-ray lines, and to establish their true identity and production mechanism.

Part of the recombination energy absorbed in the CB produces a multiband flare up of the afterglow, whose energy is mainly in X-rays. These flares should be a common feature of the early afterglow of GRBs. In the case of GRB 970508, the flare energy in X-rays is measured (Piro et al. 1998) and it agrees with the expectation.

The CB model of GRBs gives a good description of the properties of the γ-ray light curves, and of their energy distributions (Dar and De Rújula 2000b). It is also succesful in explaining the properties of their afterglows (Dar and De Rújula 2000a). Plaga (2000) has argued that the empirical relations between GRB variability, luminosity, resdhift and “spike” intensity (Fenimore and Ramirez-Ruiz, 2000) are consequences of the CB model. In this paper we have shown that the model also offers an interesting and predictive explanation of the X-ray lines and multiband flares in the afterglow of GRBs.

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Table I - GRB afterglows with X-ray lines

GRB	z	D_L	F_γ	E_γ	M
970508	0.835	5.70	0.31	0.066	25.7
970828	0.957	6.74	7.4	2.06	—
991216	1.020	7.30	25.6	8.07	24.5
000214	—	—	1.4	—	—

Comments: Redshift z. Luminosity distance D_L in Gpc (for $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$). Fluences F_γ in the 40–700 keV band, in 10^{-5} erg cm^{-2} units. Equivalent isotropic energy E_γ in units of 10^{53} ergs. R-magnitude M of the host galaxy.

Table II - Observed X-Ray line properties

GRB	E_{line}	Width	t_{obs}	Δt_{obs}	I_{line}
970508	3.4	≤ 0.50	26	11	5.0 ± 2.0
970828	5.04	$0.31^{+0.38}_{-0.31}$	128	24	1.9 ± 1.0
991216	3.49	0.23 ± 0.07	139	12	3.2 ± 0.8
991216	4.4	—	139	12	3.8 ± 2.0

Comments: Line energy E_{line} and line width, in keV. Line intensity I_{line} in $\mathrm{cm^{-2}\,s^{-1}}$. Mean observation time t_{obs} and observed line emission period Δt_{obs} in ks.

Table III - CB Properties from the X-Ray lines

GRB	ΔE_L	δ	$N_b^{(a)}$	$N_b^{(b)}$	R_{CB}^{rec}
970508	0.48	612	1.7	8.5	1.0
970828	0.31	967	0.7	4.0	1.1
991216	0.10	691	1.3	15.7	1.6
991216	0.13	800	1.3	15.7	1.8
000214	0.36	≥ 460	—	—	—

Comments: Line drift ΔE_L in keV. Doppler factor δ for the hydrogen Ly_α-line interpretation. Baryon number N_b in 10^{51} units, for: (a) $\Delta t_{Ly\alpha} \sim \Delta t_{obs}$ (b) $\Delta t_{Ly\alpha} \sim t_{obs}$. CB's radius at recombination R_{CB}^{rec}, in 10^{15} cm units.