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Coherent dipole modes for multiple interaction regions.

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Abstract

In the Large Hadron Collider (LHC) two proton beams of similar strength will collide at several interaction points. For a single interaction point it is known that the head-on collision of two equally strong beams with the same betatron tune, excites two coherent dipole modes whose frequencies are different from the frequencies of oscillation of individual particles in the beam. Because of this frequency difference Landau damping does not act on the dipole modes and the beams can be unstable.

In this paper we extend these studies to several interaction points and explore the possibility of cancellation of the dipole coherent modes by carefully adjusting the phase difference between the beams from one collision to the next. We also study the collision of the two beams with LHC optics V 6.1. Special attention should be paid to coherent resonances that are excited due to local phase advance correlations. It will be shown also that a tune split of 0.03 between the two beams suppresses these coherent dipole modes.

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1 Introduction

In a circular collider the motion of particles of each beam is strongly perturbed at the interaction points (IPs) by the electromagnetic field associated with the counter-rotating beam. This beam-beam force excites coherent oscillations of the bunches, i.e. oscillations in which *all* particles in the bunch participate in azimuthal oscillating modes of order m (dipole, quadrupole, sextupole, etc). In addition, the beam-beam force induces an amplitude dependent spread of the oscillation frequencies of individual particles.

For a single head on collision of two proton beams with small beam-beam parameter ξ, the incoherent tune spectrum extends from 0 to $-\xi$, for particles at large and small betatron amplitudes respectively. This tune spread of the oscillation frequencies of individual particles is called the *continuum*. If the colliding beams have approximately equal tunes, sizes and current, two coherent dipole modes exist: the σ-mode at the unperturbed tune and the π-mode whose frequency lies outside of the continuum tune spread. For these modes Landau damping is not possible [1, 2].

Loss of Landau damping of these coherent modes can result in an instability driven by any small impedance component of the vacuum chamber. In particular slow instabilities, such as the resistive wall transverse instability will not be damped and its rise time at top energy can be estimated to be $\tau_{r.w} = 0.2$ s [3]. If the dipole mode is excited a feedback must act on the centroid position to prevent the loss of the beam. Part of the kick of the feedback would be transferred into the incoherent oscillation of individual particles inducing an emittance growth [1]. In the LHC filling scheme the 3000 bunches are very close (with a 25 ns distance). The self-consistent calculation of their closed orbits including beam-beam interaction shows that the closed orbit position will have an unavoidable bunch to bunch offset with rms of 0.1σ [4]. The finite response time of the feedback will introduce errors when correcting the centroid position of a bunch and, in turn, will induce an additional emittance growth.

In general, a coherent motion of a bunch or an ensemble of bunches can only persist, when the particles or bunches can 'organize' and maintain a collective motion for a significant amount of time. Therefore an appropriate choice of the beam parameters could inhibit such a collective and organized motion. A potential cure is to decouple the oscillations of the two beams by separating their fractional tunes by at least $1.5 \times \xi$ [2, 5]. It can be shown [2, 5], that in this case the dipole mode frequencies are inside the continuum and Landau damping is restored. Since the two proton beams in the LHC circulate in separate rings, this can be achieved by operating the two beams at different working points (WP). However, this opens the possibility for another type of resonances, i.e. the two beams coupled together to a two beam coherent resonance [6]. Multiparticle tracking studies have shown that, for the presently proposed LHC working points, all combinations of working points are inside or too close to the stopband of coherent resonances [6]. As desired, Landau damping acts on the coherent modes and the instability is suppressed but at the price of a significant emittance growth. Furthermore, colliding the beams with significantly different fractional tunes could induce a mismatch of the transverse size of the beams (flip-flop effect) that increases with increasing ξ.

Up to this point we have considered only a single, well localized head-on interaction point, with the only option to change the phase relationship of the two beams between two collisions (tunes). For the organization of a collective motion this is the simplest and easiest situation since the same bunches always meet and under the same conditions. Fortunately, the situation at the LHC is much more complicated. First we have to consider four interaction points with experiments. Secondly, these interaction points are not evenly distributed around the circumference, neither considering the collision schedules, nor the phase advances between collisions. Thirdly, due to the large number of bunches, all bunches experience a large number of parasitic, so-called long-range interactions, around the interaction points which further break the symmetry [7]. In this report we want to study the consequences of these complications. Starting with the simplest case, we introduce more symmetry breaking effects in each step of the simulation, i.e. multiple interaction points, phase advance differences, long range effects in different combinations until we arrive at a realistic scenario. The purpose of this exercise is to explore the role of symmetry between the four interaction points, looking for a convenient and robust solution to suppress the coherent dipole modes.

A general introduction of the problem under consideration is given in section 2 and a description of the model used for the simulations is given in section 3. For the presently proposed optics the working points of the two planes are ($Q_x = 64.31, Q_y = 59.32$) for both beams. In section 4, we assume equal phase advance between IPs and explore the possibility of operating one of the beams at ($Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32$) and the other one at ($Q_x^{(2)} = 59.31, Q_y^{(2)} = 64.32$), *i.e.* for the second beam, we interchange the horizontal and vertical integer part in such a way the the beams have different parity of the integer part of the tune for both planes. This solution suppresses the coherent dipole modes. For a realistic optics the phase advance per sector is not equal. In section 5, using LHC optics V 6.0, we explore the suppression of the dipole modes due to the difference in local phase advance. We show that, for both cases, the spectra still show some coherent modes.

In the next step we also include long range interactions in the model. The complete system is studied for LHC optics V 6.1 in section 6, where we see that the local phase advances are too close to a coherent resonance and the beams are unstable. In section 7 we propose as an alternative to introduce a small tune split in the two beams. Finally, in section 8 we summarize the main results of this work.

2 Interaction regions.

In the LHC we have to consider two different types of beam-beam interactions: head on and long range collisions.

2.1 Head on collisions

Head on collisions take place in the centre of the experimental areas. The LHC ring is devided into 8 octants, and if the interaction points were all located in the middle of the octants 1,2,5 and 8, see Fig.1-left, the system would have a 8-fold symmetry. In reality the interaction point 8 is moved by 11.22 m (1.5 times the interbunch spacing of 25 ns) away from the symmetry point of the machine to allow the installation of an asymmetric experiment (LHCb). As a consequence, the LHC no longer has an 8-fold symmetry. The advantage of an 8-fold symmetry is that 8 bunches are sufficient to represent the collision scheme since they form an equivalent class that includes all the bunches that are coupled through their collisions. If this symmetry is broken a complete simulation would require a very large number of bunches. However, for the present study the number of bunches involved is not important. What will be relevant is the phase advance between collision points, which is given by the optics, and the number of collisions that each bunch will suffer which is determined by the number of interaction points. Assuming that IP8 is at the symmetry point is therefore valid and does not change the argumentation.

The part of the ring between two successive interaction points is called a sector, e.g. sector 1-2 is situated between IP1 and IP2. In the nominal configuration for proton collisions the interaction points 1 and 5 house the low-beta experiments with $\beta^* = 0.5$ m, the interaction points 2 and 8 have $\beta^* = 10$ m. The interaction points 1 and 5 are opposite in azimuth and therefore the same pairs of bunches collide there head-on. At IP2 the beams will be separated by about 4σ to produce halo collisions during proton operation since the ALICE experiment is designed for ion operation and cannot handle high event rates.

Let us first consider only the low β^* experiments IP1 and IP5 and denote the phase advance from IP1 to IP5 $\frac{2\pi Q}{2} + \delta\mu$ and from IP5 to IP1 $\frac{2\pi Q}{2} - \delta\mu$, where $\delta\mu$ is the phase advance difference. It has been shown that a phase advance difference between 2 IPs of the order of $\pm\delta\mu = \pm(0.42 \to 0.5) \times \pi$ such that $\cos\delta\mu = (0.25 \to 0)$ can bring the coherent dipole modes back into the continuum [8]. At $\pm\delta\mu = \pm 0.5 \times \pi$ the total difference in the phase advance between the two beams is π and this suppresses the coherent dipole modes. A similar study can be done for any two IPs where the same pairs of bunches collide.

If we assume the phase advance per sector to be always $\Delta\mu = 2\pi Q/8$ an equivalent scenario can be achieved by colliding two beams with different parity of the integer part of the tune. We can get exactly a total phase advance difference of $\delta\mu = \pi$ between the two colliding beams by setting their integer tunes at different parities. Starting at IP1, the beam with odd integer tune will have an extra betatron phase of $\delta\mu = 0.5 \times 2\pi$ at IP5 with respect to the beam with even integer tune. The same will happen from IP5 to IP1.

Both possibilities use the difference in the phase advance between the two beams. Using a realistic optics, one should either correct the asymmetries in the phase advance difference and collide beams with different integer tune parity, or enhance the asymmetries in the phase advance difference to get to $\pm\delta\mu_{x,y} = 0.5 \times \pi$ colliding with equal tunes. We shall study these two possibilities in the first part of the work. To implement any of these solutions one needs a perfect control on local phase advances, including errors, which in practice turns out to be very difficult.

In order to get a better understanding of the role of symmetries we shall first consider the most symmetric situation (most pessimistic case), with equal phase advance per sector and ring, and explore the relevance of the integer tune. Then we shall consider some practical cases with irregular phase advance using LHC optics V 6.0 and V 6.1.

2.2 Long range collisions

The second type of interactions are "long range" or "parasitic" interactions which occur in the common part of the vacuum chamber around the collision points where the two beams are separated but the bunches still experience the electromagnetic force of the opposing beam. For the present bunch spacing and geometrical layout there are about 32 long range encounters at each interaction point where the closed orbits are only about 7.5 σ apart. In Fig.1-right we see an schematic representation of the head-on and long range encounters in the interaction region. Although each of these long range interactions is relatively weak, their large number can have strong effects on the oscillation frequencies of individual particles. The bunches do not form a continuous train but leave gaps for injection and beam abort. This implies a rather complex collision scheme where not all bunches experience the same beam-beam forces [4]. Due to their collision scheme, the long range interactions break the symmetry completely. However, we are interested in the coherent modes from head-on collisions taking place in the symmetry points and it has been shown that different parts of the bunches participate in the coherent modes excited by the head-on or long range collisions respectively [8]. Their contribution to the head-on coherent modes is not that important, however they significantly change the incoherent

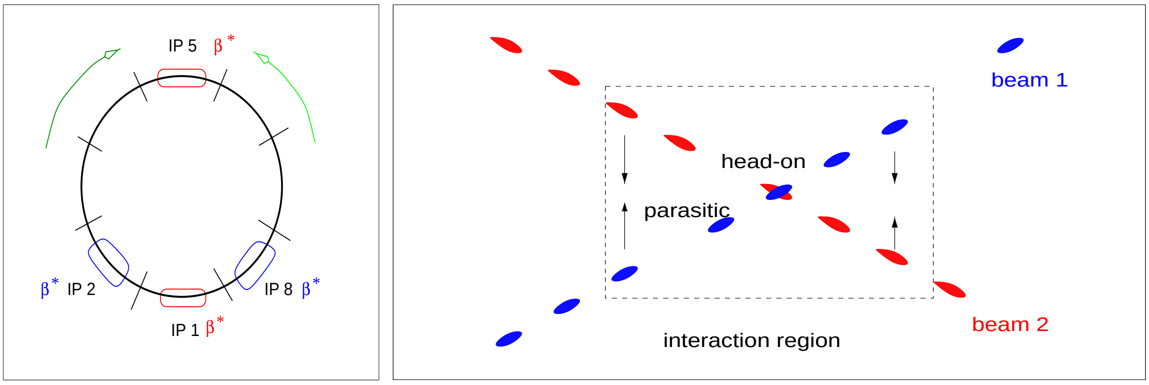

Figure 1: *Left: schematic representation of LHC 4 interaction points, with* $\beta^*_{1,5} = 0.5$ *m and* $\beta^*_{2,8} = 10$ *m. Right: in the interaction region there are about 16 parasitic or long range interactions at each side of the interaction point.*

frequency spectrum and therefore affect the Landau damping.

In this work we shall ignore the filling pattern of the 2808 bunches [4], and study the collisions of a simpler system of up to 8 bunches per beam with various patterns. This small number of bunches is enough to study and understand the dominant effects in the strong-strong collision of the two beams at the 4 IPs, including long range interactions and the offset collision at IP2. We shall study several cases with different levels of approximation and present the results in increasing order of complexity.

3 Simulation model

We simulate the transverse betatron motion of two colliding proton beams of equal strength. Our system of normalized variables is $x = X/\sigma_{oX}$, $v_x = \beta X'/\sigma_{oX}$, $y = Y/\sigma_{oY}$, $v_y = \beta Y'/\sigma_{oY}$ where $\sigma_{oX} = \sigma_{oY} = \sigma$ are the nominal horizontal and vertical rms sizes, and β the beta function at the interaction point. The prime denotes the derivative with respect to the longitudinal direction s, *e.g.* X' is the slope of the horizontal trajectory.

In our simulation, each of the beams has one or more bunches, depending on the symmetry of the problem under investigation. Each bunch is represented by a set of $N = 10^4$ macro-particles, and their trajectories are followed over n turns. We assume linear betatron motion along the sectors and apply the beam-beam head-on and long-range collisions at the IPs. At the IP, each particle in the bunch experiences a deflection caused by the electromagnetic field of the counter-rotating beam. If the beam distribution is Gaussian in the two planes with barycentres at $(<x>^*, <y>^*)$ and squared transverse sizes $M^*_{xx} = <(x - <x>)^2>^*$ and $M^*_{yy} = <(y - <y>)^2>^*$, the beam-beam force can be expressed analytically. We apply a horizontal beam-beam kick at the IP i [9]

$$\Delta v_x(i) = \frac{2 r_p N_p^*}{\gamma} \frac{\beta_x}{\sigma_x^2} F_x(x - <x>^*, y - <y>^*, M^*_{xx}, M^*_{yy}) \quad (1)$$

with r_p the classical proton radius, N_p^* the bunch population (* indicates the counter-rotating beam) , γ the relativistic Lorentz factor, β_x the horizontal betatron function at the IP, σ_x the horizontal rms size and F_x (or equivalently, F_y, for the vertical beam-beam kick) given by

$$F_{\{x,y\}}(x - <x>^*, y - <y>^*, M^*_{xx}, M^*_{yy}) = \{x, y\} \frac{1}{(x^2 + y^2)} \left[1 - \exp\left(-\frac{x^2 + y^2}{M^*_{xx} + M^*_{yy}} \right) \right]. \quad (2)$$

which is the expression for round beams when $M_{xx} \approx M_{yy}$. In the horizontal plane, the beam-beam kick at the IP is

$$\begin{pmatrix} \hat{x}(i) \\ \hat{v}_x(i) \end{pmatrix} = \begin{pmatrix} x(i) \\ v_x(i) + \Delta v_x(i) \end{pmatrix} \tag{3}$$

the linear map from one IP to the next is

$$\begin{pmatrix} x(j) \\ v_x(j) \end{pmatrix} = \begin{pmatrix} \cos(\Delta\mu^x_{ij}) & \sin(\Delta\mu^x_{ij}) \\ -\sin(\Delta\mu^x_{ij}) & \cos(\Delta\mu^x_{ij}) \end{pmatrix} \begin{pmatrix} \hat{x}(i) \\ \hat{v}_x(i) \end{pmatrix}. \tag{4}$$

where $\Delta\mu^x_{ij}$ is the phase advance between two interaction points. Assuming an optics with smooth phase advance then $\Delta\mu^x_{ij} = 2\pi Q_x/8$ per sector, otherwise the phase advance is determined by the optics. An equivalent map is applied for the vertical plane. No linear coupling is assumed.

For the simulation of parasitic (long range) collisions, the same model is employed. The two beams collide with a horizontal separation $L_x = 7.5$ (in units of σ_x). The phase advance between the IP and the long range interactions is approximately 90° for a low-β collision region. Since the betatron phase advances between the long range collisions on one side of an interaction region are very small, and the nominal separation in units of σ is almost the same, we can lump all parasitic collisions to reduce computing time. This slightly overestimates the effect because in general the bunches oscillate with different phases.

A static dipole kick would induce a change of the closed orbit. In our simulation the static kick from the long range collision is not important and must be subtracted to maintain the correct reference system. The long range beam-beam kick becomes

$$\begin{aligned} \Delta v_x(i) = & + n_{par}\frac{2r_pN_p^*}{\gamma}\frac{\beta}{\sigma^2}\left\{\frac{(x- <x>^* -L_x)}{R^2}\left[1-\exp\left(-\frac{R^2}{M^*_{xx}+M^*_{yy}}\right)\right]\right\} \\ & - n_{par}\frac{2r_pN_p^*}{\gamma}\frac{\beta}{\sigma^2}\left\{-\frac{1}{L_x}\left[1-\exp\left(-\frac{L_x^2}{M^*_{xx}+M^*_{yy}}\right)\right]\right\} \end{aligned} \tag{5}$$

where

$$R^2 = (x- <x>^* -L_x)^2 + (y- <y>^*)^2. \tag{6}$$

An equivalent expression is used for vertical long range collisions with separation L_y.

The effective number of parasitic collisions per side of each IP is n_{par} (in our simulations we use $n_{par} = 16$). The kick is the same on both sides of the IP because the betatron phase advance of 180° compensates for the opposite direction of the beam-beam separation and therefore the long range collisions *before* and *after* the IP add up.

The beam-beam parameters are defined by

$$\xi^{(i)}_{x,y} = \frac{N_p^{(i)} r_p \beta_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)} \tag{7}$$

with $i = 1$ for beam 1, and $i = 2$ for beam 2. With the nominal LHC parameters we have $\xi \approx 0.0034$.

In the simulation, the initial coordinates (x, v_x, y, v_y) of two groups of macro-particles representing the two beams are set following two uncorrelated Gaussian distributions with $<x>=<v_x>=<y>=<v_y>=0$ and $<x^2>=<v_x^2>=<y^2>=<v_y^2>=1$.

In our simulation we track $N = 10^4$ macroparticles per bunch up to 2^{15} turns colliding at 4 IPs. We shall consider 4 bunches per beam when we study only the head-on collisions and 4 trains of 2 bunches when we include long range interactions. We apply a linear rotation between IPs (assuming no coupling between the planes) with the phase advances given by the optics and nominal beta functions at the interaction points $\beta^*_{1,5} = 0.5$ m and $\beta^*_{2,8} = 10.0$ m. Fourier analyzing the barycentre of one of the bunches, as calculated turn by turn, we obtain the tune spectrum of the dipole mode. The spectra of the other bunches are equivalent since the motion of all bunches is coupled.

4 Results for equal phase advance per sector

In this section we shall assume that the total phase advance is equally distributed with $\Delta\mu^{x,y}_{ij} = 2\pi Q_{x,y}/8$ per sector. Assuming the most symmetric case of four equally distributed bunches and identical time of flight between the 4 interaction points, we arrive at a collision schedule as shown in Tab.1. Each bunch in one beam collides with

Bunch beam 1	1	2	3	4
IP 1	1	2	3	4
IP 2	2	3	4	1
IP 5	1	2	3	4
IP 8	4	1	2	3

Table 1: *Collision schedule for 4 head-on collision of 4 bunches. Table shows pairs of bunches colliding in the four interaction points.*

three bunches of the other beam due to the asymmetry of the scenario, but all bunches are eventually coupled together, although indirectly.

4.1 Head-on collisions at the 4 LHC interaction points

First we study the collision of 4 bunches against 4 bunches. The collision scheme considered still exhibits a high degree of symmetry with our simplification of head-on collision in the symmetry point for interaction region 8.

- Collision with equal parity in the integer part:
  $(Q^{(1)}_x = 64.31, Q^{(1)}_y = 59.32)$ $(Q^{(2)}_x = 64.31, Q^{(2)}_y = 59.32)$. For beams that experience 4 head-on collisions, the incoherent tune spread (also called the continuum) extends from 0 to $-4 \times \xi$. This can be seen by plotting the spectrum of the dipole mode $S(w)$ as a function of the distance to the unperturbed tune in units of 4ξ with $w = (\nu - q)/(4\xi)$ with q the fractional tune, see Fig.2-left. Two coherent dipole modes can be distinguished, the other coherent dipole modes are degenerated due to the symmetry of the system. The mode located at the unperturbed tune $w = 0$ (σ mode) corresponds to a mode where the colliding bunches oscillate in phase at the IP, the one visible at $w = -1.12$ (π mode) corresponds to a mode where all bunches oscillate with the maximum possible phase difference. The Gaussian approximation

underestimates the tune shift of this mode, for a more realistic tracking model the π-mode frequency would be further outside the continuum at $w = -1.21$. Any coherent mode whose frequency lies outside the continuum cannot be Landau damped. The existence of two coherent modes out of the continuum is dangerous for operation.

- Collision with unequal parity in the integer part: $(Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32)$ $(Q_x^{(2)} = 59.31, Q_y^{(2)} = 64.32)$. Due to the different parity of the integer part of the tune no coherent dipole modes are visible, see Fig.2-right. If the integer tunesplit is an odd number then the contribution from two IP's are exactly out of phase and compensate each other, only the incoherent spectrum remains.

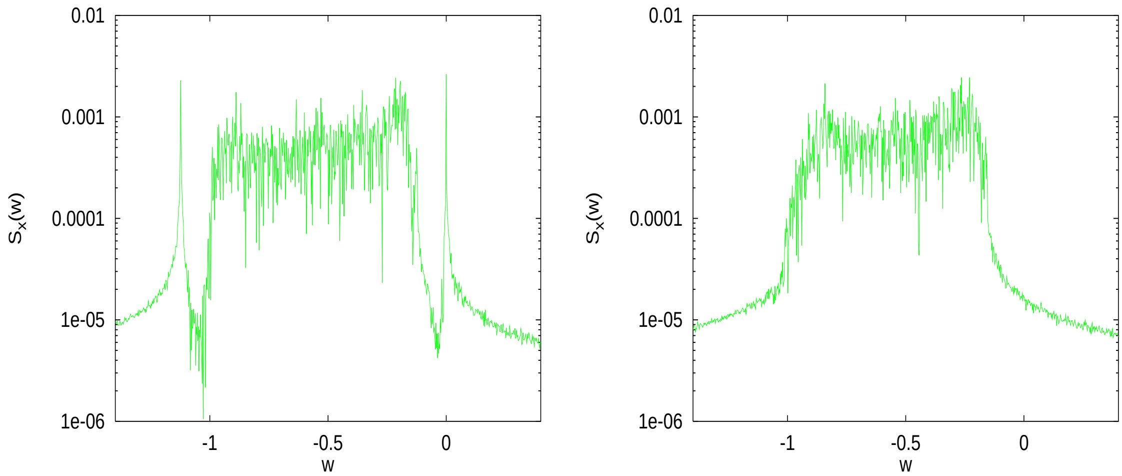

Figure 2: *Collision at 4 interaction points located as in LHC: spectrum of the horizontal oscillations of one bunch as a function of the distance to the unperturbed tune in units of 4 times the beam-beam parameter* $w = (\nu - .31)/4\xi$. *Left: collision with* $(Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32)$ $(Q_x^{(2)} = 64.31, Q_y^{(2)} = 59.32)$. *Right:collision with swapped tunes* :$(Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32)$ $(Q_x^{(2)} = 59.31, Q_y^{(2)} = 64.32)$. *The coherent modes disappear when the beams are operated at integer tunes with different parity.*

4.2 Head-on collision at IP1,IP5,IP8 and halo collisions at IP2

Next we shall consider the collisions of 4 bunches against 4 bunches but with only three head-on collisions at IP1, IP5 and IP8. At IP2 the beams collide with a horizontal offset of 4σ for halo collisions (low luminosity). This offset strongly breaks the symmetry of the system.

- Collisions with equal parity in the integer part: $(Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32)$ $(Q_x^{(2)} = 64.31, Q_y^{(2)} = 59.32)$
  The tune shift induced on the single particles by the 3 head-on collisions alone ranges from 0 to $-3 \times \xi$. Collisions at IP2 take place with an offset of 4σ, so that the tune shift induced by this collision is smaller than the beam-beam parameter ξ. Furthermore, in the horizontal plane this tune shift has the opposite sign of the one induced by head-on collisions and in the vertical plane the same sign. Plotting the spectrum in units of $w = (\nu - q)/3\xi$ we distinguish now 4 coherent modes (originally degenerated) in either plane because of the symmetry breaking between IP2 and IP8. Due to the symmetry between IP1 and IP5 the modes are still degenerated.

Without any symmetry, for the collision of 4 bunches against 4 bunches, one would expect 8 modes per plane.

– Collisions with unequal parity in the integer part:
($Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32$) ($Q_x^{(2)} = 59.31, Q_y^{(2)} = 64.32$). Again, all the coherent modes disappear when the beams are operated at integer tunes with different parity. However, with the presently foreseen powering scheme of the LHC this is not possible.

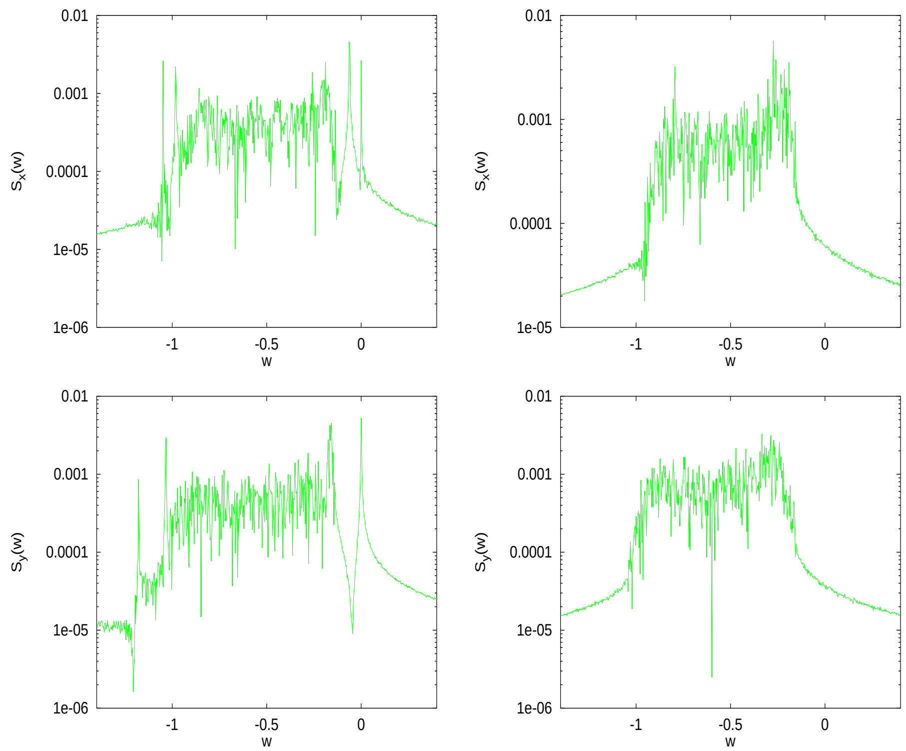

Figure 3: *Collision at 4 LHC interaction points with offset collisions at IP2: spectrum of the horizontal (top) and vertical (bottom) oscillations of one of the bunches, as a function of* $w = (\nu - .31)/3\xi$ *and* $w = (\nu - .32)/3\xi$ *respectively. Left: collision of two beams with* ($Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32$) ($Q_x^{(2)} = 64.31, Q_y^{(2)} = 59.32$)*. Right: collision of two beams with swapped integer tunes in the second beam* ($Q_x^{(1)} = 64.31, Q_y^{(1)} = 59.32$) ($Q_x^{(2)} = 59.31, Q_y^{(2)} = 64.32$)*. The coherent modes disappear if the beams are operated at integer tunes with different parity.*

5 Results for LHC optics V 6.0

In a realistic situation the phase advance per sector will not be the same. It is well known that for a finite number of local and azimuthally distributed distortions like the beam-beam interactions the harmonic contents of these distortions becomes important and these harmonics are calculated to evaluate the strength of incoherent resonances. They are very sensitive to the phase advance between the interaction points and we therefore expect also an effect on our problem. As an example we shall check two optics versions of the LHC.

The first studies were performed with the phase advance given by the LHC optics V 6.0, see Tab. 2. We consider the collisions of 4 bunches against 4 bunches and assume the same phase advance for both beams as if they were in a common ring (*i.e* $\Delta\mu_{ij} = \Delta\mu_{ji}$).

Table 2: *LHC optics V 6.0 Phase advance from IPi to IPj.*

$i \to j$	$\Delta\mu^x_{ij}[2\pi]$	$\Delta\mu^y_{ij}[2\pi]$
$1 \to 2$	8.461	7.446
$2 \to 5$	23.879	22.324
$5 \to 8$	23.954	21.527
$8 \to 1$	8.016	8.023
total $1 \to 1$	64.31	59.32

5.1 Head-on collisions at the 4 LHC interaction points

We collide 4 against 4 bunches with 4 head-on collisions. For round beams it has been predicted that a phase difference $\delta\mu$ such that $\cos\delta\mu = (0.25 \to 0)$ [8] is sufficient to shift the coherent modes into the continuum. In the horizontal plane, and for this particular optics, between IPs 1 and 5 we have a phase difference of $\Delta\mu^x_{15} - \Delta\mu^x_{51} = 2\delta\mu_x = 0.37 \times 2\pi$ so that $\cos\delta\mu_x = 0.39$ and the IP's 2-8 have a phase difference of $\Delta\mu^x_{28} - \Delta\mu^x_{82} = 2\delta\mu_x = 0.36 \times 2\pi$ so that $\cos\delta\mu_x = 0.42$ which, with this soft-Gaussian model that understimates the modes tune shift, is sufficient to suppress all the modes, see Fig.4. In the vertical plane $\Delta\mu^y_{28} - \Delta\mu^y_{82} = 2\delta\mu_y = 0.38 \times 2\pi$ so that $\cos\delta\mu_x = 0.37$ suppresses the IP2-8 modes but the phase difference is not sufficient to suppress the IP1-5 modes: $\Delta\mu^y_{15} - \Delta\mu^y_{51} = 2\delta\mu_y = 0.22 \times 2\pi$ with $\cos\delta\mu_x = 0.77$. One can therefore still distinguish 2 coherent modes in the vertical plane. This is an example that carefully adjusting the phase between two IPs can suppress the coherent dipole modes.

5.2 Head-on collisions at IP1,IP5,IP8 and offset collision at IP2

We shall consider the collisions of 4 bunches against 4 bunches with 3 head-on collisions and 1 halo collision. Now due to the offset introduced at IP2, IP8 and IP2 are no longer equivalent and therefore the two modes are not suppressed in the horizonal plane (IP1 and IP5 still suppress one another). We see 2 of the 4 existing modes in the horizontal plane. In the vertical plane where there is no suppression, due to the symmetry breaking, 4 modes are visible instead of 2, see Fig.5.

The mechanism of dipole suppression by local phase difference is equivalent to the one of the odd integer tune difference. Both require a tight control of local phase advances and would suppress only half the coherent modes.

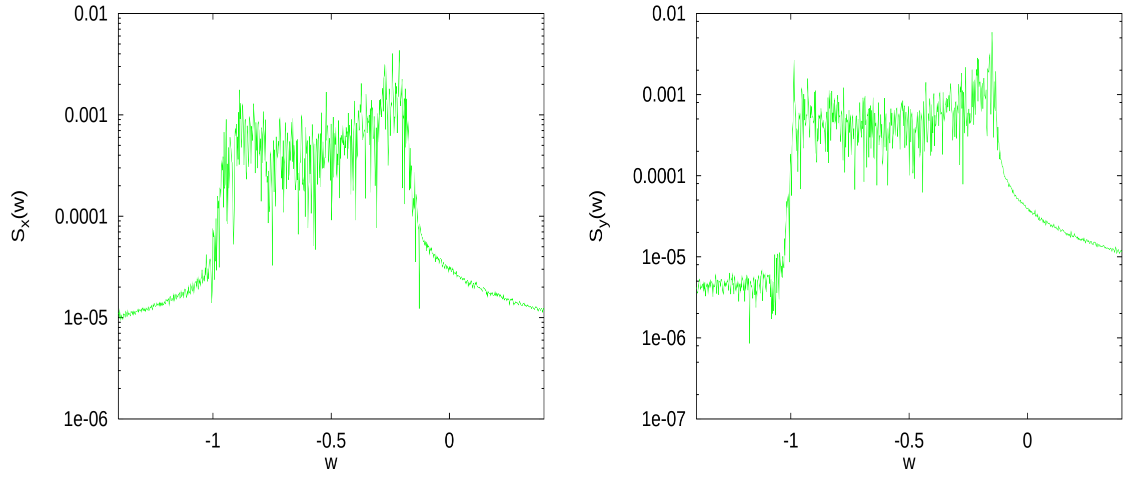

Figure 4: *Head-on collision at the 4 LHC interaction points: spectrum of horizontal (left) and vertical (right) oscillations of one of the bunches, as a function of* $w = (\nu - .31)/4\xi$ *(left) and* $w = (\nu - .32)/4\xi$ *(right) using the phase advance of LHC optics V 6.0 beam 1 for both beams. The coherent modes can be seen in the vertical plane, but disappear in the horizontal plane because of phase advance differences.*

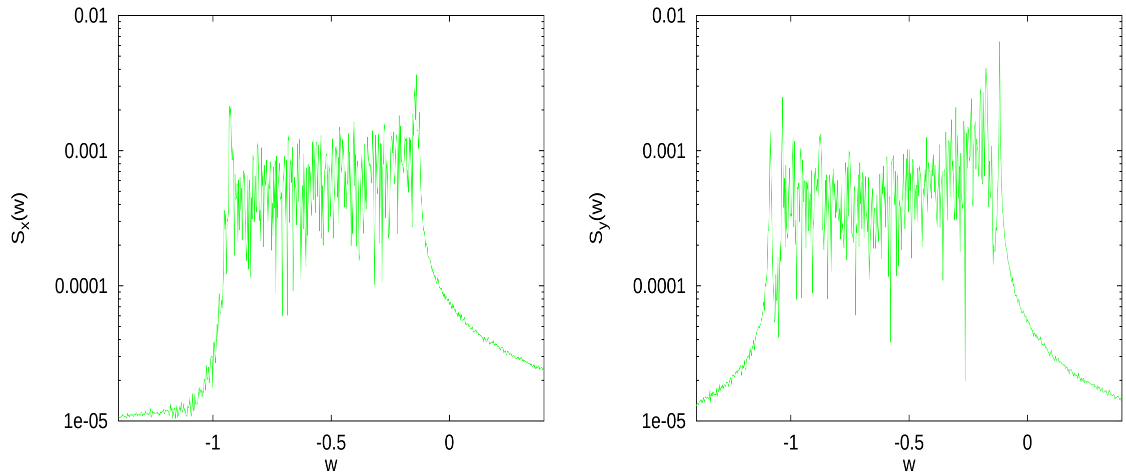

Figure 5: *Head-on collision at IP1,IP5,IP8 and offset collision at IP2: spectrum of horizontal (left) and vertical (right) oscillations of one of the bunches, as a function of* $w = (\nu - .31)/3\xi$ *(left) and* $w = (\nu - .32)/3\xi$ *(right) using the phase advance of LHC optics V 6.0 for both beams. As the symmetry IP2-8 is broken more coherent modes can be seen in both planes.*

6 LHC optics V 6.1 of beam 1 and 2, halo collisions at IP2 and long range interactions.

For the LHC optics version 6.0 we have assumed identical theoretical optics for ring 1 and 2, as it would be the case for a single aperture collider with opposite sign particles. However, in the LHC this is not the case and from optics version 6.1 the optical parameters for both rings are available separately. Phase advance differences are therefore design features rather than avoidable imperfections or technical constraints.

In the collision of the two LHC beams it will be unavoidable to have long-range interactions. These collisions will considerably change the continuum (frequencies of oscillations of individual particles) and will therefore have consequences on the coherent modes frequencies. This has to be taken into account for any realistic representation of the collision scheme. For the following studies we shall consider the next version of the LHC optics, V 6.1, this time taking into account that the two beams have different optics between collision points.

To study long range collisions we track 4 trains per beam each with 2 bunches, see Fig.6. Due to the symmetry of the IPs we expect up to 8 modes per plane. Whenever two trains of opposite beams are in an interaction region, we first collide head-on the leading bunches of the two trains at the IP. Then the bunches advance by $\pm 90°$ with respect to the IP and we apply the long range interaction between the leading bunch of one train and the second bunch of the other. Another phase advance of 90° brings the bunches at the end of the trains to the interaction point where these undergo a head-on collision. With this scheme all bunches suffer long range interactions only either *after* the IP or *before*. We therefore lump all the parasitic collisions whenever a bunch experiences this interaction.

Since the tune shifts from long range collisions have opposite sign in the two transverse planes, an alternating crossing scheme was proposed where the beams are separated in orthogonal planes at different IPs. Therefore long range collisions have vertical separation at IP1 and IP2, and horizontal separation at IP5 and IP8. We have applied the

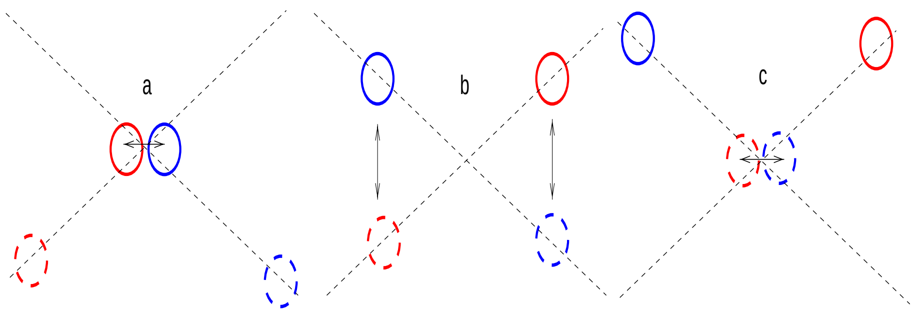

Figure 6: *View of the head-on and long range interaction scheme for a train of 2 bunches. First the leading bunches of the train collide head-on (a), then the bunches have a long range interaction with a separation of* 7.5σ *in the vertical or horizontal plane and a phase advance of* $\pm 90°$ *with respect to the IP (b) and then the bunches in the tails of the train collide head-on.*

previous collision scheme using the LHC optics V 6.1 now taking also into account the different phase advance of each beam as shown in Tab. 3. Our system consists of two beams which are coupled by the beam-beam interaction. Not only we have to avoid resonances exciting incoherent motion of single particles and the coherent modes of one beam

Table 3: *LHC optics V 6.1 with different phase advance for each beam. Phase advance from IPi to IPj*

beam 1 $i \to j$	$\Delta\mu^x_{ij}[2\pi]$	$\Delta\mu^y_{ij}[2\pi]$	beam 2 $i \to j$	$\Delta\mu^x_{ij}[2\pi]$	$\Delta\mu^y_{ij}[2\pi]$
$1 \to 2$	8.5200	7.5298	$2 \to 1$	8.1044	8.0984
$2 \to 5$	23.653	22.2152	$5 \to 2$	24.0687	21.6465
$5 \to 8$	23.9325	21.3819	$8 \to 5$	23.5695	21.9604
$8 \to 1$	8.2046	8.1931	$1 \to 8$	8.5676	7.6147
total $1 \to 1$	64.31	59.32	total $1 \to 1$	64.31	59.32

(dipolar, quadrupolar, sextupolar, etc), but also the ones exciting the coupled coherent modes. For a single IP these types of resonances have been studied by multiparticle tracking to check the possibility of using different working points for the two beams [6]. We need a parameter to measure the proximity of a coupled coherent mode to the resonance. Let Q_a be the frequency of a single beam coherent mode, and Q_b the frequency of a coherent mode in the other beam, then we define Δ as the distance to the resonance of order m ($m = 1, 2, 3..$) in units of ξ, $\Delta = \frac{n-(Q_a+Q_b)}{-m|\xi|}$ where n is an integer number. For each resonance a stopband can be calculated. Within this stopband the coupled coherent mode $Q_a + Q_b$ will be excited. For one IP, it was empirically found that the stopband of instability lies typically in the range $0.2 < \Delta < 0.8$.

The same considerations can be applied to 4 interaction points, taking instead the combinations of local phase advances. Unfortunately some of the combinations of the local phase advances of the LHC optics V 6.1 are too close to a coherent resonance. In particular $\Delta\mu^y_{18}[2\pi] + \Delta\mu^y_{58}[2\pi] = 0.9966$ seems to be inside the stopband of the coupled dipole modes $\Delta = \frac{29-(\Delta\mu_{18}[2\pi]+\Delta\mu_{58}[2\pi])}{2(3\times\xi)} = 0.17$. We observe the signature of a resonance: a strong excitation of the dipole mode and simultaneously a strong emittance growth can be seen in Fig.7 (together with another case which will be explained later). Due to that the coherent dipole modes are not completely Landau damped by the continuum. If these modes are excited their amplitude increases and drives the beam to instability. The observed growth time of the dipole mode is about 0.18 s. We show in Fig.8 the spectrum as obtained from a Fourier analysis of the first 4096 turns.

We can avoid the resonance and move into a stable situation by shifting some of the local vertical phase advances (leaving the total tunes unchanged at $Q_x = 64.31$ and $Q_y = 59.32$). For example using in beam 2 $\Delta\mu^y_{21} = 8.1284$ and $\Delta\mu^y_{18} = 7.5847$ ($\Delta = \frac{29-(\Delta\mu_{18}[2\pi]+\Delta\mu_{58}[2\pi])}{2(3\times\xi)} = 1.66$) we see that the emittance growth is much weaker and the dipole mode amplitude no longer grows. By comparing the spectrum in Fig.9, evaluated now over 2^{14} turns, with the previous one we can see the dipole modes that were excited vertically. We show the emittance and dipole mode evolution for these two cases in Fig.7. This "modified" situation is stable but notice that the difference in the local phase advance is of the order 2×10^{-2}. It is very difficult to measure and control local phase advances to this precision. This is a clear demonstration how sensitive the resonance excitation is to phase changes bewteen interaction points.

Therefore, special attention should be paid to the excitation of these resonances. It is almost unavoidable to have phase advance errors that can move a carefully adjusted optics close to a resonance. Irregular phase advances introduce the risk of hitting coherent resonances.

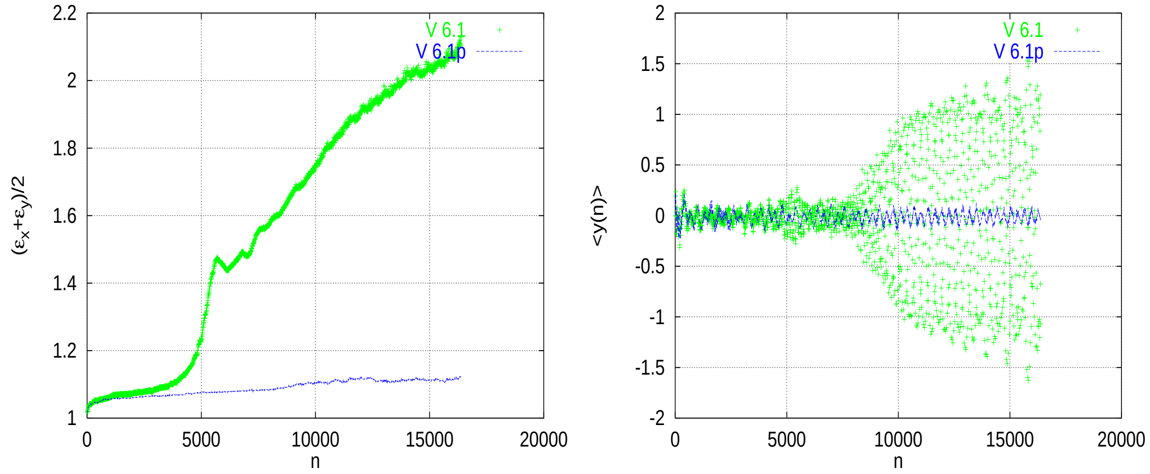

Figure 7: *Comparison of the evolution of the emittance (left) and dipole amplitude (right) for one bunch colliding with LHC optics V 6.1 and V 6.1p. LHC optics V 6.1 seems to be too close to a resonance and the emittance and amplitude of the dipole mode increase strongly.*

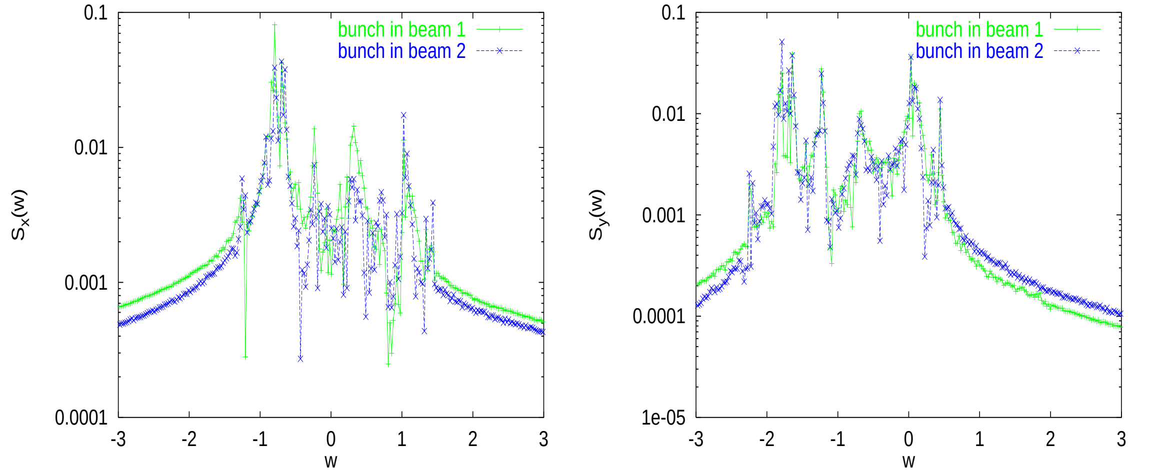

Figure 8: *For LHC optics V 6.1. with offset collision at IP2 and long range interactions: horizontal (left) and vertical (right) spectrum of dipole motion of a bunch in beam 1 and a bunch in beam 2 in units of* $w = (\nu - .31)/3\xi$ *and* $w = (\nu - .32)/3\xi$ *respectively. The spectrum is evaluated using the first 4096 turns only since the strong resonance increases the emittance and changes* ξ*.*

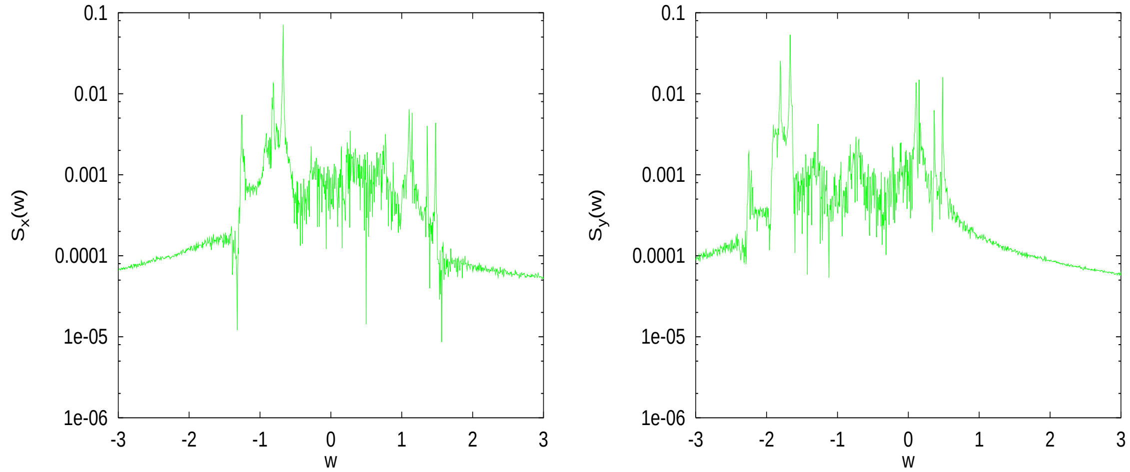

Figure 9: *For LHC optics V 6.1p (with a local phase variation) with offset collision at IP2 and long range interactions: horizontal (left) and vertical (right) spectrum of dipole motion of a bunch in beam 1 in units of* $w = (\nu - .31)/3\xi$ *and* $w = (\nu - .32)/3\xi$ *respectively. The dominant dipole modes can be clearly seen and are similar to those of LHC V 6.0 but are not excited by a resonance.*

7 LHC optics V 6.1 of beam 1 and 2, halo collisions at IP2, long range interactions and tune split.

Another option to suppress coherent dipole modes is to introduce a tune split. It is known that for 1 head-on collision a tune split bigger than $1.5 \times \xi$ decouples the two beams. For the case where all the head-on and long-range collisions are included we see in Fig.9 that the spectrum covers a range of almost $3 \times 3\xi \approx 0.03$, and the tune split that would uncouple both beams completely should be bigger than this. The option to chose two of the three working points (**WP 1** $(Q_{x,1}, Q_{y,1}) = (0.232, 0.242)$, **WP 2** $(Q_{x,2}, Q_{y,2}) = (0.310, 0.320)$, **WP 3** $(Q_{x,3}, Q_{y,3}) = (0.385, 0.395)$) has been ruled out because all combinations are too close to a coherent resonance in the horizontal plane [6]. If we keep close to one of the working points, the footprint of the beam must still fit into a safe area of the tune diagram.

We have repeated the previous studies for LHC optics V 6.1 with different phase advances per beam and introducing a variation in the total tune by changing some local phase advance in such a way that the total tune is also different, see Tab. 4.

Table 4: *Modified LHC optics V 6.1p phase advance from IPi to IPj and new total tune.*

beam 1 $i \rightarrow j$	$\Delta\mu^x_{ij}[2\pi]$	$\Delta\mu^y_{ij}[2\pi]$	beam 2 $i \rightarrow j$	$\Delta\mu^x_{ij}[2\pi]$	$\Delta\mu^y_{ij}[2\pi]$
$1 \rightarrow 2$	**8.5100**	**7.5398**	$2 \rightarrow 1$	**8.1144**	**8.0884**
$2 \rightarrow 5$	23.653	22.2152	$5 \rightarrow 2$	24.0687	21.6465
$5 \rightarrow 8$	23.9325	21.3819	$8 \rightarrow 5$	23.5695	21.9604
$8 \rightarrow 1$	8.2046	8.1931	$1 \rightarrow 8$	8.5676	7.6147
total $1 \rightarrow 1$	**64.30**	**59.33**	total $1 \rightarrow 1$	**64.32**	**59.31**

The drawback of operating the beams with different fractional tunes is that this can induce a flip-flop effect on their transverse sizes. The size variation found for these parameters is up to 10%, see Fig.10. For a tune split of 0.02 most of the coherent dipole modes have been moved back into the continuum although still two modes per plane can be seen, see Fig.11. A tune split bigger than of 0.03 (with fractional tunes 0.29 vs. 0.32) restores Landau damping of the remaining coherent dipole modes, see Fig.12.

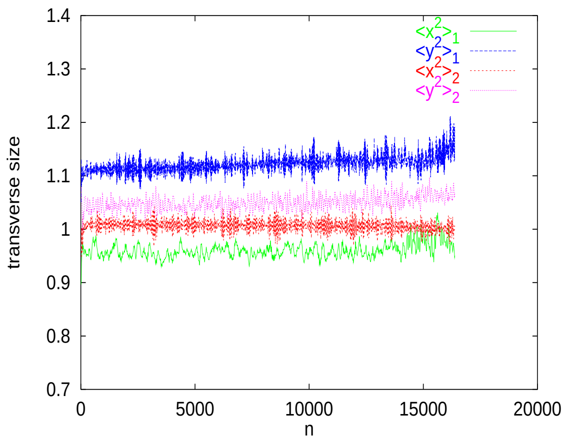

Figure 10: *For LHC optics V 6.1p: Transverse size evolution for a bunch in beam 1 and a bunch in beam 2. The tune split of* 0.02 *induces a size variation of the order of* 10%.

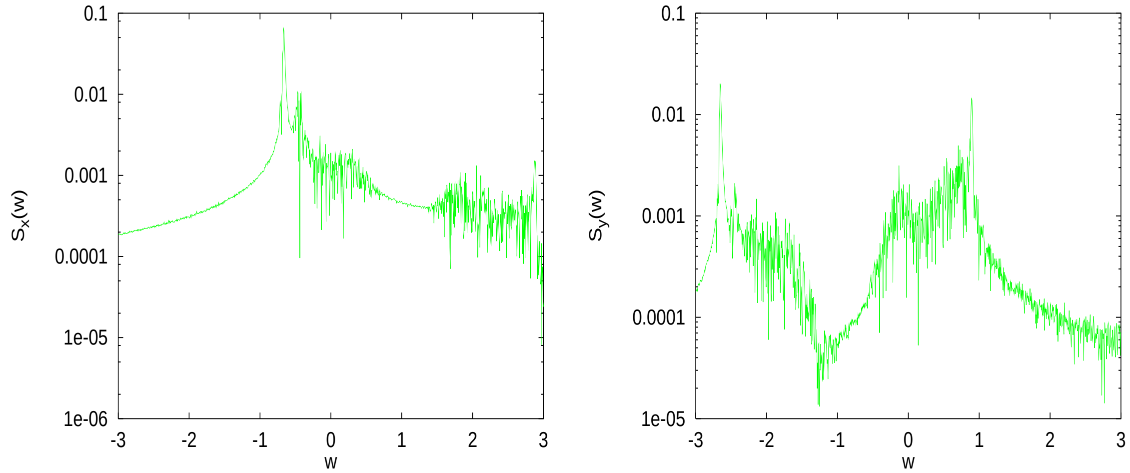

Figure 11: *For LHC optics V 6.1p with offset collision at IP2 , long range interactions and an additional tune split of* 0.02*: horizontal (left) and vertical (right) spectrum of dipole motion of a bunch in beam 1 in units of* $w = (\nu - .30)/3\xi$ *and* $w = (\nu - .32)/3\xi$ *respectively. Some of the coherent modes merge in the continuum.*

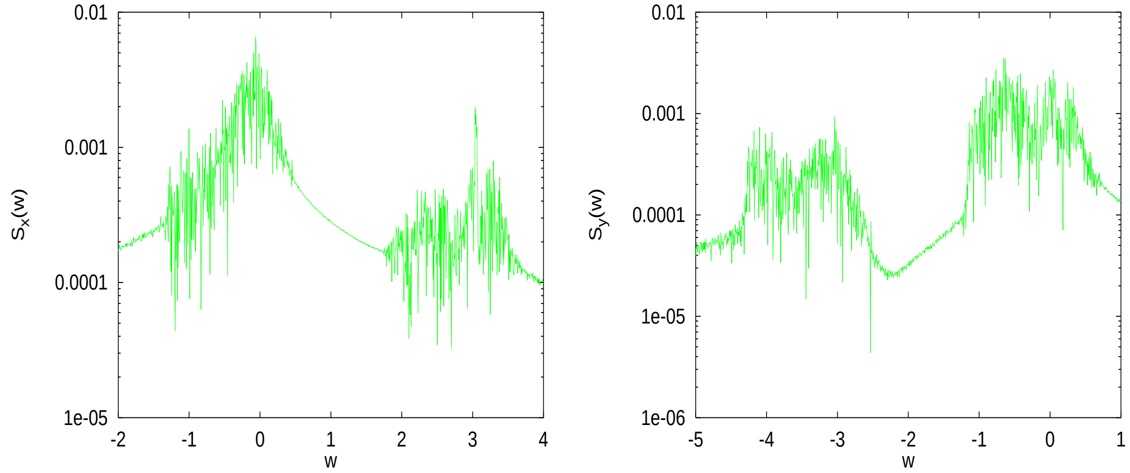

Figure 12: *For LHC optics V 6.1p with offset collision at IP2 , long range interactions and an additional tune split of* 0.03*: horizontal (left) and vertical (right) spectrum of dipole motion of a bunch in beam 1 in units of* $w = (\nu - .29)/3\xi$ *and* $w = (\nu - .32)/3\xi$ *respectively. All the coherent modes merge the continuum.*

8 Conclusions

We have studied the strong-strong collision of two beams at the 4 IPs of the LHC and evaluated the role of symmetry and phase advance. Breaking the symmetry of the collision scheme by changing the parity of the integer tune in the case of equal phase advance per sector, or by adjusting the phase advance per sector, suppresses some of the coherent modes. Cancelation of the dipole modes by carefully adjusting the local phase advance may be delicate and cannot suppress all coherent modes. Operating with horizontally and vertically swapped interger tunes is not possible with the presently foreseen powering scheme of the main quadrupoles.

These studies have been extended to include long range collisions. If LHC optics V 6.1 is chosen to collide the beams, we distinguish all the coherent dipole modes in the spectrum and due to the proximity of a resonance the system is unstable. The growth time of the amplitude of oscillations of the beam centroid is $\tau = 0.18$ s. Having irregular phase advances introduces the risk to hit coherent resonances. We propose, as a possible solution to decouple the system and restore Landau damping on these modes, to induce a tune split of at least 0.03, provided that the footprint fits into a stable area of the tune diagram. However, in case a flip-flop effect is induced this could lead to a small loss of luminosity. It is recommended to perform similar checks for any proposed LHC optics.

Additional effects such as the crossing angle, the synchrotron motion, or the jitter in the bunch population and sizes, might help to avoid the build-up of the coherent oscillation modes. These topics should be included in further studies.

9 Acknowledgement

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